# Extracting CKM phases from angular distributions of $B_{d,s}$ decays into admixtures of *CP* eigenstates

Robert Fleischer
*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
(Received 27 April 1999; published 8 September 1999)

The time-dependent angular distributions of certain $B_{d,s}$ decays into final states that are admixtures of $CP$-even and $CP$-odd configurations provide valuable information about CKM phases and hadronic parameters. We present the general formalism to accomplish this task, taking also into account penguin contributions, and illustrate it by considering a few specific decay modes. We give particular emphasis to the decay $B_d \to J/\psi\rho^0$, which can be combined with $B_s \to J/\psi\phi$ to extract the $B_d^0$-$\overline{B_d^0}$ mixing phase and—if penguin effects in the former mode should be sizeable—also the angle $\gamma$ of the unitarity triangle. As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the $B_s^0$-$\overline{B_s^0}$ mixing phase from $B_s \to J/\psi\phi$. Moreover, a discrete ambiguity in the extraction of the CKM angle $\beta$ can be resolved, and valuable insights into $SU(3)$-breaking effects can be obtained. Other interesting applications of the general formalism presented in this paper, involving $B_d \to \rho\rho$ and $B_{s,d} \to K^* \overline{K^*}$ decays, are also briefly noted. [S0556-2821(99)03619-X]

PACS number(s): 12.15.Hh, 13.25.Hw

## I. INTRODUCTION

Studies of $CP$ violation in the $B$-meson system and the determination of the three angles $\alpha$, $\beta$ and $\gamma$ of the usual non-squashed unitarity triangle [1] of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2] are among the central targets of future $B$-physics experiments. During recent years, several strategies were proposed to accomplish this task [3]. In this context, also quasi-two-body modes $B_q \to X_1 X_2$ of neutral $B_q$-mesons ($q \in \{d,s\}$), where both $X_1$ and $X_2$ carry spin and continue to decay through $CP$-conserving interactions, are of particular interest [4,5]. In this case, the time-dependent angular distribution of the decay products of $X_1$ and $X_2$ provides valuable information. For an initially, i.e. at time $t=0$, present $B_q^0$-meson, it can be written as

$$f(\Theta,\Phi,\Psi;t) = \sum_k \mathcal{O}^{(k)}(t) g^{(k)}(\Theta,\Phi,\Psi), \qquad (1)$$

where we have denoted the angles describing the kinematics of the decay products of $X_1$ and $X_2$ generically by $\Theta$, $\Phi$ and $\Psi$. Note that we have to deal, in general, with an arbitrary number of such angles. The observables $\mathcal{O}^{(k)}(t)$ describing the time evolution of the angular distribution (1) can be expressed in terms of real or imaginary parts of certain bilinear combinations of decay amplitudes. In the applications discussed in this paper, we will focus on $B_q \to [X_1 X_2]_f$ decays, where $X_1$ and $X_2$ are both vector mesons, and $f$ denotes a final-state configuration with $CP$ eigenvalue $\eta_f$. It is convenient to analyze such modes in terms of the linear polarization amplitudes $A_0(t)$, $A_\parallel(t)$ and $A_\perp(t)$ [6]. Whereas $A_\perp(t)$ describes a $CP$-odd final-state configuration, both $A_0(t)$ and $A_\parallel(t)$ correspond to $CP$-even final-state configurations, i.e. to the CP eigenvalues $-1$ and $+1$, respectively. The observables of the corresponding angular distribution are given by

$$|A_f(t)|^2 \quad \text{with} \quad f \in \{0,\parallel,\perp\}, \qquad (2)$$

as well as by the interference terms

$$\Re\{A_0^*(t) A_\parallel(t)\} \quad \text{and} \quad \Im\{A_f^*(t) A_\perp(t)\} \quad \text{with} \quad f \in \{0,\parallel\}. \qquad (3)$$

This formalism is discussed in more detail in [7], where several explicit angular distributions can be found and appropriate weighting functions to extract their observables in an efficient way from the experimental data are given.

In the following considerations, the main role is played by neutral $B_q \to [X_1 X_2]_f$ decays, where the "unevolved" decay amplitudes can be expressed, with the help of the unitarity of the CKM matrix, as

$$A_f = \mathcal{N}_f [1 - b_f e^{i\rho_f} e^{+i\omega}] \qquad (4)$$

$$\overline{A_f} = \eta_f \mathcal{N}_f [1 - b_f e^{i\rho_f} e^{-i\omega}], \qquad (5)$$

where $\omega$ denotes a $CP$-violating weak phase and $\mathcal{N}_f \equiv |\mathcal{N}_f| e^{i\delta_f}$. Both $\rho_f$ and $\delta_f$ are $CP$-conserving strong phases. In this case, the observables (2) and (3) allow us to probe the $B_q^0$-$\overline{B_q^0}$ mixing phase $\phi_q$ and the weak phase $\omega$, as we will show in this paper. Concerning practical applications, $\omega$ is given by one of the angles of the unitarity triangle. However, the observables specified in Eqs. (2) and (3) are not independent from one another and do not provide sufficient information to extract $\phi_q$ and $\omega$, as well as the corresponding hadronic parameters, simultaneously. To this end, we have to use an additional input.

The reason for this feature is the fact that the parametrizations given in Eqs. (4) and (5) are not unique. In order to illustrate this point in more detail, let us consider the amplitude $P$ of a non-leptonic $\overline{b} \to \overline{d}$ penguin decay into two pseudoscalar mesons, such as $B_d^0 \to K^0 \overline{K^0}$:

$$P = \lambda_u^{(d)} P_u + \lambda_c^{(d)} P_c + \lambda_t^{(d)} P_t. \qquad (6)$$

Here the

 

$$\lambda_q^{(d)} \equiv V_{qd} V_{qb}^* \tag{7}$$

are the usual CKM factors and the $P_q$ denote hadronic matrix elements, which are related to penguin topologies with internal $q$-quark exchanges ($q \in \{u,c,t\}$). The unitarity of the CKM matrix implies

$$\lambda_u^{(d)} + \lambda_c^{(d)} + \lambda_t^{(d)} = 0, \tag{8}$$

allowing us to eliminate one of the CKM factors in Eq. (6). These quantities can be expressed in terms of the angles of the unitarity triangle as follows:

$$\lambda_u^{(d)} = \lambda^3 A R_b e^{i\gamma}, \quad \lambda_c^{(d)} = -\lambda^3 A, \quad \lambda_t^{(d)} = \lambda^3 A R_t e^{-i\beta}, \tag{9}$$

where

$$\lambda \equiv |V_{us}| = 0.22, \quad A \equiv \frac{1}{\lambda^2}|V_{cb}| = 0.81 \pm 0.06, \tag{10}$$

and

$$R_b \equiv \frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| = 0.41 \pm 0.07, \quad R_t \equiv \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| = \mathcal{O}(1). \tag{11}$$

If we eliminate the $\lambda_t^{(d)}$ term in Eq. (6), we obtain

$$P = -\lambda^3 A (P_c - P_t)\left[1 - R_b e^{i\gamma}\left(\frac{P_u - P_t}{P_c - P_t}\right)\right]. \tag{12}$$

On the other hand, if we prefer to eliminate the $\lambda_u^{(d)}$ term, we arrive at

$$P = -\lambda^3 A (P_c - P_u)\left[1 - R_t e^{-i\beta}\left(\frac{P_t - P_u}{P_c - P_u}\right)\right]. \tag{13}$$

Both parametrizations are related to each other through the unitarity of the CKM matrix. If we compare Eqs. (12) and (13) with the parametrization given in Eq. (4), we observe that $\omega = \gamma$ in the former case, whereas $\omega = -\beta$ in the latter one. Consequently, if it were possible to extract $\omega$ by using *only* the observables provided by our considered penguin decay, we would obtain different values for $\omega$ for different parametrizations of the decay amplitude, as in general $-\beta \neq \gamma$. This point is worked out in more detail in a recent paper [8], where also some of the other issues discussed here are addressed.

The kind of reasoning given in the previous paragraph applies also to the observables of the angular distributions of quasi-two-body $B_q \to X_1 X_2$ modes. In order to extract information from these observables, the ambiguity in the definition of the decay amplitudes (4) and (5) has to be resolved. To accomplish this goal, an additional input is required, which is usually provided by flavor-symmetry or dynamical arguments. The particular input we are using determines the appropriate parametrization of decay amplitudes, which will always be of the same form as that of Eqs. (4) and (5).

In this context, there is an interesting feature, as we will show in this paper: if we fix the mixing phase $\phi_q$ separately, it is possible to express $\omega$—and interesting hadronic quantities—as a function of a *single* hadronic parameter in a *theoretically clean* way. If we determine this quantity, for instance, by comparing $B_q \to X_1 X_2$ with an $SU(3)$-related mode, all remaining parameters, including $\omega$, can be extracted. As noted above, the input used to fix this parameter determines the appropriate parametrization of decay amplitudes, in particular also the CKM phase $\omega$. If we are willing to make more extensive use of flavor-symmetry arguments, it is possible to determine the $B_q^0$-$\overline{B_q^0}$ mixing phase $\phi_q$ as well. An example for such a strategy is given by the decay $B_d \to J/\psi \rho^0$, which can be combined with $B_s \to J/\psi \phi$ to extract the $B_d^0$-$\overline{B_d^0}$ mixing phase $\phi_d = 2\beta$ and—if penguin effects in the former mode should be sizeable—also the angle $\gamma$ of the unitarity triangle. As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the $B_s^0$-$\overline{B_s^0}$ mixing phase from $B_s \to J/\psi \phi$, which is an important issue for ''second-generation'' $B$-physics experiments at hadron machines. Moreover, we may resolve a discrete ambiguity in the extraction of the CKM angle $\beta$, and may obtain valuable insights into $SU(3)$-breaking effects. Other interesting applications of the general formalism presented in this paper, involving $B_d \to \rho\rho$ and $B_{s,d} \to K^* \overline{K^*}$ decays, are also briefly noted.

As the extraction of CKM phases with the help of these modes involves ''penguin,'' i.e., flavor-changing neutral-current (FCNC) processes and relies moreover on the unitarity of the CKM matrix, it may well be affected by new physics. In such a case, discrepancies would show up with other strategies to determine these phases, for example with the theoretically clean extractions of $\gamma$ making use of pure ''tree'' decays such as $B \to DK$ or $B_s \to D_s^{\pm} K^{\mp}$. Since no FCNC processes contribute to the decay amplitudes of these modes, it is quite unlikely that they—and the extracted value of $\gamma$—are significantly affected by new physics.

The outline of this paper is as follows: in Sec. II, the time-dependent observables of the $B_q \to X_1 X_2$ angular distribution are given. The strategies to extract CKM phases, as well as interesting hadronic parameters, with the help of these observables are discussed in Sec. III. In Sec. IV, we focus on the extraction of $\beta$ and $\gamma$ from $B_d \to J/\psi \rho^0$ and $B_s \to J/\psi \phi$. Further applications of the formalism developed in Secs. II and III are discussed in Sec. V, and the conclusions are summarized in Sec. VI.

## II. THE TIME EVOLUTION OF THE ANGULAR DISTRIBUTIONS

In this section, we consider the general case of a neutral quasi-two-body decay $B_q \to [X_1 X_2]_f$ into a final-state configuration $f$ with $CP$ eigenvalue $\eta_f$ that exhibits ''unmixed'' decay amplitudes of the same structure as those given in Eqs. (4) and (5). If we use linear polarization states to characterize the final-state configurations as, for example, in [7], we have $f \in \{0, \parallel, \perp\}$.

At this point a comment on the angular distribution of the $CP$-conjugate decay $\overline{B_q^0} \to X_1 X_2$, which is given by

$$\overline{f}(\Theta,\Phi,\Psi;t)=\sum_{k}\overline{\mathcal{O}}^{(k)}(t)g^{(k)}(\Theta,\Phi,\Psi), \tag{14}$$

is in order. Since the meson content of the $X_1X_2$ states is the same whether these result from the $B_q^0$ or $\overline{B_q^0}$ decays, we may use the same generic angles $\Theta$, $\Phi$ and $\Psi$ to describe the angular distribution of their decay products. Within this formalism, the effects of $CP$ transformations relating $B_q^0\to[X_1X_2]_f$ to $\overline{B_q^0}\to[X_1X_2]_f$ are taken into account by the $CP$ eigenvalue $\eta_f$ appearing in Eq. (5), and do not affect the form of $g^{(k)}(\Theta,\Phi,\Psi)$. Therefore the same functions $g^{(k)}(\Theta,\Phi,\Psi)$ are present in Eqs. (1) and (14) (see also [5]).

In view of applications to $B_s$ decays, we allow for a non-vanishing width difference $\Delta\Gamma_q\equiv\Gamma_{\rm H}^{(q)}-\Gamma_{\rm L}^{(q)}$ between the $B_q$ mass eigenstates $B_q^{\rm H}$ (''heavy'') and $B_q^{\rm L}$ (''light''). In contrast to the $B_d$ case, this width difference may be sizeable in the $B_s$ system [9]; it may allow studies of $CP$ violation with ''untagged'' $B_s$ data samples, where one does not distinguish between initially, i.e. at time $t=0$, present $B_s^0$ or $\overline{B_s^0}$ mesons [10]. The time evolution of the observables corresponding to Eq. (2) takes the following form:

$$|A_f(t)|^2=\frac{1}{2}[R_{\rm L}^f e^{-\Gamma_{\rm L}^{(q)}t}+R_{\rm H}^f e^{-\Gamma_{\rm H}^{(q)}t}+2e^{-\Gamma_q t}\{A_{\rm D}^f\cos(\Delta M_q t)+A_{\rm M}^f\sin(\Delta M_q t)\}] \tag{15}$$

$$|\overline{A}_f(t)|^2=\frac{1}{2}[R_{\rm L}^f e^{-\Gamma_{\rm L}^{(q)}t}+R_{\rm H}^f e^{-\Gamma_{\rm H}^{(q)}t}-2e^{-\Gamma_q t}\{A_{\rm D}^f\cos(\Delta M_q t)+A_{\rm M}^f\sin(\Delta M_q t)\}], \tag{16}$$

where $\Delta M_q\equiv M_{\rm H}^{(q)}-M_{\rm L}^{(q)}>0$ denotes the mass difference between the $B_q$ mass eigenstates, and $\Gamma_q\equiv[\Gamma_{\rm L}^{(q)}+\Gamma_{\rm H}^{(q)}]/2$. The quantities $R_{\rm L}^f$, $R_{\rm H}^f$, $A_{\rm D}^f$ and $A_{\rm M}^f$, which are not independent from one another and satisfy the relation

$$(A_{\rm D}^f)^2+(A_{\rm M}^f)^2=R_{\rm L}^f R_{\rm H}^f, \tag{17}$$

are given by

$$R_{\rm L}^f=|\mathcal{N}_f|^2[(1+\eta_f\cos\phi_q)-2b_f\cos\rho_f\times\{\cos\omega+\eta_f\cos(\phi_q+\omega)\}+b_f^2\{1+\eta_f\cos(\phi_q+2\omega)\}] \tag{18}$$

$$R_{\rm H}^f=|\mathcal{N}_f|^2[(1-\eta_f\cos\phi_q)-2b_f\cos\rho_f\times\{\cos\omega-\eta_f\cos(\phi_q+\omega)\}+b_f^2\{1-\eta_f\cos(\phi_q+2\omega)\}] \tag{19}$$

$$A_{\rm D}^f=2|\mathcal{N}_f|^2 b_f\sin\rho_f\sin\omega \tag{20}$$

$$A_{\rm M}^f=\eta_f|\mathcal{N}_f|^2[\sin\phi_q-2b_f\cos\rho_f\sin(\phi_q+\omega)+b_f^2\sin(\phi_q+2\omega)]. \tag{21}$$

Here the phase $\phi_q$ denotes the $CP$-violating weak $B_q^0$–$\overline{B_q^0}$ mixing phase:

$$\phi_q=\begin{cases}2\beta & \text{for } q=d\\ -2\delta\gamma & \text{for } q=s,\end{cases} \tag{22}$$

where $2\delta\gamma\approx0.03$ is tiny in the standard model because of a Cabibbo suppression of $\mathcal{O}(\lambda^2)$. This phase cancels in

$$S_f\equiv\frac{1}{2}(|A_f(0)|^2+|\overline{A}_f(0)|^2)=\frac{1}{2}(R_{\rm L}^f+R_{\rm H}^f)=|\mathcal{N}_f|^2(1-2b_f\cos\rho_f\cos\omega+b_f^2). \tag{23}$$

It is also interesting to note that there are no $\Delta M_q t$ terms present in the ''untagged'' combination

$$|A_f(t)|^2+|\overline{A}_f(t)|^2=R_{\rm L}^f e^{-\Gamma_{\rm L}^{(q)}t}+R_{\rm H}^f e^{-\Gamma_{\rm H}^{(q)}t}, \tag{24}$$

whereas

$$|A_f(t)|^2-|\overline{A}_f(t)|^2=2e^{-\Gamma_q t}[A_{\rm D}^f\cos(\Delta M_q t)+A_{\rm M}^f\sin(\Delta M_q t)]. \tag{25}$$

Because of Eq. (17), each of the $|A_f(t)|^2$ or $|\overline{A}_f(t)|^2$ ($f\in\{0,\|,\perp\}$) terms of the $B_q\to X_1X_2$ angular distribution provides three independent observables, which we may choose as $A_{\rm D}^f$, $A_{\rm M}^f$ and $S_f$.

The time evolution of the interference terms (3) is analogous to Eqs. (24) and (25). Let us first give the expressions for the observables corresponding to Eq. (23):

$$R\equiv\frac{1}{2}[\Re\{A_0^*(0)A_\|(0)\}+\Re\{\overline{A}_0^*(0)\overline{A}_\|(0)\}]=|\mathcal{N}_0||\mathcal{N}_\||[\cos\Delta_{0,\|}-\{b_0\cos(\rho_0-\Delta_{0,\|})+b_\|\cos(\rho_\|+\Delta_{0,\|})\}\cos\omega+b_0b_\|\cos(\rho_0-\rho_\|-\Delta_{0,\|})] \tag{26}$$

$$I_{\rm D}^f\equiv\frac{1}{2}[\Im\{A_f^*(0)A_\perp(0)\}+\Im\{\overline{A}_f^*(0)\overline{A}_\perp(0)\}]=|\mathcal{N}_f||\mathcal{N}_\perp|[b_f\cos(\rho_f-\Delta_{f,\perp})-b_\perp\cos(\rho_\perp+\Delta_{f,\perp})]\sin\omega, \tag{27}$$

where the

$$\Delta_{\tilde f,f}\equiv\delta_f-\delta_{\tilde f} \tag{28}$$

denote the differences of the $CP$-conserving strong phases of the amplitudes $\mathcal{N}_f\equiv e^{i\delta_f}|\mathcal{N}_f|$ and $\mathcal{N}_{\tilde f}\equiv e^{i\delta_{\tilde f}}|\mathcal{N}_{\tilde f}|$. On the other hand, the rate differences corresponding to Eq. (25) take the following form:

$$\Re\{A_0^*(t)A_\|(t)\}-\Re\{\overline{A}_0^*(t)\overline{A}_\|(t)\}=2e^{-\Gamma_q t}[R_{\rm D}\cos(\Delta M_q t)+R_{\rm M}\sin(\Delta M_q t)] \tag{29}$$

$$\Im\{A_f^*(t)A_\perp(t)\}-\Im\{\overline{A}_f^*(t)\overline{A}_\perp(t)\}$$
$$=2e^{-\Gamma_q t}[I_f\cos(\Delta M_q t)-I_{\rm M}^f\sin(\Delta M_q t)], \quad (30)$$

where

$$R_{\rm D}=|\mathcal{N}_0||\mathcal{N}_\|| [b_0\sin(\rho_0-\Delta_{0,\|})+b_\|\sin(\rho_\|+\Delta_{0,\|})]\sin\omega \quad (31)$$

$$R_{\rm M}=|\mathcal{N}_0||\mathcal{N}_\||[\cos\Delta_{0,\|}\sin\phi_q-\{b_0\cos(\rho_0-\Delta_{0,\|}) + b_\|\cos(\rho_\|+\Delta_{0,\|})\}\sin(\phi_q+\omega)+b_0b_\| \times\cos(\rho_0-\rho_\|-\Delta_{0,\|})\sin(\phi_q+2\omega)] \quad (32)$$

and

$$I_f=|\mathcal{N}_f||\mathcal{N}_\perp|[\sin\Delta_{f,\perp}+\{b_f\sin(\rho_f-\Delta_{f,\perp}) -b_\perp\sin(\rho_\perp+\Delta_{f,\perp})\}\cos\omega -b_fb_\perp\sin(\rho_f-\rho_\perp-\Delta_{f,\perp})] \quad (33)$$

$$I_{\rm M}^f=|\mathcal{N}_f||\mathcal{N}_\perp|[\cos\Delta_{f,\perp}\cos\phi_q-\{b_f\cos(\rho_f-\Delta_{f,\perp}) +b_\perp\cos(\rho_\perp+\Delta_{f,\perp})\}\cos(\phi_q+\omega) +b_fb_\perp\cos(\rho_f-\rho_\perp-\Delta_{f,\perp})\cos(\phi_q+2\omega)]. \quad (34)$$

Note that $f\in\{0,\|\}$ in Eqs. (27) and (30). The minus sign in the latter expression is due to the different $CP$ eigenvalues of $f\in\{0,\|\}$ and $f=\perp$. If we set ''$0=\|$'' in Eqs. (31) and (32), we get expressions taking the same form as Eqs. (20) and (21), which provides a nice cross check. The expressions given above generalize those derived in [7] in two respects: they take into account penguin contributions, and they allow for a sizeable value of the $B_q^0$-$\overline{B_q^0}$ mixing phase $\phi_q$. In the discussion of $B_s\to J/\psi\phi$ in [7], it was assumed that $\phi_s$ is a small phase, and terms of $\mathcal{O}(\phi_s^2)$ were neglected.

Unfortunately, not all of the observables $S_f$, $A_{\rm D}^f$ and $A_{\rm M}^f$ are independent from those of the interference terms (3). This can be seen by considering two different final-state configurations $f$ and $\tilde{f}$. In this case, the time-dependent angular distribution provides nine observables. To be definite, let us consider the case $f=0$ and $\tilde{f}=\|$. Then we have six observables, corresponding to $S_f$, $A_{\rm D}^f$ and $A_{\rm M}^f$ ($f\in\{0,\|\}$), as well as the three observables $R$, $R_{\rm D}$ and $R_{\rm M}$, which are due to the real parts in Eq. (3). The measurement of $S_f$ and $A_{\rm D}^f$ allows us to fix the magnitudes $|A_0|$, $|A_\||$ and $|\overline{A}_0|$, $|\overline{A}_\||$. Using in addition the observables $R$ and $R_{\rm D}$, we can determine the angle $\sigma$ between the unmixed amplitudes $A_0$ and $A_\|$, as well as the angle $\overline{\sigma}$ between $\overline{A}_0$, $\overline{A}_\|$ (see Fig. 1). So far, the relative orientation of the amplitudes ($A_0$, $A_\|$) and ($\overline{A}_0$, $\overline{A}_\|$) is not determined. However, if we use, in addition, the mixing-induced $CP$ asymmetry $A_{\rm M}^0$, we are in a position to fix $\phi_q+\psi_0$, where $\psi_0$ denotes the angle between the amplitudes $A_0$ and $\overline{A}_0$:

$$A_{\rm M}^0=|A_0||\overline{A}_0|\sin(\phi_q+\psi_0). \quad (35)$$

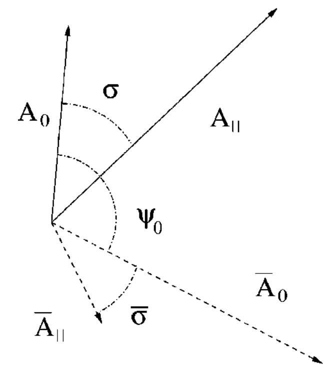


FIG. 1. The amplitudes $A_0$, $A_\|$ and $\overline{A}_0$, $\overline{A}_\|$ in the complex plane.

Since the relative orientation of the amplitudes $e^{-i\phi_q}\overline{A}_\|$ and $A_\|$ is also fixed this way, we can predict the values of the two remaining mixing-induced $CP$-violating observables $A_{\rm M}^\|$ and $R_{\rm M}$. Consequently, only seven of the nine observables are independent from one another.

It is convenient to introduce the following ''normalized'' observables:

$$\hat{A}_{\rm D}^f\equiv\frac{A_{\rm D}^f}{S_f},\quad \hat{A}_{\rm M}^f\equiv\frac{A_{\rm M}^f}{S_f}, \quad (36)$$

$$\hat{R}\equiv\frac{R}{\sqrt{S_0S_\|}},\quad \hat{R}_{\rm D}\equiv\frac{R_{\rm D}}{\sqrt{S_0S_\|}},\quad \hat{R}_{\rm M}\equiv\frac{R_{\rm M}}{\sqrt{S_0S_\|}}, \quad (37)$$

$$\hat{I}_f\equiv\frac{I_f}{\sqrt{S_fS_\perp}},\quad \hat{I}_{\rm D}^f\equiv\frac{I_{\rm D}^f}{\sqrt{S_fS_\perp}},\quad \hat{I}_{\rm M}^f\equiv\frac{I_{\rm M}^f}{\sqrt{S_fS_\perp}}, \quad (38)$$

which have the advantage that they do not depend on the overall normalization factors $|\mathcal{N}_f|$. The observables $\hat{A}_{\rm D}^f$ and $\hat{A}_{\rm M}^f$ allow us to determine the hadronic parameters $b_f$ and $\rho_f$ as functions of $\omega$ and $\phi_q$:

$$b_f=\sqrt{\frac{1}{k_f}[l_f\pm\sqrt{l_f^2-h_fk_f}]} \quad (39)$$

$$2b_f\cos\rho_f=u_f+v_fb_f^2 \quad (40)$$

$$2b_f\sin\rho_f=[(1-u_f\cos\omega)+(1-v_f\cos\omega)b_f^2]\left(\frac{\hat{A}_{\rm D}^f}{\sin\omega}\right), \quad (41)$$

where

$$h_f=u_f^2+D_f(1-u_f\cos\omega)^2 \quad (42)$$

$$k_f=v_f^2+D_f(1-v_f\cos\omega)^2 \quad (43)$$

$$l_f=2-u_fv_f-D_f(1-u_f\cos\omega)(1-v_f\cos\omega), \quad (44)$$

with

$$u_f=\frac{\eta_f\hat{A}^f_{\rm M}-\sin\phi_q}{\eta_f\hat{A}^f_{\rm M}\cos\omega-\sin(\phi_q+\omega)} \tag{45}$$

$$v_f=\frac{\eta_f\hat{A}^f_{\rm M}-\sin(\phi_q+2\omega)}{\eta_f\hat{A}^f_{\rm M}\cos\omega-\sin(\phi_q+\omega)} \tag{46}$$

and

$$D_f=\left(\frac{\hat{A}^f_{\rm D}}{\sin\omega}\right)^2. \tag{47}$$

It should be emphasized that no approximations were made in order to derive these expressions. If we consider, in addition to $\hat{A}^f_{\rm D}$ and $\hat{A}^f_{\rm M}$, either the observables specified in Eq. (37) or those given in Eq. (38), we obtain seven normalized observables, which depend on five hadronic parameters ($b_f$, $\rho_f$, $b_{\tilde f}$, $\rho_{\tilde f}$ and $\Delta_{\tilde f,f}$), as well as on the two $CP$-violating weak phases $\phi_q$ and $\omega$. However, only five of the seven observables are independent from one another, so that we do not have sufficient observables at our disposal to extract these parameters simultaneously. This feature is due to the CKM ambiguity, as we have discussed in Sec. I. In order to extract these parameters from the observables, we have to make use, for example, of another decay that can be related to $B_q\to X_1X_2$ through flavor-symmetry arguments. On the other hand, if we use $\phi_q$ and the weak phase $\omega$ of a specific parametrization (4) of the decay $B_q\to X_1X_2$ as an input, the corresponding hadronic parameters can be extracted *without* any additional assumption, thereby providing valuable insights into hadronic physics and a very fertile testing ground for model calculations of $B_q\to X_1X_2$. The measurement of the angular distributions discussed in this paper requires high statistics and can probably only be performed at "second-generation" $B$-physics experiments at hadron machines, such as LHCb or BTeV, where decays of $B_s$-mesons can also be studied. Since several promising strategies to extract the weak phases $\phi_q$ and $\omega$ at such experiments were already proposed (see, for example, [3]), it may indeed be an interesting alternative to use measurements of angular distributions not to extract CKM phases, but to explore hadronic physics.

In practical applications, the parameters $b_f$ typically measure the ratio of "penguin" to "tree" contributions. Applying the Bander–Silverman–Soni mechanism [11], and following the formalism developed in [12,13], which makes use—among other things—of the "factorization hypothesis," we obtain for various classes of $B$ decays

$$b_f\equiv b,\quad \rho_f\equiv\rho\quad \forall\ f\in\{0,\|,\perp\}, \tag{48}$$

i.e. these quantities are independent of the final-state configuration $f$ in this case. The main reason for these relations is that the form factors, which depend on the final-state configuration $f$, cancel in the ratios $b_f$ of "penguin" to "tree" contributions. Although non-factorizable contributions are expected to play an important role, thereby affecting Eq. (48), it is interesting to investigate the implications of these relations on the observables of the angular distributions in more detail. If we introduce

$$\hat{A}_{\rm D}\equiv\frac{2b\sin\rho\sin\omega}{1-2b\cos\rho\cos\omega+b^2} \tag{49}$$

$$\hat{A}_{\rm M}\equiv\frac{\sin\phi_q-2b\cos\rho\sin(\phi_q+\omega)+b^2\sin(\phi_q+2\omega)}{1-2b\cos\rho\cos\omega+b^2}, \tag{50}$$

we obtain

$$\hat{A}^f_{\rm D}=\hat{A}_{\rm D},\quad \hat{A}^f_{\rm M}=\eta_f\hat{A}_{\rm M}, \tag{51}$$

$$\hat{R}=\cos\Delta_{0,\|},\quad \hat{R}_{\rm D}=\hat{A}_{\rm D}\cos\Delta_{0,\|},\quad \hat{R}_{\rm M}=\hat{A}_{\rm M}\cos\Delta_{0,\|}, \tag{52}$$

$$\hat{I}_f=\sin\Delta_{f,\perp},\quad \hat{I}^f_{\rm D}=\hat{A}_{\rm D}\sin\Delta_{f,\perp}, \tag{53}$$

$$I\equiv\frac{\hat{I}^f_{\rm M}}{\cos\Delta_{f,\perp}}=\left[\frac{\cos\phi_q-2b\cos\rho\cos(\phi_q+\omega)+b^2\cos(\phi_q+2\omega)}{1-2b\cos\rho\cos\omega+b^2}\right], \tag{54}$$

where

$$(\hat{A}_{\rm D})^2+(\hat{A}_{\rm M})^2+I^2=1. \tag{55}$$

These relations provide an interesting test of whether Eq. (48) is realized in the decay $B_q\to X_1X_2$. Note that $\hat{I}^f_{\rm M}$ does not—in contrast to Eq. (53)—vanish for trivial values of $\Delta_{f,\perp}$.

## III. EXTRACTING CKM PHASES AND HADRONIC PARAMETERS

Let us now focus on the extraction of CKM phases from the observables of the $B_q\to X_1X_2$ angular distribution. As we have already noted, to this end, we have to employ an additional input, since we have only five independent normalized observables at our disposal, which depend on seven "unknowns." Although it would be desirable to determine $\phi_q$ and $\omega$ simultaneously, usually only the CKM phase $\omega$ is of central interest.

The $B^0_s$-$\overline{B^0_s}$ mixing phase $\phi_s\equiv-2\delta\gamma=2\arg(V^*_{ts}V_{tb})$ is negligibly small in the standard model. It can be probed—and in principle even determined—with the help of the decay $B_s\to J/\psi\phi$ (see, for example, [7]). Large $CP$-violating effects in this decay would signal that $2\delta\gamma$ is not tiny, and would be a strong indication for new-physics contributions to $B^0_s$-$\overline{B^0_s}$ mixing. On the other hand, the $B^0_d$-$\overline{B^0_d}$ mixing phase $\phi_d=2\beta$ can be fixed in a reliable way through the "gold-plated" mode $B_d\to J/\psi K_{\rm S}$ [14]. Strictly speaking, mixing-induced $CP$ violation in $B_d\to J/\psi K_{\rm S}$ probes $\sin(2\beta+\phi_K)$, where $\phi_K$ is related to the weak $K^0$-$\overline{K^0}$ mixing phase and is negligibly small in the standard model. Because of the small value of the $CP$-violating parameter $\varepsilon_K$ of the neutral kaon system, $\phi_K$ can only be affected by very contrived models of new physics [15]. A measurement of mixing-induced $CP$ violation in $B_d \to J/\psi K_S$ allows us to fix $\phi_d = 2\beta$ only up to a twofold ambiguity. Several strategies to resolve this ambiguity were proposed in the literature [16], which should be feasible for ''second-generation'' $B$-physics experiments. As we will see in the following section, also the decay $B_d \to J/\psi\rho^0$, in combination with $B_s \to J/\psi\phi$, allows us to accomplish this task.

If we use $\phi_q$ thus determined as an input and consider, in addition to $\hat{A}_{\rm D}^f$ and $\hat{A}_{\rm M}^f$, either the observables specified in Eq. (37) or those given in Eq. (38), we can determine $\omega$ as a function of a *single* hadronic parameter. Let us, for the moment, focus on the latter case, i.e. on the observables $\hat{A}_{\rm D}^f$, $\hat{A}_{\rm M}^f$, $\hat{A}_{\rm D}^\perp$, $\hat{A}_{\rm M}^\perp$ and $\hat{I}_f$, $\hat{I}_{\rm D}^f$, $\hat{I}_{\rm M}^f$ for a given final-state configuration $f \in \{0,\|\}$. Since $|\mathcal{N}_f|$ and $|\mathcal{N}_\perp|$ cancel in these quantities, they depend only on the hadronic parameters $b_f$, $\rho_f$, $b_\perp$, $\rho_\perp$, $\Delta_{f,\perp}$, as well as on the weak phases $\omega$ and $\phi_q$. Consequently, we have seven observables at our disposal, which depend on seven ''unknowns.'' However, only five of the seven observables are independent from one another, as we have discussed in the previous section. If we use $\phi_q$ as an input, we can, for instance, obtain $\omega$ and $b_f$, $\rho_f$, $b_\perp$, $\rho_\perp$ as functions of the strong phase difference $\Delta_{f,\perp}$ in a *theoretically clean* way. Although the following discussion deals with $\Delta_{f,\perp}$, we can also replace this quantity by another hadronic parameter of our choice. If we fix $\Delta_{f,\perp}$, for example, by comparing $B_q \to X_1 X_2$ with an $SU(3)$-related mode, all parameters can be extracted. Using in addition the observables $S_f$, we can also determine the normalization factors $|N_f|$. Comparing them with those of the $SU(3)$-related mode used to fix $\Delta_{f,\perp}$, we can obtain valuable insights into $SU(3)$-breaking corrections. The observables that we have not used so far can be used to resolve discrete ambiguities, arising typically in the extraction of these parameters.

Let us now give the formulas to implement this approach in a mathematical way. The general expression for the observable $\hat{I}_f$ [see Eqs. (33) and (38)] leads to the equation

$$A_f \sin\Delta_{f,\perp} + B_f \cos\Delta_{f,\perp} = C_f, \tag{56}$$

where

$$A_f = \frac{1}{N_f}[1-(b_f\cos\rho_f + b_\perp\cos\rho_\perp)\cos\omega + b_f b_\perp(\cos\rho_f\cos\rho_\perp + \sin\rho_f\sin\rho_\perp)] \tag{57}$$

$$B_f = \frac{1}{N_f}[(b_f\sin\rho_f - b_\perp\sin\rho_\perp)\cos\omega - b_f b_\perp(\sin\rho_f\cos\rho_\perp - \cos\rho_f\sin\rho_\perp)] \tag{58}$$

$$C_f = \hat{I}_f, \tag{59}$$

with

$$N_f = \sqrt{(1-2b_f\cos\rho_f\cos\omega + b_f^2)(1-2b_\perp\cos\rho_\perp\cos\omega + b_\perp^2)}. \tag{60}$$

The solution of Eq. (56) is straightforward, and is given as follows:

$$\sin\Delta_{f,\perp} = \frac{A_f C_f \pm \sqrt{(A_f^2+B_f^2-C_f^2)B_f^2}}{A_f^2+B_f^2},$$

$$\cos\Delta_{f,\perp} = \frac{C_f - A_f\sin\Delta_{f,\perp}}{B_f}. \tag{61}$$

If we insert $b_f$ and $\rho_f$, determined as functions of $\omega$ and $\phi_q$ with the help of Eqs. (39)–(41), into the expressions given above, we can—for a given value of $\phi_q$—determine $\Delta_{f,\perp}$ as a function of $\omega$. It should be emphasized that the relation between $\Delta_{f,\perp}$, $\omega$ and $\phi_q$ obtained this way is valid *exactly*. Using $\hat{I}_{\rm D}^f$ or $\hat{I}_{\rm M}^f$ instead of $\hat{I}_f$ would lead to the same relation, since these observables are not independent from $\hat{I}_f$.

Alternatively, we may use the observables (37) instead of Eq. (38). The general expression for $\hat{R}$ [see Eqs. (26) and (37)] implies an equation similar to Eq. (56), where $A_f$, $B_f$ and $C_f$ have to be replaced through

$$A = \frac{1}{N}[(b_\|\sin\rho_\| - b_0\sin\rho_0)\cos\omega + b_0 b_\|(\sin\rho_0\cos\rho_\| - \cos\rho_0\sin\rho_\|)] \tag{62}$$

$$B = \frac{1}{N}[1-(b_0\cos\rho_0 + b_\|\cos\rho_\|)\cos\omega + b_0 b_\|(\cos\rho_0\cos\rho_\| + \sin\rho_0\sin\rho_\|)] \tag{63}$$

$$C = \hat{R}, \tag{64}$$

where

$$N = \sqrt{(1-2b_0\cos\rho_0\cos\omega + b_0^2)(1-2b_\|\cos\rho_\|\cos\omega + b_\|^2)}. \tag{65}$$

Obviously, the most efficient strategy of combining the observables provided by the $B_q \to X_1 X_2$ angular distribution depends on their actually measured values.

If we are willing to make more extensive use of flavor-symmetry arguments than just to fix the strong phase difference $\Delta_{\tilde{f},f}$, it is in principle possible to determine also the $B_q^0$-$\overline{B_q^0}$ mixing phase $\phi_q$. In the following section, we will have a closer look at the decay $B_d \to J/\psi\rho^0$, which can be related to $B_s \to J/\psi\phi$ through $SU(3)$ arguments and a certain dynamical assumption concerning ''exchange'' and ''penguin annihilation'' topologies. However, before we turn to these modes, which allow the simultaneous extraction of $\phi_d = 2\beta$ and $\gamma$, let us first give two useful expressions for the observables $\hat{R}$ and $\hat{I}_f$. Since the parameters $b_f$ measure typically the importance of ''penguin'' topologies in comparison with current–current contributions, they may not be too large. If we eliminate the hadronic parameters $b_f$ and $\rho_f$ in $\hat{R}$ and $\hat{I}_f$ with the help of the observables $\hat{A}_{\rm D}^f$ and $\hat{A}_{\rm M}^f$ and keep only the leading-order terms in $b_f$, we obtain

$$\hat{R}\approx\cos\Delta_{0,\parallel}-\frac{1}{2}(\hat{A}_{\rm D}^0-\hat{A}_{\rm D}^{\parallel})\frac{\sin\Delta_{0,\parallel}}{\tan\omega} \qquad (66)$$

$$\hat{I}_f\approx\sin\Delta_{f,\perp}+\frac{1}{2}(\hat{A}_{\rm D}^f-\hat{A}_{\rm D}^{\perp})\frac{\cos\Delta_{f,\perp}}{\tan\omega}, \qquad (67)$$

allowing us to determine $\omega$ if the strong phase differences $\Delta_{0,\parallel}$ or $\Delta_{f,\perp}$ are known. Interestingly, the leading-order expressions (66) and (67) do not depend on the $B_q^0$-$\overline{B_q^0}$ mixing phase $\phi_q$. A possible disadvantage of $\hat{R}$ is that $\omega$ enters in combination with $\sin\Delta_{0,\parallel}$. Since $\Delta_{0,\parallel}$ is a difference of $CP$-conserving strong phases [see Eq. (28)], it may be small, thereby weakening the sensitivity of these observables on $\omega$. The situation concerning this point is very different in the case of the observables $\hat{I}_f$, which allow us to determine $\omega$ even in the case of $\Delta_{f,\perp}\in\{0°,180°\}$.

## IV. EXTRACTING $\beta$ AND $\gamma$ FROM $B_d\to J/\psi\rho^0$ AND $B_s\to J/\psi\phi$

If we combine the observables describing the time-dependent angular distribution of the decay $B_d\to J/\psi[\to l^+l^-]\rho^0[\to\pi^+\pi^-]$ with those of $B_s\to J/\psi[\to l^+l^-]\phi[\to K^+K^-]$, we may extract the $B_d^0$-$\overline{B_d^0}$ mixing phase $\phi_d=2\beta$ and the angle $\gamma$ of the unitarity triangle. The $B_d\to J/\psi\rho^0$ angular distribution can be obtained straightforwardly from the $B_s\to J/\psi\phi$ case, which has been discussed in detail in [7], by performing appropriate replacements of kinematical variables.

The decay $B_d^0\to J/\psi\rho^0$ originates from $\overline{b}\to\overline{c}c\overline{d}$ quark-level transitions; the structure of its decay amplitude is completely analogous to the one of $B_s^0\to J/\psi K_{\rm S}$ (see [17]). For a given final-state configuration $f$ with $CP$ eigenvalue $\eta_f$, we have

$$A(B_d^0\to[J/\psi\rho^0]_f)=\lambda_c^{(d)}[A_{\rm cc}^{(c)f}+A_{\rm pen}^{(c)f}]+\lambda_u^{(d)}A_{\rm pen}^{(u)f}+\lambda_t^{(d)}A_{\rm pen}^{(t)f}, \qquad (68)$$

where $A_{\rm cc}^{(c)f}$ is due to current–current contributions, and the amplitudes $A_{\rm pen}^{(q)f}$ describe penguin topologies with internal $q$ quarks ($q\in\{u,c,t\}$). These penguin amplitudes take into account both QCD and electroweak penguin contributions. Employing the unitarity of the CKM matrix and the Wolfenstein parametrization [18], generalized to include nonleading terms in $\lambda$ [19], we obtain

$$A(B_d^0\to[J/\psi\rho^0]_f)=-\lambda\mathcal{A}_f[1-a_fe^{i\theta_f}e^{i\gamma}], \qquad (69)$$

where

$$\mathcal{A}_f\equiv\lambda^2A[A_{\rm cc}^{(c)f}+A_{\rm pen}^{(ct)f}], \qquad (70)$$

with $A_{\rm pen}^{(ct)f}\equiv A_{\rm pen}^{(c)f}-A_{\rm pen}^{(t)f}$, and

$$a_fe^{i\theta_f}\equiv R_b\left(1-\frac{\lambda^2}{2}\right)\left[\frac{A_{\rm pen}^{(ut)f}}{A_{\rm cc}^{(c)f}+A_{\rm pen}^{(ct)f}}\right]. \qquad (71)$$

The quantity $A_{\rm pen}^{(ut)f}$ is defined in analogy to $A_{\rm pen}^{(ct)f}$, and the relevant CKM factors are given in Eqs. (10) and (11). It should be emphasized that the standard-model parametrization (69) of the $B_d^0\to[J/\psi\rho^0]_f$ decay amplitude relies *only* on the unitarity of the CKM matrix. In particular, it takes also into account final-state-interaction effects, which can be considered as long-distance penguin topologies with internal up- and charm-quark exchanges [20]. Comparing Eq. (69) with Eq. (4), we observe that

$$\mathcal{N}_f=-\lambda\mathcal{A}_f,\quad b_f=a_f,\quad \rho_f=\theta_f,\quad \omega=\gamma. \qquad (72)$$

In this parametrization, we have eliminated the $\lambda_t^{(d)}$ term in Eq. (68) with the help of Eq. (8), which is the appropriate choice to relate $B_d^0\to J/\psi\rho^0$ to $B_s^0\to J/\psi\phi$. Using the same notation as in Eq. (69), we have

$$A(B_s^0\to[J/\psi\phi]_f)=\left(1-\frac{\lambda^2}{2}\right)\mathcal{A}_f'[1+\epsilon a_f'e^{i\theta_f'}e^{i\gamma}], \qquad (73)$$

where $\mathcal{A}_f'$ and $a_f'e^{i\theta_f'}$ take the same form as Eqs. (70) and (71), respectively, and

$$\epsilon\equiv\frac{\lambda^2}{1-\lambda^2}. \qquad (74)$$

The primes remind us that we are dealing with a $\overline{b}\to\overline{s}$ transition. Consequently, if we compare Eq. (73) with Eq. (4), we obtain

$$\mathcal{N}_f=\left(1-\frac{\lambda^2}{2}\right)\mathcal{A}_f',\quad b_f=\epsilon a_f',\quad \rho_f=\theta_f'+180°,\quad \omega=\gamma. \qquad (75)$$

The $B_s\to J/\psi\phi$ and $B_d\to J/\psi\rho^0$ observables can be related to each other through

$$|\mathcal{A}_f'|=\sqrt{2}|\mathcal{A}_f| \qquad (76)$$

$$\Delta_{\tilde{f},f}'=\Delta_{\tilde{f},f} \qquad (77)$$

$$a_f'=a_f,\quad \theta_f'=\theta_f, \qquad (78)$$

where the factor of $\sqrt{2}$ is due to the $\rho^0$ wave function. These relations rely both on the $SU(3)$ flavor symmetry of strong interactions and on the neglect of certain ''exchange'' and ''penguin annihilation'' topologies. Although such topologies, which can be probed, for example, through $B_s\to\rho^+\rho^-$, $D^{*+}D^{*-}$ decays, are usually expected to play a very minor role, they may in principle be enhanced through final-state-interaction effects [21]. For the following considerations, it is useful to introduce the quantities

$$H_f\equiv\frac{1}{\epsilon}\left(\frac{|\mathcal{A}_f'|}{|\mathcal{A}_f|}\right)^2\frac{S_f}{S_f'}=\frac{1-2a_f\cos\theta_f\cos\gamma+a_f^2}{1+2\epsilon a_f'\cos\theta_f'\cos\gamma+\epsilon^2a_f'^2}, \qquad (79)$$

which can be fixed through the ''untagged'' $B_d \to J/\psi\rho^0$ and $B_s \to J/\psi\phi$ observables with the help of Eq. (76). Consequently, each of the linear polarization states $f \in \{0,\|,\perp\}$ provides the following three observables:

$$H_f, \quad \hat{A}_{\rm D}^f, \quad \hat{A}_{\rm M}^f. \tag{80}$$

Applying Eq. (78) to Eq. (79), these observables depend only on the hadronic parameters $a_f$ and $\theta_f$, as well as on the $B_d^0$-$\overline{B_d^0}$ mixing phase $\phi_d = 2\beta$ and the angle $\gamma$ of the unitarity triangle. If we choose two different linear polarization states, the observables (80) allow us to determine the corresponding hadronic parameters and $\beta$ and $\gamma$ simultaneously.

This approach can be implemented in a mathematical way as follows: if we consider a given final-state configuration $f$ and combine the observables $H_f$ and $\hat{A}_{\rm D}^f$, which do not depend on $\phi_d$, with each other, we can determine $a_f$ and $\theta_f$ as functions of $\gamma$:

$$a_f = \sqrt{p_f \pm \sqrt{p_f^2 - q_f}} \tag{81}$$

$$2a_f \cos\theta_f = \frac{1 - H_f + (1-\epsilon^2 H_f)a_f^2}{(1+\epsilon H_f)\cos\gamma} \tag{82}$$

$$2a_f \sin\theta_f = \left[\frac{(1+\epsilon)(1+\epsilon a_f^2)H_f}{(1+\epsilon H_f)}\right]\left(\frac{\hat{A}_{\rm D}^f}{\sin\gamma}\right), \tag{83}$$

where

$$p_f = \frac{[2(1+\epsilon H_f)^2\cos^2\gamma - (1-H_f)(1-\epsilon^2 H_f)]\sin^2\gamma - \epsilon_f E_f}{(1-\epsilon^2 H_f)^2\sin^2\gamma + \epsilon^2 E_f} \tag{84}$$

$$q_f = \frac{(1-H_f)^2\sin^2\gamma + E_f}{(1-\epsilon^2 H_f)^2\sin^2\gamma + \epsilon^2 E_f}, \tag{85}$$

with

$$E_f = [(1+\epsilon)H_f \hat{A}_{\rm D}^f \cos\gamma]^2. \tag{86}$$

These expressions allow us to eliminate the hadronic parameters $a_f$ and $\theta_f$ in the mixing-induced $CP$ asymmetry $\hat{A}_{\rm M}^f$, thereby fixing a contour in the $\gamma$–$\phi_d$ plane, which is related to

$$\tilde{A}_f \sin\phi_d + \tilde{B}_f \cos\phi_d = \tilde{C}_f, \tag{87}$$

with

$$\tilde{A}_f = 1 - 2a_f\cos\theta_f\cos\gamma + a_f^2\cos 2\gamma \tag{88}$$

$$\tilde{B}_f = -2a_f\cos\theta_f\sin\gamma + a_f^2\sin 2\gamma \tag{89}$$

$$\tilde{C}_f = (1 - 2a_f\cos\theta_f\cos\gamma + a_f^2)(\eta_f \hat{A}_{\rm M}^f). \tag{90}$$

The solution of Eq. (87) has already been given in Eq. (61). If we consider two different final-state configurations $f$ and $\tilde{f}$, we obtain two different contours in the $\gamma$–$\phi_d$ plane; their intersection allows us to determine both $\gamma$ and $\phi_d = 2\beta$. Using, in addition, the observables (37) or (38)–depending on which final-state configurations $f$ and $\tilde{f}$ we consider—we may resolve discrete ambiguities, arising typically in the extraction of $\phi_d$ and $\gamma$.

Because of the strong suppression of $a_f'$ through $\epsilon = 0.05$ in Eq. (79), this approach is essentially unaffected by possible corrections to Eq. (78), and relies predominantly on the relation (76). If we insert the values of $\phi_d$ and $\gamma$ thus determined into the expressions for the observables of the third linear polarization state $f'$, which has not been used so far, its hadronic parameters $|\mathcal{A}_{f'}|$, $a_{f'}$ and $\theta_{f'}$ can also be determined. Comparing $|\mathcal{A}_{f'}|$ with the $B_s \to J/\psi\phi$ parameter $|\mathcal{A}_{f'}'|$, we can obtain valuable insights into the validity of Eq. (76). Moreover, several other interesting cross checks can be performed with the many observables of the angular distributions. Because of our poor understanding of the hadronization dynamics of non-leptonic $B$ decays, only the ''factorization'' approximation can be used for the time being to estimate factorizable $SU(3)$-breaking corrections to Eq. (76). Explicit expressions for the $B_s \to J/\psi\phi$ observables can be found in [7], and $SU(3)$-breaking effects in the corresponding form factors were studied in [22]. However, also non-factorizable effects are expected to play an important role, and experimental insights into these issues would be very helpful to find a better theoretical description.

The simultaneous extraction of $\phi_d$ and $\gamma$ discussed above works only if the hadronic parameters $a_f$ and $\theta_f$ are sufficiently different from each other for two different final-state configurations $f$. If, for example, Eq. (48) should apply to $B_d \to J/\psi\rho^0$—which seems to be quite unlikely—the $B_d^0$-$\overline{B_d^0}$ mixing phase has to be fixed separately in order to determine $\gamma$. In this case, each linear polarization state $f \in \{0,\|,\perp\}$ provides a strategy to extract $\gamma$ that is completely analogous to the one proposed in [17], which makes use of $B_{s(d)} \to J/\psi K_{\rm S}$ decays. If we combine $H_f$ with $\hat{A}_{\rm M}^f$, we obtain

$$a_f = \sqrt{\frac{H_f - 1 + u_f(1+\epsilon H_f)\cos\gamma}{1 - v_f(1+\epsilon H_f)\cos\gamma - \epsilon^2 H_f}}. \tag{91}$$

The intersection of the contours in the $\gamma$–$a_f$ plane described by this expression with those related to Eq. (39) allows us to determine $\gamma$ and $a_f$.

If we use $\phi_d$ as an input in order to extract $\gamma$ from $B_d \to J/\psi\rho^0$, it is, however, more favorable to follow the approach discussed in the previous section, i.e. to use Eq. (61), and to fix $\Delta_{f,\perp}$ (or $\Delta_{0,\|}$) through the the $B_s \to J/\psi\phi$ observables with the help of Eqs. (77) and (78). Using in addition the observables involving the third linear polarization state $f'$ that we have not employed so far, we can also fix its hadronic parameters $a_{f'}$ and $\theta_{f'}$, as well as the strong phase difference $\Delta_{f',f}$. Comparing $\Delta_{f',f}$ with its $B_s \to J/\psi\phi$ counterpart $\Delta_{f',f}'$, we may obtain valuable insights into possible corrections to (77).

As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the $B_s^0$-$\overline{B_s^0}$ mixing phase $\phi_s$ from $B_s \to J/\psi\phi$. Although the penguin contributions are strongly suppressed in this mode because of the tiny parameter $\epsilon = 0.05$ [see Eq. (73)], they may well lead to uncertainties of the extracted value of $\phi_s$ at the level of 10%, since $\phi_s = \mathcal{O}(0.03)$ within the standard model. A measurement of $\phi_s = -2\lambda^2\eta$ would allow us to determine the Wolfenstein parameter $\eta$ [18], thereby fixing the height of the unitarity triangle. Since the decay $B_s \to J/\psi\phi$ is very accessible at ''second-generation'' $B$-physics experiments performed at hadron machines, for instance at LHCb, it is an important issue to think about the hadronic uncertainties affecting the determination of $\phi_s$ from the corresponding angular distribution. The approach discussed above allows us to control these uncertainties with the help of $B_d \to J/\psi\rho^0$.

The experimental feasibility of the determination of $\gamma$ from the $B_d \to J/\psi\rho^0$ angular distribution depends strongly on the ''penguin parameters'' $a_f$. It is very difficult to estimate these quantities theoretically. In contrast to the ''usual'' QCD penguin topologies, the QCD penguins contributing to $B_d \to J/\psi\rho^0$ require a color-singlet exchange, i.e. are ''Zweig-suppressed.'' Such a comment does not apply to the electroweak penguins, which contribute in ''color-allowed'' form. The current–current amplitude $A_{\rm cc}^{(c)f}$ originates from ''color-suppressed'' topologies, and the ratio $A_{\rm pen}^{(ut)f}/[A_{\rm cc}^{(c)f} + A_{\rm pen}^{(ct)f}]$, which governs $a_f$, may be sizeable. It would be very important to have a better theoretical understanding of the quantities $a_f e^{i\theta_f}$. However, such analyses are far beyond the scope of this paper, and are left for further studies.

If the parameters $a_f$ should all be very small, which would be indicated by $A_{\rm D}^f = R_{\rm D} = I_{\rm D}^f = 0$, we could still determine the $B_d^0$-$\overline{B_d^0}$ mixing phase from the observables $\hat{A}_{\rm M}^f = \eta_f \sin\phi_d$. If we use, in addition, $\hat{I}_{\rm M}^f = \cos\Delta_{f,\perp}\cos\phi_d$ and fix $\cos\Delta_{f,\perp}$ through the corresponding $B_s \to J/\psi\phi$ observable, $\cos\phi_d$ can be determined as well. Consequently, the $B_d^0$-$\overline{B_d^0}$ mixing phase $\phi_d$ can be fixed *unambiguously* this way, thereby resolving a twofold ambiguity, which arises in the extraction of $\phi_d$ from $B_d \to J/\psi K_{\rm S}$. This mode probes only $\sin\phi_d$. Since $\phi_d = 2\beta$, we are left with a twofold ambiguity for $\beta \in [0°,360°]$. If we assume that $\beta \in [0°,180°]$, as implied by the measured value of $\varepsilon_K$, we can fix $\beta$ unambiguously. For alternative methods to deal with ambiguities of this kind, see [16].

Before we turn to $B_d \to \rho\rho$ and $B_s \to K^*\overline{K^*}$ decays, let us note that the approach presented in this section can also be applied to the angular distributions of the decay products of $B_{s(d)} \to J/\psi[\to l^+l^-]K^*[\to \pi^0 K_{\rm S}]$ and $B_{d(s)} \to D_{d(s)}^{*+} D_{d(s)}^{*-}$. For the $B_{s(d)} \to J/\psi K_{\rm S}$ and $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ variants of these strategies, see [17].

## V. FURTHER APPLICATIONS

In this section, we discuss further applications of the general strategies presented in Sec. III. All of the methods discussed below have counterparts using $B_{d,s}$ decays into two pseudoscalar mesons. If we replace the pseudoscalars by higher resonances, for example, by vector mesons, as in the following discussion, the angular distributions of their decay products provide interesting alternative ways to extract CKM phases and hadronic parameters, going beyond the $B_{d,s} \to PP$ strategies. Because of the many observables provided by the angular distributions, we can, moreover, perform many interesting cross checks, for example, of certain flavor-symmetry relations.

### A. The decays $B_d \to \rho^+\rho^-$ and $B_s \to K^{*+}K^{*-}$

The decay $B_d^0 \to \rho^+\rho^-$ originates from $\bar{b} \to \bar{u}u\bar{d}$ quark-level processes. Using the same notation as in Eq. (69), we have

$$A(B_d^0 \to [\rho^+\rho^-]_f) = \left(1 - \frac{\lambda^2}{2}\right)\mathcal{C}_f e^{i\gamma}[1 - d_f e^{i\Theta_f} e^{-i\gamma}], \tag{92}$$

where

$$\mathcal{C}_f \equiv \lambda^3 A R_b [\tilde{A}_{\rm cc}^{(u)f} + \tilde{A}_{\rm pen}^{(ut)f}] \tag{93}$$

and

$$d_f e^{i\Theta_f} \equiv \frac{1}{(1-\lambda^2/2)R_b}\left[\frac{\tilde{A}_{\rm pen}^{(ct)f}}{\tilde{A}_{\rm cc}^{(u)f} + \tilde{A}_{\rm pen}^{(ut)f}}\right]. \tag{94}$$

In order to distinguish the $B_d^0 \to \rho^+\rho^-$ amplitudes from the $B_d^0 \to J/\psi\rho^0$ case discussed in the previous section, we have introduced the tildes. The phase structure of the $B_d^0 \to \rho^+\rho^-$ decay amplitude given in Eq. (92), which is an exact parametrization within the standard model, is completely analogous to the one for the $B_d^0 \to \pi^+\pi^-$ amplitude given in [23], where a more detailed discussion can be found.

The expressions for the observables describing the time evolution of the angular distribution of the decay products of $B_d^0 \to \rho^+[\to \pi^+\pi^0]\rho^-[\to \pi^-\pi^0]$ can be obtained straightforwardly from the formulas given in Sec. II, by performing the following substitutions:

$$\mathcal{N}_f = \left(1 - \frac{\lambda^2}{2}\right)\mathcal{C}_f, \quad b_f = d_f, \quad \rho_f = \Theta_f, \quad \omega = -\gamma. \tag{95}$$

Because of the factor of $e^{i\gamma}$ in front of the square brackets on the right-hand side of Eq. (92), we have to deal with a small complication. Since the observables are governed by

$$\xi_f = e^{-i\phi_d}\frac{\overline{A_f}}{A_f} = \eta_f e^{-i(\phi_d + 2\gamma)}\left[\frac{1 - d_f e^{i\Theta_f} e^{+i\gamma}}{1 - d_f e^{i\Theta_f} e^{-i\gamma}}\right], \tag{96}$$

we have to do the following replacement, in addition:

$$\phi_d \to \phi_d + 2\gamma. \tag{97}$$

Let us now turn to the decay $B_s^0 \to K^{*+}K^{*-}$, which is due to $\bar{b} \to \bar{u}u\bar{s}$ quark-level transitions. For a given final-state configuration $f$ of the $K^{*+}K^{*-}$ pair, its decay amplitude can be parametrized as follows:

$$A(B_s^0\to[K^{*+}K^{*-}]_f)=\lambda\mathcal{C}_f' e^{i\gamma}\left[1+\left(\frac{1-\lambda^2}{\lambda^2}\right)d_f' e^{i\Theta_f'}e^{-i\gamma}\right], \tag{98}$$

where $\mathcal{C}_f'$ and $d_f' e^{i\Theta_f'}$ take the same form as Eqs. (93) and (94), respectively, and the primes have been introduced to remind us that we are dealing with a $\overline{b}\to\overline{s}$ mode. The phase structure of Eq. (98) is completely analogous to the $B_s^0\to K^+K^-$ decay amplitude [23]. The observables of the time-dependent $B_s\to K^{*+}[\to\pi K]K^{*-}[\to\overline{\pi K}]$ angular distribution can be obtained straightforwardly from the formulas given in Sec. II by simply using the replacements:

$$\mathcal{N}_f=\lambda\mathcal{C}_f',\quad b_f=\left(\frac{1-\lambda^2}{\lambda^2}\right)d_f',\quad \rho_f=\Theta_f'+180°,$$
$$\omega=-\gamma. \tag{99}$$

Moreover, we have to perform the substitution

$$\phi_s\to\phi_s+2\gamma \tag{100}$$

because of the factor of $e^{i\gamma}$ in front of the square brackets in Eq. (98).

Explicit expressions for the $B_d\to\rho^+\rho^-$ and $B_s\to K^{*+}K^{*-}$ angular distributions in terms of helicity amplitudes can be found in [13]. Since $B_d^0\to\rho^+\rho^-$ and $B_s^0\to K^{*+}K^{*-}$ are related to each other by interchanging all down and strange quarks, the *U*-spin flavor symmetry of strong interactions implies

$$|\mathcal{C}_f'|=|\mathcal{C}_f|,\quad d_f'=d_f,\quad \Theta_f'=\Theta_f, \tag{101}$$

as well as

$$\Delta_{\tilde f,f}'=\Delta_{\tilde f,f}. \tag{102}$$

In contrast to Eqs. (76)–(78), these relations do not rely on any dynamical assumption—just on the *U*-spin flavor symmetry. They can be used to combine the $B_d\to\rho^+\rho^-$ and $B_s\to K^{*+}K^{*-}$ observables with each other, thereby allowing the extraction of the CKM angle $\gamma$ and of the $B_{d,s}^0$-$\overline{B}_{d,s}^0$ mixing phases $\phi_d=2\beta$ and $\phi_s=-2\delta\gamma$. In contrast to the $B_d\to\pi^+\pi^-$, $B_s\to K^+K^-$ variant of this approach proposed in [23], *both* mixing phases and the CKM angle $\gamma$ can in principle be determined simultaneously. However, for the extraction of $\gamma$, it is more favorable to fix $\phi_d$ and $\phi_s$ separately. Then we are in a position to determine two contours in the $\gamma$–$\Delta_{\tilde f,f}$ and $\gamma$–$\Delta_{\tilde f,f}'$ planes in a *theoretically clean* way with the help of Eq. (61). Using now the *U*-spin relation (102), $\gamma$ and all hadronic parameters describing the decays $B_d\to\rho^+\rho^-$ and $B_s\to K^{*+}K^{*-}$ can be determined. As we have already noted, the hadronic parameters provide a very fertile testing ground for model calculations of the decays $B_d\to\rho^+\rho^-$ and $B_s\to K^{*+}K^{*-}$. In particular, the penguin parameters $d_f e^{i\Theta_f}$ and $d_f' e^{i\Theta_f'}$ would be very interesting; comparing their values with each other, we could obtain valuable insights into *U*-spin-breaking corrections. Moreover, there is one strong phase difference $\Delta_{f',f}$ left, which can be compared with its *U*-spin counterpart $\Delta_{f',f}'$. If we should find a small difference between these phases, it would be quite convincing to assume that our *U*-spin input (102) is also not affected by large corrections.

Whereas the parametrization of $B_d^0\to\rho^+\rho^-$ given in Eq. (92) is the appropriate one to relate this mode to $B_s^0\to K^{*+}K^{*-}$ through the *U*-spin flavor symmetry of strong interactions, there is another interesting way to parametrize the $B_d\to\rho^+\rho^-$ decay amplitudes (see also [24]). If we eliminate $\lambda_c^{(d)}$ through the unitarity of the CKM matrix—instead of $\lambda_t^{(d)}$, as done in Eq. (92)—we obtain

$$A(B_d^0\to[\rho^+\rho^-]_f)=\left(1-\frac{\lambda^2}{2}\right)\lambda^3AR_be^{i\gamma}[\tilde A_{\rm cc}^{(u)f}+\tilde A_{\rm pen}^{(uc)f}]$$
$$\times[1+r_fe^{i\sigma_f}e^{-i(\beta+\gamma)}], \tag{103}$$

where

$$r_fe^{i\sigma_f}\equiv\frac{R_t}{(1-\lambda^2/2)R_b}\left[\frac{\tilde A_{\rm pen}^{(tc)f}}{\tilde A_{\rm cc}^{(u)f}+\tilde A_{\rm pen}^{(uc)f}}\right]. \tag{104}$$

The CKM factor $R_t$ has been introduced in Eq. (11). Taking into account that we have $\phi_d=2\beta$ and $\beta+\gamma=180°-\alpha$ within the standard model, we arrive at

$$b_f=r_f,\quad \rho_f=\sigma_f,\quad \omega=\alpha, \tag{105}$$

and at the ''effective'' mixing phase $\phi=\phi_d+2\gamma=-2\alpha$. Consequently, using the strategy presented in Sec. III, the $B_d\to\rho^+\rho^-$ angular distribution allows us to probe also the combination $\alpha=180°-\beta-\gamma$ directly, i.e. to determine $\alpha$ as a function of a *CP*-conserving strong phase difference $\Delta_{\tilde f,f}$ [see also Eqs. (66) and (67)]. Needless to note that the decay $B_d\to\rho^0\rho^0$ may also be interesting in this respect. Since the normalization factors $\mathcal{N}_f$ of the parametrization (103) are proportional to

$$\tilde A_{\rm cc}^{(u)f}+\tilde A_{\rm pen}^{(uc)f}, \tag{106}$$

which is governed by ''color-allowed tree-diagram-like'' topologies, it may well be that $\Delta_{\tilde f,f}\approx 0$. This relation would allow us to extract $\alpha$, as well as the $B_d\to\rho^+\rho^-$ hadronic parameters, which include also another strong phase difference $\Delta_{f',f}$, providing an important cross check.

### B. The decays $B_d\to K^{*0}\overline{K}^{*0}$ and $B_s\to K^{*0}\overline{K}^{*0}$

The decays $B_d^0\to K^{*0}\overline{K}^{*0}$ and $B_s^0\to K^{*0}\overline{K}^{*0}$ are pure ''penguin'' modes, originating from $\overline{b}\to\overline{d}s\overline{s}$ and $\overline{b}\to\overline{s}d\overline{d}$ quark-level transitions, respectively. They do not receive contributions from current–current operators at the ''tree'' level, and can be parametrized within the standard model in complete analogy to Eqs. (69) and (73). We have just to set the current–current amplitudes equal to zero in these expressions. The decays $B_d^0\to K^{*0}\overline{K}^{*0}$ and $B_s^0\to K^{*0}\overline{K}^{*0}$ are related to each other by interchanging all down and strange quarks, i.e. through the $U$-spin flavor symmetry of strong interactions, and the strategies to probe $\gamma$ and the $B^0_{d,s}$-$\overline{B}^0_{d,s}$ mixing phases are analogous to those discussed in Sec. IV. Since the $B_{d,s}\to K^{*0}\overline{K}^{*0}$ decays are pure ''penguin'' modes, they represent a particularly sensitive probe for new physics.

An interesting alternative to parametrize the $B^0_d \to[K^{*0}\overline{K}^{*0}]_f$ decay amplitudes within the standard model is given as follows:

$$A(B^0_d\to[K^{*0}\overline{K}^{*0}]_f)=\lambda^3 A R_t \mathcal{A}^{(tu)f}_{\rm pen} e^{-i\beta}[1-g_f e^{i\varphi_f}e^{i\beta}], \tag{107}$$

where

$$g_f e^{i\varphi_f}\equiv\frac{1}{R_t}\frac{\mathcal{A}^{(cu)f}_{\rm pen}}{\mathcal{A}^{(tu)f}_{\rm pen}} \tag{108}$$

may well be sizeable due to the presence of final-state-interaction effects [25]. Consequently, we have

$$b_f=g_f,\quad \rho_f=\varphi_f,\quad \omega=\beta. \tag{109}$$

Because of the factor of $e^{-i\beta}$ in front of the square brackets on the right-hand side of Eq. (107), the ''effective'' mixing phase is given by $\phi=\phi_d-2\beta$. Consequently, the strategy presented in Sec. III allows us to probe the $CP$-violating weak phase $\beta$ of the CKM element $V_{td}=|V_{td}|e^{-i\beta}$. Within the standard model, we have $\phi=\phi_d-2\beta=0$. However, this relation may well be affected by new physics, and represents a powerful test of the standard-model description of $CP$ violation (for a recent discussion, see [26]). Therefore it would be very important to determine this combination of CKM phases experimentally. The observables of the $B_d \to K^{*0}[\to\pi^-K^+]\overline{K}^{*0}[\to\pi^+K^-]$ angular distribution may provide an important step towards this goal.

## VI. CONCLUSIONS

The angular distributions of certain quasi-two-body modes $B_{d,s}\to X_1X_2$, where both $X_1$ and $X_2$ carry spin and continue to decay through $CP$-conserving interactions, provide valuable information about CKM phases and hadronic parameters. We have presented the general formalism to accomplish this task, taking into account also penguin contributions, and have illustrated it by having a closer look at a few specific decay modes. In comparison with strategies using non-leptonic $B_{d,s}$ decays into two pseudoscalar mesons, an important advantage of the angular distributions is that they provide much more information, thereby allowing various interesting cross checks, for instance, of certain flavor-symmetry relations. Moreover, they provide a very fertile testing ground for model calculations of the $B_{d,s}\to X_1X_2$ modes.

We have pointed out that the decay $B_d\to J/\psi\rho^0$ can be combined with $B_s\to J/\psi\phi$ to extract the $B^0_d$-$\overline{B}^0_d$ mixing phase $\phi_d=2\beta$ and—if penguin effects in the former mode should be sizeable—also the angle $\gamma$ of the unitarity triangle. As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the $B^0_s$-$\overline{B}^0_s$ mixing phase from $B_s\to J/\psi\phi$. If penguin effects should be very small in $B_d\to J/\psi\rho^0$, $\phi_d$ could still be determined and it would even be possible to resolve a twofold ambiguity, arising in the extraction of this CKM phase from $B_d\to J/\psi K_S$. Other interesting applications, involving $B_d \to\rho\rho$ and $B_{s,d}\to K^*\overline{K}^*$ decays, were also noted. Within the standard model, these modes are expected to exhibit branching ratios at the $10^{-5}$ level; also the one for $B_d\to K^{*0}\overline{K}^{*0}$ may well be enhanced, from its ''short-distance'' expectation of $\mathcal{O}(10^{-6})$ to this level, by final-state-interaction effects.

Since the formalism presented in this paper is very general, it can of course be applied to many other decays. Detailed studies are required to explore which channels are most promising from an experimental point of view. Although the $B_d$ modes listed above may already be accessible at the asymmetric $e^+$–$e^-$ $B$-factories operating at the $\Upsilon(4S)$ resonance, which will start taking data very soon, the strategies presented in this paper appear to be particularly interesting for ''second-generation'' experiments at hadron machines, such as LHCb or BTeV, where also the very powerful physics potential of the $B_s$ system can be exploited.

## ACKNOWLEDGMENTS

I am very grateful to Jerome Charles for interesting discussions.

[1] L. L. Chau and W.-Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984); C. Jarlskog and R. Stora, Phys. Lett. B **208**, 268 (1988).

[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] For reviews, see, for instance, *The BaBar Physics Book*, edited by P. F. Harrison and H.R. Quinn, SLAC Report 504, 1998; Y. Nir, in the *Proceedings of the 18th International Symposium on Lepton-Photon Interactions (LP '97)*, Hamburg, Germany, 1997, edited by A. De Roeck and A. Wagner (World Scientific, Singapore, 1998), p. 295; M. Gronau, Nucl. Phys. B (Proc. Suppl.) **65**, 245 (1998); R. Fleischer, Int. J. Mod. Phys. A **12**, 2459 (1997).

[4] I. Dunietz, H. Quinn, A. Snyder, W. Toki, and H. J. Lipkin, Phys. Rev. D **43**, 2193 (1991).

[5] R. Fleischer and I. Dunietz, Phys. Rev. D **55**, 259 (1997); Phys. Lett. B **387**, 361 (1996).

[6] J. L. Rosner, Phys. Rev. D **42**, 3732 (1990).

[7] A. S. Dighe, I. Dunietz, and R. Fleischer, Eur. Phys. J. C **6**, 647 (1999); see also A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B **369**, 144 (1996).

[8] D. London, N. Sinha and R. Sinha, this issue, Phys. Rev. D **60**, 0740 ■■ (1999).

[9] For a recent calculation of $\Delta\Gamma_s$, see M. Beneke, G. Buchalla, C. Greub, A. Lenz, and U. Nierste, Report No. CERN-TH/98-261, hep-ph/9808385.

[10] I. Dunietz, Phys. Rev. D **52**, 3048 (1995).
[11] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).
[12] R. Fleischer, Z. Phys. C **58**, 483 (1993); **62**, 81 (1994); G. Kramer, W. F. Palmer, and H. Simma, *ibid.* **66**, 429 (1995).
[13] G. Kramer and W. F. Palmer, Phys. Rev. D **45**, 193 (1992); **46**, 3197 (1992); G. Kramer, W. F. Palmer, and H. Simma, Nucl. Phys. **B428**, 77 (1994).
[14] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981); I. I. Bigi and A. I. Sanda, Nucl. Phys. **B193**, 85 (1981).
[15] Y. Nir and D. Silverman, Nucl. Phys. **B345**, 301 (1990).
[16] See, for example, Y. Grossman and H. R. Quinn, Phys. Rev. D **56**, 7259 (1997); J. Charles, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Lett. B **425**, 375 (1998); Phys. Rev. D **58**, 114021 (1998); A. S. Dighe, I. Dunietz, and R. Fleischer, Phys. Lett. B **433**, 147 (1998).
[17] R. Fleischer, Eur. Phys. J. C (to be published), Report No. CERN-TH/99-78, hep-ph/9903455.
[18] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[19] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D **50**, 3433 (1994).
[20] A. J. Buras, R. Fleischer, and T. Mannel, Nucl. Phys. **B533**, 3 (1998).
[21] See, for instance, L. Wolfenstein, Phys. Rev. D **52**, 537 (1995); B. Blok and I. Halperin, Phys. Lett. B **385**, 324 (1996); B. Blok, M. Gronau, and J. L. Rosner, Phys. Rev. Lett. **78**, 3999 (1997); J.-M. Gérard and J. Weyers, Eur. Phys. J. C **7**, 1 (1999); M. Neubert, Phys. Lett. B **424**, 152 (1998); A. F. Falk, A. L. Kagan, Y. Nir, and A. A. Petrov, Phys. Rev. D **57**, 4290 (1998); D. Atwood and A. Soni, *ibid.* **58**, 036005 (1998).
[22] P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).
[23] R. Fleischer, Phys. Lett. B **459**, 306 (1999).
[24] J. Charles, Phys. Rev. D **59**, 054007 (1999).
[25] R. Fleischer, Phys. Lett. B **341**, 205 (1994); Phys. Rev. D **58**, 093001 (1998).
[26] L. Wolfenstein, Phys. Rev. D **57**, 6857 (1998).