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Four-neutrino mass spectra and the Super-Kamiokande atmospheric up-down asymmetry

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In the framework of schemes with mixing of four massive neutrinos, which can accommodate the atmospheric, solar and LSND ranges of Δm^2 and contain three active neutrinos and a sterile one, we show that, in the whole region of $\Delta m^2_{\rm LSND}$ allowed by LSND, the Super-Kamiokande up-down asymmetry excludes all mass spectra with a group of three close neutrino masses separated from the fourth mass by the LSND gap of order 1 eV. Only two schemes with mass spectra in which two pairs of close masses are separated by the LSND gap can describe the Super-Kamiokande up-down asymmetry and all other existing neutrino oscillation data. [S0556-2821(99)03419-0]

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The observation of a significant up-down asymmetry of atmospheric high-energy $\overset{(-)}{\nu}_\mu$- induced events in the Super-Kamiokande experiment [1] is considered as the first model-independent evidence in favor of neutrino oscillations. Such indications were also obtained in other atmospheric neutrino experiments: Kamiokande, IMB, Soudan-2 and MACRO [2]. In addition, evidence in favor of neutrino masses and mixing is provided by all solar neutrino experiments: Homestake, Kamiokande, GALLEX, SAGE and Super-Kamiokande [3]. Finally, observation of $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillations have been claimed by the LSND Collaboration [4]. For the explanation of all these data three different scales of neutrino mass-squared differences are required: $\Delta m^2_{\rm sun} \sim 10^{-10}$ eV^2 (vacuum oscillations) or $\Delta m^2_{\rm sun} \sim 10^{-5}$ eV^2 (MSW), $\Delta m^2_{\rm atm} \sim 10^{-3}$ eV^2, $\Delta m^2_{\rm LSND} \sim 1$ eV^2. Thus, at least four neutrinos with definite mass are needed to describe all data. In the following we will confine ourselves to the minimal scenario of four massive neutrinos. Thus, in addition to the three active neutrinos, one sterile neutrino is required. In our analysis the possibility of transitions into sterile neutrinos will be intrinsically included through the unitarity of the 4×4 mixing matrix. Note that having more than one sterile neutrino such that the additional massive neutrinos are degenerate with one or more of the first four neutrinos does not influence our conclusions.

Four-neutrino schemes have been considered in many papers. For early works see Ref. [5] and for a more comprehensive list of four-neutrino papers consult, e.g., Ref. [6]. In Refs. [7,8] it was shown that from the results of all existing experiments, including short-baseline (SBL) reactor and accelerator experiments in which no indications of neutrino oscillations have been found, information on the four-neutrino mass spectrum can be inferred. In the case of three different scales of Δm^2, there are two different classes of neutrino mass spectra (see Fig. 1) that satisfy the inequalities $\Delta m^2_{\rm sun} \ll \Delta m^2_{\rm atm} \ll \Delta m^2_{\rm LSND}$. In the spectra of class 1 there is a group of three close masses which is separated from the fourth mass by the LSND gap of around 1 eV. It contains the spectra (I)-(IV) in Fig. 1. Note that spectrum (I) corresponds to a mass hierarchy, spectrum (III) to an inverted mass hierarchy, whereas (II) and (IV) are non-hierarchical spectra. In the spectra of class 2 there are two pairs of close masses which are separated by the LSND gap. The two possible spectra in this class are denoted by (A) and (B) in Fig. 1.

It was shown in Ref. [7] that, in the case of the spectra of class 1, from the existing data one can obtain constraints on the amplitude of SBL $\nu_\mu \to \nu_e$ oscillations that are not compatible with the results of the LSND experiment in the allowed region 0.2 eV$^2 \lesssim \Delta m^2_{\rm LSND} \lesssim 2$ eV^2 with the exception of the small interval from 0.2 to 0.3 eV^2. In Ref. [7] the double ratio R of μ-like over e-like events has been used as input from atmospheric neutrino measurements, whereas in the present article we consider what constraints on neutrino mixing can be inferred from the up–down asymmetry of multi-GeV muon-like events measured in the Super-Kamiokande experiment [9], i.e., from

$$A_\mu = \frac{U-D}{U+D} = -0.311 \pm 0.043 \pm 0.01, \tag{1}$$

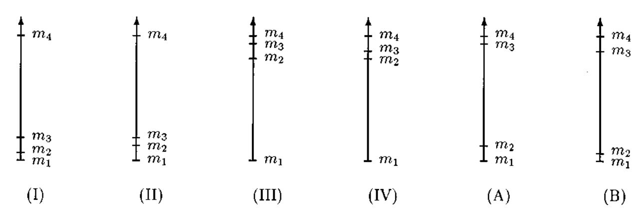

FIG. 1. The six types of neutrino mass spectra that can accommodate the solar, atmospheric and LSND scales of Δm^2. The different distances between the masses on the vertical axes symbolize the different scales of Δm^2. The spectra (I)-(IV) define class 1, whereas class 2 comprises (A) and (B).

where U and D denote the number of events in the zenith angle ranges $-1<\cos\theta<-0.2$ and $0.2<\cos\theta<1$, respectively. We will show that with this input the conclusion of Ref. [7] will be strengthened and that now the neutrino mass spectra of class 1 are disfavored for any value of $\Delta m^2_{\rm LSND}$ in the allowed range. In addition, we will also derive a constraint on the mixing matrix for the neutrino mass spectra (A) and (B).

The general case of mixing of four massive neutrinos is described by $\nu_{\alpha L}=\Sigma_{j=1}^4 U_{\alpha j}\nu_{jL}$, where U is the 4×4 unitary mixing matrix, $\alpha=e,\mu,\tau,s$ denotes the three active neutrino flavors and the sterile neutrino, respectively, and $j=1,\ldots,4$ enumerates the neutrino mass eigenfields. For definiteness, we will consider the spectrum of type I with a neutrino mass hierarchy $m_1\ll m_2\ll m_3\ll m_4$, but the results that we will obtain in this case will apply to all spectra of class 1.

The probability of SBL $\nu_\mu\to\nu_e$ transitions is given by the two-neutrino-like formula [7]

$$P_{\nu_\mu\to\nu_e}=P_{\bar\nu_\mu\to\bar\nu_e}=A_{\mu;e}\sin^2\frac{\Delta m^2_{41}L}{4E}, \tag{2}$$

where $\Delta m^2_{41}\equiv\Delta m^2_{\rm LSND}$, L is the distance between source and detector and E is the neutrino energy. We use the abbreviation $\Delta m^2_{kj}\equiv m_k^2-m_j^2$. The oscillation amplitude $A_{\mu;e}$ is given by

$$A_{\mu;e}=4(1-c_e)(1-c_\mu) \tag{3}$$

with

$$c_\alpha=\sum_{j=1}^{3}|U_{\alpha j}|^2 \quad (\alpha=e,\mu). \tag{4}$$

From the results of reactor and accelerator disappearance experiments it follows that [7]

$$c_\alpha\leq a_\alpha^0 \quad \text{or} \quad c_\alpha\geq 1-a_\alpha^0 \tag{5}$$

with $a_\alpha^0=\frac{1}{2}(1-\sqrt{1-B^0_{\alpha;\alpha}})$, where $B^0_{\alpha;\alpha}$ is the upper bound for the amplitude of $\nu_\alpha\to\nu_\alpha$ oscillations. The exclusion plots obtained from the Bugey and CDHS and CCFR experiments [10] imply that $a_e^0\lesssim 4\times 10^{-2}$ for $\Delta m^2_{\rm LSND}\gtrsim 0.1$ eV^2 and $a_\mu^0\lesssim 0.2$ for $\Delta m^2_{\rm LSND}\gtrsim 0.4$ eV^2 [11]. Below $\Delta m^2\simeq 0.3$ eV^2, the survival amplitude $B_{\mu;\mu}$ is not restricted by experimental data, i.e., $B^0_{\mu;\mu}=1$.

The survival probability of solar ν_e's is bounded by $P^{\odot}_{\nu_e\to\nu_e}\geq(1-c_e)^2$ [7]. Therefore, to be in agreement with the results of solar neutrino experiments we conclude that from the two ranges of c_e in Eq. (5) only

$$c_e\geq 1-a_e^0 \tag{6}$$

is allowed.

We will address now the question of what information on the parameter c_μ can be obtained from the asymmetry A_μ (1). As a first step we derive an upper bound on the number of downward-going μ-like events D. The probability of $\nu_\alpha\to\nu_\alpha$ and $\bar\nu_\alpha\to\bar\nu_\alpha$ transitions of atmospheric neutrinos is given by

$$P_{\nu_\alpha\to\nu_\alpha}=P_{\bar\nu_\alpha\to\bar\nu_\alpha}=\left|\sum_{j=1,2}|U_{\alpha j}|^2+|U_{\alpha 3}|^2\exp\left(-i\frac{\Delta m^2_{31}L}{2E}\right)\right|^2+|U_{\alpha 4}|^4, \tag{7}$$

where we have taken into account that $\Delta m^2_{41}\gg\Delta m^2_{31}$ and $\Delta m^2_{21}L/2E\ll 1$ (Δm^2_{21} is relevant for solar neutrinos). Because of the small value of $\Delta m^2_{\rm atm}\equiv\Delta m^2_{31}$, it is well fulfilled that downward-going neutrinos do not oscillate with the atmospheric mass-squared difference.[1] Therefore, we obtain for the survival probability of downward-going neutrinos

$$P^D_{\nu_\alpha\to\nu_\alpha}=c_\alpha^2+(1-c_\alpha)^2. \tag{8}$$

Furthermore, conservation of probability, Eq. (6) and the experimental limit $a_e^0\lesssim 4\times 10^{-2}$ allow us to deduce the upper bound

$$P^D_{\nu_e\to\nu_\mu}\leq 1-P^D_{\nu_e\to\nu_e}=2c_e(1-c_e)\leq 2a_e^0(1-a_e^0). \tag{9}$$

Note that all arguments hold for neutrinos and antineutrinos. Denoting the number of muon (electron) neutrinos and antineutrinos produced in the atmosphere by n_μ (n_e), from Eqs. (8) and (9) we have the upper bound

$$D\leq n_\mu[c_\mu^2+(1-c_\mu)^2]+2n_e a_e^0(1-a_e^0). \tag{10}$$

Taking into account only the part of D which is determined by the $\overset{(-)}{\nu}_\mu$ survival probability, we immediately obtain the lower bound

$$D\geq n_\mu[c_\mu^2+(1-c_\mu)^2]. \tag{11}$$

Considering only $|U_{\mu 4}|^4$ in Eq. (7), we readily arrive at a lower bound on U as well:

$$U\geq n_\mu(1-c_\mu)^2. \tag{12}$$

This inequality is analogous to the above inequality for the survival of solar neutrinos and is valid also with matter effects in the earth.

Now we can assemble the inequalities (10), (11) and (12) and the main result of this work follows:

$$-A_\mu\leq\frac{c_\mu^2+2a_e^0(1-a_e^0)/r}{c_\mu^2+2(1-c_\mu)^2}, \tag{13}$$

[1] This is not completely true for neutrino directions close to the horizon with $\Delta m^2_{\rm atm}\gtrsim 3\times 10^{-3}$ eV^2. Taking into account the result of the CHOOZ experiment [12], we have checked, however, that numerically this has a negligible impact on the following discussion.

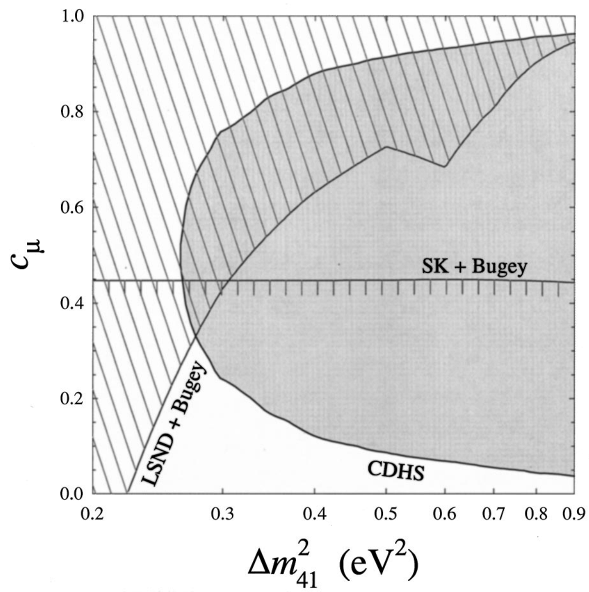

FIG. 2. Regions in the Δm^2_{41}–c_μ plane disfavored by the results of the CDHS, LSND, Super-Kamiokande and Bugey experiments in the case of the spectra of class 1. The shaded region is excluded by the inequalities (15) and the hatched region by the bound (16). The nearly horizontal curve labelled SK + Bugey represents the lower bound (13) derived from the Super-Kamiokande up-down asymmetry. Since this bound lies above the white region allowed by inequalities (15) and (16), the spectra of class 1 are disfavored by the data.

where we have defined $r \equiv n_\mu / n_e$. For the numerical evaluation of Eq. (13) we use $-A_\mu \geqslant 0.254$ at 90% C.L., the 90% C.L. bound a_e^0 from the result of the Bugey experiment and $r=2.8$ read off from Fig. 3 in Ref. [1] of the Super-Kamiokande Collaboration. As a result we get

$$c_\mu \geqslant a_{\rm SK} \simeq 0.45, \tag{14}$$

as can be seen from the horizontal line in Fig. 2. Note that the dependence of this lower bound on $\Delta m^2_{\rm LSND} \equiv \Delta m^2_{41}$ is almost negligible due to the smallness of the second term in the numerator on the right-hand side of Eq. (13). Consequently, also the exact value of r is not important numerically.

In Fig. 2 we have also depicted the bounds

$$c_\mu \leqslant a_\mu^0 \quad \text{and} \quad c_\mu \geqslant 1 - a_\mu^0 \tag{15}$$

that were obtained from the exclusion plot of the CDHS ν_μ disappearance experiment. For $\Delta m^2_{\rm LSND} \simeq 0.24$ eV^2 these two bounds meet at $c_\mu = 0.5$. Below 0.24 eV^2 there are no restrictions on c_μ from SBL experiments.

Finally, we take into account the result of the LSND experiment, from which information on the SBL $\bar\nu_\mu \to \bar\nu_e$ transition amplitude $A_{\mu;e}$ (3) is obtained. Using Eq. (6) and the lower bound $A_{\mu;e}^{\rm min}$, which can be inferred from the region allowed by LSND, we derive the further bound on c_μ [13]

$$c_\mu \leqslant a_{\rm LSND} \equiv 1 - A_{\mu;e}^{\rm min}/4a_e^0. \tag{16}$$

This bound is represented by the curve in Fig. 2 labeled LSND + Bugey.

Figure 2 clearly shows that a four-neutrino mass hierarchy is strongly disfavored because no allowed region for c_μ is left in this plot. A four-neutrino mass hierarchy is also strongly disfavored for $\Delta m^2_{\rm LSND} \gtrsim 0.4$ eV^2 as was shown in Ref. [7]. We want to stress that all bounds are derived from 90% C.L. plots and that the bound (16) is quite sensitive to the actual values of $A_{\mu;e}^{\rm min}$ and a_e^0. This has to be kept in mind in judging the result derived here. As was noticed before [7], the procedure discussed here applies to all four-neutrino mass spectra of class 1 where a group of three neutrino masses is close together and separated from the fourth neutrino mass by a gap needed to explain the result of the LSND experiment. The reason is that all arguments presented here remain unchanged if one defines c_α (3) by a summation over the indices of the three close masses for each of the mass spectra of class 1 (see Fig. 1), i.e., $j=1,2,3$ for the spectra I and II and $j=2,3,4$ for the spectra III and IV.

To give an intuitive understanding that the data disfavor all spectra of class 1 we note that c_μ cannot be too close to 1 in order to explain the non-zero LSND $\bar\nu_\mu \to \bar\nu_e$ oscillation amplitude (3). On the other hand, if c_μ is too close to zero, the atmospheric ν_μ oscillations are suppressed [see Eq. (7), taking into account that $|U_{\mu 4}|^2 = 1 - c_\mu$]. For $\Delta m^2_{\rm LSND} \lesssim 0.3$ eV^2 these two requirements contradict each other. For $\Delta m^2_{\rm LSND} \gtrsim 0.3$ eV^2 they are in contradiction to the results of the CDHS and CCFR ν_μ disappearance experiments requiring c_μ to be either close to zero or 1 [see Eq. (5)].

According to the previous discussion, only the mass spectra of class 2 remain. They can be characterized in the following way:

$$\text{(A)} \qquad \underbrace{\overbrace{m_1 < m_2}^{\text{atm}} \ll \overbrace{m_3 < m_4}^{\text{solar}}}_{\text{LSND}}, \tag{17}$$

$$\text{(B)} \qquad \underbrace{\overbrace{m_1 < m_2}^{\text{solar}} \ll \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{LSND}}. \tag{18}$$

Let us now discuss which impact the up-down asymmetry A_μ has on these mass schemes. We consider first scheme (A) and go through the same steps as in the case of the mass hierarchy. Now we define

$$c_\alpha = \sum_{j=1,2} |U_{\alpha j}|^2. \tag{19}$$

Then the results of reactor experiments and the energy-dependent suppression of the solar neutrino flux lead to

$$c_e \leqslant a_e^0. \tag{20}$$

Repeating the derivation of Eq. (13) with c_α as defined in Eq. (19), it is easily seen that the inequality (13) holds also for scheme (A). On the other hand, the bound that takes into account the LSND result now has the form

$$c_\mu \geqslant A_{\mu;e}^{\min}/4a_e^0 . \tag{21}$$

The corresponding curve in the $\Delta m_{41}^2 - c_\mu$ plane is given by a reflection of the curve labeled LSND + Bugey in Fig. 2 at the horizontal line $c_\mu = 0.5$. Therefore, in the case of scheme (A) the allowed region of c_μ is determined by the bound (21) and by $c_\mu \geqslant 1 - a_\mu^0$. This region is allowed and not restricted by $c_\mu \gtrsim 0.45$ obtained from the Super-Kamiokande up-down asymmetry.

A discussion of scheme (B) with $c_e \geqslant 1 - a_e^0$ leads to the bound (13) with c_μ replaced by $1 - c_\mu$ in this formula and to Eq. (16). Therefore, the bounds for scheme (B) are obtained from those of scheme (A) by a reflection of the curves at the line $c_\mu = 0.5$. In summary, the white area in Fig. 2 represents the allowed region for $1 - c_\mu$ in scheme (A) and for c_μ in scheme (B).

In this paper we have shown that the existing neutrino oscillation data allow us to draw definite conclusions about the nature of the possible four-neutrino mass spectra. We have demonstrated that the spectra (I)-(IV) in Fig. 1, including the hierarchical one, are all disfavored by the data in the whole range 0.2 $\text{eV}^2 \lesssim \Delta m_{\text{LSND}}^2 \lesssim 2$ eV^2 of the mass-squared difference determined by LSND and other SBL neutrino oscillation experiments. With the Super-Kamiokande result on the atmospheric up-down asymmetry it has also been possible to investigate the region $\Delta m_{\text{LSND}}^2 \lesssim 0.3$ eV^2 which was not explored in previous publications. The only four-neutrino mass spectra that can accommodate all the existing neutrino oscillation data are the spectra (A) and (B) in Fig. 1 in which two pairs of close masses are separated by the LSND mass gap. The analysis introduced in this paper enables us in addition to obtain information on the mixing matrix U via a rather stringent bound on the quantity c_μ (19) for the allowed schemes (A) and (B).

In the framework of schemes (A) and (B) it is possible to make some predictions for reactor and accelerator long-baseline experiments, for ^{3}H β-decay experiments and experiments on the search for neutrinoless double β-decay. In particular, from the negative results of reactor short-baseline experiments and the results of solar neutrino experiments strong constraints can be obtained on the $\bar{\nu}_e$ survival probability in long-baseline reactor experiments and the probability of $\nu_\mu \to \nu_e$ transitions in long-baseline accelerator experiments [14].

In conclusion let us stress that we considered here the impact of the Super-Kamiokande up-down asymmetry for the minimal scenario of mixing of four massive neutrinos, which includes one sterile neutrino in addition to the three flavor neutrinos. For a discussion of schemes with three active and three sterile neutrinos see citations in Ref. [6].

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