Reply to "Comment on 'Ponderomotive force due to neutrinos' "

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We acknowledge an algebraic error in our previous calculation of the ponderomotive force due to neutrinos, though we disagree strongly with the other criticisms of the previous Comment. Our calculational error leads to a factor of four change in our quantitative results concerning the strength of the ponderomotive force. It does not, however, change our qualitative result that the ponderomotive force due to neutrinos is completely negligible in core-collapse supernovae. [S0556-2821(99)03716-9]

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There has been a great deal of controversy surrounding the importance of collective neutrino-plasma interactions in core-collapse supernovae and other physical systems. The calculation of the ponderomotive force due to neutrinos forms an important measure of the importance of such interactions. The results of Hardy and Melrose [1], (HM) demonstrated that the ponderomotive force due to neutrinos is unimportant in core collapse supernovae. This was in contradiction with the results of some of the authors of the Comment [2] in Ref. [3]. In this response we explain this contradiction, and show that it is not the result of a misinterpretation of the concept of a ponderomotive force, rather that it is due to additional physical assumptions made in Ref. [3] which are not justified. We reiterate our statement that the ponderomotive force is unimportant in core-collapse supernova.

Unfortunately, as correctly pointed out in the previous Comment there was an algebraic error in the calculation of the ponderomotive force presented in HM. In Eq. (27) of HM, the ponderomotive force per unit volume on the background plasma due to a distribution of neutrinos was stated as

$$\tilde{F} = -\frac{G_F}{\sqrt{2}} 2 \sin^2 \theta_W \boldsymbol{\nabla}[(n_\nu - n_{\bar{\nu}})(n_e - n_{\bar{e}})], \tag{1}$$

where G_F is the Fermi constant, θ_W is the Weinberg angle, n_ν, $n_{\bar{\nu}}$ are the neutrino and anti-neutrino number densities, respectively, n_e, $n_{\bar{e}}$ are the electron and positron number densities, respectively, and $\boldsymbol{\nabla}$ represents the usual spatial derivative. Equation (1) is incorrect, and should be replaced by

$$\tilde{F} = -\sqrt{2} G_F \mathcal{A} \boldsymbol{\nabla}[(n_\nu - n_{\bar{\nu}})(n_e - n_{\bar{e}})], \tag{2}$$

where

$$\mathcal{A} = \frac{1}{2} + 2 \sin^2 \theta_W \approx 1 \quad \text{for} \quad \nu_e . \tag{3}$$

Comparison between Eqs. (1) and (2) shows them to be different by a factor of approximately 1/4. Throughout HM, the replacement $\sin^2\theta_W \rightarrow \mathcal{A}$ should be made, and the numerical results multiplied by a factor of approximately 4. This in no way changes the qualitative results of HM, where it is shown that the ponderomotive force is far too small to be of importance physically.

It is straightforward to reproduce the expression given in the Comment for the ponderomotive force due to neutrinos on a single electron from Eq. (2). One proceeds by setting the positron number density to zero, $n_{\bar{e}} = 0$, and assuming an electron density constant in space. Then the ponderomotive force felt by a single electron is given by

$$\mathbf{F} = \frac{\tilde{F}}{n_e} = -\sqrt{2} G_F \mathcal{A} \boldsymbol{\nabla}(n_\nu - n_{\bar{\nu}}), \tag{4}$$

which is identical to Eq. (3) of the comment, when the approximation $\mathcal{A} = 1$ is made.

Thus, apart from an erroneous factor of approximately 4, there is no difference between the results for the ponderomotive force given in the Comment and the appropriate limit of our results. The criticisms leveled in the Comment that we have made a flawed analysis of the ponderomotive force concept and that we have somehow included the ponderomotive force due to electrons in our calculation are completely unjustified.

This factor of four error is in no way responsible for the huge difference in the ponderomotive force as calculated by Bingham *et al.* [3]. This is due to the entirely different assumptions made in the two papers about the nature of the neutrino distributions.

To illustrate this point we will make a straightforward calculation of the ponderomotive force on an electron due to the neutrinos free streaming from the core of the supernova (SN). From Eq. (3) of the Comment, the force due to electron neutrinos is written

$$F = -\sqrt{2} G_F \boldsymbol{\nabla} n_\nu . \tag{5}$$

The neutrino number density may be related to the neutrino luminosity, L_ν, through

$$L_\nu = 4\pi r^2 n_\nu E_\nu , \tag{6}$$

where r is the distance to the center of the SN ($\approx 100\,\mathrm{km}$), and E_ν is the average neutrino energy ($\approx 15\,\mathrm{MeV}$). Hence the ponderomotive force on a single electron due to these neutrinos is given by

$$F=\frac{\sqrt{2}G_F L_\nu}{2\pi c E_\nu r^3}\approx 2\times 10^{-28}\,\mathrm{N}, \tag{7}$$

for $L_\nu = 4\times 10^{46}\,\mathrm{J\,s^{-1}}$. This represents an acceleration of an electron of $200\,\mathrm{m\,s^{-2}}$ or a change in velocity of the electron over the 5 ms of the prompt electron neutrino burst of $1\,\mathrm{m\,s^{-1}}$, which is insignificant. This force is around 5 orders of magnitude smaller than the force due to neutrino-electron scattering which is approximately $3\times 10^{-23}\,\mathrm{N}$.

On the other hand, the authors of the Comment claim that the ponderomotive force due to neutrinos on a single electron is ten orders of magnitude *greater* than the force due to neutrino-electron scattering (see also [3]). Thus, there is fifteen orders of magnitude difference between our calculation of the ponderomotive force and that of [3]. This wild discrepancy is accounted for by the extreme variation in the neutrino number density assumed in [3]. This variation is generated through a two stage process: The neutrinos introduce very strong number density perturbation in the electron distribution through a plasma instability (see [4]); the neutrinos then ''bunch'' in reaction to these electron number density variations leading to a great variation in the neutrino number density, and hence a much greater ponderomotive force.

The neutrino driven plasma instability referred to in the first point has been shown not to operate, both by ourselves [5] and by Tsytovich *et al.* [6]. Thus it is clear that the conditions for creating the variations in the neutrino number density required for a large ponderomotive force due to neutrinos to exist are not met. The ponderomotive force due to neutrinos has no dynamical significance for core-collapse SN.

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