Chronology protection in generalized Gödel spacetime

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The effective action of a free scalar field propagating in the generalized Gödel spacetime is evaluated by the ζ-function regularization method. From the result we show that the renormalized stress energy tensor may be divergent at the chronology horizon. This gives support to the chronology protection conjecture. [S0556-2821(99)02018-4]

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Many people have considered the problem of time travel [1–13]. Recently, Hawking proposed the ''chronology protection conjecture'' [6] which states that the laws of physics will always prevent a spacetime from forming closed timelike curves (CTC's). In his argument, the conjecture may be due to the divergence of the stress tensor near the chronology horizon where CTC's are beginning to form. Several model spacetimes have been considered: the wormhole spacetime with a ''time machine'' [1–4], Gott's two-string spacetime [5], Grant space [7], and Misner spacetime [8,12–14]. In a number of papers it is concluded that a time machine is quantum unstable, as the stress tensor becomes divergent near the chronology horizon. However, some authors [7–13] claim that the stress tensor is finite everywhere and it poses a problem for chronology protection.

For instance, despite the divergent behavior shown in the calculation by Hiscock and Konkowski [14], a specific choice of the vacuum state in the Misner spacetime [11,13] could make the renormalized stress tensor vanish at the chronology horizon. Boulware, Tanaka, and Hiscock [8] also found that for sufficiently massive fields in Gott space and Grant space, respectively, the renormalized stress tensor could remain regular at the chronology horizon. In [13] it is also argued that the renormalized stress tensor may be smoothed out by introducing absorption material, so that the spacetime with a time machine may be stable against vacuum fluctuation. The investigation of the Roman ring of a traversable wormhole in [10] also found that the vacuum fluctuation can be made arbitrarily small. However, in [12] it was argued that, even though a self-consistent vacuum state in the Misner spacetime could be constructed, it must ultimately be unstable against a slight perturbation. Therefore, the chronology protection conjecture may be right. Some people [6,10] have further argued that solving the problem of chronology protection is impossible within the semiclassical theory of gravitation and it requires a fully developed theory of quantum gravity. In any case, a general proof of the chronology protection conjecture does not yet exist and it is useful to study the quantum stress tensor of physical matter near a chronology horizon in various spacetimes with CTC's.

In this paper we will consider the generalized Gödel spacetime [15]

$$ds^2=-dt^2+dx^2-2\sqrt{2}\,\frac{\Omega}{\alpha}e^{\alpha x}dy\,dt+\frac{1}{2}\left(1-4\frac{\Omega^2}{\alpha^2}\right)e^{2\alpha x}dy^2+a^2dz^2. \tag{1}$$

Note that we have introduced a constant metric $g_{zz}=a^2$ in the above equation as this could help us to find the renormalized stress tensor in the later calculation. The case of $\alpha=\sqrt{2}\Omega$ is a rotating spacetime originally proposed by Gödel [16]. As analyzed in [15], there are no CTC's in the spacetimes with $\alpha^2\geqslant 4\Omega^2$. But the spacetimes with $\alpha^2<4\Omega^2$ admit CTC's. We will use the ζ-function regularization method [17] to evaluate the renormalized effective action of a free scalar field propagating in the generalized Gödel spacetime. The result shows that the action is divergent at chronology horizon, i.e., $\alpha^2=4\Omega^2$. From the result we can show that the stress energy may be divergent at the chronology horizon and this thus gives a new example to support the chronology protection conjecture.

In a system with a Hamiltonian H the renormalized effective action W evaluated by the ζ-function regularization method [17] can be expressed as

$$W=-\frac{i}{2}[\zeta'(0)+\zeta(0)\ln\mu^2], \tag{2}$$

where

$$\zeta(\nu)=\mathrm{Tr}(H)^{-\nu}. \tag{3}$$

To evaluate the above ζ function exactly we shall first find the eigenvalue of the Hamiltonian H. It is easy to see that the eigenvalue Δ of the Hamiltonian H for a massive (m) scalar field with curvature coupling (ξ) to the generalized Gödel spacetime can be easily found through the same method [18,19] as that in the old Gödel spacetime. The method is that we first assume the following form of the eigenfunction:

$$\Psi(t,x,y,z)=h(x)\exp(-i\omega t+ik_y y+ik_z z). \tag{4}$$

Then the function $h(x)$ will satisfy the Whittaker's function and the condition to have an everywhere-bounded solution is that

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$$\Delta = a^{-2}k_z{}^2 + m^2 + \xi R + \left(\frac{\alpha}{2}\right)^2 + (4\Omega^2 - \alpha^2)\left(n + \frac{1}{2}\right)^2 - \left[\omega - 2\Omega\epsilon\left(n + \frac{1}{2}\right)\right]^2, \tag{5}$$

where the scalar curvature $R = 2(\alpha^2 - \Omega^2)$, $\epsilon = k_z/|k_z|$, and $n = 0,1,2,\ldots$. The ζ function can then be calculated by the formula [19]

$$\zeta(v) = \int d^4x\sqrt{-g}\int dk_z \int d\omega \sum_{n=0}^{\infty} (\Delta)^{-v}. \tag{6}$$

Note that in [19] we have evaluated the finite-temperature effective potential for a scalar field in the old Gödel spacetime, also with the help of ζ-function regularization method and thus Eq. (6).

Now, denoting $M^2 = m^2 + \xi R + (\alpha/2)^2$ and rotating ω through $\pi/2$ in the complex plane to become $i\omega_E$ [17], we have the relation

$$\begin{aligned}\zeta(v) &= i\int d^4x\sqrt{-g}\int dk_z\int d\omega_E \sum_{n=0}^{\infty}\left[a^{-2}k_z{}^2 + \omega_E{}^2 + M^2 + (4\Omega^2-\alpha^2)\left(n+\frac{1}{2}\right)^2\right]^{-v} \\ &= ia\pi(v-1)^{-1}\int d^4x\sqrt{-g}\sum_{n=0}^{\infty}\left[M^2 + (4\Omega^2-\alpha^2)\left(n+\frac{1}{2}\right)^2\right]^{1-v}.\end{aligned} \tag{7}$$

Next, using the relation

$$\sum_{n=0}^{\infty} F[(n+1/2)^2] = \sum_{n=1}^{\infty} F[(n/2)^2] - \sum_{n=1}^{\infty} F[n^2], \tag{8}$$

in which F is an arbitrary function, we have

$$\begin{aligned}\zeta(v) &= i\pi a(v-1)^{-1}\int d^4x\sqrt{-g} \\ &\quad\times\sum_{n=1}^{\infty}[M^2 + (4\Omega^2-\alpha^2)(n/2)^2]^{1-v} - i\pi a(v-1)^{-1} \\ &\quad\times\int d^4x\sqrt{-g}\sum_{n=1}^{\infty}[M^2+(4\Omega^2-\alpha^2)n^2]^{1-v}.\end{aligned} \tag{9}$$

To proceed, let us quote a formula [20]

$$\begin{aligned}\sum_{n=1}^{\infty}(An^2+B)^{-S} &= \frac{1}{2}A^{-1/2}B^{1/2-S}\pi\Gamma(S-1/2)/\Gamma(S) \\ &\quad + \frac{1}{2}(A+B)^{-S} - \int_0^l (Ax^2+B)^{-S}dx \\ &\quad + i\int_0^{\infty}\{[A(1+ix)^2+B]^{-S} \\ &\quad - [A(1-ix)^2+B]^{-S}\}(e^{2\pi x}-1)^{-1}dx.\end{aligned} \tag{10}$$

Then, after the calculations the ζ function becomes

$$\begin{aligned}\zeta(v) &= -iv\int d^4x\sqrt{-g}\frac{7}{12}a\pi\frac{1}{\sqrt{4\Omega^2-\alpha^2}} \\ &\quad\times[m^2+\xi R+(\alpha/2)^2]^{3/2} + O(v^2) \\ &\quad + (\text{finite terms when } 4\Omega^2 \text{ approaches } \alpha^2).\end{aligned} \tag{11}$$

Using this result and from Eq. (2) we see that

$$\begin{aligned}\langle T^{zz}\rangle &= \frac{2}{\sqrt{-g}}\frac{\delta W}{\delta g_{zz}} = \frac{7}{12}\pi\frac{1}{\sqrt{4\Omega^2-\alpha^2}}[m^2+\xi R+(\alpha/2)^2]^{3/2} \\ &\quad + (\text{finite terms with } 4\Omega^2 \text{ approaches } \alpha^2),\end{aligned} \tag{12}$$

after letting $a = \mu = 1$. We thus see that, at least one component of the renormalized stress energy tensor is divergent at the chronology horizon and the investigation thus gives a new support to the chronology protection conjecture. The evaluation of all components of the stress energy tensor is more difficult and is yet to be found.

Finally, let us mention that the spacetime considered in this paper is not like those that will form CTC's after the cosmological evolution [1–13]. Thus our result does not directly relate to the chronology protection conjecture initially suggested therein. However, as a geometry with the causality violation, such as the Gödel spacetime, can be taken as a part of a globally causal spacetime [21]. Thus our investigation does suggest that the physical law of requiring a finite stress tensor, will forbid a geometry with the causality violation to be enclosed in a region which maintains the required condition of global causality. This means that the physical law will protect a would-be advanced civilization to create a noncausal geometry to violate the chronology protection conjecture.

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