Superconducting cosmic string in Brans-Dicke theory

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In the present work, the gravitational field of a superconducting cosmic string has been investigated in the context of Brans-Dicke (BD) theory of gravity. We have presented two kinds of solutions for the spacetime in the far field zone of the string. When the BD scalar field is switched off, one of the solutions reduces to the solution earlier obtained by Moss and Poletti in general relativity. [S0556-2821(99)06016-6]

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Spontaneous symmetry breaking in gauge theories may give rise to some topologically trapped regions of a false vacuum, namely, domain walls, cosmic strings, or monopoles, depending on the dimension of the region [1]. Among these, cosmic strings have gained a lot of attention in recent years as a possible seed for galaxy formation. Strings arising from the breaking of a U(1) local symmetry are called local strings whereas those arising from the breaking of U(1) global symmetry are called the global strings. The gravitational fields of both local and global strings have been investigated by many authors in the recent past [2–8].

It has been demonstrated by Witten [9] that under certain conditions of local gauge symmtery breaking, cosmic strings might behave as superconductors whose motion through astronomical magnetic fields can produce interesting effects. Peter [10] has emphasized that superconductivity is a rather generic feature of cosmic string models. Massive superconducting strings may have an important role to play in the generation of large scale magnetic fields [11,12]. For the superconducting string, there is a current along the symmtery axis and consequently a magnetic field in the transverse direction. The geometry is no longer boost invariant. Moss and Poletti [13] first investigated the gravitational field of such a string assuming that at large distances from the core of the string, the energy stress tensor is dominated by the magnetic field alone. Later, Demiansky [14] investigated the particle motion in the spacetime given by Moss and Poletti.

Scalar tensor theories, especially the Brans-Dicke (BD) theory of gravity [15], which is compatible with the Mach's principle, have been considerably revived in recent years. It was shown by La and Steinhardt [16] that because of the interaction of the BD scalar field and the Higgs type sector, which undergoes a strongly first-order phase transition, the exponential inflation as in the Guth model [17] could be slowed down to power law one. The ''graceful exit'' in inflation is thus resolved as the phase transition completes via bubble nucleation.

On the other hand, it seems likely that, in the high energy scales, gravity is not governed by the Einstein action, and is modified by the superstring terms which are scalar tensor in nature. In the low-energy limit of this string theory, one recovers Einstein's gravity along with a scalar dilaton field which is nonminimally coupled to the gravity [18]. Although dilaton gravity and BD theory arise from entirely different motivations, it can be shown by a simple transformation of the scalar field that the former is a special case of the latter at least formally.

The dilaton gravity is given by the action

$$A=\frac{1}{16\pi}\int \sqrt{-g}e^{-2\phi}(R+4g^{\mu\nu}\phi_{,\mu}\phi_{,\nu})d^4X, \tag{1a}$$

where no matter field is present except the massless dilaton field ϕ. Now if one defines a variable

$$\psi=e^{-2\phi}, \tag{1b}$$

then the action (1a) looks similar to

$$A=\frac{1}{16\pi}\int \sqrt{-g}\left(\psi R+\frac{\psi_{,\mu}\psi_{,\nu}}{\psi}g^{\mu\nu}\right)d^4X, \tag{1c}$$

which is indeed a special case of BD theory given by the action

$$A=\frac{1}{16\pi}\int \sqrt{-g}\left(\psi R-\omega\frac{\psi_{,\mu}\psi_{,\nu}}{\psi}g^{\mu\nu}\right)d^4X, \tag{1d}$$

for the parameter $\omega=-1$. But when matter field is present, then the action in the dilaton gravity is given by

$$A=\frac{1}{16\pi}\int \sqrt{-g}e^{-2\phi}(R+4g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}+e^{2a\phi}\mathcal{L})d^4X, \tag{1e}$$

where $\mathcal{L}$ is the Lagrangian for the matter field present. The action (1e) cannot be reduced to the corrsponding action for the BD theory which is given by

$$A=\frac{1}{16\pi}\int \sqrt{-g}\left(\psi R-\omega\frac{\psi_{,\mu}\psi_{,\nu}}{\psi}g^{\mu\nu}+\mathcal{L}\right)d^4X, \tag{1f}$$

by the transformation (1b) and putting $\omega=-1$ because the nonminimal coupling between the matter Lagrangian $\mathcal{L}$ and the dilaton field ϕ in the action (1e) which is not present in Eq. (1f) for the BD theory.

But for $a=1$ in Eq. (1e), there is no coupling between the $\mathcal{L}$ and the dilaton field ϕ and one can reduce Eq. (1e) to the corresponding BD action (1f). The renewed interest in BD theory acquires more points of interest when we observe that

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the topological defects such as domain walls, cosmic strings, and monopoles are formed during inflation in the early phase of the universe and hence can interact with the BD scalar field with remarkable change in their properties [19,20]. The implications of the dilaton gravity for the defects have also been explored by several authors [21,22].

In the present work, we have investigated the gravitational field of a superconducting string in Brans-Dicke (BD) theory of gravity. Similar to Moss and Poletti, we have assumed that in the far field zone, the energy momentum tensor for the string is dominated by the magnetic field. We have found a family of solutions for the spacetime in the far field zone depending upon the value of some arbitary constants. One of our solutions reduces to the solution given by Moss and Poletti [13] when the BD scalar field becomes constant, but for the other case one cannot recover the corresponding general relativity (GR) solution.

The gravitational field equations in BD theory are given by

$$G_{\mu\nu}=8\pi\frac{T_{\mu\nu}}{\phi}+\frac{\omega}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu}-\frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right)+\frac{1}{\phi}(\phi_{,\mu;\nu}-g_{\mu\nu}\Box\phi) \tag{2a}$$

and

$$\Box\phi=\frac{8\pi T}{(2\omega+3)} \tag{2b}$$

in units where $c=1$. $T_{\mu\nu}$ is the energy momentum tensor representing the contribution from any other field except the BD scalar field ϕ and ω is the constant parameter. T represents the trace of $T_{\mu\nu}$.

To describe the spacetime geometry due to an infinitely long static cosmic string, the line element is taken to be general static cylindrically symmetric one given by

$$ds^2=e^{2(K-U)}(-dt^2+dr^2)+e^{2U}dz^2+e^{-2U}W^2d\theta^2, \tag{3}$$

where K,U,W are functions of the radial coordinate r.

A model for the fields of superconducting cosmic string can be described by a $\mathrm{U}(1)\times\mathrm{U}(\tilde{1})$ gauge theory with the gauge fields A_μ and B_μ are coupled to the scalar fields σ and ψ [9,13]. The string is represented by the Nielsen-Olesen [23] vortex solution with

$$\psi=\psi(r)e^{i\theta},\quad B_\mu=(1/e)[B(r)-1]\delta_\mu^\theta, \tag{4a}$$

where e is the coupling parameter. Outside the core of the string, the $\mathrm{U}(\tilde{1})$ symmetry is broken and ψ attains a nonzero expectation value at the minimum of the potential $V(\sigma,\psi)$.

The $\mathrm{U}(1)$ symmetry is unbroken away from the string with A_μ representing photon. We have chosen [13]

$$\sigma=\sigma_0(r)e^{i\chi},\quad A_\mu=A(r)\delta_\mu^z \tag{4b}$$

with the phase χ being a function of z.

With Eq. (4b), there is an electric current density J along the z axis, given by

$$J=2ej\sigma^2e^{-2U}, \tag{5}$$

where

$$j=\frac{\partial\chi}{\partial z}+eA. \tag{6}$$

The Maxwell equation which we will need later is

$$(\sqrt{-g}e^{-2K}A')'=8\pi ej\sigma_0^2e^{-2U}\sqrt{-g},$$

which on integration yields a first integral

$$A'=\frac{2Ie^{2U}}{W} \tag{7}$$

with

$$I(r)=\int J2\pi\sqrt{-g}dr \tag{8}$$

being the electric current along the symmetry axis of the string.

As one moves away from the string, all the other string fields drop off rapidly leaving the magnetic field A_μ on its own. The energy momentum tensor then takes the form

$$T_t^t=-T_r^r=-T_z^z=T_\theta^\theta=\frac{1}{2}A'^2e^{-2K}. \tag{9}$$

With Eqs. (9) and (3), the BD field equations (2) take the form

$$(W'\phi)'=(\phi'W)'=0, \tag{10a}$$

$$(K'W\phi)'=0, \tag{10b}$$

$$2U''+\frac{2U'W'}{W}-2K''-2U'^2=\omega\frac{\phi'^2}{\phi^2}+\frac{2W''}{W}, \tag{10c}$$

$$(U'W\phi)'=-4\pi A'^2e^{-2U}W. \tag{10d}$$

Here a prime denotes differentiation with respect to r.

Solving Eqs. (10a) and (10b) we get

$$\phi=\phi_0r^p, \tag{11a}$$

$$W=W_0r^{1/n}, \tag{11b}$$

and

$$e^K=\alpha r^{b/n}, \tag{11c}$$

where ϕ_0, W_0, b, α, and n are constants of integration and $p=(n-1)/n$.

Using Eqs. (11a), (11b), and (11c), one gets, from Eq. (10c),

$$U'' = U'^2 - \frac{U'}{nr} + \frac{M}{r^2}, \tag{12a}$$

where

$$M = \frac{1}{2}\left[\omega p^2 + \frac{2(1-n)}{n^2} - \frac{2b}{n}\right]. \tag{12b}$$

One can have three solutions of (12a) depending on $(1-4C)$ is positive, zero, or negative, where $C = M + 1/4n^2$. In the follwing calculations, we have concentrated on the cases where $(1-4C)$ is positive and zero.

Case 1: $1-4C>0$. In this case the solution of Eq. (12a) becomes

$$U = -\ln[C_1 r^{i_1} + C_2 r^{i_2}], \tag{13}$$

where C_1 and C_2 are arbitary constants and

$$i_1 = \frac{1}{2} + \sqrt{\frac{1-4C}{2}} - \frac{1}{2n}, \tag{14a}$$

$$i_2 = \frac{1}{2} - \sqrt{\frac{1-4C}{2}} - \frac{1}{2n}. \tag{14b}$$

Hence the complete line element in the far field zone of a superconducting string in BD theory becomes

$$ds^2 = \alpha^2 r^{2b/n} H^2(-dt^2 + dr^2) + H^{-2} dz^2 + W_0^2 r^{2/n} H^2 d\theta^2, \tag{15}$$

where $H = C_1 r^{i_1} + C_2 r^{i_2}$.

Putting Eqs. (11a), (11b), (11c), and (13) in Eq. (10d) and using Eq. (7) one can get the relation

$$C_1 C_2 (i_1 - i_2)^2 \phi_0 W_0^2 = 16\pi I^2. \tag{16}$$

Now if one puts $n=1$ and $\phi_0 = 1/G$ where G is the gravitational constant, it is easy to verify that the BD scalar field becomes constant and

$$ds^2 = \alpha^2 r^{2b} F^2(-dt^2 + dr^2) + F^{-2} dz^2 + W_0^2 r^2 F^2 d\theta^2, \tag{17}$$

where $F = C_1 r^{\sqrt{b}} + C_2 r^{-\sqrt{b}}$ and Eq. (16) becomes

$$C_1 C_2 = \frac{4\pi G I^2}{b W_0^2}. \tag{18}$$

Equations (17) and (18) are the result earlier obtained by Moss and Poletti [13] in GR. Comparing our result to the result earlier obtained by Moss and Ploetti one can identify the two integration constants b,n to be related to the Mass per unit length of the string and the field energy density of the string, respectively.

One can also calculate in the spacetime given by Eq. (15), the radial accelaration $\dot{v}^1$ of a particle that remains stationary (i.e., $v^1 = v^2 = v^3 = 0$) in the field of the string. Now

$$\dot{v}^1 = v^1_{;0} v^0 = \Gamma^1_{00} v^0 v^0. \tag{19a}$$

In our spacetime (15),

$$v^0 = \frac{1}{\alpha r^{b/n}[C_1 r^{i_1} + C_2 r^{i_2}]}. \tag{19b}$$

Using Eq. (19b), we get from Eq. (19a)

$$\dot{v}^1 = \frac{b(C_1 r^{i_1} + C_2 r^{i_2}) + n(C_1 i_1 r^{i_1} + C_2 i_2 r^{i_2})}{\alpha^2 n r^{(2b/n+1)}[C_1 r^{i_1} + C_2 r^{i_2}]^3}. \tag{20}$$

Now if one assumes $b>0$ and $n>0$, then $\dot{v}^1>0$. So the particle has to accelarate away from the string, which implies that the gravitational force due to the string itself is attractive. It is similar to the case of superconducting string in GR [13]. But for other values of b,n, the string may have a repulsive effect.

Case 2: $1-4C=0$. In this case the solution of (12) becomes

$$e^{-U} = r^{1/2(1-1/n)}[C_1 + C_2 \ln r]. \tag{21}$$

Hence the line element becomes

$$ds^2 = \alpha^2 r^{2b/n} r^{(1-1/n)} P^2(-dt^2 + dr^2) + r^{-(1-1/n)} P^{-2} dz^2 + W_0^2 r^{(1+1/n)} P^2 d\theta^2, \tag{22}$$

with $P = C_1 + C_2 \ln r$ and $4bn = 2\omega(n^2 - 2n + 1) - 4n - n^2 + 5$. In this case one can get after some straightforward calculations, the result

$$16\pi I^2 = -C_2^2 W_0^2 \phi_0 n. \tag{23}$$

Now as ϕ_0 should be positive to ensure the positivity of G, the gravitational constant, n must be negative. But one can check from Eq. (11a) that in order to make the BD scalar field ϕ a constant, i.e., to get the corresponding GR solution one should put $n=1$. Hence in this case one cannot recover the corresponding GR solution.

In conclusion, this work extends the earlier work by Moss and Poletti regarding the gravitational field of a superconducting cosmic string to the Brans-Dicke theory of gravity. The main feature of our solutions in BD theory is that one can have a family of solutions for the spacetime of the superconducting string depending on the choices of arbitary constants. The author is grateful to Dr. Narayan Banerjee for valuable suggestions.

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