Constructive algebraic renormalization of the Abelian Higgs-Kibble model

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We propose an algorithm, based on algebraic renormalization, that allows the restoration of Slavnov-Taylor invariance at every order of perturbative expansion for an anomaly-free Becchi-Rouet-Stora (BRS) invariant gauge theory. The counterterms are explicitly constructed in terms of a set of one-particle-irreducible Feynman amplitudes evaluated at zero momentum (and derivatives of them). The approach is discussed here in the case of the Abelian Higgs-Kibble model (in $3+1$ dimensions), where the zero momentum limit can be safely performed. The normalization conditions are imposed by means of the Slavnov-Taylor invariants. A judicious choice of the normalization conditions greatly simplifies the calculation of the counterterms. In particular within this model all counterterms involving BRS external sources (antifields) can be put to zero with the exception of the fermion sector. [S0556-2821(99)01916-5]

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I. INTRODUCTION

Few gauge models of physical interest enjoy a symmetrical regularization of Feynman amplitudes (such as QCD in dimensional regularization). In particular for the standard model the difficulty comes from the endemic presence of γ_5 and of the complete antisymmetric tensor. Thus, if the regularization breaks the desired symmetries, one has to recover the correct Green's functions by finite renormalization in order to satisfy the Slavnov-Taylor identities (STIs). The algebraic Renormalization (AR) [1–5] theory gives the conditions under which this strategy is possible: in particular there should be no anomalies in the STI. Thus in principle the renormalization program can be performed. However, it is not an easy task beyond the one-loop approximation, since a high number of vertex functions at lower order must be evaluated for generic external momenta in order to restore the STI.

In this paper we propose a strategy for the evaluation of the counterterms, based on zero-momentum subtraction [6]. The final result is an explicit solution of the STI where the counterterms are given in terms of a set of finite vertex functions and their derivatives evaluated at zero momentum. Our strategy is based on various results taken from the Bogoliubov-Parasiuk-Hepp-Zimmermann-Lowenstein (BPHZL) renormalization scheme and from the algebraic renormalization theory. We show that the zero momentum subtraction and a judicious use of the normalization conditions permits a practical evaluation of the counterterms by means of a relevant set of finite vertex functions. In particular the choice of the normalization conditions entails a diagonal block structure of the matrices that fix the counterterms. This approach is here discussed in the Abelian Higgs-Kibble model (in $3+1$ dimensions) [1,7,8], where the zero momentum limit can be safely performed.

As a starting point we assume that a consistent subtraction procedure allows the evaluation of the n-loop vertex functions $\Gamma^{(n)}$ when the correct vertex functional Γ^j is given for any $j<n$. That is we assume that our procedure has been successfully worked out for the lower orders and we proceed to restore Slavnov-Taylor (ST) invariance on $\Gamma^{(n)}$. The n-order vertex functions are constructed by iterative use of subgraphs and counterterms according to the scheme of Bogoliubov [9,10].

The regularization can be any, provided it respects the quantum action principle (QAP) [11] (i.e., it is correct up to counterterms in the action). In order to make the discussion simpler we assume also that the regularization procedure respects some basic symmetries of the classical action, such as Lorentz covariance, Faddeev-Popov (FP) charge conservation, and any possible further symmetry (such as charge conservation C). Thus, if the regularization is not invariant, we expect that STIs are broken

$$S(\Gamma)^{(n)}=\int d^4x\left[\partial^\mu c\Gamma^{(n)}_{A^\mu}+\left(\partial^\mu A_\mu+\frac{ev}{\alpha}\phi_2\right)\Gamma^{(n)}_{c}\right]+(\Gamma,\Gamma)^{(n)}=\Delta^{(n)}, \tag{1}$$

where the brackets represent

$$(X,Y)=\frac{\delta X}{\delta J_1}\frac{\delta Y}{\delta\phi_1}+\frac{\delta X}{\delta J_2}\frac{\delta Y}{\delta\phi_2}-\frac{\delta X}{\delta\psi}\frac{\delta Y}{\delta\bar\eta}+\frac{\delta X}{\delta\bar\psi}\frac{\delta Y}{\delta\eta}. \tag{2}$$

$J_1,J_1,\eta,\bar\eta$ are the sources coupled to the Becchi-Rouet-Stora (BRS) variations, i.e., the antifields [see Eq. (A9)]. We use the convention that derivatives are always from left and fields with Fermi character anticommute. Although STIs are

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broken, the QAP guarantees that at every order $\Delta^{(n)}$ is a local insertion (provided that STIs are valid at the lower orders), has the correct invariance properties under exact symmetries (e.g., Lorentz, Faddeev-Popov (FP) charge, etc.), and is consistent with the power counting. Thus we can expand Δ on a suitable basis:

$$\Delta=\sum_i c_i\mathcal{M}_i=\sum_i c_i\int d^4x f_i(\phi,\partial\phi)(x), \tag{3}$$

where $\mathcal{M}_i$ is any Lorentz scalar monomial f_i in the fields and their derivatives, integrated over the Minkowski space. Residual symmetries restrict the basis of $\mathcal{M}_i$; for instance, for the present Higgs-Kibble model C invariance constrains the possible breaking terms. By construction the canonical dimension of $\mathcal{M}_i$ is less than or equal to 5.

The renormalization of the model consists in finding the finite counterterms in the action that restore the validity of the STIs and consequently the physical unitarity (algebraic renormalization). Let us denote by $\mathbf{\Gamma}$ the vertex functions resulting from this procedure. The locality and covariance of Δ suggests that we consider the Taylor expansion in momentum space. Let t^{δ} be the projector of the polynomials of degree δ (the Taylor expansion in the independent external momenta up to degree δ). The symbol $\delta_{\rm pc}$ denotes the superficial degree of a given amplitude.

These facts suggest a strategy in the evaluation of the counterterms. The first step consists in the zero momentum subtraction compatible with the power counting

$$(1-t^{\delta_{\rm pc}})\Gamma. \tag{4}$$

The above expression $t^{\delta}\Gamma$ is a short-hand notation of the following procedure: at first we consider the relevant amplitude (the functional derivatives respect to fields are denoted by subscripts)

$$\Gamma_{\phi_1(p_1)\phi_2(p_2)\,\dots\,\phi_m(p_m)}\big|_{p_m=-\Sigma_{j=1}^{m-1}p_j} \tag{5}$$

and then its Taylor expansion t^{δ} in the independent momenta up to degree δ. Formally $t^{\delta}\Gamma$ can be written in the form [14]

$$t^{\delta}\Gamma=\sum_{m=1}^{\infty}\int\prod_{i=1}^{m}d^4p_i\phi_i(p_i)\times\delta^4\left(\sum_{j=1}^{m}p_j\right)t^{\delta}\Gamma_{\phi_1(p_1)\phi_2(p_2)\cdots\phi_m(p_m)}\big|_{p_m=-\Sigma_{j=1}^{m-1}p_j}. \tag{6}$$

Thus we consider the lowest $\delta_{\rm D}$ such that

$$(1-t^{\delta_{\rm D}})S(\Gamma)^{(n)}=(1-t^{\delta_{\rm D}})\Delta^{(n)}=0 \tag{7}$$

at every order in the perturbative expansion ($\delta_{\rm D}$ does not depend on n). Also the expression $t^{\delta_{\rm D}}\Delta^{(n)}$ has to be understood in the above sense (6) and $\delta_D=5-\Sigma_i d_{\phi_i}$ (d_{ϕ_i} are the naive dimensions of the fields, entering in the expansion of $\Delta^{(n)}$ in terms of polynomial of fields. In the above equation the relevant term for a recursive construction of the invariant vertex functions is the linear operator on $\Gamma^{(n)}$:

$$S_0(\Gamma^{(n)})\equiv\int d^4x\left[\partial^{\mu}c\Gamma^{(n)}_{A^{\mu}}+\left(\partial^{\mu}A_{\mu}+\frac{ev}{\alpha}\phi_2\right)\Gamma^{(n)}_{\bar{c}}\right]+(\Gamma^{(0)},\Gamma^{(n)})+(\Gamma^{(n)},\Gamma^{(0)}), \tag{8}$$

where S_0 is the linearized ST operator. We assume that zero momentum subtraction is possible and focus our attention on other effects of the subtraction. In general S_0 is not homogeneous in the dimensions of the fields (e.g., in presence of a spontaneous breaking of symmetry). As a consequence the action of $(1-t^{\delta_D})$ on each single terms of $S_0(\Gamma^{(n)})$ induces some oversubtractions of $\Gamma(n)$. These oversubtractions manifest theirselves as new local breaking terms Ψ, obtained by reshuffling the Eq. (7) in the form

$$(1-t^{\delta_{\rm D}})S_0(\Gamma^{(n)})=S_0[(1-t^{\delta_{\rm pc}})\Gamma^{(n)}]+S_0(t^{\delta_{\rm pc}}\Gamma^{(n)})-t^{\delta_{\rm D}}S_0(\Gamma^{(n)}). \tag{9}$$

The last terms show that the zero momentum subtraction does not give ST invariant vertex functions and that order-by-order we have to introduce counterterms in the action. Let us make explicit the STI. The recursive procedure gives

$$S_0[(1-t^{\delta_{\rm pc}})\Gamma^{(n)}]+\sum_{j=1}^{n-1}(\mathbf{\Gamma}^{(j)},\mathbf{\Gamma}^{(n-j)})=[t^{\delta_{\rm D}}S_0-S_0t^{\delta_{\rm pc}}]\Gamma^{(n)}+t^{\delta_{\rm D}}\sum_{j=1}^{n-1}(\mathbf{\Gamma}^{(j)},\mathbf{\Gamma}^{(n-j)})\equiv\Psi^{(n)}. \tag{10}$$

The $\mathbf{\Gamma}$ terms are computed at the lower orders in the perturbative expansion. They are supposed to satisfy STI at every order less than n. In our strategy one of the criterion in the choice of the normalization conditions is the suppression of the above bilinear contributions.

If the model has no anomalies the problem is then to find the counterterms $\Xi^{(n)}$ which satisfy

$$S_0(\Xi^{(n)})=-\Psi^{(n)} \tag{11}$$

or

$$S_0(\Xi^{(n)})=-[t^{\delta_{\rm D}}S_0-S_0t^{\delta_{\rm pc}}]\Gamma^{(n)}-t^{\delta_{\rm D}}\sum_{j=1}^{n-1}(\mathbf{\Gamma}^{(j)},\mathbf{\Gamma}^{(n-j)}). \tag{12}$$

Finally the correct vertex functions are

$$\mathbf{\Gamma}^{(n)}=(1-t^{\delta_{\rm pc}})\Gamma^{(n)}+\Xi^{(n)}. \tag{13}$$

The zero momentum subtraction, as intermediate renormalization, has the advantage to reduce the renormalization in any subtraction procedure to a common ground: the algorithm is then the same and it consists in the evaluation of a set of finite amplitudes and their derivatives at zero momenta. Moreover, as we will discuss later, it suggests a natural choice of the normalization conditions. Finally in the zero momentum subtraction the contributions of the lower orders of perturbation to Ψ is consistently reduced [Eq. (10)].

A limit of the present approach is that the vertex functions and their derivatives with respect to external momenta must have regular behavior at zero momenta. In the presence of massless and massive fields, this requirement implies the introduction of infrared cutoffs and the Taylor operator t^{δ_D} has to be modified (see Ref. [12]); however, this possibility will not be explored in the present work.

There is a fairly large amount of freedom in the choice of the counterterms Ξ [Eq. (13)]. This is due to the presence of a certain number of ST invariant terms explicitly given in Appendix C. This freedom will be exploited in order to obtain the most efficient strategy in the evaluation of Ξ and in order to reduce the contribution to Ψ [Eq. (12)] due to the lower perturbative terms. Any choice of Ξ fixes automatically the normalization conditions.

The use of ST invariants and the normalization conditions is organized by introducing a hierarchy for the counterterms (choice of a basis of noninvariant counterterms). They will be grouped into disjoint sets: the S_0 variation of the elements of two different sets have no common elements. In addition the elements of a single set can be organized with a nesting structure. By following this hierarchy decomposition, in the present model it is possible to avoid all counterterms involving the external sources J_i, tadpoles and out-of-diagonal bilinear expressions. As a consequence the mass counterterms turn out to be zero with the exception of the fermions. The ghost equation, which guarantees the nilpotency of the ST operator, also plays an important role in the control of some of the counterterms.

By construction the functional Ψ contains only finite vertex functions, i.e., at every order of the perturbative expansion n it can be evaluated and it is independent from the regularization procedure (once $\Gamma^{(j<n)}$ is correctly constructed). The counterterm functional Ξ is determined by Eq. (11). In general there are more equations than unknowns (overdetermined problem). However, the system of equations has a solution since there are consistency conditions [13]. Most of them are consequence of the nilpotency of the ST operator

$$S_0(\Psi^{(n)})=0. \tag{14}$$

The evaluation of Ξ can be performed either by imposing the consistency conditions on $\Psi^{(n)}$ or by a choice of the linearly independent equations. It should be remarked that the expression of $\Xi^{(n)}$ in terms of $\Psi^{(n)}$ is a simple linear relation independent from the order of the perturbative expansion.

The really hard work is the evaluation of $\Psi^{(n)}$. It consists in the computation of vertex functions and of some of their derivatives at zero momenta. The number of graphs turns out to be very large (especially for amplitudes involving scalars). For this reason it is important to find possible relations among the amplitudes, e.g., the Callan-Symanzik equation (CSE), and to use automatic calculus to generate and evaluate the graphs. Particularly interesting is the CSE (see for example Refs. [14,15,8]). The consistency conditions imposed by the CSE on the breaking terms $\Psi^{(n)}$ allows the evaluation of some amplitudes in terms of simpler vertex functions. Moreover some amplitude can be obtained as the result of mass insertions on vertex functions with less external legs. The automatic calculus is particularly useful since the external momenta are zero.

It is important to reduce the contributions to $\Psi^{(n)}$ of the lower terms in the perturbation expansion. Equation (10) allows the direct control of the consequences of any particular choice for the basis of the noninvariant counterterms, i.e., of the choice of the normalization conditions. This point of view is at variance with the on-shell conditions, which cannot dispose this particular problem. For instance, it is clear that by dropping external sources counterterms one can eliminate most of the terms coming from the lower order in the perturbative expansion [see Eq. (2)].

The physical amplitudes necessitate the study of the zeros of the two-point functions. Then the free parameters of the action have to be tuned in order to obtain the physical masses and the correct coupling constants.

The Higgs-Kibble model has the advantage of admitting dimensional regularization (if there is no fermion sector). It is nontrivial, since the presence of γ_5 requires the full generality of the Algebraic Renormalization. Moreover the model has no anomalies: the Adler-Bardeen-Jackiw anomaly is zero due to C conjugation.

In the following to make the formalism simpler and more direct we use to give a compact notation for the breaking terms $\Psi^{(n)}$ and its coefficients [see Eq. (3)]:

$$\Psi^{(n)}=\sum_i \psi^{(n)}_{\mathcal{M}_i}\mathcal{M}_i \tag{15}$$

and in the same way we will denote the counterterms $\Xi^{(n)}$ by

$$\Xi^{(n)}=\sum_k \xi^{(n)}_{\mathcal{P}_k}\mathcal{P}_k, \tag{16}$$

where $\mathcal{P}_i$ is a Lorentz invariant monomial with dimension less than or equal to 4, null Faddeev-Popov charge and C even. We may omit also the sign of integral $\int d^4x$, when not necessary, e.g., $\xi^{(n)}_{\phi_1^2}\equiv\xi^{(n)}_{\int d^4x\phi_1^2}$. Moreover the evaluation of Ψ in terms of finite Feynman amplitudes, according to the Eq. (10), requires the expansion of Γ in terms of effective amplitudes by using Eq. (6)

$$\Gamma^{(n)}=\sum_k \gamma_k\mathcal{P}_k, \tag{17}$$

where the dimension of $\mathcal{P}_k$ is now unlimited. However, we shall need only the amplitudes of dimension 5 and 6, due to the structure of $\mathcal{S}_0$ as given in Eq. (10).

Equation (13) can be looked from the point of view of a different renormalization scheme. Let $\Gamma^{(n)}$ be the result of any (nonsymmetric) renormalization. One needs to introduce a set of counterterms $\Gamma^{(n)}_{\mathrm{CT}}$ order-by-order:

$$\Gamma^{(n)}+\Gamma^{(n)}_{\mathrm{CT}}. \tag{18}$$

By comparing with our procedure we have

$$\Gamma_{\rm CT}^{(n)} = -t^{\delta_{\rm pc}}\Gamma^{(n)} + \Xi^{(n)} + \sum_j v_j \mathcal{I}_j . \tag{19}$$

The first term is just a Taylor expansion of the action-like amplitudes. The second term is evaluated in terms of finite amplitudes and of some of their derivatives at zero momenta (this computation can be possibly performed by automatic calculus). The last term contains the ST invariants and accounts for the differences between the normalization conditions in the two schemes.

Section II is devoted to the separation of the counterterms into sectors. By a judicious choice of the normalization conditions we can drop the tadpole and most of the external source counterterms. Only in the fermion sector the external source terms are modified by the renormalization procedure. Moreover we can identify a bosonic, a kinetic-gauge sector and a fermionic sector. At the end of Sec. II we discuss the computation of S-matrix elements.

Section III contains a study of the breaking term functional Ψ. In particular the ST linearized operator S_0 of Eq. (8), which enters in expression for Ψ, is modified in order to keep track of the ghost equation.

Section IV provides the complete list of the counterterms in terms of finite amplitudes. The solution contains the contribution of the lower terms of the perturbative expansion. Moreover some consistency conditions are shown to be present among the finite amplitudes.

Technical detail are in the Appendixes. In Appendix A we give the essential elements of the BRS transformations and of the model. In Appendix B we list all possible counterterms and their ST transforms. In Appendix C we discuss the important issue of the linearly independent ST invariants. Finally Appendix D contains the expansion of the breaking terms Ψ in terms of local Lorentz invariant monomials [Eq. (15)]. The expansion of the functional Ψ in terms of Lorentz invariant amplitudes allows the evaluation of the solutions given in Sec. IV.

II. HIERARCHY OF COUNTERTERMS AND BREAKING TERMS

The complexity of the problem is somehow distributed on two different steps. The evaluation of the breaking-term functional Ψ is probably the most complex part. Once Ψ is given, one has to evaluate the counterterms Ξ by Eq. (11). The present section is devoted to this last problem. In order to reduce the problem of managing the complete set of STI simultaneously, we introduce a hierarchy for the counterterms Ξ and breaking terms Ψ. This problem has been already discussed in previous works (see Refs. [1] and [4]) on algebraic renormalization.

S_0 is a mapping of $\mathcal{V}_\Xi$ on $\mathcal{V}_\Psi$,

$$S_0 : \mathcal{V}_\Xi \to \mathcal{V}_\Psi , \tag{20}$$

where the vector spaces are given by the relevant monomials

$$\mathcal{V}_\Xi \equiv \left\{ \sum_k x_k \mathcal{P}_k | x_k \in \mathcal{C}, \dim(\mathcal{P}_k) \leqslant 4, \right.$$
$$\left. \text{FP charge } (\mathcal{P}_k) = 0 \right\} \tag{21}$$

and

$$\mathcal{V}_\Psi \equiv \left\{ \sum_i x_i \mathcal{M}_i | x_i \in \mathcal{C}, \dim(\mathcal{M}_i) \leqslant 5, \right.$$
$$\left. \text{FP charge } (\mathcal{M}_i) = 1 \right\}. \tag{22}$$

The set of all actionlike functionals $\{\mathcal{I}_i\}$ which are invariant under ST transformations forms the kernel of S_0,

$$\ker(S_0) = \left\{ \sum_i v_i \mathcal{I}_i | v_i \in \mathcal{C}, \dim(\mathcal{I}_i) \leqslant 4, \right.$$
$$\left. \text{FP charge } (\mathcal{I}_i) = 0 \right\}. \tag{23}$$

Some of the ST invariants are genuine BRS invariants. The trivial ST invariants are given by all elements which are S_0 variations of local functionals of dimension $\leqslant 3$ and FP charge $= -1$. The subspace $\ker(S_0)$ induces an equivalence relation among the counterterms. The freedom of the choice of the representative of the equivalence classes will be used as one of the tools to organize the counterterms in a hierarchy, according to a strategy aiming to reduce the complexity of AR. This choice amounts to fix the normalization conditions; in fact in this way we select a basis on which we write the counterterm functional Ξ. Therefore all monomials outside the basis do not appear as counterterms. It should be mentioned here that the subspace $\ker(S_0)$ is further restricted by the condition imposed by the ghost equation of motion. The necessity to impose this condition as a first step comes from the fact that the ghost equation of motion is the statement of the nilpotency of S_0.

The image of $\mathcal{V}_\Xi$ is a proper subspace of $\mathcal{V}_\Psi$,

$$S_0(\mathcal{V}_\Xi) \subset \mathcal{V}_\Psi . \tag{24}$$

By construction

$$\Psi \in S_0(\mathcal{V}_\Xi) \tag{25}$$

since there are no anomalies. It is convenient to use a basis

$$\mathcal{M}_i e_{ik} = S_0(\mathcal{P}_k), \tag{26}$$

where k labels the chosen representatives of the equivalence classes in $\mathcal{V}_\Xi$. Finally we have

$$\Xi = \sum_k \xi_k \mathcal{P}_k , \tag{27}$$

where ξ_k are determined from

$$\Psi = \sum_{ki} \mathcal{M}_i e_{ik} \xi_k , \tag{28}$$

i.e.

$$\psi_i = \sum_k e_{ik} \xi_k . \tag{29}$$

The matrix e_{ki} is fixed by the model and by the choice of the basis $\{\mathcal{P}_k\}$. It can be evaluated solely by using the ST transformations given in Appendix B. In particular it does not depend on the order of the perturbative expansion.

In general the number of ψ_i is higher than the number of the unknowns ξ_k. The solution exists since the theory is assumed to satisfy STI (no anomalies). Most of the consistency conditions can be derived from the nilpotency of S_0 (if the ghost equation is satisfied),

$$S_0(\Psi) = 0. \tag{30}$$

The choice of the representatives and of the linearly independent equations in Eq. (11) is performed according to the following strategy, which aims to reduce the complexity of AR. First, we look for a block or triangular structure of the matrix e_{ki} (hierarchy). Second, we reduce the number of terms coming from the lower perturbation expansion [see Eq. (12)]. Third, the choice of the linearly independent equations is done by preferring the breaking terms with lower number of external legs and higher derivatives in the external momenta. In this way the number of graphs is reduced at the cost of some derivatives on external momenta. This strategy might look unnecessary in the present simple model. However, it will be useful in a more complicated situation as in the standard model.

Two A,B subspaces of $\mathcal{V}_\Xi$ are disjoint if

$$S_0(A) \cap S_0(B) = \{0\}. \tag{31}$$

Practically, this means that the ST transforms of A,B do not shear any monomial $\mathcal{M}_i$. A includes B if

$$S_0(B) \subset S_0(A). \tag{32}$$

These definitions are the guide for the hierarchy structure of the counterterms. If they can be grouped into disjoint sets then we have a block diagonalization of e_{ki}. If we get an including structure then the matrix is triangular. In both cases the task is consistently reduced. Moreover we can use the ST invariants in order to improve the structure of the matrices e_{ki} by choosing appropriate normalization conditions. This is performed by exploiting the invariance of Eq. (11) under the transformation

$$\Xi \rightarrow \Xi + \sum_j v_j \mathcal{I}_j . \tag{33}$$

The coefficients v_j will be determined by excluding some monomials $\mathcal{P}_k$ from the basis for Ξ.

A. Ghost equation and invariant counterterms

The proof of physical unitarity relies on the property of S_0 of being nilpotent. In the present on-shell formalism the ghost equation guarantees the above requirement

$$\alpha \Box c + e v \Gamma_{J_2} = \Gamma_{\bar{c}}. \tag{34}$$

This requirement excludes a mass term in $\Gamma^{(0)}$ of the form

$$M^2 \left[\frac{A^2}{2} + \bar{c} c - \frac{1}{2\alpha} (\phi_1^2 + \phi_2^2) \right]. \tag{35}$$

The present approach is equivalent to the Nakanishi-Lautrup formulation of the gauge fixing.[1] The ghost equation must be valid after the renormalization procedure. For $n > 1$ we have

$$\begin{aligned} &ev\mathbf{\Gamma}^{(n)}_{J_2 c} = \mathbf{\Gamma}^{(n)}_{\bar{c}c} , \\ &ev\mathbf{\Gamma}^{(n)}_{J_2 c \phi_1} = \mathbf{\Gamma}^{(n)}_{\bar{c}c\phi_1} , \\ &ev\Gamma^{(n)}_{J_2 c \phi_1^2} = \mathbf{\Gamma}^{(n)}_{\bar{c}c\phi_1^2} , \\ &ev\Gamma^{(n)}_{J_2 c \phi_2^2} = \mathbf{\Gamma}^{(n)}_{\bar{c}c\phi_2^2} , \\ &ev\Gamma^{(n)}_{J_2 c A_\mu^2} = \mathbf{\Gamma}^{(n)}_{\bar{c}cA_\mu^2} . \end{aligned} \tag{36}$$

These equations fix the counterterms

$$\xi_{\bar{c}\Box c}, \xi_{\bar{c}c\phi_1^2}, \xi_{\bar{c}c\phi_2^2}, \xi_{\bar{c}cA_\mu^2} \tag{37}$$

since they are related to superficially finite vertex functions. The remaining counterterms

$$\xi_{\bar{c}c}, \xi_{\bar{c}c\phi_1} \tag{38}$$

are related to counterterms involving external sources

$$\xi_{J_2 c}, \xi_{J_2 c \phi_1}. \tag{39}$$

In Appendix C we list the linearly independent ST invariants with charge conjugation $+1$. Any linear combination of ST invariants

$$\Xi \rightarrow \Xi + \sum_{j=1,\ldots,11} v_j \mathcal{I}_j \tag{40}$$

can be added to the vertex functional. A straightforward analysis shows that the ghost equation is preserved provided

$$\begin{aligned} &v_7 = 0 \\ &v_8 = -(ev)^2 v_9 - e^2 v v_{11} \end{aligned} \tag{41}$$

[1] The Nakanishi-Lautrup formulation requires a Lagrange multiplier b coupled to the gauge fixing function $\mathcal{F}(A,\phi)$ [see Eq. (A7)] and whose BRS transformation is simply given by $sb = \bar{c}, s\bar{c} = 0$. This provides an off-shell nilpotent BRS transformations avoiding the constraints (34) in order to guarantee the nilpotency of S_0.

and moreover that, under such circumstances, the monomial $\int d^4x\bar{c}\Box c$ is absent in the rest of the ST invariants in Eq. (40).

For further use we notice that the remaining constants $\{v_j\}$ can be determined by fixing the coefficients of the following nine monomials:

$$\phi_1, \phi_2^2\phi_1, A^2\phi_1, F_{\mu\nu}^2, i\bar{\psi}\gamma_5\psi\phi_2, \bar{\psi}\gamma_5 A\!\!\!/\psi, J_2c, J_2c\phi_1, J_1c\phi_2 \tag{42}$$

as can be seen from the matrix given in Appendix C.

B. Sector 0

The counterterms containing external sources $J_i, \eta, \bar{\eta}$ are the right group to start with

$$\xi_{J_2c}, \xi_{J_2c\phi_1}, \xi_{J_1c\phi_2}, \xi_{\bar{\eta}c\psi}, \xi_{\bar{\psi}c\eta}. \tag{43}$$

The ST transforms of their corresponding monomials (see Appendix B) contain the equations of motion and therefore the external source counterterms are present in most of the subspaces of $\mathcal{V}_\Xi$ as defined in Eq. (31) and in Eq. (32). Moreover in the recursive equation (12) the counterterms which contain external sources are present in almost every terms. Thus it is advantageous to set all possible loop corrections to the BRS external sources to zero by using the freedom in the choice of the coefficients $\{v_j\}$ in Eq. (33). By using the ST invariants $\mathcal{I}_{9-11}$ given in Appendix C we impose the normalization conditions $(n>0)$

$$\Gamma^{(n)}_{J_2c}(0)=\xi^{(n)}_{J_2c}=0,$$

$$\Gamma^{(n)}_{J_2c\phi_1}(0)=\xi^{(n)}_{J_2c\phi_1}=0,$$

$$\Gamma^{(n)}_{J_1c\phi_2}(0)=\xi^{(n)}_{J_1c\phi_2}=0. \tag{44}$$

As a consequence of this choice Eq. (36) now fixes the counterterms in Eq. (38). By using the relation

$$\Gamma=(1-t^{\delta_{\rm pc}})\Gamma+\Xi \tag{45}$$

one gets $(n>0)$

$$\xi^{(n)}_{\bar{c}c}=\Gamma^{(n)}_{\bar{c}c}(0)=0,$$

$$8\xi^{(n)}_{\bar{c}\Box c}=\partial_{p_\mu}\partial_{p^\mu}\Gamma^{(n)}_{\bar{c}c}(0)=ev\,\partial_{p_\mu}\partial_{p^\mu}\Gamma^{(n)}_{J_2c}(0),$$

$$\xi^{(n)}_{\bar{c}c\phi_1}=\Gamma^{(n)}_{\bar{c}c\phi_1}(0)=0,$$

$$\xi^{(n)}_{\bar{c}c\phi_1^2}=-ev\Gamma^{(n)}_{J_2c\phi_1^2}(0),$$

$$\xi^{(n)}_{\bar{c}c\phi_2^2}=-ev\Gamma^{(n)}_{J_2c\phi_2^2}(0),$$

$$\xi^{(n)}_{\bar{c}cA_\mu^2}=-ev\Gamma^{(n)}_{J_2cA_\mu^2}(0). \tag{46}$$

Since the ghost equation fixes all counterterms involving the ghost field, we drop the analysis of the ghost sector. The ghost part of Ξ is

$$\Xi_{\rm ghost}=\int d^4x[\xi_{\bar{c}\Box c}\bar{c}\Box c+\xi_{\bar{c}c\phi_1^2}\bar{c}c\phi_1^2+\xi_{\bar{c}c\phi_2^2}\bar{c}c\phi_2^2 + \xi_{\bar{c}cA_\mu^2}\bar{c}cA_\mu^2]. \tag{47}$$

C. Sector I

The next sector is selected by the condition

$$\mathcal{N}_\phi\leq 4,\quad \mathcal{N}_A=\mathcal{N}_\psi=\mathcal{N}_{\bar{\psi}}=0, \tag{48}$$

where $\mathcal{N}_\phi$, $\mathcal{N}_A$, $\mathcal{N}_\psi$, and $\mathcal{N}_{\bar{\psi}}$ respectively count the number of $\phi, A, \psi, \bar{\psi}$. The coefficients of the monomial of this sector are mass terms (3)[2]: $\xi_{\phi_1}, \xi_{\phi_1\phi_1}, \xi_{\phi_2\phi_2}$, trilinear self-interacting terms (2): $\xi_{\phi_1\phi_2\phi_2}, \xi_{\phi_1\phi_1\phi_1}$, and, quadrilinear interacting terms (3): $\xi_{\phi_1\phi_1\phi_2\phi_2}, \xi_{\phi_1\phi_1\phi_1\phi_1}, \xi_{\phi_2\phi_2\phi_2\phi_2}$. The sector can be further decomposed into two sub-sectors with $\mathcal{N}_\phi\leq 2$ and $\mathcal{N}_\phi>2$. These two sub-sectors turn out to be disjoint if we put to zero the coefficient $\xi_{\phi_1\phi_2\phi_2}$ (see Appendix B). This can be achieved by using the ST invariant $\mathcal{I}_2$. The contribution from the lower orders of perturbation are reduced if we put equal to zero the coefficient ξ_{ϕ_1} of the tadpole. This condition can be imposed by using the ST invariant $\mathcal{I}_1$. Finally six coefficients have to be evaluated. A direct inspection of the ST transforms of the corresponding monomial shows that the breaking terms to be evaluated are

$$\psi^I_i=\{\psi_{c\phi_2}, \psi_{c\phi_2\phi_1}, \psi_{c\phi_2\phi_1^2}, \psi_{c\phi_2\phi_1^3}, \psi_{c\phi_2^3}, \psi_{c\phi_2^3\phi_1}\}. \tag{49}$$

With the above conventions it is straightforward, with the help of the ST transformations in Appendix B, to construct the reduced matrix in Eq. (29):

$$\begin{pmatrix} & \xi_{\phi_1^2} & \xi_{\phi_2^2} & \xi_{\phi_1^3} & \xi_{\phi_1^2\phi_2^2} & \xi_{\phi_1^4} & \xi_{\phi_2^4} \\ \psi_{c\phi_2} & 0 & -2ev & 0 & 0 & 0 & 0 \\ \psi_{c\phi_2\phi_1} & 2e & -2e & 0 & 0 & 0 & 0 \\ \psi_{c\phi_2\phi_1^2} & 0 & 0 & 3e & -2ev & 0 & 0 \\ \psi_{c\phi_2\phi_1^3} & 0 & 0 & 0 & -2e & 4e & 0 \\ \psi_{c\phi_2^3} & 0 & 0 & 0 & 0 & 0 & -4ev \\ \psi_{c\phi_2^3\phi_1} & 0 & 0 & 0 & 2e & 0 & -4e \end{pmatrix}. \tag{50}$$

D. Sector II

This sector (selected by the condition $\mathcal{N}_\phi\leq 2$, $\mathcal{N}_\psi=0$, $\mathcal{N}_{\bar{\psi}}=0$, $\mathcal{N}_A+\mathcal{N}_\partial=2$) deals with the kinetic terms of

[2]The number in brackets counts the number of counterterms of the corresponding sub-sector.

the scalar fields and the corresponding terms coming form the covariant derivatives, that is the interaction terms of the scalar fields and the gauge fields. It also contains the mass of the gauge boson. The coefficients ξ^{II} are mass term for gauge field (1), $\xi_{A_\mu^2}$, kinetic terms for scalar fields (2), $\xi_{\partial^\mu\phi_1\partial_\mu\phi_1},\xi_{\partial^\mu\phi_2\partial_\mu\phi_2}$, mixing terms between scalar field and gauge field (1), $\xi_{\partial_\mu A^\mu\phi_2}$, coupling scalar-gauge fields (2), $\xi_{A_\mu\partial^\mu\phi_2\phi_1},\xi_{A_\mu\phi_2\partial^\mu\phi_1}$, trilinear term (1), $\xi_{A_\mu^2\phi_1}$, and quadrilinear terms (2), $\xi_{A_\mu^2\phi_1^2},\xi_{A_\mu^2\phi_2^2}$. The bilinear out-of-diagonal counterterm can be put to zero,

$$t^2\mathbf{\Gamma}_{A^\mu\phi_2}(0)=\Xi_{A^\mu\phi_2}=0, \tag{51}$$

by using the ST invariant $\mathcal{I}_3$. Finally one has to evaluate eight coefficients

$$\xi^{II}\equiv\{\xi_{A_\mu^2},\xi_{\partial^\mu\phi_1\partial_\mu\phi_1},\xi_{\partial^\mu\phi_2\partial_\mu\phi_2},\xi_{A_\mu\phi_2\partial^\mu\phi_1}, \times\xi_{A_\mu\partial^\mu\phi_2\phi_1},\xi_{A_\mu^2\phi_1},\xi_{A_\mu^2\phi_1^2},\xi_{A_\mu^2\phi_2^2}\} \tag{52}$$

in terms of the following breaking terms:

$$\psi_i^{II}=\{\psi_{\partial_\mu c\partial^\mu\phi_2},\psi_{c\phi_2\Box\phi_1},\psi_{c\partial_\mu\phi_2\partial^\mu\phi_1}, \times\psi_{c\Box\phi_2\phi_1},\psi_{c\partial^\mu A_\mu},\psi_{c\partial^\mu A_\mu\phi_1},\psi_{cA^\mu\partial_\mu\phi_1}, \times\psi_{c\partial^\mu A_\mu\phi_1^2},\psi_{cA^\mu\partial_\mu\phi_1^2},\psi_{c\partial^\mu A_\mu\phi_2^2},\psi_{cA^\mu\partial_\mu\phi_2^2}, \times\psi_{cA_\mu A^\mu\phi_1^2},\psi_{cA_\mu A^\mu\phi_2^2},\}. \tag{53}$$

The transformation matrix e_{ik} [Eq. (29)] is

	$\xi_{A_\mu^2}$	$\xi_{\partial^\mu\phi_1\partial_\mu\phi_1}$	$\xi_{\partial^\mu\phi_2\partial_\mu\phi_2}$	$\xi_{A_\mu\phi_2\partial^\mu\phi_1}$	$\xi_{A_\mu\partial^\mu\phi_2\phi_1}$	$\xi_{A_\mu^2\phi_1}$	$\xi_{A_\mu^2\phi_1^2}$	$\xi_{A_\mu^2\phi_2^2}$
$\psi_{c\Box\phi_2}$	0	0	$2ev$	0	0	0	0	0
$\psi_{c\phi_2\Box\phi_1}$	0	$-2e$	0	1	0	0	0	0
$\psi_{c\partial_\mu\phi_2\partial^\mu\phi_1}$	0	0	0	1	1	0	0	0
$\psi_{c\Box\phi_2\phi_1}$	0	0	$2e$	0	1	0	0	0
$\psi_{c\partial^\mu A_\mu}$	2	0	0	0	0	0	0	0
$\psi_{c\partial^\mu A_\mu\phi_1}$	0	0	0	0	ev	2	0	0
$\psi_{cA^\mu\partial_\mu\phi_1}$	0	0	0	$-ev$	ev	2	0	0
$\psi_{c\partial^\mu A_\mu\phi_1^2}$	0	0	0	0	e	0	2	0
$\psi_{cA^\mu\phi_1\partial_\mu\phi_1}$	0	0	0	$-e$	e	0	4	0
$\psi_{c\partial^\mu A_\mu\phi_2^2}$	0	0	0	$-e$	0	0	0	2
$\psi_{cA^\mu\phi_2\partial_\mu\phi_2}$	0	0	0	$-e$	e	0	0	4
$\psi_{cA_\mu^2\phi_2}$	0	0	0	0	0	e	0	$-2ev$
$\psi_{cA_\mu^2\phi_2\phi_1}$	0	0	0	0	0	0	$2e$	$-2e$

. (54)

E. Sector III

In the present model the kinetic terms for the gauge fields are trivial because of the Abelianity of the gauge group. The eigenvalues of the counting operators are given by

$$\mathcal{N}_\phi=\mathcal{N}_\psi=0,\quad(\mathcal{N}_A+\mathcal{N}_\partial)=4. \tag{55}$$

The sector contains kinetic terms for gauge fields (2): $\xi_{\partial_\mu A^\mu\partial_\nu A^\nu},\xi_{\partial_\nu A_\mu\partial^\nu A^\mu}$, and interacting terms (1): $\xi_{A_\mu^4}$. The corresponding breaking-terms are given by

$$\psi_i^{III}=\{\psi_{c\Box\partial_\mu A^\nu},\psi_{c\partial^\mu A_\mu A_\nu^2},\psi_{c\partial^\nu A_\mu A_\nu A^\mu}\}. \tag{56}$$

The ST invariant $\mathcal{I}_4$ can be used in order to put equal zero the counterterm corresponding to the transverse part:

$$\xi_{F_{\mu\nu}^2}=0. \tag{57}$$

By looking at Eq. (B2) we have the following relations:

$$\psi_{c\Box\partial_\mu A^\nu}=-2\xi_{\partial_\mu A^\mu\partial_\nu A^\nu},$$

$$\psi_{c\partial^\mu A_\mu A_\nu^2}=4\xi_{A_\mu^4}-\xi_{\bar{c}cA_\mu^2}^{-},$$

$$\psi_{c\partial^\nu A_\mu A_\nu A^\mu}=8\xi_{A_\mu^4}, \tag{58}$$

where the coefficient $\xi_{\bar{c}cA_\mu^2}$ is known from the ghost equation (46).

F. Sector IV

This sector contains the Green's functions with fermion fields, and it can be further divided into the sector of mass terms of fermion fields and their coupling with the scalar fields and the sector of the kinetic terms and the interaction with the gauge fields. The present sector is completely decoupled for the previous sectors, it is specified by the following eigenvalues:

$$\mathcal{N}_\phi \leqslant 1, \quad \mathcal{N}_\psi = 2, \quad \mathcal{N}_A + \mathcal{N}_\partial \leqslant 1, \tag{59}$$

and the counterterms are mass term (1): $\xi_{\bar{\psi}\psi}$, Yukawa term (2): $\xi_{\bar{\psi}\psi\phi_1}, \xi_{i\bar{\psi}\gamma_5\psi\phi_2}$, kinetic term and interaction with the gauge field (2): $\xi_{i\bar{\psi}\partial\!\!\!/\psi}, \xi_{\bar{\psi}A\!\!\!/\psi}$. The breaking-terms are given by

$$\psi_i^{IV} = \{\psi_{c\bar{\psi}\gamma_5\psi}, \psi_{c\phi_1\bar{\psi}\gamma_5\psi}, \psi_{c\phi_2\bar{\psi}\psi}, \psi_{c\bar{\psi}\gamma_5\partial\!\!\!/\psi}, \psi_{c\partial^\mu\bar{\psi}\gamma_\mu\gamma_5\psi}, \}. \tag{60}$$

There are two invariants ($\mathcal{I}_5$ and $\mathcal{I}_6$), pertinent to this sector. They are used to impose the following normalization conditions:

$$\xi_{\bar{\psi}A\!\!\!/\psi} = 0,$$
$$\xi_{i\bar{\psi}\gamma_5\psi\phi_2} = 0. \tag{61}$$

The matrix e_{ik} which express the functional $\psi^{(IV)}$ in terms of $\xi^{(IV)}$ is given by

$$\begin{pmatrix} & \xi_{\bar{\psi}\psi} & \xi_{\bar{\psi}\psi\phi_1} & \xi_{i\bar{\psi}\partial\!\!\!/\psi} \\ \psi_{c\bar{\psi}\gamma_5\psi} & -1 & 0 & 0 \\ \psi_{c\phi_1\bar{\psi}\gamma_5\psi} & 0 & -1 & 0 \\ \psi_{c\phi_2\bar{\psi}\psi} & 0 & 1 & 0 \\ \psi_{c\bar{\psi}\gamma_5\partial\!\!\!/\psi} & 0 & 0 & -\frac{1}{2} \\ \psi_{c\partial^\mu\bar{\psi}\gamma_\mu\gamma_5\psi} & 0 & 0 & -\frac{1}{2} \end{pmatrix}. \tag{62}$$

G. Summary of the normalization conditions

For $n>0$ we have imposed the normalization conditions

$$\xi_{\phi_1}^{(n)} = 0, \quad \xi_{\phi_2^2\phi_1}^{(n)} = 0, \quad \xi_{\partial^\mu A_\mu\phi_2}^{(n)} = 0, \quad \xi_{F_{\mu\nu}^2}^{(n)} = 0, \quad \xi_{i\bar{\psi}\gamma_5\psi\phi_2}^{(n)} = 0,$$
$$\xi_{\bar{\psi}A\!\!\!/\gamma_5\psi}^{(n)} = 0, \quad \xi_{J_2c}^{(n)} = 0, \quad \xi_{J_2c\phi_1}^{(n)} = 0, \quad \xi_{J_1c\phi_2}^{(n)} = 0. \tag{63}$$

This set of normalization conditions should be considered as an intermediate step in the evaluation of physical amplitudes. Furthermore normalization conditions at zero momentum is a standard procedure in the BPHZ formalism. However, it might seem unfamiliar for those involved in the explicit computation of physical processes. For this reason, we outline the construction of the physical amplitudes.

By denoting $\mathbf{\Gamma}_{ab}(p)$ the elements of the two-point function matrix, for generic fields a,b the masses of the asymptotic fields are given by the solutions of the equation

$$\det[\mathbf{\Gamma}_{ab}(p)] = 0. \tag{64}$$

The physical values are recovered by tuning the free parameters e,v,λ,G of Γ_0. The wave functions of the asymptotic fields coincide with the eigenvectors $\boldsymbol{\epsilon}_a^{(i)}$

$$[\mathbf{\Gamma}_{ab}(p)]\boldsymbol{\epsilon}_b^{(i)}(p)|_{p^2=m_i^2} = 0. \tag{65}$$

Finally the LSZ formalism provides the correct physical amplitudes. For instance, the reduction formula for an outgoing particle will be

$$z_i^{1/2}\boldsymbol{\epsilon}_b^{(i)*} \lim_{p^2\to m_i^2} \mathbf{\Gamma}_{bc}(p)\frac{\delta}{\delta Y_c(p)} Z[Y]\bigg|_{Y=0}, \tag{66}$$

where $z_i^{1/2}$ are the residual at the pole of the propagators, $Z[Y]$ is the generating functional of connected amplitudes, and Y_a is the source of a. After the reduction formula the S-matrix elements depend only on the parameter e since λ,G,v are fixed by the masses. This last parameter has to be tuned on a physical process.

III. ST BREAKING TERMS

In the strategy outlined before the counterterm functional Ξ is obtained by solving a set of linear equations (29). The restoration of ST invariance consists in the evaluation of a certain number of (finite) vertex functions (the functional Ψ). This fact is clear evidence that it is the finite part of the perturbative expansion that fixes the counterterms in the action.

In this section we discuss some aspects of this procedure. The first step consists in the evaluation of the functional derivatives of Ψ. It is of some help to remember that, in absence of anomalies, Ψ is the image through S_0 of noninvariant counterterms (Ξ). Therefore it has a FP charge equal $+1$, C-even and dimensionless or equal to 5. The next step is to find the coefficients ψ_i in the expansion in terms of Lorentz scalar monomials

$$\Psi = \sum_i \psi_i \mathcal{M}_i. \tag{67}$$

Let us write explicitly, for $n>0$, the operator S_0, where we impose the ghost equation of motion given in Eq. (34), i.e.,

$$ev\Gamma_{J_2}^{(n)} = \Gamma_c^{(n)} \quad \text{for} \quad n>0, \tag{68}$$

$$\hat{S}_0(\Gamma^{(n)}) \equiv \int d^4x \Bigg\{ \partial_\mu c \Gamma^{(n)}_{A_\mu} - ec\phi_2 \Gamma^{(n)}_{\phi_1} + ec(\phi_1+v)\Gamma^{(n)}_{\phi_2} + i\frac{e}{2} c\bar{\psi}\gamma_5 \Gamma^{(n)}_{\bar{\psi}} + i\frac{e}{2} c\gamma_5 \psi \Gamma^{(n)}_{\psi} + \Gamma^{(0)}_{\phi_1}\Gamma^{(n)}_{J_1} + \left[\Gamma^{(0)}_{\phi_2} + ev\left(\partial^\mu A_\mu + \frac{ev}{\alpha}\phi_2\right)\right]\Gamma^{(n)}_{J_2} - \Gamma^{(0)}_{\psi}\Gamma^{(n)}_{\bar{\eta}} + \Gamma^{(0)}_{\bar{\psi}}\Gamma^{(n)}_{\eta} \Bigg\}. \tag{69}$$

By imposing the condition (68) the breaking term Ψ changes. We denote this change with the notation $\Psi \to \hat{\Psi}$.

In the linearized form (S_0) one of the factors in each monomial contains the vertex function at zero loop $\Gamma^{(0)}$. All these facts have some interesting consequences.

(1) The expansion of Ψ in terms of polynomial of fields, which is relevant for evaluating the counterterms, can be read directly from the ST transforms of all actionlike terms (see Appendix B).

(2) Let δ be the total dimension of the fields we use for the functional derivative of Ψ. Then the order of the Taylor operator $\delta_{\rm D}$ is [see Eq. (7)]

$$\delta_{\rm D} = 5 - \delta. \tag{70}$$

(3) Let us consider a generic term of S_0 for instance $\Gamma^{(0)}_{J_i}\Gamma^{(n)}_{\phi_i}$ or $\Gamma^{(n)}_{J_i}\Gamma^{(0)}_{\phi_i}$. If $\Gamma^{(0)}$ does not contain any dimensional parameter, then

$$t^{\delta_{\rm D}}\Gamma^{(0)}_{J_i}\Gamma^{(n)}_{\phi_i} = \Gamma^{(0)}_{J_i} t^{\delta_{\rm pc}}\Gamma^{(n)}_{\phi_i}. \tag{71}$$

In the above equation we use a rather short-hand writing and to be more explicit we give an example: by taking the functional derivative of the $\Gamma^{(0)}_{J_1}\Gamma^{(n)}_{\phi_1}$ term with respect to $c\phi_2$, the $\delta_{\rm D}$ is equal to 3 and we get

$$t^3\Gamma^{(0)}_{J_1 c\phi_2}\Gamma^{(n)}_{\phi_1} = -et^3\Gamma^{(n)}_{\phi_1}. \tag{72}$$

where $\delta_{\rm pc}=3$ is the superficial degree of divergence of $\Gamma^{(n)}_{\phi_1}$. On the contrary, if $\Gamma^{(0)}$ contains a dimensional parameter, namely v, Eq. (71) is not valid.

For instance, considering the functional derivative of $\Gamma^{(n)}_{J_1}\Gamma^{(0)}_{\phi_1}$ with respect to $c\phi_2\phi_1$, the δ_D is equal to 2 and we obtain

$$t^2(\Gamma^{(n)}_{J_1 c\phi_2}\Gamma^{(0)}_{\phi_1\phi_1}) = -m_1^2 t^2 \Gamma^{(n)}_{J_1 c\phi_2} + p^2 t^0 \Gamma^{(n)}_{J_1 c\phi_2}. \tag{73}$$

The first terms on the right hand side of the above equation, namely $-m_1^2 t^2 \Gamma^{(n)}_{J_1 c\phi_2}$, contains oversubtractions of the three-point function $\Gamma^{(n)}_{J_1 c\phi_2}$. As discussed in the Introduction these are the sources of the ST breaking terms.

(4) The above point implies that Ψ [Eq. (10)] gets contributions only from those terms of $\Gamma^{(0)}$ which carry a dimensioned parameter (v and masses).

The functional derivatives of Ψ are listed in Appendix D. It should be noticed that a few other counterterms turn out to be zero at every order:

$$\int d^4x\phi_1^2, \quad \int d^4x\phi_2^2, \quad \int d^4x(\partial_\mu\phi_2)^2, \quad \int d^4x A_\mu^2. \tag{74}$$

This is due to the combined effects of our choice of normalization conditions and of the zero momentum subtraction procedure. Moreover the contribution to STI from the lower order amplitudes appear only in few functional derivatives of Ψ. One can describe this fact by saying that the set of STI becomes *almost* linear in Γ.

IV. SOLUTION FOR COUNTERTERMS

The relations obtained in Appendix D can be expanded in terms of Lorentz invariant amplitudes. Thus one can express the invariant amplitude for Ξ in terms of the invariant amplitude for Ψ. This amounts to solve the linear algebra problem given in Eq. (29) where the matrices are given in Eqs. (50), (54), (50), (58), and (62).

We remind our notations [see Eqs. (15), (16), and (17)] where the small letters ξ, γ, and ψ denote the coefficients of the Lorentz invariant monomials, respectively, of Ξ (counterterms), Γ, and Ψ (breaking terms) indicated by the subscript. The order of perturbation theory is not shown and it is understood to be n, unless explicitly exhibited. The ghost equation (34) fixes the kinetic counterterms of the ghost [see Eq. (46)]

$$\xi_{\bar{c}\Box c} = ev\gamma_{\Box J_2 c}. \tag{75}$$

A. Counterterms of sector I

In this sector we have the same number of equations and unknowns. The solution is (including the normalization conditions)

$$\xi_{\phi_1}=0, \;\; \xi_{\phi_2^2}=0, \;\; \xi_{\phi_1^2}=0, \;\; \xi_{\phi_2^2\phi_1}=0, \;\; \xi_{\phi_2^4}=0,$$

$$\xi_{\phi_1^3} = \frac{1}{3e}\left\{ -m_1^2\gamma_{J_2 c\phi_2^2} + 4ev^2\gamma_{\phi_2^4\phi_1} - m_1^2 v\gamma_{J_1 c\phi_2^3} - \frac{3}{2}m_1^2\gamma_{J_1 c\phi_2\phi_1}\right\},$$

$$\xi_{\phi_2^2\phi_1^2} = \frac{1}{2e}\{-2\lambda v\gamma_{J_2 c\phi_2^2} - v\lambda\gamma_{J_1 c\phi_2\phi_1} + 4ev\gamma_{\phi_2^4\phi_1} - m_1^2\gamma_{J_1 c\phi_2^3}\},$$

$$\xi_{\phi_1^4} = \frac{1}{4e}\Bigg\{ -2v\lambda\gamma_{J_2 c\phi_1^2} - 4v\lambda\gamma_{J_1 c\phi_2\phi_1} - m_1^2\gamma_{J_1 c\phi_2\phi_1^2} + 2ev\gamma_{\phi_2^2\phi_1^3} - 2\lambda v\gamma_{J_2 c\phi_2^2} + 4ev\gamma_{\phi_2^4\phi_1} - m_1^2\gamma_{J_1 c\phi_2^3} + 3\sum_{j=1}^{n-1}\gamma^{(j)}_{J_1 c\phi_2\phi_1}\xi^{(n-j)}_{\phi_1^3}\Bigg\}. \tag{76}$$

B. Counterterms of sector II

In this sector the problem is overdetermined. We use the first six and the last two rows of the matrix (54). The solution is (including the normalization conditions)

$$\xi_{A^2}=0, \quad \xi_{A^\mu \partial_\mu \phi_2}=0, \quad \xi_{\partial_\mu \phi_2 \partial^\mu \phi_2}=0,$$

$$\xi_{A^\mu \phi_1 \partial_\mu \phi_2}=-m_1^2 \gamma_{J_1 c \Box \phi_2}-2ev\,\gamma_{\partial_\mu \phi_2 \partial^\mu \phi_2 \phi_1}-2\lambda v\,\gamma_{\Box J_2 c},$$

$$\xi_{A^\mu \partial_\mu \phi_1 \phi_2}=m_1^2 \gamma_{J_1 c \Box \phi_2}-m_1^2 \gamma_{\partial_\mu J_1 c \partial^\mu \phi_2}-2\lambda v\,\gamma_{\Box J_2 c},$$

$$\xi_{\partial_\mu \phi_1 \partial^\mu \phi_1}=\frac{1}{2e}\{m_1^2 \gamma_{\Box J_1 c \phi_2}+ev\,\gamma_{\partial_\mu \phi_2 \phi_2 \partial^\mu \phi_1} - m_1^2 \gamma_{\partial_\mu J_1 c \partial^\mu \phi_2}+m_1^2 \gamma_{J_1 c \Box \phi_2}\},$$

$$\xi_{A^2 \phi_1}=-\frac{1}{2} m_1^2 \gamma_{J_1 c \partial_\mu A^\mu}+\frac{1}{2} m_1^2 ev\,\gamma_{J_1 c \Box \phi_2} +(ev)^2 \gamma_{\partial_\mu \phi_2 \partial^\mu \phi_2 \phi_1}+\frac{1}{2} m_1^2 e\,\gamma_{\Box J_2 c},$$

$$\xi_{A^2 \phi_2^2}=\frac{1}{2v}\xi_{A^2 \phi_1},$$

$$\xi_{A^2 \phi_1^2}=\frac{1}{2e}\Bigg\{-m_1^2 \gamma_{J_1 c \phi_2 A^2}-2v\lambda\,\gamma_{J_2 c A^2}+e^2 v\,\gamma_{J_1 c \phi_2 \phi_1} -\lambda ev\,\gamma_{J_1 c \partial_\mu A^\mu}+\lambda(ev)^2 \gamma_{J_1 c \Box \phi_2}+e^3 v\,\gamma_{\partial_\mu \phi_2 \partial^\mu \phi_2 \phi_1} +\lambda e^2 v\,\gamma_{\Box J_2 c}+\sum_{j=1}^{n-1} \gamma^{(j)}_{J_1 c \phi_2 \phi_1} \xi^{(n-j)}_{\phi_1 A^2}\Bigg\}. \tag{77}$$

The rest of the equations provided by the matrix (54) gives consistency conditions. However, not all of them are linear independent, in fact one can easily check that the linear combination

$$-2\psi_{cA^2 \phi_2}+e\psi_{cA^\mu \partial_\mu \phi_1}-ev\,\psi_{cA^\mu \partial_\mu \phi_2 \phi_2} \tag{78}$$

is identically zero. It should be reminded that this peculiar property is a consequence of our normalization conditions. Then the consistency conditions are

$$(1) \quad ev[\gamma_{\partial_\mu J_1 c \partial^\mu \phi_2}-\gamma_{J_1 c \Box \phi_2}]+e\gamma_{\Box J_2 c}+\gamma_{\partial_\mu J_1 c A^\mu} -\gamma_{J_1 c \partial_\mu A^\mu}=0, \tag{79}$$

$$(2) \quad 6\lambda v(\gamma_{J_1 c \partial_\mu A^\mu}-\gamma_{\partial_\mu J_1 c A^\mu})-m_1^2 \gamma_{J_1 c A^\mu \partial_\mu \phi_1} -m_1^2 \gamma_{\partial_\mu J_1 c A^\mu \phi_1}+2m_1^2 \gamma_{J_1 c \partial_\mu A^\mu \phi_1}-m_1^2 e\,\gamma_{\partial_\mu J_1 c \partial^\mu \phi_2} -2e^2 v\,\gamma_{\partial_\mu \phi_2 \partial^\mu \phi_2 \phi_1}-4\lambda ev\,\gamma_{\Box J_2 c}-2ev\,\gamma_{J_2 c \phi_1^2} +2ev(\gamma_{\phi_2 \partial_\mu \phi_1^2 A^\mu}-\gamma_{\phi_2 \phi_1^2 \partial_\mu A^\mu})=0, \tag{80}$$

$$(3) \quad \lambda v^2 e^2 \gamma_{J_1 c \Box \phi_2}+e^3 v\,\gamma_{\partial_\mu \phi_2 \partial^\mu \phi_2 \phi_1}+\lambda ve^2 \gamma_{\Box J_2 c} -2\lambda ve\,\gamma_{J_1 c \partial_\mu A^\mu}+e^2 v\,\gamma_{\phi_2 \phi_1^2 \partial_\mu A^\mu}+2\lambda v^2 \gamma_{J_1 c \phi_2 A^2} +2v\lambda\,\gamma_{J_2 c A^2}-e^2 v\,\gamma_{J_1 c \phi_2 \phi_1}-2\lambda v^2 e\,\gamma_{J_1 c \partial_\mu A^\mu \phi_1} +e^2 v\,\gamma_{J_2 c \phi_1^2}-\sum_{j=1}^{n-1} \gamma^{(j)}_{J_1 c \phi_2 \phi_1} \xi^{(n-j)}_{\phi_1 A^2} +3e\sum_{j=1}^{n-1} \gamma^{(j)}_{J_1 c \partial^\mu A_\mu} \xi^{(n-j)}_{\phi_1^3}=0, \tag{81}$$

$$(4) \quad 3\gamma_{\phi_2^3 \partial_\mu A^\mu}+\lambda v\,\gamma_{J_1 c \Box \phi_2}-2\lambda v\,\gamma_{\partial_\mu J_1 c \partial^\mu \phi_2}-3\lambda\,\gamma_{\Box J_2 c} +\gamma_{J_2 c \phi_2^2}-e\,\gamma_{\partial_\mu \phi_2 \partial^\mu \phi_2 \phi_1}=0. \tag{82}$$

C. Counterterms of sector III

The counterterms of the sector III, together with the normalization condition, are

$$\xi_{F^2_{\mu\nu}}=0,$$

$$\xi_{\partial_\mu A_\nu \partial^\nu A^\mu}=\frac{1}{2}\{ev\,\gamma_{A_\mu \partial^\mu \Box \phi_2}-ev\,\gamma_{\Box J_2 c}\},$$

$$\xi_{A^4}=\frac{1}{4}\Bigg\{ev\,\gamma_{\phi_2 \partial^\mu A_\mu A^2}+e^2 v\,\gamma_{\partial_\mu J_1 c A^\mu}+\sum_{j=1}^{n-1} \gamma^{(j)}_{\partial^\mu J_1 c A_\mu} \xi^{(n-j)}_{\phi_1 A^2}\Bigg\}. \tag{83}$$

In this sector there is one consistency condition

$$\gamma_{\phi_2 A_\mu \partial^\mu A^2}+e\,\gamma_{J_1 c \partial_\mu A^\mu}+\gamma_{J_2 c A^2}+\sum_{j=1}^{n-1} \gamma^{(j)}_{J_1 c \partial^\mu A_m u} \xi^{(n-j)}_{\phi_1 A^2} =\gamma_{\phi_2 \partial^\mu A_\mu A^2}+e\,\gamma_{\partial_\mu J_1 c A^\mu}+\sum_{j=1}^{n-1} \gamma^{(j)}_{\partial^\mu J_1 c A_\mu} \xi^{(n-j)}_{\phi_1 A^2}. \tag{84}$$

It is remarkable that contribution from the lower order terms appears only in three counterterms [see Eqs. (76), (77), and (83)].

D. Counterterms of the fermion sector

The expansion of Ψ in terms of the Lorentz invariant amplitudes performed in Appendix D reveals that the fermion source counterterms are nonvanishing. The counterterms of this sector are

$$\xi_{i/2(\bar{\eta}\gamma_5\psi c+c\bar{\psi}\gamma_5\eta)}=-\frac{1}{2G}\Big\{2ev\gamma_{\phi_2\phi_2\bar{\psi}\psi}-2Gv\gamma_{c\phi_2\bar{\eta}\psi}+ev\gamma_{i\phi_2\phi_1\bar{\psi}\gamma_5\psi}-2Gv\gamma_{ic\phi_1\bar{\eta}\gamma_5\psi}-m_1^2\gamma_{iJ_1c\bar{\psi}\gamma_5\psi}-\sum_{j=1,n-1}[2\xi^{(j)}_{\bar{\psi}\psi}(\gamma^{(n-j)}_{c\phi_2\bar{\psi}\eta}+\gamma^{(n-j)}_{ic\phi_1\bar{\psi}\gamma_5\eta})+\xi^{(j)}_{\phi_1\bar{\psi}\psi}\xi^{(n-j)}_{i/2(\bar{\eta}\gamma_5\psi c+c\bar{\psi}\gamma_5\eta)}]\Big\}. \tag{85}$$

The other counterterms are

$$\xi_{\phi_1\bar{\psi}\psi}=\frac{1}{2e}\Big\{2ev\gamma_{\phi_2\phi_2\bar{\psi}\psi}-2Gv\gamma_{c\phi_2\bar{\eta}\psi}-ev\gamma_{i\phi_2\phi_1\bar{\psi}\gamma_5\psi}+2Gv\gamma_{ic\phi_1\bar{\eta}\gamma_5\psi}+m_1^2\gamma_{iJ_1c\bar{\psi}\gamma_5\psi}-\sum_{j=1,n-1}[2\xi^{(j)}_{\bar{\psi}\psi}(-\gamma^{(n-j)}_{c\phi_2\bar{\psi}\eta}+\gamma^{(n-j)}_{ic\phi_1\bar{\psi}\gamma_5\eta})+\xi^{(j)}_{\phi_1\bar{\psi}\psi}\xi^{(n-j)}_{i/2(\bar{\eta}\gamma_5\psi c+c\bar{\psi}\gamma_5\eta)}]\Big\}, \tag{86}$$

$$\xi_{\bar{\psi}\psi}=-\frac{1}{e}\Big[Gv\xi_{i/2(\bar{\eta}\gamma_5\psi c+c\bar{\psi}\gamma_5\eta)}-\sum_{j=1,n-1}\xi^{(j)}_{\bar{\psi}\psi}\xi^{(n-j)}_{i/2(\bar{\eta}\gamma_5\psi c+c\bar{\psi}\gamma_5\eta)}\Big], \tag{87}$$

and

$$\xi_{i\bar{\psi}\gamma_\mu\partial^\mu\psi}=\frac{2}{e}\Big\{-\frac{1}{2}\xi_{i/2(\bar{\eta}\gamma_5\psi c+c\bar{\psi}\gamma_5\eta)}+Gv\gamma_{\bar{\eta}c\gamma_\mu\gamma_5\partial^\mu\psi}+Gv\gamma_{\bar{\psi}c\gamma_\mu\gamma_5\partial^\mu\eta}-ev\gamma_{\phi_2\bar{\psi}\gamma_\mu\gamma_5\partial^\mu\psi}-\sum_{j=1,n-1}\Big[\xi^{(j)}_{\bar{\psi}\psi}(\gamma^{(n-j)}_{c\bar{\psi}\gamma_\mu\gamma_5\partial^\mu\eta}-\gamma^{(n-j)}_{c\bar{\eta}\gamma_\mu\gamma_5\partial^\mu\psi})+\frac{1}{2}\xi^{(j)}_{i\bar{\psi}\gamma_\mu\partial^\mu\psi}\xi^{(n-j)}_{i/2(\bar{\eta}\gamma_5\psi c+c\bar{\psi}\gamma_5\eta)}\Big]\Big\}. \tag{88}$$

Since we require Hermiticity and charge conjugation invariance, there is only one consistency condition, given by the equation

$$0=-ev\gamma_{\phi_2\gamma^\mu A_\mu\bar{\psi}\psi}+Gv\gamma_{c\bar{\eta}\gamma^\mu\gamma^5\psi A_\mu}-Gv\gamma_{c\bar{\psi}\gamma^\mu\gamma^5\eta A_\mu}+\sum_{j=1,n-1}(\xi^{(j)}_{\bar{\psi}\psi}\gamma^{(n-j)}_{c\bar{\eta}\gamma^\mu\gamma^5\psi A_\mu}-\xi^{(j)}_{\bar{\psi}\psi}\gamma^{(n-j)}_{c\bar{\psi}\gamma^\mu\gamma^5\eta A_\mu}). \tag{89}$$

V. CONCLUSIONS

The absence of anomalies in the Higgs-Kibble model allows the explicit construction of counterterms which reestablish the Slavnov-Taylor invariance of the model. Therefore any regularization procedure which preserves the Lorentz covariance and the relevant discrete symmetries can be corrected by finite counterterms. In the present work we explicitly gave the counterterms in terms of a set finite vertex functions. Our strategy relied on two essential ingredients. One was the possibility to perform subtraction at zero momentum. The second consists of the use of normalization conditions which simplify the construction of explicit solutions. Quite a few counterterms turn out to be zero and moreover the contribution of the lower terms in the perturbative expansion is highly reduced. Although the solution look cumbersome we believe that it makes possible the automatic evaluation of the counterterms.

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APPENDIX A: CLASSICAL ACTION AND BRS

Feynman rules. The Lagrangian density is

$$\mathcal{L}=-\frac{1}{4}F^2_{\mu\nu}-\frac{\alpha}{2}(\partial A)^2+|D_\mu\phi|^2-\lambda\left(|\phi|^2-\frac{v^2}{2}\right)^2+\bar{\psi}i\mathcal{D}\psi+\frac{G}{\sqrt{2}}\bar{\psi}(1-\gamma_5)\psi\phi+\frac{G}{\sqrt{2}}\bar{\psi}(1+\gamma_5)\psi\phi^*, \tag{A1}$$

where

$$D_\mu=\partial_\mu-ieA_\mu,\qquad \mathcal{D}-\partial_\mu-i\frac{e}{2}\gamma_5A_\mu. \tag{A2}$$

The BRS transformations are

$$\delta A_\mu=\partial_\mu c,\quad \delta\phi=iec\phi,\quad \delta\phi^*=-iec\phi^*,\quad \delta\psi=-i\frac{e}{2}\gamma_5\psi c,\quad \delta\bar{\psi}=i\frac{e}{2}c\bar{\psi}\gamma_5. \tag{A3}$$

Now we consider the spontaneous symmetry breaking

$$\phi=\frac{\phi_1+v+i\phi_2}{\sqrt{2}}. \tag{A4}$$

The bilinear parts give a out-of-diagonal term

$$ev\phi_2\partial A, \tag{A5}$$

thus we need gauge fixing ('t Hooft)

$$-\frac{\alpha}{2}\left(\partial A+\frac{ev}{\alpha}\phi_2\right)^2. \tag{A6}$$

Thus we complete the BRS

$$\delta\phi_1=-ec\phi_2,$$

$$\delta\phi_2=ec(\phi_1+v),$$

$$\delta\bar{c}=\mathcal{F}=\partial A+\frac{ev}{\alpha}\phi_2. \tag{A7}$$

Then the gauge fixing term is

$$\Gamma^{(0)}_{\rm GF}=\int d^4x\left[-\frac{\alpha}{2}\mathcal{F}^2+\alpha\bar{c}\delta\mathcal{F}\right]$$
$$=\int d^4x\left[-\frac{\alpha}{2}\mathcal{F}^2+\alpha\bar{c}\Box c+(ev)^2\bar{c}c+e^2v\bar{c}c\phi_1\right]. \tag{A8}$$

and the zero-loop action is

$$\Gamma^{(0)}=\int d^4x\left[-\frac{1}{4}F^2_{\mu\nu}+\frac{e^2v^2}{2}A^2_\mu-\frac{\alpha}{2}\partial A^2+\alpha\bar{c}\Box c+(ev)^2\bar{c}c+e^2v\bar{c}c\phi_1+\frac{1}{2}(\partial_\mu\phi_1^2+\partial_\mu\phi_2^2)-\lambda v^2\phi_1^2-\frac{(ev)^2}{2\alpha}\phi_2^2\right.$$
$$+eA_\mu(\phi_2\partial^\mu\phi_1-\partial^\mu\phi_2\phi_1)+e^2v\phi_1A^2+\frac{e^2}{2}(\phi_1^2+\phi_2^2)A^2-\lambda v\phi_1(\phi_1^2+\phi_2^2)-\frac{\lambda}{4}(\phi_1^2+\phi_2^2)^2+\bar{\psi}i\partial\!\!\!/\psi+Gv\bar{\psi}\psi$$
$$\left.+\frac{e}{2}\bar{\psi}\gamma_\mu\gamma_5\psi A^\mu+G\bar{\psi}\psi\phi_1-iG\bar{\psi}\gamma_5\psi\phi_2+J_1[-ec\phi_2]+J_2ec(\phi_1+v)+i\frac{e}{2}\bar{\eta}\gamma_5\psi c+i\frac{e}{2}c\bar{\psi}\gamma_5\eta\right]. \tag{A9}$$

The action is C invariant if the fields $\phi_2, A_\mu, i\bar{\psi}\gamma_5\psi, c, \bar{c}, J_2$ are C odd and the fields $\phi_1, J_1, \bar{\psi}\psi$ are C even. This invariance can be extended to $\eta, \bar{\eta}$ by requiring

$$C\psi C^{-1}=B\bar{\psi}^T,\quad C\eta C^{-1}=B\bar{\eta}^T,\quad B^\dagger\gamma_\mu B=-\gamma_\mu^T,$$
$$B^2=1,\ B^*=B,\ B^T=-B,\ B^\dagger=B^{-1}. \tag{A10}$$

Moreover we impose Hermiticity for the low momentum expansion of the vertex amplitude Γ by requiring

$$c^\dagger=c,\ \ \bar{c}^\dagger=-\bar{c},$$
$$\bar{\eta}^\dagger=\gamma_0\eta. \tag{A11}$$

APPENDIX B: ST TRANSFORMATION OF COUNTERTERMS

The ST of the counterterms. The scalar boson sectors:

$$S_0\left[\int d^4x\phi_1\right]=-e\int d^4x(c\phi_2),$$

$$S_0\left[\int d^4x\phi_1^2\right]=-2e\int d^4x(c\phi_1\phi_2),$$

$$S_0\left[\int d^4x\phi_2^2\right]=2e\int d^4x[c\phi_2(\phi_1+v)],$$

$$S_0\left[\int d^4x\phi_1^3\right]=-3e\int d^4x(c\phi_1^2\phi_2),$$

$$S_0\left[\int d^4x\phi_2^2\phi_1\right]=e\int d^4xc[2\phi_2(\phi_1+v)\phi_1-\phi_2^3],$$

$$S_0\left[\int d^4x\phi_1^4\right]=-4e\int d^4x(c\phi_1^3\phi_2),$$

$$S_0\left[\int d^4x\phi_2^4\right]=4e\int d^4x[c\phi_2^3(\phi_1+v)],$$

$$S_0\left[\int d^4x\phi_2^2\phi_1^2\right]=e\int d^4x[2c\phi_2\phi_1^2(\phi_1+v)$$
$$-2c\phi_2^2\phi_2\phi_1]. \tag{B1}$$

The kinetic boson sector:

$$S_0\left[\int d^4xA^2\right]=-2\int d^4x(c\partial_\mu A^\mu),$$

$$S_0\left[\int d^4x\partial_\mu A^\mu\phi_2\right]=\int d^4xc[\Box\phi_2+e\partial_\mu A^\mu(\phi_1$$
$$+v)],$$

$$S_0\left[\int d^4x(\partial_\mu\phi_1)^2\right]=2e\int d^4x(c\Box\phi_1\phi_2),$$

$$S_0\left[\int d^4x(\partial_\mu\phi_2)^2\right]=-2e\int d^4x[c\Box\phi_2(\phi_1+v)],$$

$$S_0\left[\int d^4x A^\mu \partial_\mu \phi_1 \phi_2\right] = \int d^4x c[-\Box\phi_1\phi_2 - \partial_\mu\phi_1\partial^\mu\phi_2 + e\partial_\mu A^\mu \phi_2^2 + eA^\mu\phi_2\partial_\mu\phi_2 + eA^\mu\partial_\mu\phi_1(\phi_1+v)],$$

$$S_0\left[\int d^4x A^\mu \phi_1(\partial_\mu\phi_2)\right] = \int d^4x c[-\phi_1\Box\phi_2 - \partial_\mu\phi_1\partial^\mu\phi_2 - eA^\mu\phi_2\partial_\mu\phi_2 - e\partial_\mu A^\mu\phi_1(\phi_1 + v) - eA^\mu\partial_\mu\phi_1(\phi_1+v)],$$

$$S_0\left[\int d^4x A^2\phi_1\right] = \int d^4x c[-2\partial^\mu(A_\mu\phi_1) - eA^2\phi_2],$$

$$S_0\left[\int d^4x A^2\phi_1^2\right] = 2\int d^4x c[-\partial^\mu(A_\mu\phi_1^2) - eA^2\phi_1\phi_2],$$

$$S_0\left[\int d^4x A^2\phi_2^2\right] = 2\int d^4x c[-\partial^\mu(A_\mu\phi_2^2) + eA^2\phi_2(\phi_1+v)],$$

$$S_0\left[\int d^4x \partial_\mu A^\nu\partial^\mu A_\nu\right] = 2\int d^4x c\Box\partial^\nu A_\nu,$$

$$S_0\left[\int d^4x \partial^\mu A_\mu\partial^\nu A_\nu\right] = 2\int d^4x c\Box\partial^\nu A_\nu,$$

$$S_0\left[\int d^4x A^4\right] = -4\int d^4x c\partial^\mu(A_\mu A^2),$$

$$S_0\left[\int d^4x \bar{c}cA^2\right] = \int d^4x[\mathcal{F}cA^2 - 2\bar{c}c(\partial^\mu c)A_\mu]. \tag{B2}$$

Fermion sectors:

$$S_0\left[\int d^4x \bar{\psi} i\gamma_\mu\partial^\mu\psi\right] = \frac{e}{2}\int d^4x c[\bar{\psi}\gamma_\mu\gamma_5\partial^\mu\psi + (\partial^\mu\bar{\psi})\gamma_\mu\gamma_5\psi],$$

$$S_0\left[\int d^4x \bar{\psi}\gamma_\mu\gamma_5 A^\mu\psi\right] = -\int d^4x c[\bar{\psi}\gamma_\mu\gamma_5\partial^\mu\psi + (\partial^\mu\bar{\psi})\gamma_\mu\gamma_5\psi],$$

$$S_0\left[\int d^4x \bar{\psi}\psi\right] = ie\int d^4x c[\bar{\psi}\gamma_5\psi],$$

$$S_0\left[\int d^4x \bar{\psi}\psi\phi_1\right] = e\int d^4x c[i\bar{\psi}\gamma_5\psi\phi_1 - \bar{\psi}\psi\phi_2],$$

$$S_0\left[\int d^4x \bar{\psi} i\gamma_5\psi\phi_2\right] = e\int d^4x c[-\bar{\psi}\psi\phi_2 + i\bar{\psi}\gamma_5\psi(\phi_1 + v)]. \tag{B3}$$

The ghost sector:

$$S_0\left[\int d^4x \bar{c}c\right] = \int d^4x \mathcal{F}c,$$

$$S_0\left[\int d^4x \bar{c}\Box c\right] = \int d^4x \mathcal{F}\Box c,$$

$$S_0\left[\int d^4x \bar{c}c\phi_1\right] = \int d^4x \mathcal{F}c\phi_1,$$

$$S_0\left[\int d^4x \bar{c}c\phi_1^2\right] = \int d^4x \mathcal{F}c\phi_1^2,$$

$$S_0\left[\int d^4x \bar{c}c\phi_2^2\right] = \int d^4x \mathcal{F}c\phi_2^2. \tag{B4}$$

Fermion sources sector:

$$S_0\left[\int d^4x \frac{i}{2}(\bar{\eta}\gamma_5\psi c + c\bar{\psi}\gamma_5\eta)\right] = c\left[\frac{1}{2}\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) + iGv\bar{\psi}\gamma_5\psi + iG\bar{\psi}\gamma_5\psi\phi_1 + G\bar{\psi}\psi\phi_2\right]. \tag{B5}$$

APPENDIX C: ST INVARIANTS

We have two classes of ST invariants: the BRS invariants where the sources do not intervene

$$\mathcal{I}_1 = \int d^4x(\phi_1^2 + \phi_2^2 + 2v\phi_1),$$

$$\mathcal{I}_2 = \int d^4x(\phi_1^4 + \phi_2^4 + 2\phi_1^2\phi_2^2 + 4v\phi_1^3 + 4v\phi_1\phi_2^2 + 4v^2\phi_1^2),$$

$$\mathcal{I}_3 = \int d^4x|D_\mu\phi|^2,$$

$$\mathcal{I}_4 = \int d^4x(F_{\mu\nu})^2,$$

$$\mathcal{I}_5 = \int d^4x\bar{\psi} i\gamma_\mu\mathcal{D}^\mu\psi,$$

$$\mathcal{I}_6 = \int d^4x\bar{\psi}[(\phi_1+v) - i\gamma_5\phi_2]\psi,$$

$$\mathcal{I}_7 \equiv \mathcal{I}_7 = \int d^4x\left(\frac{1}{2}\mathcal{F}^2 + \bar{c}\delta_{\mathrm{BRS}}\mathcal{F}\right),$$

$$\mathcal{I}_8 \equiv \mathcal{I}_8 = \int d^4x\left(\frac{1}{2}A^2 + \bar{c}c + \frac{v}{\alpha}\phi_1\right) \tag{C1}$$

and ST invariants with external sources

$$
\begin{aligned}
\mathcal{I}_9 &= \int d^4x [A^\mu \Gamma^{(0)}_{A^\mu} + c\Gamma^{(0)}_c + \alpha(\mathcal{F}\partial^\mu A_\mu - \bar{c}\Box c)],\\
\mathcal{I}_{10} &= S_0\left(\int d^4x J_1\right) = \int d^4x \Gamma^{(0)}_{\phi_1},\\
\mathcal{I}_{11} &= S_0\left(\int d^4x J_1\phi_1\right) = \int d^4x (\phi_1 \Gamma^{(0)}_{\phi_1} + eJ_1 c\phi_2).
\end{aligned}
\tag{C2}
$$

There are other invariants which are linearly dependent from the previous ones:

$$
\begin{aligned}
\mathcal{I}_{12} &= \int d^4x \{\phi_2 \Gamma^{(0)}_{\phi_2} - eJ_2 c(\phi_1 + v)\\
&\quad + ev[\mathcal{F}\phi_2 - e\bar{c}c(\phi_1 + v)]\}\\
&= -\lambda v^2 \mathcal{I}_1 - \lambda \mathcal{I}_2 + 2\mathcal{I}_3 + G\mathcal{I}_6 - ev\mathcal{I}_{10} - \mathcal{I}_{11},\\
\mathcal{I}_{13} &= \int d^4x \left(\Gamma^{(0)}_{\psi}\psi - i\frac{e}{2}\bar{\eta}c\gamma_5\psi\right) = -\mathcal{I}_5 - G\mathcal{I}_6,\\
\mathcal{I}_{14} &= \int d^4x \left(\bar{\psi}\Gamma^{(0)}_{\bar{\psi}} + i\frac{e}{2}\bar{\psi}c\gamma_5\eta\right) = -\mathcal{I}_{13}.
\end{aligned}
\tag{C3}
$$

The coefficients of the invariant counterterms can be fixed by choosing the normalization conditions on some monomials. The following matrix provides an example of the linear dependence of the ST invariants from a set of monomials (for comparison an extra row is added involving the Fermi external source):

$$
\left(
\begin{array}{llllllllll}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_9 & v_{10} & v_{11}\\
\phi_1 & 2v & 0 & 0 & 0 & 0 & 0 & 0 & -2\lambda v^2 & 0\\
\phi_2^2\phi_1 & 0 & 4v & 0 & 0 & 0 & 0 & 0 & -\lambda & -\lambda v\\
A^2\phi_1 & 0 & 0 & e^2 v & 0 & 0 & 0 & 2e^2 v & e^2 & e^2 v\\
F^2_{\mu\nu} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
\bar{\psi}\gamma_\mu\gamma_5\psi A^\mu & 0 & 0 & 0 & 0 & \frac{e}{2} & 0 & \frac{e}{2} & 0 & 0\\
i\bar{\psi}\gamma_5\psi\phi_2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\
J_2 c & 0 & 0 & 0 & 0 & 0 & 0 & ev & e & 0\\
J_2 c\phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & e & 0 & e\\
J_1 c\phi_2 & 0 & 0 & 0 & 0 & 0 & 0 & -e & 0 & e\\
i\bar{\eta}\gamma_5\psi c & 0 & 0 & 0 & 0 & 0 & 0 & \frac{e}{2} & 0 & 0
\end{array}
\right).
\tag{C4}
$$

APPENDIX D: EXPANSION OF Ψ IN TERMS OF LOCAL POLYNOMIALS OF FIELDS

The expansion of $\hat{\Psi}$ in terms of local polynomials of fields and their derivatives can be achieved by the functional derivatives. The relevant terms are selected by the charge and ghost number conservation and by the naive dimension ($\leqslant 5$) of $\hat{\Psi}$. Each functional derivative $\hat{\Psi}_c\cdots$ is, in general, a linear combination of Lorentz invariant amplitudes [denoted by $\psi_c\cdots$ in Eq. (15)].

(1) At first we consider the derivative of $\hat{\Psi}$ with respect to the ghost field c and to the Goldstone ϕ_2. Its naive dimension is three and, therefore, according to Eq. (9), we use the Taylor operator $(1-t^3)$. Moreover, by applying the normalization conditions (63), we get

$$
\begin{aligned}
\hat{\Psi}_{c\phi_2} &= -ev\,\Xi^{(n)}_{\phi_2\phi_2}\\
&= \left(\frac{(ev)^2}{\alpha} - m_2^2\right)(t^3 - t^1)\mathbf{\Gamma}^{(n)}_{cJ_2} + t^3 \sum_{j=1}^{n-j} \mathbf{\Gamma}^{(j)}_{cJ_2}\mathbf{\Gamma}^{(n-j)}_{\phi_2^2}\\
&= t^3 \sum_{j=1}^{n-j} \mathbf{\Gamma}^{(j)}_{cJ_2}\mathbf{\Gamma}^{(n-j)}_{\phi_2^2}.
\end{aligned}
\tag{D1}
$$

In the Taylor expansion denoted by t^3, the odd-number derivative of the vertex functions at zero momentum are zero, by Lorentz covariance. Moreover the constant term is zero due to the normalization condition $\Gamma^{(j)}_{cJ_2}(0)=0$ thus only $\xi^{(n)}_{\partial_\mu\phi_2\partial^\mu\phi_2}$ can be nonzero. However, we get

$$-ev\Xi^{(n)}_{\phi_2\phi_2}=\sum_{j=1}^{n-j}\left[(t^2-t^1)\mathbf{\Gamma}^{(j)}_{cJ_2}\mathbf{\Gamma}^{(n-j)}_{\phi_2^2}(0)+\mathbf{\Gamma}^{(j)}_{cJ_2}(0)(t^2-t^1)\mathbf{\Gamma}^{(n-j)}_{\phi_2^2}\right]=0. \tag{D2}$$

Thus finally $\Xi_{\phi_2\phi_2}$ is zero.

(2) The second coefficient is computed by taking the derivative of Ψ in Eq. (69) with respect to the ghost c , to the Goldstone ϕ_2, and to the Higgs ϕ_1. Moreover by using the normalization conditions (63) and $\Xi_{\phi_2\phi_2}=0$, one gets

$$\begin{aligned}\hat{\Psi}^{(n)}_{c\phi_2(p)\phi_1(q)}&=-[i(p+q)^\mu\Xi^{(n)}_{A^\mu\phi_2(p)\phi_1(q)}-e\Xi^{(n)}_{\phi_1\phi_1(q)}]\\&=-m_1^2(t^2-t^1)\Gamma^{(n)}_{cJ_1(q)\phi_2(p)}+ev(t^2-t^1)\\&\quad\times\Gamma^{(n)}_{\phi_2\phi_2(p)\phi_1(q)}-2\lambda v(t^2-t^1)\Gamma^{(n)}_{cJ_2(p+q)}.\end{aligned} \tag{D3}$$

The lower order contributions are zero by the normalization conditions. Since $\Psi^{(n)}_{c\phi_2\phi_1}$ contains only terms quadratic in the momenta, then there is no counterterm as $\int d^4x\,\phi_1^2$. It should be reminded that we have already chosen $\Xi_{\phi_2^2\phi_1}=0$.

$$(3)\quad \hat{\Psi}^{(n)}_{c\phi_2\phi_1^2}=-[-e\Xi^{(n)}_{\phi_1^3}+ev\Xi^{(n)}_{\phi_2^2\phi_1^2}]=-2m_1^2\Gamma^{(n)}_{cJ_1\phi_2\phi_1}(0). \tag{D4}$$

$$\begin{aligned}(4)\quad \hat{\Psi}^{(n)}_{c\phi_2\phi_1^3}&=-[-e\Xi^{(n)}_{\phi_1^4}+3e\Xi^{(n)}_{\phi_2^2\phi_1^2}]\\&=-18\lambda v\Gamma^{(n)}_{cJ_1\phi_2\phi_1}(0)-3m_1^2\Gamma^{(n)}_{cJ_1\phi_2\phi_1^2}(0)+ev\Gamma^{(n)}_{\phi_2^2\phi_1^3}(0)-6\lambda v\Gamma^{(n)}_{cJ_2\phi_1^2}(0)+3\sum_{j=1}^{n-1}(\mathbf{\Gamma}^{(n-j)}_{\phi^3}(0)\mathbf{\Gamma}^{(j)}_{cJ_1\phi_1\phi_2}).\end{aligned} \tag{D5}$$

$$(5)\quad \hat{\Psi}^{(n)}_{c\phi_2^3}=0. \tag{D6}$$

$$(6)\quad \hat{\Psi}^{(n)}_{c\phi_2^3\phi_1}=-[-3e\Xi^{(n)}_{\phi_1^2\phi_2^2}+e\Xi^{(n)}_{\phi_2^4}]=-6\lambda v\Gamma^{(n)}_{cJ_1\phi_2\phi_1}(0)+ev\Gamma^{(n)}_{\phi_2^4\phi_1}(0)-m_1^2\Gamma^{(n)}_{cJ_1\phi_2^3}(0)-6\lambda v\Gamma^{(n)}_{cJ_2\phi_2^2}(0). \tag{D7}$$

$$(7)\quad \hat{\Psi}^{(n)}_{cA_\nu}=-ip^\mu\Xi^{(n)}_{A^\mu A_\nu}=ev(t_p^3-t_p^2)\Gamma^{(n)}_{\phi_2A_\nu}-ievp^\nu(t_p^2-t_p^1)\Gamma^{(n)}_{cJ_2}. \tag{D8}$$

Then there is no contribution to A^2 and to the transverse part of A_μ, i.e., $\int d^4xF_{\mu\nu}^2$; only to $(\partial_\mu A^\mu)^2$.

$$(8)\quad \hat{\Psi}^{(n)}_{cA_\nu(p)\phi_1(q)}=-[i(p+q)^\mu\Xi^{(n)}_{A^\mu A_\nu(p)\phi_1(q)}+ev\Xi^{(n)}_{\phi_2A_\nu(p)\phi_1(q)}]=-m_1^2(t^2-t^0)\Gamma^{(n)}_{cJ_1(q)A_\nu(p)}+ev(t^2-t^1)\Gamma^{(n)}_{\phi_2A_\nu(p)\phi_1(q)}. \tag{D9}$$

The last term is zero because of covariance.

$$\begin{aligned}(9)\quad \hat{\Psi}^{(n)}_{cA_\nu(p)\phi_1(q_1)\phi_1(q_2)}&=-[i(p+q_1+q_2)^\mu\Xi^{(n)}_{A^\mu A_\nu(p)\phi_1(q_1)\phi_1(q_2)}+e\Xi^{(n)}_{\phi_2A_\nu(p)\phi_1(q_1)}+e\Xi^{(n)}_{\phi_2A_\nu(p)\phi_1(q_2)}]\\&=-m_1^2(t^1\Gamma^{(n)}_{cJ_1A_\nu(p)\phi_1(q_1)}+t^1\Gamma^{(n)}_{J_1cA_\nu(p)\phi_1(q_2)})-6\lambda v(t^1-t^0)\Gamma^{(n)}_{cJ_1(q_1+q_2)A_\nu(p)}\\&\quad+ev(t^1-t^0)\Gamma^{(n)}_{\phi_2A_\nu(p)\phi_1(q_1)\phi_1(q_2)}-ievp^\nu\Gamma^{(n)}_{cJ_2(p)\phi_1(q_1)\phi_1(q_2)}+\sum_{j=1}^{n-1}[\mathbf{\Gamma}^{(n-j)}_{\phi_1^3}(0)t^1\mathbf{\Gamma}^{(j)}_{cJ_1A_\nu}].\end{aligned} \tag{D10}$$

Notice that the breaking term $\Gamma^{(0)}_{\phi_2A_\nu\phi_1}(t^1-t^0)\Gamma^{(n)}_{cJ_2\phi_1}$ is zero and therefore it has been omitted.

$$\begin{aligned}(10)\quad \hat{\Psi}^{(n)}_{cA_\nu(p)\phi_2(q_1)\phi_2(q_2)}&=-[i(p+q_1+q_2)^\mu\Xi^{(n)}_{A^\mu A_\nu(p)\phi_2(q_1)\phi_2(q_2)}-e\Xi^{(n)}_{\phi_1A_\nu(p)\phi_2(q_1)}-e\Xi^{(n)}_{\phi_1A_\nu(p)\phi_2(q_2)}]\\&=-2\lambda v(t^1-t^0)\Gamma^{(n)}_{cJ_1(q_1+q_2)A_\nu(p)}+ev(t^1-t^0)\Gamma^{(n)}_{\phi_2A_\nu(p)\phi_2(q_1)\phi_2(q_2)}+ip^\nu v\Gamma^{(n)}_{cJ_2\phi_2^2}(0).\end{aligned} \tag{D11}$$

(11)
$$\hat{\Psi}^{(n)}_{cA_\mu(p_1)A_\nu(p_2)\phi_2(q)}=-[-e\Xi^{(n)}_{\phi_1A_\mu(p_1)A_\nu(p_2)}+ev\Xi^{(n)}_{\phi_2A_\mu A_\nu\phi_2(q)}]$$
$$=ev(t^1-t^0)\Gamma^{(n)}_{\phi_2A_\mu(p_1)A_\nu(p_2)\phi_2(q)}+2ev^2(t^1-t^0)g^{\mu\nu}\Gamma^{(n)}_{cJ_1(p_1+p_2)\phi_2(q)}iev[p_1^\mu\Gamma^{(n)}_{cJ_1(p_1)A_\nu(p_2)\phi_2(q)}(0)$$
$$+p_2^\nu\Gamma^{(n)}_{cJ_1(p_2)A_\mu(p_1)\phi_2(q)}(0)]+\sum_{j=1}^{n-1}[\boldsymbol{\Gamma}^{(n-j)}_{\phi_1A_\mu A_\nu}(0)t^1\boldsymbol{\Gamma}^{(j)}_{cJ_1(p_1+p_2)\phi_2(q)}]. \quad \text{(D12)}$$

From the Lorentz structure we see that all terms are zero and therefore

$$\hat{\Psi}^{(n)}_{cA_\mu(p_1)A_\nu(p_2)\phi_2(q)}=e\Xi^{(n)}_{\phi_1A_\mu(p_1)A_\nu(p_2)}-ev\Xi^{(n)}_{\phi_2A_\mu A_\nu\phi_2(q)}=0. \quad \text{(D13)}$$

(12)
$$\hat{\Psi}^{(n)}_{cA_\mu A_\nu\phi_2\phi_1}=-[-e\Xi^{(n)}_{\phi_1A_\mu A_\nu\phi_1}+e\Xi^{(n)}_{\phi_2A_\mu A_\nu\phi_2}]$$
$$=ev\Gamma^{(n)}_{\phi_2A_\mu A_\nu\phi_2\phi_1}(0)-m_1^2\Gamma^{(n)}_{cJ_1A_\mu A_\nu\phi_2}(0)+2ev^2g^{\mu\nu}\Gamma^{(n)}_{cJ_1\phi_2\phi_1}(0)$$
$$-2\lambda v\Gamma^{(n)}_{cJ_2A_\mu A_\nu}(0)+\sum_{j=1}^{n-1}(\boldsymbol{\Gamma}^{(j)}_{cJ_1\phi_2\phi_1}\boldsymbol{\Gamma}^{(n-j)}_{\phi_1A_\mu A_\nu})(0). \quad \text{(D14)}$$

(13)
$$\hat{\Psi}^{(n)}_{cA_\nu A_\rho A_\sigma}=-[i(p_1+p_2+p_3)_\mu\Xi^{(n)}_{A_\mu A_\nu A_\rho A_\sigma}]$$
$$=evt^1\Gamma^{(n)}_{\phi_2A_\nu A_\rho A_\sigma}+2ev^2g^{\rho\sigma}(t^1-t^0)\Gamma^{(n)}_{cJ_1(p_2+p_3)A_\nu(p_1)}+2ev^2g^{\nu\sigma}(t^1-t^0)\Gamma^{(n)}_{cJ_1(p_1+p_3)A_\rho(p_2)}$$
$$+2ev^2g^{\nu\rho}(t^1-t^0)\Gamma^{(n)}_{cJ_1(p_1+p_2)A_\sigma(p_3)}+ievp_1^\nu\Gamma^{(n)}_{cJ_2(p_1)A_\rho A_\sigma}(0)+ievp_2^\rho\Gamma^{(n)}_{cJ_2(p_2)A_\mu A_\sigma}(0)+ievp_3^\sigma\Gamma^{(n)}_{cJ_2(p_3)A_\mu A_\rho}(0)$$
$$+\sum_{j=1}^{n-1}(t^1\boldsymbol{\Gamma}^{(j)}_{cJ_1A_\rho})\boldsymbol{\Gamma}^{(n-j)}_{\phi_1A_\sigma A_\nu}(0)+\sum_{j=1}^{n-1}(t^1\boldsymbol{\Gamma}^{(j)}_{cJ_1A_\nu})\boldsymbol{\Gamma}^{(n-j)}_{\phi_1A_\sigma A_\rho}(0)+\sum_{j=1}^{n-1}(t^1\boldsymbol{\Gamma}^{(j)}_{cJ_1A_\sigma})\boldsymbol{\Gamma}^{(n-j)}_{\phi_1A_\rho A_\nu}(0). \quad \text{(D15)}$$

(14)
$$\hat{\Psi}^{(n)}_{c\bar{\psi}(p_1)\psi(p_2)}=-\left[i\frac{e}{2}\gamma_5\Xi^{(n)}_{\psi(p_2)\bar{\psi}}+i\frac{e}{2}\Xi^{(n)}_{\psi\bar{\psi}(p_1)}\gamma_5-Gv\Xi^{(n)}_{\bar{\eta}(p_1)c\psi(p_2)}-Gv\Xi^{(n)}_{\eta(p_2)\bar{\psi}(p_1)c}\right]$$
$$=-ev(t^1-t^0)\Gamma^{(n)}_{\phi_2\psi(p_2)\bar{\psi}(p_1)}-Gv(t^1-t^0)\Gamma^{(n)}_{\bar{\eta}(p_1)c\psi(p_2)}-Gv(t^1-t^0)\Gamma^{(n)}_{\eta(p_2)\bar{\psi}(p_1)c}$$
$$-t^1\sum_{j=1}^{n-1}(\boldsymbol{\Gamma}^{(j)}_{\eta(p_2)\bar{\psi}(p_1)c}\boldsymbol{\Gamma}^{(n-j)}_{\psi(p_2)\bar{\psi}}+\boldsymbol{\Gamma}^{(j)}_{\psi\bar{\psi}(p_1)}\boldsymbol{\Gamma}^{(n-j)}_{\bar{\eta}(p_1)c\psi(p_2)}). \quad \text{(D16)}$$

(15)
$$\hat{\Psi}^{(n)}_{c\bar{\psi}(p_1)\psi(p_2)\phi_1(q)}=-\left[i\frac{e}{2}\gamma_5\Xi^{(n)}_{\psi\bar{\psi}(p_2)\phi_1(q)}+i\frac{e}{2}\Xi^{(n)}_{\psi\bar{\psi}(p_1)\phi_1(q)}\gamma_5-e\Xi^{(n)}_{\psi(p_2)\bar{\psi}(p_1)\phi_2(q)}-G\Xi^{(n)}_{\bar{\eta}(p_1)c\psi(p_2)}-G\Xi^{(n)}_{\eta(p_2)\bar{\psi}(p_1)c}\right]$$
$$=-ev\Gamma^{(n)}_{\phi_2\bar{\psi}(p_1)\psi(p_2)\phi_1(q)}-m_1^2\Gamma^{(n)}_{cJ_1\bar{\psi}(p_1)\psi(p_2)}-Gv\Gamma^{(n)}_{\bar{\eta}(p_1)c\psi(p_2)\phi_1(q)}-Gv\Gamma^{(n)}_{\eta(p_2)\bar{\psi}(p_1)c\phi_1(q)}$$
$$-t^0\sum_{j=1}^{n-1}(\boldsymbol{\Gamma}^{(j)}_{\eta(p_2)\bar{\psi}(p_1)c}\boldsymbol{\Gamma}^{(n-j)}_{\psi(p_2)\bar{\psi}\phi_1(q)}+\boldsymbol{\Gamma}^{(j)}_{\psi\bar{\psi}(p_1)\phi_1(q)}\boldsymbol{\Gamma}^{(n-j)}_{\bar{\eta}(p_1)c\psi(p_2)})-t^0\sum_{j=1}^{n-1}(\boldsymbol{\Gamma}^{(j)}_{\eta(p_2)c\bar{\psi}(p_1)\phi_1(q)}\boldsymbol{\Gamma}^{(n-j)}_{\psi(p_2)\bar{\psi}}$$
$$+\boldsymbol{\Gamma}^{(j)}_{\psi\bar{\psi}(p_1)}\boldsymbol{\Gamma}^{(n-j)}_{\bar{\eta}(p_1)c\psi(p_2)\phi_1(q)}). \quad \text{(D17)}$$

(16)
$$\hat{\Psi}^{(n)}_{c\bar{\psi}(p_1)\psi(p_2)\phi_2(q)}=-[+e\Xi^{(n)}_{\psi(p_2)\bar{\psi}\phi_1(q)}-iG\Xi^{(n)}_{\bar{\eta}(p_1)c(-p_1-p_2-q)\psi(p_2)}-iG\Xi^{(n)}_{\eta(p_2)\bar{\psi}(p_1)c(-p_1-p_2-q)}]$$
$$=-ev\Gamma^{(n)}_{\phi_2\psi(p_2)\bar{\psi}(p_1)\phi_2(q)}-Gv\Gamma^{(n)}_{\bar{\eta}(p_1)c\psi(p_2)\phi_2(q)}-Gv\Gamma^{(n)}_{\eta(p_2)\bar{\psi}(p_1)c\phi_2(q)}$$
$$-t^0\sum_{j=1}^{n-1}(\boldsymbol{\Gamma}^{(j)}_{\eta(p_2)\bar{\psi}(p_1)c\phi_2(q)}\boldsymbol{\Gamma}^{(n-j)}_{\psi(p_2)\bar{\psi}}+\boldsymbol{\Gamma}^{(j)}_{\psi\bar{\psi}(p_1)}\boldsymbol{\Gamma}^{(n-j)}_{\bar{\eta}(p_1)c\psi(p_2)\phi_2(q)}). \quad \text{(D18)}$$

$\hat{\Psi}_{c\bar{\psi}\gamma_\mu\psi A^\mu}=0$. In fact the only possible counterterm is $\overline{\psi}\gamma_\mu\gamma_5\psi A^\mu$ and this is excluded by the normalization conditions. The above analysis shows that at every order the following counterterms are absent to all orders:

$$\int d^4x\phi_1^2,\ \int d^4x\phi_2^2,\ \int d^4x(\partial_\mu\phi_2)^2,\ \int d^4xA_\mu^2. \tag{D19}$$

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