Gravitational mass in asymptotically de Sitter space-times with compactified dimensions

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We define the gravitational mass in asymptotically de Sitter space-times with compactified dimensions. It is shown that the mass can be negative for a space-time with matter spreading beyond the cosmological horizon scale or having a large outward ''momentum'' in four dimensions. We give simple examples with negative energy in higher dimensions even if the matter is not beyond the horizon or the system does not have a large ''momentum.'' They do not have the lower bound on the mass. We also give a positive energy argument in higher dimensions and realize that the elementary fermion cannot exist in our examples.
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I. INTRODUCTION

Superstring or M theory may offer the proper theory of gravity [1]. Such kinds of theories are formulated in higher dimensions and it is believed that the extra dimension space will be compactified to be less than the Planck length.

The stability of such space-times is important and has been discussed. It was shown that there is an instanton which may indicate the decay of Kaluza-Klein vacuum [2–4]. As Witten pointed out, the decay mode is excluded by the existence of a (massless) elementary fermion related to supersymmetry [5].

On the other hand, the stability of the asymptotically anti–de Sitter (AdS) space-time with compactified dimensions has been focused on recently by Horowitz and Myers [6] because the AdS/CFT correspondence [7] links the stability of super Yang-Mills theory to that of AdS space-time. So they suggested a positive energy conjecture in locally asymptotically AdS space-times.

In the actual cosmological context, the same argument of stability is important. In this paper, for simplicity, we consider D-dimensional Einstein gravity with a positive cosmological constant. A positive cosmological constant is essential for the inflationary universe [8] and the acceleration of the universe confirmed gradually by recent observations of supernovae [9]. In the study of Nakao *et al.*, it was shown that the gravitational mass can be negative in four-dimensional space-times when the matter distributes beyond the cosmological horizon scale [10,11]. In other words, the mass is negative if the space-time has a large outward ''momentum.'' Evaluating the electric part of Weyl tensor, they also checked that the mass is related to gravitational tidal force. In this paper we find an initial data set whose mass can be negative and does not have the lower bound even if the space-time does not have momentum. Furthermore, we give another dynamical solution which also does not have momentum.

In asymptotically flat space-times with compactified extra dimensions, there are examples of momentarily static initial slices such that the energy can be negative regardless of the size of the compactified dimensions [4]. A parallel argument helps us discuss the lower bound of the energy in asymptotically de Sitter space-times. Since a positive cosmological constant prompts the rapid expansion of the universe, we might be able to obtain an implication about the effect to the energy from the dynamics of the compactified dimensions.

The rest of the present paper is organized as follows. In the next section we give the definition of gravitational mass in asymptotically de Sitter space-time with extra dimensions and write down the expression in terms of canonical quantities on the hypersurfaces. Since a part of this is a straightforward extension of the work by Nakao *et al.* [10] based on Refs. [12,13], we give a brief discussion. We will point out that the expression is a special case of the gravitational Hamiltonian defined by Hawking and Horowitz [14]. In Sec. III we show two examples which have negative energy and no naked singularity. In Sec. IV, we consider the positive energy theorem to discuss the stability of de Sitter space-time. As a result we confirm that elementary fermion cannot exist in the examples given by us. Finally we give a summary in Sec. IV.

II. GRAVITATIONAL MASS IN ASYMPTOTICALLY DE SITTER SPACE-TIMES WITH HIGHER DIMENSIONS

We consider D-dimensional space-times which satisfy the Einstein equation with a positive cosmological constant Λ:

$$R_{IJ}-\frac{1}{2}g_{IJ}R+\Lambda g_{IJ}=8\pi G_D T_{IJ}, \qquad (2.1)$$

where suffices I,J runs over $0,1,\ldots,D-1$, T_{IJ} is the energy-momentum tensor, and G_D is D-dimensional Newton's constant. We decompose the space-time metric into the D-dimensional de Sitter metric $\bar{g}_{IJ}$ and the rest h_{IJ};

$$g_{IJ}=\bar{g}_{IJ}+h_{IJ}. \qquad (2.2)$$

Hereafter the overbar indicates quantities of the background de Sitter space-time. We remember that h_{IJ} is not necessarily small, but we impose that it vanishes at infinity. We will give a further description below.

Here we note that Einstein equation is written as

$$R_l^{IJ} - \frac{1}{2}\overline{g}^{IJ}R_l - \Lambda h^{IJ} = (-\overline{g})^{-1/2}\mathcal{T}^{IJ}, \tag{2.3}$$

where $h^{IJ} = \overline{g}^{IK}\overline{g}^{JL}h_{KL}$, R_l^{IJ} is the linear part of Ricci tensor R_{IJ} with respect to h_{IJ}, and all the higher order terms are included in $\mathcal{T}^{IJ}$ of the right-hand side. As the left-hand side of Eq. (2.3) satisfies the Bianchi identity, we see that

$$\overline{\nabla}_I \mathcal{T}^{IJ} = 0 \tag{2.4}$$

holds, where $\overline{\nabla}_I$ is the covariant derivative with respect to $\overline{g}_{IJ}$. Contracting $\mathcal{T}^{IJ}$ with the Killing vector of de Sitter space-time

$$\mathcal{T}^I = \mathcal{T}^{IJ}\overline{\xi}_J, \tag{2.5}$$

we obtain the local conservation low

$$\partial_I \mathcal{T}^I = 0. \tag{2.6}$$

Thus, $\int d^{D-1}x\mathcal{T}^0$ is conserved if the surface term $\int d^{D-2}S_i\mathcal{T}^i$ vanishes. We define

$$E_{\rm AD} = \frac{1}{8\pi G_D}\int d^{D-1}x\mathcal{T}^{0K}\overline{\xi}_K$$
$$= \frac{1}{8\pi G_D}\int d^{D-2}S_i[\overline{\nabla}_I K^{0iJM} - K^{0jJi}\overline{\nabla}_j]\overline{\xi}_J, \tag{2.7}$$

where $K^{IJKL} = (1/2)(\overline{g}^{IL}H^{KJ} + \overline{g}^{KJ}H^{IL} - \overline{g}^{IK}H^{JL} - \overline{g}^{JL}H^{IK})$, $H^{IJ} = h^{IJ} - (1/2)\overline{g}^{IJ}h_K^K$, and $i = 1,2,\ldots,D-1$. In the case that $\overline{\xi}^I$ is a timelike Killing vector $E_{\rm AD}$ is regarded as the Killing energy, the so called Abbott-Deser (AD) mass.

Next, we rewrite Eq. (2.7) in a more familiar form in order to obtain its physical meaning. First of all, we need to take the asymptotic region carefully. In asymptotic flat cases, the extrinsic curvature K_{ij} of slices has the behavior $K_j^i \to 0$ toward the spatial infinity i^0. As we know, we can expect that such a slice does not have an asymptotic region in asymptotically de Sitter space-time because static slices with $K_{ij} = 0$ have no boundary in de Sitter space-time. So we remember that the flat chart of de Sitter space-time has a boundary and its spatial metric is conformally flat (see Fig. 2 in Ref. [10]). Thus, we realize that the most natural condition is given by $K_j^i \to H\delta_j^i$ or $K \to (D-1)H$ [15], where $H = \sqrt{2\Lambda/(D-1)(D-2)}$. In this chart the metric of the background de Sitter space-time is written as

$$\overline{g} = -dt^2 + a(t)^2\sum_{i=1}^{D-1} dx_i^2, \tag{2.8}$$

where $a(t) = e^{Ht}$. The Killing vector has the component $\overline{\xi}^M = (-1, Hx^i)$. After simple calculations we obtain

$$E_{AD} = a(t)[E_{\rm ADM} + \Delta P_{\rm ADM}(-\overline{\xi})]$$
$$= \frac{a(t)}{16\pi G_D}\int d\overline{S}_i(\partial_j h^{ij} - \overline{g}^{ij}\partial_j h_k^k)$$
$$- \frac{a(t)}{8\pi G_D}\int d\overline{S}_i[K_j^i - K\delta_j^i + (D-2)H\delta_j^i]\overline{\xi^j}, \tag{2.9}$$

where K_{ij} is the extrinsic curvature of t=constant $(D-1)$-dimensional hypersurface. By using the momentum $\pi_{ij} = K_{ij} - q_{ij}K$, where q_{ij} is the metric of t=const hypersurface, the second term in the second line of Eq. (2.9) can be written as

$$\Delta P_{\rm ADM}(-\overline{\xi}) = P_{\rm ADM}(-\overline{\xi}) - \overline{P}_{\rm ADM}(-\overline{\xi})$$
$$= -\frac{1}{8\pi G_D}\int d\overline{S}_i(\pi_j^i - \overline{\pi}_j^i)\overline{\xi^j}. \tag{2.10}$$

As a result, $E_{\rm AD}$ is written in terms of the sum of the ADM energy and net momentum. We note that the momentum of background de Sitter space-time is subtracted automatically. The above net momentum is the term ''momentum'' used in the Introduction. The above argument does not depends on whether the extra dimensions are compactified or not.

Now we compare the above expression with the gravitational energy defined via a ''physical Hamiltonian'' [14]. The gravitational energy has the expression

$$E_{\rm HH} = -\frac{1}{8\pi G_D}\int dS(Nk - \overline{Nk})$$
$$+ \frac{1}{8\pi G_D}\int dS(N^i\pi_{ij} - \overline{N^i}\overline{\pi}_{ij})r^j, \tag{2.11}$$

where k and r^i are the trace of the extrinsic curvature and the unit normal vector of $(D-2)$-dimensional surface at the infinity, respectively. N and N^i are the lapse function and the shift vector.

The ''physical Hamiltonian'' is defined by the substraction the Hamiltonian of a background space-time from the original Hamiltonian. As Hawking and Horowitz showed, the first term of Eq. (2.11) is just the Arnowitt-Desir-Misner (ADM) energy. Comparing Eq. (2.9) with Eq. (2.11), one can see that they have the same expression except for scale factor, i.e., $E_{\rm AD} = a(t)E_{\rm HH}$, if one chooses the lapse function and the shift vector as follows:

$$N = \overline{N} = 1 \quad \text{and} \quad N^i = \overline{N^i} = -\overline{\xi^i}. \tag{2.12}$$

It is likely that the priority is given to our definition in asymptotically de Sitter space-times because the Hawking and Horowitz construction of the physical Hamiltonian has the operation of artificial substraction to keep it finite.

In asymptotically flat cases, the ADM energy is defined by view point of a static observer with $N = 1$ and $N^i = 0$ at spatial infinity i^0. The naturalness of observer selection to define the energy is related to the timelike translation symmetry at infinity. On the other hand, in asymptotically de Sitter space-time, we must consider hypersurfaces which have infinity in order to define the nonzero energy. As we have done, the most convenient slices is one which corresponds to the flat slices in de Sitter space-time. This slices reach into timelike infinity $\mathcal{I}^+$ (see Fig. 2 in Ref. [10]). However, the slices is not associated with the timelike translation symmetry which de Sitter space-time possesses, that is, the slice is not orthogonal to timelike Killing vector. Thus, it may be natural that the ADM momentum term enters into the expression of the mass.

In four dimensions the AD mass can be negative when system has a large outward momentum [10]. On the other hand, the positivity of the AD or ADM mass is guaranteed for systems without the net momentum, $\Delta P_{\rm ADM}=0$, in four dimensions [16].

III. EXAMPLES IN FIVE DIMENSIONS

In the same way as four-dimensional cases, the AD mass can be negative regardless of the compactification of the extra dimension if the system has a large momentum. However, the physical reason of the negativity is the dynamics of the four-dimensional part of space-time, rather than the (quantum) stability. Now we are only interested in stability. So, we consider the situation only where the contribution of the net ADM momentum does not exist. We give two examples in five dimensions which are regular everywhere and have negative energy.

Let us consider an initial slice with $K_{ij}=\pm Hg_{ij}$ and $H=\sqrt{\Lambda/6}$. In this slice the Hamiltonian constraint becomes ${}^{(4)}R=0$. Thus one can use the argument on the momentarily static slices in asymptotically flat cases because the Hamiltonian constraint is just same one.

One can see easily that the Euclidian Reissner-Nordström metric with imaginary "charge" satisfies the Hamiltonian constraint [4]. This metric of the hypersurface is given by

$$ {}^{(4)}g=V(r)d\chi^2+\frac{dr^2}{V(r)}+r^2d\Omega_2^2, \tag{3.1}$$

where $V(r)=1-2m/r-e^2/r^2$ and $r\geqslant r_+:=m+\sqrt{m^2+e^2}$. To avoid a conical singularity at $r=r_+$, we assume the period $\chi_p=4\pi/V'(r_+)=2\pi r_+^2/(r_+-m)$ along the χ direction.

On the present slice, the mass is constructed by only the ADM energy component[1] $E_{\rm AD}=E_{\rm ADM}=m\chi_p/2G_5=m/2$. By the same argument as Brill and Horowitz [4], it is shown that the mass becomes negative and does not have the lower bound. The mass can be set to be arbitrary negative regardless of the radius of the compactified space. However, it is not obvious whether the AD energy is conserved or not in the course of time development because of the existence of the cosmological constant and momentarily nonstatic slices. In fact we will see that the energy is not conserved in the next example.

[1]Here we used the fact that the five-dimensional Newton's constant is written as $G_5=\chi_pG_4$ in terms of the four-dimensional one.

As a second example, we take a dynamical solution [17]. The metric is

$$ds^2=-dt^2+a(t)^2\left[\frac{d\chi^2}{\Delta}+\Delta dr^2+r^2\Delta^2d\Omega_2^2\right], \tag{3.2}$$

where $\Delta(r)=1-m/r$, $a(t)=e^{\pm Ht}$, and $H=\sqrt{\Lambda/6}$. The $t=$const hypersurface has the extrinsic curvature $K^i_j=\pm H\delta^i_j$. However, this space-time has timelike naked singularity at $r=m$. Since the expansion of outgoing null geodesics congruence is

$$\theta_+=\frac{1}{\sqrt{2}}\left(\pm 2H+\frac{4r-m}{2a\Delta^{3/2}r^2}\right), \tag{3.3}$$

the apparent horizon does not exist in the expanding chart with $a=e^{Ht}$. In collapsing chart with $a=e^{-Ht}$, apparent horizon also does not exist although surfaces such that $\theta_+=0$ exists. The curvature invariant is given by

$$R_{IJKL}R^{IJKL}=5H^4+3\left[H^2-\frac{m}{2a^2(r-m)^3}\right]^2+\left[H^2-\frac{m}{a^2(r-m)^3}\right]^2. \tag{3.4}$$

To avoid the naked singularity at $r=m$, we change the sign of the mass parameter, $m\to -m$. After that the radial coordinate r can run up to $r=0$ and the conical singularity occurs at $r=0$ in general cases. Near $r=0$, the metric is written as

$$ds^2\simeq -dt^2+a(t)^2\left[\frac{r}{m}d\chi^2+\frac{m}{r}dr^2+m^2d\Omega_2^2\right]. \tag{3.5}$$

Here we introduce a new coordinate $R=(rm)^{1/2}$ and the metric is

$$ds^2\simeq -dt^2+a^2\left[4\left\{dR^2+R^2d\left(\frac{\chi}{2m}\right)^2\right\}+m^2d\Omega_2^2\right]. \tag{3.6}$$

Hence, the metric is regular everywhere except for $a=0$ singularity if one assumes the period $\chi_p=4\pi m$ along the χ direction. The physical size of the compactified dimensions is given by $4\pi a(t)m$. Thus, it decreases or increases if one takes the collapsing or expanding chart. A cosmological constant make the compactified space dynamical as well as the four-dimensional part. The AD mass is

$$E=E_{\rm ADM}=-\frac{a(t)^3m\chi_p}{G_5}=-a^3(t)m. \tag{3.7}$$

This is not conserved due to the expansion of the universe! This comes from the nonvanishing boundary term $\int d^{D-2}S_i\mathcal{T}^i$ which vanishes for asymptotically flat cases. At first glance, best we can do might be choosing the collapsing chart and we can keep the radius of the compactified space to be less than Planck length. In the chart, we observe the value of the energy approaches zero. One may think that a large cosmological constant stabilizes the space-times because the extra dimension shrinks rapidly and the energy becomes zero before space-time decays. At the same time the shrinking, however, means the big crunch of the space-time and then this case does not give attractive model. Since there is no reason why one has to choose the collapsing chart *a priori*, we need to consider the expanding chart too. In this chart, the compactified space expands till the end of the inflation and the absolute value of the AD energy increases. Naively speaking, the decay rate of the de Sitter space-time into the space-time with the large compactified space is suppressed.

IV. POSITIVE ENERGY THEOREM, ELEMENTARY FERMION, AND STABILITY

In locally asymptotically flat cases with compactified dimensions, the break down of positive energy theorem means the Witten spinor does not exist [5]. In supergravity, that means there is no supersymmetry because the spinor is related to the infinitesimal generator of local supersymmetry [18].

In asymptotically de Sitter cases, the situation is different from above. As we stated, the energy can be negative even if the extra dimension is not compactified and the topology is trivial. In this section, we discuss the positive energy theorem, based on Refs. [16,19],[2] in cases where the extra dimension is not compactified. Conversely, we can see easily that the Witten spinor cannot exist for examples given in the previous section.

Following Kastor and Traschen [19], we define the cosmological supercovariant derivative operator on a spinor ϵ as

$$\hat{\nabla}_I\epsilon=\left(\nabla_I+\frac{i}{2}H\gamma_I\right)\epsilon=\left(\partial_I+\Gamma_I+\frac{i}{2}H\gamma_I\right)\epsilon=:(\partial_I+\Gamma'_I)\epsilon, \tag{4.1}$$

where Γ_I is the spin connection,

$$\Gamma_I=-\frac{1}{8}e^{J\hat{K}}\nabla_I e_J^{\hat{L}}[\gamma_{\hat{L}},\gamma_{\hat{K}}], \tag{4.2}$$

and e_J^I is the quasiorthogonal basis. The cosmological Witten equation[3] is defined by

$$\gamma^i\hat{\nabla}_i\epsilon=0. \tag{4.3}$$

The solution is given by a constant spinor ϵ_0 satisfying $\gamma^{\hat{0}}\epsilon_0=-i\epsilon_0$ in the expanding flat slice of de Sitter space-time.

By using the Bianchi identity, the Hamiltonian and momentum constraints, we obtain the identity

$$\int dS^i\epsilon^\dagger\hat{\nabla}_i\epsilon=\int dV[|\hat{\nabla}_i\epsilon|^2+4\pi G_D(\epsilon^\dagger T_{\hat{0}\hat{0}}\epsilon+\epsilon^\dagger T_{i\hat{0}}\gamma^i\gamma^{\hat{0}}\epsilon)]. \tag{4.4}$$

Let us evaluate the left-hand side. We follow Witten's argument [21] carefully. First, the constant spinor satisfies

$$\gamma^i\hat{\nabla}_i\epsilon_0=\gamma^i\Gamma'_i\epsilon_0=\gamma^{i(D-1)}\Gamma_i\epsilon_0+\frac{i}{2}[-K+(D-1)H]\epsilon_0, \tag{4.5}$$

where ${}^{(D-1)}\Gamma_i=-(1/8)e^{j\hat{k}}\,{}^{(D-1)}\nabla_i e_k^{\hat{l}}[\gamma_{\hat{l}},\gamma_{\hat{k}}]$ and we used

$$\Gamma'_i={}^{(D-1)}\Gamma_i+\frac{1}{2}K_{ij}\gamma^j\gamma^{\hat{0}}+\frac{i}{2}H\gamma_i. \tag{4.6}$$

If we suppose $K=(D-1)H+O(1/r^{D-1})$, the same argument of the existence of solution as asymptotic flat holds and we obtain

$$\int dS^i\epsilon^\dagger\hat{\nabla}_i\epsilon=\int dS^i\epsilon_0^\dagger(\Gamma'_i-\gamma_i\gamma^j\Gamma'_j)\epsilon_0. \tag{4.7}$$

Inserting the decomposition (4.6) of the spin connection into the above, we obtain the familiar expression

$$\begin{aligned}\int dS^i\epsilon^\dagger\hat{\nabla}_i\epsilon&=\int dS^i\epsilon_0^\dagger({}^{(D-1)}\Gamma_i-\gamma_i\gamma^j\,{}^{(D-1)}\Gamma_j)\epsilon_0\\&\quad+\frac{1}{2}\int dS_i\epsilon_0^\dagger[K^i_j-\delta^i_jK+(D-2)H\delta^i_j]\gamma^j\gamma^{\hat{0}}\epsilon_0\\&=\frac{1}{4}\int dS^i\epsilon_0^\dagger(\partial_jh^j_i-\partial_ih^j_j)\epsilon_0-\frac{1}{2}\int dS_i\tilde{K}^i_j\overline{\epsilon_0}\gamma^j\epsilon_0\\&=4\pi G_D[E_{\rm ADM}|\epsilon_0|^2+\Delta P_{\rm ADM}(\overline{\epsilon_0}\gamma\epsilon_0)],\end{aligned} \tag{4.8}$$

where $\tilde{K}^i_j$ is the traceless part of K^i_j. Here we note that $\overline{\epsilon_0}\gamma^i\epsilon_0=-e^i_{\hat{0}}|\epsilon_0|^2=O(1/r^{D-3})$ due to $\gamma^{\hat{0}}\epsilon_0=-i\epsilon_0$. This and $\tilde{K}^i_j=O(1/r^{D-2})$ lead us that the net momentum term $\Delta P_{\rm ADM}(\overline{\epsilon_0}\gamma\epsilon_0)$ vanishes. Finally we obtain the inequality

$$E=\frac{a(t)}{4\pi G_D}\int dS^i\epsilon^\dagger\hat{\nabla}_i\epsilon=a(t)E_{\rm ADM}|\epsilon_0|^2\geqslant 0, \tag{4.9}$$

under the dominant energy condition [22] on the energy-momentum tensor T_{IJ}. Unfortunately, we cannot say some-

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[2]The argument in Ref. [16] is a prototype of the proof. Rigorously speaking, the ‘‘supercovariant derivative’’ defined there is not covariant for full space-times. An excellent approach bearing supergravity in mind was given in Ref. [19]. However, both approaches were not presented in the present refined form. The explicit evaluation of the left-hand side of Eq. (4.4) was not done in Ref. [19]. See Ref. [20] for asymptotically AdS space-times.

[3]The vector $\xi^I=\overline{\epsilon}\gamma^I\epsilon$ defined by spinor ϵ satisfying $\hat{\nabla}_I\epsilon=0$ is a conformal Killing vector, not a Killing vector in de Sitter space-time. That is, ϵ is a conformal Killing spinor, not a Killing spinor. This point is the main reason why one cannot prove the positivity of the AD mass for asymptotically de Sitter space-times having the net momentum because the left-hand side of Eq. (4.4) cannot be the AD mass.

thing about $E=0$ case because Witten spinor approaching to ϵ_0 at infinity is uniquely determined.[4] Thus, there is not contradiction with the existence of nontrivial solution which has zero ADM mass and is given in Ref. [10].

We can see that E is not equal to the AD mass, $E_{\rm AD}$. When $\tilde{K}^i_j = O(1/r^{D-1})$ holds and the momentum term of the AD mass vanishes, the AD mass equals to the ADM energy and the positivity is guaranteed. For our purpose, it is worth imposing $\tilde{K}^i_j = O(1/r^{D-1})$ because we are interest in the stability of space-time, not its dynamics. As with Eq. (4.9), this implies the positivity of the AD mass. The apparent contradiction with examples given in the previous section indicates that the Witten spinor does not exist in such examples.

V. SUMMARY

In this paper we defined the gravitational mass in asymptotically de Sitter space-time with extra dimensions and obtained a refined expression related to the ADM energy and momentum associated to the timelike Killing vector of the background de Sitter space-time. Furthermore, we gave one dynamical solution and one initial data with the negative energy in five dimensions. Do these solutions indicate the quantum decay of de Sitter space-time? We cannot reply to the question instantly because we do not know whether the instanton exists or not. Naively speaking, we can guess from the previous section that the decay occurs unless one imposes the existence of an elementary fermion or supersymmetry.

Finally, we should stress the fact that the energy seems to depend on the time in general although the definition is reasonable. This means that the contribution from the boundary is not negligible.

[4]In asymptotically flat space-times, ϵ_0 is arbitrary constant spinor. This means that there is an independent solution of the D's of the Witten equation. As a result, we obtain $R_{IJKL}=0$ from $0 = [\nabla_i, \nabla_j]\epsilon = (1/4)R_{ijKL}[\gamma^K, \gamma^L]\epsilon$.

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