Green-Schwarz string action on $AdS_3 \times S^3$ with Ramond-Ramond charge

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(Received 11 March 1999; published 20 August 1999)

We derive the classical κ-symmetric type IIB string action on $AdS_3 \times S^3$ by employing the $SU(1,1|2)^2$ algebra. We then gauge fix the κ symmetry in the background adapted Killing spinor gauge and present the action in a very simple form. [S0556-2821(99)01216-3]

PACS number(s): 04.65.+e, 11.25.Mj

I. INTRODUCTION

There has been great interest recently in string theory on $AdS_5 \times S^5$ [1,2,3] due to its possible relation to $\mathcal{N}=4, d=4$ Yang-Mills theory. Whereas the large g^2N limit is conjectured to be dual to type IIB supergravity on this manifold, for which there is by now mounting evidence, stringy effects are supposed to correspond to $1/g^2N$ corrections [1] in the Yang-Mills theory. It is of great interest, therefore to construct string theory in this background. Although there has been significant progress in this direction [4,5,6,7,8], the action (so far) has proven too difficult to quantize. In this note, we will try to analyze a simpler case, that of string theory on $AdS_3 \times S^3$.

One interesting aspect of this background is that the compactification of $D=6$ supergravity on S^3 can be achieved in two fundamentally different ways: the charged three-form field strength can either be of Neveu-Schwarz (NS) or Ramond-Ramond (RR) type. In [9] the NS field was charged and a significant understanding of a string propagating in this background was achieved. In this paper we focus on a string in the nontrivial RR background and construct the string action in the Green-Schwarz (GS) formulation [10]. The hope is that eventually this case can be better understood, maybe by relating it to results of [9]. Various other aspects of this background have been studied in [11,12,13,14,15].

We shall follow the approach of [4] which requires a description of the background as a supercoset manifold. The $AdS_3 \times S^3$ background is the near-horizon geometry of the D1–D5 brane system and is a solution of chiral $N=2(2,0)$ supergravity in six dimensions [16] preserving all 16 supersymmetries. By essentially straightforward extension of the arguments given in [17] it can be shown that the solution does not get any α' corrections which is a necessity to formulate string theory in this background. In [18] it was noted that the isometry group of D1–D5 system is $SU(1,1|2)^2$, and hence the background can be viewed as the supercoset space $SU(1,1|2)^2/SO(1,2)\times SO(3)$. The construction of the action following [4] is then straightforward except for the construction of the Wess-Zumino term, which requires some trial and error.

This is done in Sec. II, where we start with the algebra of $SU(1,1|2)^2$ [which we derive very explicitly in 6D covariant form from the $SU(1,1|2)$ algebra in the Appendix] and construct the Wess-Zumino (WZ) term from first principles following [4]. We find, in fact a continuous family of WZ terms interpolating between the pure NS background and the RR background.

The resulting GS action is then given in terms of supervielbeins which we also solve for in Sec. III. In Sec. IV we gauge fix κ symmetry in the ‘‘background adapted Killing spinor gauge’’ [19,6,7] which simplifies the action considerably.

Finally we present our conclusions and some open questions.

II. FROM THE ALGEBRA $SU(1,1|2)^2$ TO THE STRING ACTION ON $AdS_3 \times S^3$

The target space of string theory on $AdS_3 \times S^3$ with 16 supersymmetry generators is the supercoset manifold $SU(1,1|2)^2/SO(1,2)\times SO(3)$ whose bosonic part is $SO(2,2)\times SO(4)/SO(1,2)\times SO(3)$. The generators of this supergroup are the momenta and Lorentz transformations on AdS_3 and S^3

$$P_a, J_{ab}, \quad \text{and} \quad P_{a'}, J_{a'b'}, \tag{2.1}$$

where $a=0,1,2,$ and $a'=3,4,5,$ plus 2 complex chiral 6D spinors

$$Q_{I\alpha\alpha'} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{2.2}$$

with $I=1,2$, $\alpha=1,2$, $\alpha'=1,2$. Our conventions are

$$\Gamma^a = \gamma^a \otimes 1 \otimes \sigma_1, \quad \Gamma^{a'} = 1 \otimes \gamma^{a'} \otimes \sigma_2, \tag{2.3}$$

where $\gamma^0 = i\sigma^3$, $\gamma^{1,2} = \sigma^{1,2}$, $\gamma^{a'} = \sigma^{a'-2}$. In the following we will freely use γ^a short for $\gamma^a \otimes 1$ (and the same for primed indices). With these definitions it is clear that Q_I defined as above is indeed chiral. The conjugate supercharge $\overline{Q}^{I\alpha\alpha'}$ is defined by

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0556-2821/99/60(6)/064014(8)/$15.00

$$\overline{Q^I}=(Q^I)^\dagger\gamma^0. \tag{2.4}$$

Crucial for the construction of the action are the (anti-Hermitian) supervielbeins L^a, $L^{a'}$, L^I and $\overline{L^I}$ and the super-connection L^{ab} and $L^{a'b'}$. Being a σ model with a super-coset as the target space, the action is only allowed to contain the supervielbeins, and will be of the general structure

$$S=S_{kin}+S_{WZ}. \tag{2.5}$$

The kinetic term is next to trivial to write down, the more subtle issue is to construct the Wess-Zumino term, needed for κ invariance, which is an integral over a closed 3-form. To find this form we need the superalgebra, derive from there the Maurer-Cartan equations and identify a unique closed three-form built from the supervielbeins.

It should be apparent by now that an important ingredient is the $SU(1,1|2)^2$ algebra

$$\begin{aligned}
\{Q_I,\overline{Q_J}\}&=2\delta_{IJ}(iP_a\gamma^a-P_{a'}\gamma^{a'})+\epsilon_{IJ}(J_{ab}\gamma^{ab}-J_{a'b'}\gamma^{a'b'}),\\
[P_a,Q_I]&=-\frac{i}{2}\epsilon_{IJ}\gamma_aQ_J,\quad [P_{a'},Q_I]=\frac{1}{2}\epsilon_{IJ}\gamma_{a'}Q_J,\\
[J_{ab},Q_I]&=-\frac{1}{2}\gamma_{ab}Q_I,\quad [J_{a'b'},Q_I]=-\frac{1}{2}\gamma_{a'b'}Q_I,\\
[P_a,\overline{Q_I}]&=\frac{i}{2}\overline{Q}_J\epsilon_{JI}\gamma_a,\quad [P_{a'},\overline{Q_I}]=-\frac{1}{2}\overline{Q}_J\epsilon_{JI}\gamma_{a'},\\
[J_{ab},\overline{Q_I}]&=\frac{1}{2}\overline{Q}_I\gamma_{ab},\quad [J_{a'b'},\overline{Q_I}]=\frac{1}{2}\overline{Q}_I\gamma_{a'b'},\\
[M_{AB},M_{CD}]&=\eta_{BC}M_{AD}+\eta_{AD}M_{BC}-\eta_{AC}M_{BD}-\eta_{BD}M_{AC},\\
[M_{A'B'},M_{C'D'}]&=\delta_{B'C'}M_{A'D'}+\delta_{A'D'}M_{B'C'}-\delta_{A'C'}M_{B'D'}\\
&\quad-\delta_{B'D'}M_{AC},
\end{aligned} \tag{2.6}$$

where we defined

$$P_a=M_{0a},\quad J_{ab}=M_{ab},\quad P_{a'}=M_{0'a'},\quad J_{a'b'}=M_{a'b'} \tag{2.7}$$

and where $\eta=(-++-)$. Note that the bosonic generators are taken to be anti-Hermitian. The defining equations of the background can be obtained in the standard way by defining the group derivative

$$\mathcal{D}=d+L^aP_a+\frac{1}{2}L^{ab}J_{ab}+L^{a'}P_{a'}+\frac{1}{2}L^{a'b'}J_{a'b'}+\frac{1}{2}(\overline{Q^I}L_I+\overline{L^I}Q_I) \tag{2.8}$$

and requiring $\mathcal{D}^2=0$. This leads to the Maurer-Cartan equations

$$\begin{aligned}
dL^a&=-\frac{i}{2}\overline{L^I}\gamma^a\wedge L^I-L^b\wedge L^{ba},\\
dL^{a'}&=+\frac{1}{2}\overline{L^I}\gamma^{a'}\wedge L^I-L^{b'}\wedge L^{b'a'},\\
dL^I&=-\frac{i}{2}\epsilon^{IJ}L^a\wedge\gamma_aL^J+\frac{1}{2}\epsilon^{IJ}L^{a'}\wedge\gamma_{a'}L^J\\
&\quad-\frac{1}{4}L^{ab}\wedge\gamma_{ab}L^I-\frac{1}{4}L^{a'b'}\wedge\gamma_{a'b'}L^I,\\
d\overline{L^I}&=\frac{i}{2}\epsilon^{IJ}\overline{L^J}\gamma_a\wedge L^a-\frac{1}{2}\epsilon^{IJ}\overline{L^J}\gamma_{a'}\wedge L^{a'}\\
&\quad-\frac{1}{4}\overline{L^I}\gamma_{ab}\wedge L^{ab}-\frac{1}{4}\overline{L^I}\gamma_{a'b'}\wedge L^{a'b'}
\end{aligned} \tag{2.9}$$

plus the nonrelevant ones for dL^{ab} and $dL^{a'b'}$. The Wess-Zumino term can be constructed in terms of the vielbeins without actually solving these equations. In the background at hand this is only slightly more subtle than in the $AdS_5\times S^5$ background, since the L^I do not obey any Majorana conditions. We find that the unique form satisfying the requirements is

$$\begin{aligned}
\mathcal{H}_3&=As^{IJ}(\overline{L^I}\gamma_a\wedge L^J\wedge L^a+i\overline{L^I}\gamma_{a'}\wedge L^J\wedge L^{a'})+\text{c.c.}\\
&=As^{IJ}(\overline{L^I}\Gamma_a\wedge L^J\wedge L^a+\overline{L^I}\Gamma_{a'}\wedge L^J\wedge L^{a'})+\text{c.c.},
\end{aligned} \tag{2.10}$$

where $s^{IJ}=\sigma_3^{IJ}$. In proving that $d\mathcal{H}_3=0$ one has to apply the identities (A25), and has to use

$$s^{IJ}(\overline{L^I}\gamma_aL^J\overline{L^K}\gamma^aL^K-\overline{L^I}\gamma_{a'}L^J\overline{L^K}\gamma^{a'}L^K)=0. \tag{2.11}$$

It remains to find the coefficient in front of the Wess-Zumino term. For this we consider the flat-space limit, where the vielbeins read, in our notation [See Eq. (3.7) with $\mathcal{M}=0$ and $s=1$,],

$$\begin{aligned}
L^I&=d\theta^I,\\
\overline{L^I}&=d\overline{\theta^I},\\
L^a&=dx^a-\frac{i}{4}(\overline{\theta^I}\gamma^ad\theta^I-d\overline{\theta^I}\gamma^a\gamma^a\theta^I),\\
L^{a'}&=dx^{a'}+\frac{1}{4}(\overline{\theta^I}\gamma^{a'}d\theta^I-d\overline{\theta^I}\gamma^{a'}\theta^I).
\end{aligned} \tag{2.12}$$

Therefore,

$$\begin{aligned}WZ&=As^{IJ}\overline{L^I}\Gamma_a\wedge L^J\wedge L^a+\text{c.c.}\\&=A(d\overline{\theta^1}\Gamma_a d\theta^1-d\overline{\theta^2}\Gamma_a\wedge d\theta^2)\wedge dx^a+\text{c.c.}+\cdots\\&=Ad([(\overline{\theta^1}\Gamma_a d\theta^1-d\overline{\theta^1}\Gamma_a\theta^1)\\&\quad-(\overline{\theta^2}\Gamma_a\wedge d\theta^2-d\overline{\theta^2}\Gamma_a\wedge\theta^2)]\wedge dx^a+\cdots)\partial_\beta\theta^1-\partial_\alpha\hat{\theta}^2\end{aligned}\tag{2.13}$$

and hence

$$\int_{M^3}WZ=A\int_{M^2}d^2\sigma\epsilon^{ij}(\overline{\theta^1}\Gamma_a\partial_i\theta^1-\partial_i\overline{\theta^1}\Gamma_a\theta^1)\partial_j x^a+\cdots.\tag{2.14}$$

By comparison with standard literature (see for example [20]) one finds

$$A=\frac{i}{4}.\tag{2.15}$$

Therefore, the 6D superstring action is given by

$$\begin{aligned}S=&-\frac{1}{2}\int_{M^2}d^2\sigma(L^aL^a+L^{a'}L^{a'})\\&+\frac{i}{4}\int_{M^3}s^{IJ}[(\overline{L^I}\gamma_a\wedge L^J\wedge L^a+i\overline{L^I}\gamma_{a'}\wedge L^J\wedge L^{a'})+\text{c.c.}]\\=&-\frac{1}{2}\int_{M^2}d^2\sigma(L^aL^a+L^{a'}L^{a'})\\&+\frac{i}{4}\int_{M^3}s^{IJ}[(\overline{L^I}\Gamma_a\wedge L^J\wedge L^a+\overline{L^I}\Gamma_{a'}\wedge L^J\wedge L^{a'})+\text{c.c.}],\end{aligned}\tag{2.16}$$

which is the main result of this section.

This WZ term, however, should not really be unique, since there exists also the string in the same geometry, but charged under the NS B field, and there must be a different WZ term for it. The answer suggested by the work of [21] answer is that the general WZ term should be given by

$$\begin{aligned}\mathcal{H}\sim&s^{IJ}[(\overline{L^I}\gamma_a\wedge L^J\wedge L^a+i\overline{L^I}\gamma_{a'}\wedge L^J\wedge L^{a'})+\text{c.c.}]\\&+L^a\wedge L^b\wedge L^c H^+_{abc},\end{aligned}\tag{2.17}$$

where H^+_{abc} is one of the five components of the self-dual superfield [22]. This is to be understood from the point of view of compactifying the $D=10,N=2$ type IIB theory on $K3$ (and truncating the matter fields). Of the five self-dual field strengths that arise [23], three find their origin in the self-dual five-form field strength, one from the RR three-form (plus its dual) and one (H^+_{abc}) from the NS three-form and its dual in $D=10$.

III. THE SUPERGEOMETRY

It remains to actually solve the Maurer-Cartan equations and obtain the supervielbeins. The general method is standard and was outlined for example in [4] where for the $AdS_5\times S^5$ case the vielbeins were constructed up to quartic order. In [5] it was observed that the equations can in fact be integrated and the supergeometry can be found in closed form.

To do so we have to play the usual trick and introduce $\theta_s=s\theta$ to solve for a generalized vielbein L_s from which one obtains eventually the standard vielbein as $L=L_{s=1}$. In the process we also find following [24] a convenient form of the Wess-Zumino as a two-dimensional worldsheet integral, integrated once more over the parameter s.

Let us denote the general structure of the algebra by

$$\begin{aligned}\{Q_I,\overline{Q_J}\}&=f^A_{I\bar{J}}B_A,\\ [B_A,Q_I]&=f^J_{AI}Q_J,\\ [B_A,\overline{Q_I}]&=f^{\bar{J}}_{A\bar{I}}\overline{Q_J},\\ [B_A,B_B]&=f^C_{AB}B_C,\end{aligned}\tag{3.1}$$

where the distinction between I and $\bar{I}$ serves only the purpose to keep track of Q and $\bar{Q}$. With D being the standard covariant (bosonic) derivative

$$D=d+\frac{1}{4}\omega^{ab}J_{ab}+\frac{1}{4}\omega^{a'b'}J_{a'b'}+e^aP_a+e^{a'}P_{a'}\tag{3.2}$$

we find from

$$e^{-(s/2)(\bar{\theta}Q+\bar{Q}\theta)}De^{(s/2)(\bar{\theta}Q+\bar{Q}\theta)}=L^A_sB_A+\frac{1}{2}(\overline{L_s}Q+\bar{Q}L_s)\tag{3.3}$$

the differential equations

$$\begin{aligned}\partial_sL^A_s&=-\frac{1}{4}\overline{\theta}^I f^A_{I\bar{J}}L^{\bar{J}}_s+\frac{1}{4}\overline{L}^I_s f^A_{I\bar{J}}\theta^{\bar{J}},\\ \partial_sL^{\bar{I}}_s&=d\theta^{\bar{I}}+L^B_s f^{\bar{I}}_{B\bar{J}}\theta^{\bar{J}},\\ \partial_s\overline{L^I_s}&=d\overline{\theta^I}-\overline{\theta^J}f^I_{JB}L^B_s.\end{aligned}\tag{3.4}$$

These equations can easily be integrated since

$$\partial^2_s\begin{pmatrix}L_s\\L^*_s\end{pmatrix}^{\bar{I}}=(\mathcal{M}^2)^{\bar{I}}_{\bar{J}}\begin{pmatrix}L_s\\L^*_s\end{pmatrix}^{\bar{J}},\tag{3.5}$$

with

$$(\mathcal{M}^2)^{\bar{I}}_{\bar{J}}=\frac{1}{4}\begin{pmatrix}f^{\bar{I}}_{B\bar{K}}\theta^{\bar{K}}\overline{\theta}^L f^B_{L\bar{J}} & -f^{\bar{I}}_{B\bar{K}}\theta^{\bar{K}}\overline{\theta}^{*L}f^{*B}_{L\bar{J}}\\ -f^{*\bar{I}}_{B\bar{K}}\theta^{*\bar{K}}\overline{\theta}^L f^B_{L\bar{J}} & f^{*\bar{I}}_{B\bar{K}}\theta^{*\bar{K}}\overline{\theta}^{*L}f^{*B}_{L\bar{J}}\end{pmatrix}.\tag{3.6}$$

The solution to Eq. (3.4) is then given by

$$\begin{pmatrix} L \\ L^* \end{pmatrix}_s^{\bar{I}} = \left(\frac{\sinh s\mathcal{M}}{\mathcal{M}} \right)^{\bar{I}}_{\bar{J}} \begin{pmatrix} D\theta \\ D\theta^* \end{pmatrix}^{\bar{J}},$$

$$L^A = e^A - \frac{1}{2}(\bar{\theta}^I f^A_{I\bar{J}}, -\bar{\theta}^{*I} f^{*A}_{I\bar{J}}) \times \left(\frac{\sinh^2(s\mathcal{M}/2)}{(\mathcal{M})^2} \right)^{\bar{I}}_{\bar{K}} \begin{pmatrix} D\theta \\ D\theta^* \end{pmatrix}^{\bar{K}}, \tag{3.7}$$

with

$$D^{IJ} = \delta^{IJ}\left(d + \frac{1}{4}\omega^{ab}\gamma_{ab} + \frac{1}{4}\omega^{a'b'}\gamma_{a'b'} \right) + \epsilon^{IJ}\frac{i}{2}(e^a\gamma_a - ie^{a'}\gamma_{a'}). \tag{3.8}$$

Here, we used the initial conditions

$$L^a(\theta=0) = e^a, \quad L^{a'}(\theta=0) = e^{a'},$$
$$L^{ab}(\theta=0) = \omega^{ab}, \quad L^{a'b'}(\theta=0) = \omega^{a'b'}. \tag{3.9}$$

The real vielbeins are then obtained by setting $s=1$.

Another virtue of above procedure is that one can obtain the Wess-Zumino term as a world-sheet integral of an expression which is itself integrated over s [24]. The important point is that

$$\partial_s \mathcal{H}_{3s} = d\Omega_{2s}, \tag{3.10}$$

where $\mathcal{H}_s$ is obtained from $\mathcal{H}$ by replacing all L by L_s, and where

$$\Omega_{2s} = \frac{i}{2} s^{IJ}(\bar{\theta}^I \gamma_a \wedge L^J_s \wedge L^a_s + i\bar{\theta}^I \gamma_{a'} \wedge L^J_s \wedge L^{a'}_s) + \text{c.c.} \tag{3.11}$$

This can be verified with the differential equations (2.9) and (3.4). Hence

$$S_{WZ} = \int_{M^3} \mathcal{H}_{3s}|_{s=1} = \int_{M^2}\int_{s=0}^{1} \Omega_{2s}. \tag{3.12}$$

IV. SIMPLIFICATION OF THE ACTION

We now turn to the very important aspect of simplifying the action. We will follow here the ideas of [6,7] and fix κ symmetry in the background adapted way. The procedure consists of two steps:

Choosing the gauge

$$\theta^I_- \equiv \mathcal{P}^{IJ}_- \theta^J \equiv \frac{1}{2}(\delta^{IJ} - i\epsilon^{IJ}\Gamma^0\Gamma^1)\theta^J = 0 \tag{4.1}$$

and redefining the remaining fermions θ_+ to be space-time dependent as

$$\theta^I_+(x) = g_{tt}^{1/4} \vartheta^I_+, \tag{4.2}$$

where ϑ^I_+ are constant spinors which also satisfies $\mathcal{P}^{IJ}_- \vartheta^J_+ = 0$.

This gauge is motivated by the observation that $D\theta$ as defined in Eq. (3.8) is essentially simply the Killing equation on $AdS_3 \times S^3$ augmented by a fermionic differential operator $d\vartheta\partial_\vartheta + \bar{d}\bar{\vartheta}\partial_{\bar{\vartheta}}$. Hence, choosing the fermionic coordinates θ in Eq. (3.3) to be space-time dependent Killing spinors, i.e.,

$$\theta^{I\alpha\alpha'}(x) = e^{I\alpha\alpha'}_{\underline{J\beta\beta'}}(x)\vartheta^{\underline{J\beta\beta'}}, \tag{4.3}$$

where $e^{I\alpha\alpha'}_{\underline{J\beta\beta'}}$ is a known space-time dependent matrix and $\vartheta = \text{const}$, leads to

$$D\theta^I = e^I_{\underline{J}}(x) d\vartheta^{\underline{J}}. \tag{4.4}$$

The Killing spinors on $AdS_3 \times S^3$ in horospherical coordinates can, for example, be found in [25], and it can be easily verified, as first noted in [19], that using κ symmetry to project on half of them precisely via Eq. (4.1) leads to the fact that θ_+ and $D\theta_+$ obey the same projection, i.e.,

$$\mathcal{P}_- \theta_+ = \mathcal{P}_- D\theta_+ = 0, \tag{4.5}$$

since Eq. (4.3) reduces for this component to[1]

$$\theta^I_+(x) = g_{tt}^{1/4} \vartheta^J_+. \tag{4.6}$$

Since this gauge is based on the isometry of the background, it is called the killing spinor gauge and was proposed in [19] as a procedure to gauge-fix κ symmetry of extended objects in their own background. In [7] it was shown that it could also be used to simplify dramatically the GS string action on $AdS_5 \times S^5$. Since the arguments given there for admissibility of the gauge are exactly the same needed here we refer the reader to that publication.

What we will show now is that with this gauge we have

$$\mathcal{M}^2_+ \begin{pmatrix} D\theta_+ \\ D\theta^*_+ \end{pmatrix} = 0, \tag{4.7}$$

which clearly simplifies Eq. (3.7) and therefore the action dramatically. The important fact to use is that terms of the form

$$\bar{\theta}^I_+ \hat{\Gamma} D\theta^I_+, \quad \text{with } [\Gamma^{01}, \hat{\Gamma}] = 0 \tag{4.8}$$

vanish. This implies that

$$f^{\bar{I}}_{B\bar{K}} \theta^{\bar{K}}_+ \bar{\theta}^{\bar{L}}_+ f^B_{L\bar{K}} d\theta^{\bar{K}}_+ = f^{\bar{I}}_{i\bar{K}} \theta^{\bar{K}}_+ \bar{\theta}^{\bar{L}}_+ f^i_{L\bar{K}} D\theta^{\bar{K}}_+ + f^{\bar{I}}_{(i2)\bar{K}} \theta^{\bar{K}}_+ \bar{\theta}^{\bar{L}}_+ f^{(i2)}_{L\bar{K}} D\theta^{\bar{K}}_+, \tag{4.9}$$

with $i=0,1$, i.e., in the sum over the bosonic generators B only the two momenta P_i and the two Lorentz generators J_{i2}

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[1] Incidentally, the surviving $\theta_+(x)$ spinor is nothing but the Killing spinor of the full D1-D5 geometry, in the near horizon region. This might have some so-far-not-understood implications.

can contribute. Then, with a little algebra and using the explicit form of the structure constants we find that in fact the contributions from P_i and J_{i2} arise with opposite sign and cancel. The same happens for the other term, i.e.,

$$f^{\bar{I}}_{B\bar{K}}\theta^{\bar{K}}_{+}\overline{\theta}^{*L}_{+}f^{*B}_{L\bar{J}}D\theta^{*\bar{J}}_{+}=0. \tag{4.10}$$

Putting all this together we see indeed that

$$(\mathcal{M}^2_{+})^{\bar{I}}_{\bar{J}}\begin{pmatrix} D\theta^{\bar{J}}_{+} \\ D\theta^{*\bar{J}}_{+}\end{pmatrix}=\begin{pmatrix} f^{\bar{I}}_{B\bar{K}}\theta^{\bar{K}}_{+}\overline{\theta}^{L}_{+}f^{B}_{L\bar{J}}D\theta^{\bar{J}}_{+}-f^{\bar{I}}_{B\bar{K}}\theta^{\bar{K}}_{+}\overline{\theta}^{*L}_{+}f^{*B}_{L\bar{J}}D\theta^{*\bar{J}}_{+} \\ -f^{*\bar{I}}_{B\bar{K}}\theta^{*\bar{K}}_{+}\overline{\theta}^{*L}_{+}f^{*B}_{L\bar{J}}D\theta^{*\bar{J}}_{+}+f^{*\bar{I}}_{B\bar{K}}\theta^{*\bar{K}}_{+}\overline{\theta}^{L}_{+}f^{B}_{L\bar{J}}D\theta^{\bar{J}}_{+}\end{pmatrix}=0. \tag{4.11}$$

Now, recall that one explicit form of the $AdS_3\times S^3$ metric in the "2+4"-split is

$$ds^2=y^2(dx^p dx_p)+\frac{1}{y^2}(dy^t dy^t), \tag{4.12}$$

where t and p denote coordinate transverse (y^2,y^3,y^4,y^5) and parallel (x^0,x^1) to the brane. With this form of the metric, Eq. (4.2), and

$$\vartheta\equiv\vartheta^1, \tag{4.13}$$

we find the simple supervielbeins. The supergeometry reads

$$\begin{aligned}
(L^I_s)_+&=s\sqrt{|y|}\,d\vartheta^I,\\
(L^I_s)_-&=0,\\
L^p_s&=|y|\left(dx^{\hat m}-\frac{is^2}{2}(\bar\vartheta\gamma^p d\vartheta-d\bar\vartheta\gamma^p\vartheta)\right),\\
L^t_s&=\frac{1}{|y|}dy^t.
\end{aligned} \tag{4.14}$$

Finally, inserting this into Eq. (2.16) we obtain

$$\begin{aligned}
S=-\frac{1}{2}\int d^2\sigma\Bigg[\sqrt{g}g^{ij}\Bigg(y^2\left(\partial_i x^p-\frac{i}{2}(\bar\vartheta\Gamma^p\partial_i\vartheta-\partial_i\bar\vartheta\Gamma^p\vartheta)\right)\\
\times\left(\partial_j x_p-\frac{i}{2}(\bar\vartheta\Gamma_p\partial_j\vartheta-\partial_j\bar\vartheta\Gamma_p\vartheta)\right)+\frac{1}{y^2}\partial_i y^t\partial_j y^t\Bigg)\\
-\frac{1}{2}\epsilon^{ij}\partial_i y^t(\bar\vartheta\Gamma^t\partial_j\vartheta-\partial_j\bar\vartheta\Gamma^t\vartheta)\Bigg].
\end{aligned} \tag{4.15}$$

V. CONCLUSIONS AND OPEN QUESTIONS

We presented the action of the the string in an $AdS_3\times S^3$ background. We explicitly constructed the Wess-Zumino term as a closed three-form from first principles by employing the supercoset structure of the background geometry. It was then shown that the action can be simplified significantly to contain fermionic terms only up to quadratic order. Of course, it is still non-linear and a quantization procedure is not apparent off-hand.

Since the pure NS background can be solved explicitly in the RNS formalism [9], at least in that case one should be able to quantize the GS action as well. The quantization procedure is not, however, obvious. An approach to the problem may be to construct the currents corresponding to the space-time Virasoro algebra and comparing these to those obtained from the RNS formalism.

Furthermore, from knowing the NS background, several things about the RR background can be deduced, e.g., the spectrum of chiral primaries. It is of great interest to see if these can be computed directly from the string action.

Note added. After completion of this work we became aware of the paper by Pesando [27] which has some overlap with the present publication.

ACKNOWLEDGMENTS

We had stimulating discussions with Renata Kallosh. J.R. is supported by NSF grant PHY-9219345 and A.R. is supported by DOE grant DE-FG02-96ER40559.

APPENDIX

We start with the SU(1,1|2) algebra in the form of [26]:

$$\begin{aligned}
&[D,P]=P,\quad [D,K]=-K,\quad [K,P]=2D,\\
&[N_{mn},N_{pq}]=\delta_{np}N_{mq}+\delta_{mq}N_{np}-\delta_{nq}N_{mp}-\delta_{mp}N_{nq},\\
&[D,Q_i]=\frac{1}{2}Q_i,\quad [D,S_i]=-\frac{1}{2}S_i,\\
&[N_{mn},Q_i]=-\frac{1}{4}\gamma_{mn}Q_i,\quad [N_{mn},S_i]=-\frac{1}{4}\gamma_{mn}S_i,\\
&\{Q_{i\alpha'},Q_{j\beta'}\}=-2\epsilon_{ij}C_{\alpha'\beta'}P,\\
&\{S_{i\alpha'},S_{j\beta'}\}=-2\epsilon_{ij}C_{\alpha'\beta'}K,\\
&\{Q_{i\alpha'},S_{j\beta'}\}=-2\epsilon_{ij}C_{\alpha'\beta'}D\\
&\qquad+\epsilon_{ij}(\gamma^{mn})_{\alpha'\beta'}N_{mn}.
\end{aligned} \tag{A1}$$

Here, we have $i,j=1,2$, $\alpha',\beta'=1,2$ are SO(3) spinor indices and $m,n=1,2,3$. The $AdS_3\times S^3$ geometry is the supercoset manifold $SU(1,1|2)^2/SO(1,2)\times SO(3)$ with bosonic subgroup $SO(2,2)\times SO(4)/SO(1,2)\times SO(3)\sim SO(1,2)^2\times SO(3)^2/SO(1,2)\times SO(3)$. The strategy is to combine two copies of above algebra (variables X and $\tilde X$) and combine the spinors $Q,\tilde Q,S,\tilde S$ into suitable $SO(2,2)\times SO(4)$ spinors and the bosonic operators as generators of this group. We will then convert the bosonic and fermionic generators to covariant 6D objects which results in Eq. (2.6).

We start with the bosonic SO(1,2) subalgebra:

$$[D,P]=P, \quad [K,P]=2D, \quad [D,K]=-K, \tag{A2}$$

which can be rewritten with $P_+=\frac{1}{2}(P+K)$, $P_-=\frac{1}{2}(P-K)$ as

$$[D,P_+]=P_-, \quad [D,P_-]=P_+, \quad [P_+,P_-]=D. \tag{A3}$$

These generators should be combined with their counterparts $\tilde{D}$, $\tilde{P}$ and $\tilde{K}$ satisfying the same algebra into one SO(2,2) matrix M_{AB}. One finds that with

$$M_{12}=i(D-\tilde{D}), \quad M_{23}=P_-+\tilde{P}_-, \quad M_{13}=-i(P_+-\tilde{P}_+),$$
$$M_{03}=i(D+\tilde{D}), \quad M_{01}=P_--\tilde{P}_-, \quad M_{02}=-i(P_++\tilde{P}_+), \tag{A4}$$

M_{AB} satisfies indeed the proper SO(2,2) algebra:

$$[M_{ab},M_{cd}]=\eta_{ac}M_{db}-\eta_{ad}M_{cb}-\eta_{bc}M_{da}+\eta_{bd}M_{ca}, \tag{A5}$$

with the signature $(-++-)$ for indices (0123).

We now turn to unifying the spinors $Q,\tilde{Q},S,\tilde{S}$. It is useful to keep the index structure of the γ matrices and spinors in mind:

$$\gamma_\alpha^\beta, \quad Q_\alpha, \quad \hat{Q}^\alpha, \tag{A6}$$

where $\tilde{Q}$ is the SO(2,2) conjugate spinor of Q, i.e., $\hat{Q}\equiv Q^\dagger\Gamma^0\Gamma^3$. With the following set of definitions [and the convention that we take SO(2,2) spinors to be Majorana]:

$$\Gamma^0=\begin{pmatrix}0&1\\-1&0\end{pmatrix}, \quad \Gamma^{1,2}=\begin{pmatrix}0&\sigma^{1,2}\\\sigma^{1,2}&0\end{pmatrix},$$
$$\Gamma^3=\begin{pmatrix}0&i\sigma^3\\i\sigma^3&0\end{pmatrix}, \quad \Gamma^5=\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \quad C=\Gamma^0\Gamma^2, \tag{A7}$$

we find

$$M_{AB}\Gamma^{AB}C=2\begin{pmatrix}-2iK&2iD&0&0\\2iD&-2iP&0&0\\0&0&-2i\tilde{P}&2i\tilde{D}\\0&0&2i\tilde{D}&-2i\tilde{K}\end{pmatrix}, \tag{A8}$$

which reveals that one part of the algebra is given by

$$\{q_i,q_j\}=-\frac{i}{2}\epsilon_{ij}M_{AB}\Gamma^{AB}CC'+\cdots, \tag{A9}$$

with

$$q_{i\alpha'}=\begin{pmatrix}S_{\alpha'}\\-Q_{\alpha'}\\-\tilde{Q}_{\alpha'}\\\tilde{S}_{\alpha'}\end{pmatrix}_i. \tag{A10}$$

To complete the algebra we turn to the SO(4) part, where spinors are taken to be symplectic Majorana. Our conventions are

$$\Gamma'^0=\begin{pmatrix}0&-i\\i&0\end{pmatrix}, \quad \Gamma'^a=\begin{pmatrix}0&\sigma^a\\\sigma^a&0\end{pmatrix},$$
$$C'=\Gamma'^0\Gamma'^2, \quad \Gamma'^5=\begin{pmatrix}1&0\\0&-1\end{pmatrix} \tag{A11}$$

and we find

$$M'_{A'B'}\Gamma'^{A'B'}C'=\begin{pmatrix}-2M'_{0'i'}\sigma^{i'}+M'_{i'j'}\sigma^{i'j'}&0\\0&2M'_{0'i'}\sigma^{i'}+M'_{i'j'}\sigma^{i'j'}\end{pmatrix}C', \tag{A12}$$

which implies that the $N_{i'j'}$ in Eq. (A1) are given by

$$N_{i'j'}=\frac{1}{2}(M'_{i'j'}-\epsilon_{i'j'k'}M'_{0'k'}). \tag{A13}$$

With these preliminaries the SO(2,2)×SO(4) spinors are defined as

$$q_{iI\alpha I'\alpha'}=\begin{pmatrix}S_{1'\alpha'}\\S_{2'\alpha'}\\-Q_{1'\alpha'}\\-Q_{2'\alpha'}\\-\tilde{Q}_{1'\alpha'}\\-\tilde{Q}_{2'\alpha'}\\\tilde{S}_{1'\alpha'}\\\tilde{S}_{2'\alpha'}\end{pmatrix}_i, \tag{A14}$$

where the vector components denote the $q_{I\alpha}$ elements in the natural order. The pair $I,\alpha(I',\alpha')$ is an SO(2,2)[SO(4)] index, whereas i is still the symplectic index. Counting the degrees of freedom reveals that half of the 32 components of q (''real'' by Majorana–symplectic-Majorana condition) have to be projected out. The underlying reason is that spinors transform under SO(2,2)×SO(4)∼$\mathrm{SO}(1,2)_1\times\mathrm{SO}(1,2)_2\times\mathrm{SO}(3)_1\times\mathrm{SO}(3)_2$ only under $\mathrm{SO}(1,2)_1\times\mathrm{SO}(3)_1$ or $\mathrm{SO}(1,2)_2\times\mathrm{SO}(3)_2$, since the algebra is the product $\mathrm{SU}(1,1|2)^2$. Clearly, the projector $\mathcal{P}$ has to ensure that $I=I'$ which results in

$$\mathcal{P}=\frac{1}{2}(1\otimes 1'+\Gamma^5\otimes\Gamma'^5). \tag{A15}$$

With these conventions the algebra reads

$$\{q_i,q_j\}=-\frac{i}{2}\epsilon_{ij}\mathcal{P}(M_{AB}\Gamma^{AB}C\otimes C' - C\otimes M'_{A'B'}\Gamma'^{A'B'}C'),$$
$$[M_{AB},q_i]=-\frac{1}{2}\Gamma_{AB}q_i,$$
$$[M'_{A'B'},q_i]=-\frac{1}{2}\Gamma'_{A'B'}q_i \qquad (A16)$$

plus the conventional SO(2,2) and SO(4) pieces.

In order to achieve more closeness to 6D quantities it is useful to define

$$\hat{q}_i\equiv\epsilon_{ij}q_j^TC\otimes C', \qquad (A17)$$

which are the conjugate spinors since by the symplectic-Majorana condition we have

$$(q_i)^*=\epsilon_{ij}B\otimes B'q_j, \qquad (A18)$$

and to consider as fundamental supercharges $q\equiv q_{i=1}$ and $\hat{q}\equiv\hat{q}_{i=1}$. It is also convenient for later purposes to change the basis to

$$\Gamma^0\rightarrow\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix},\quad \Gamma'^0\rightarrow\begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix}. \qquad (A19)$$

The components of q in this basis which survive the projection are

$$Q_1\equiv q_{11'}+q_{22'},\quad Q_2\equiv q_{12'}-q_{21'}, \qquad (A20)$$

where the indices denote I,I'. Although this is a source for confusion, let us denote these generators by Q_I, where I is not to be confused with the SO(2,2) index. It is of crucial importance for symmetry considerations to know that

$$\hat{Q}_I=(\hat{Q})_I=\hat{q}_{1I'}+\epsilon_{I'J'}\hat{q}_{2J'}=+\epsilon_{IJ}Q_J^\dagger\sigma_3, \qquad (A21)$$

where σ_3 acts on the α index of $Q^\dagger$. With $P_a,P_{a'},J_{ab},J_{a'b'}$ as defined in Eq. (2.7) we can write down the algebra

$$\{Q_I,\hat{Q}_J\}=-i\delta_{IJ}(J_{ab}\gamma^{ab}-J_{a'b'}\gamma^{a'b'}) +2i\epsilon_{IJ}(iP_a\gamma^a-P_{a'}\gamma^{a'}), \qquad (A22)$$

$$[P_a,Q_I]=-\frac{i}{2}\epsilon_{IJ}\gamma_aQ_J,\quad [P_{a'},Q_I]=\frac{1}{2}\epsilon_{IJ}\gamma_{a'}Q_J,$$
$$[M_{ab},Q_I]=-\frac{1}{2}\gamma_{ab}Q_I,\quad [M_{a'b'},Q_I]=-\frac{1}{2}\gamma_{a'b'}Q_I,$$
$$[P_a,\hat{Q}_I]=\frac{i}{2}\hat{Q}_J\epsilon_{JI}\gamma_a,\quad [P_{a'},\hat{Q}_I]=-\frac{1}{2}\hat{Q}_J\epsilon_{JI}\gamma_{a'},$$
$$[M_{ab},\hat{Q}_I]=\frac{1}{2}\hat{Q}_I\gamma_{ab},\quad [M_{a'b'},\hat{Q}_I]=\frac{1}{2}\hat{Q}_I\gamma_{a'b'}, \qquad (A23)$$

$$[M_{AB},M_{CD}]=\eta_{BC}M_{AD}+\eta_{AD}M_{BC}-\eta_{AC}M_{BD}-\eta_{BD}M_{AC},$$
$$[M_{A'B'},M_{C'D'}]=\delta_{B'C'}M_{A'D'}+\delta_{A'D'}M_{B'C'}-\delta_{A'C'}M_{B'D'}-\delta_{B'D'}M_{AC}.$$

In verifying the Jacobi identities, heavy use was made of the following identities:

$$(\sigma^a)_\alpha^\gamma(\sigma^a)_\beta^\delta=2\delta_\alpha^\delta\delta_\beta^\gamma-\delta_\alpha^\gamma\delta_\beta^\delta, \qquad (A24)$$

$$(\gamma^{ab})_\alpha^\gamma(\gamma_{ab})_\beta^\delta=-4\delta_\alpha^\delta\delta_\beta^\gamma+2\delta_\alpha^\gamma\delta_\beta^\delta. \qquad (A25)$$

So far, the 6D covariance of the algebra is not quite obvious. However, if we define the 6D gamma matrices as in Eq. (2.3), the chiral 6D supercharges Q as in Eq. (2.2) and $\overline{Q}$ as in Eq. (2.4) we find from Eq. (A23) precisely Eq. (2.6). To see this it is noteworthy that $\hat{Q}$ and $\overline{Q}$ are related by

$$\hat{Q}_I=-i\epsilon_{IJ}\overline{Q}_J. \qquad (A26)$$

[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).

[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).

[3] E. Witten, Adv. Theor. Phys. **2**, 253 (1998).

[4] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. **B533**, 109 (1998).

[5] R. Kallosh, J. Rahmfeld, and A. Rajaraman, J. High Energy Phys. **09**, 002 (1998).

[6] R. Kallosh, ''Superconformal actions in killing gauge,'' hep-th/9807206 (1998).

[7] R. Kallosh and J. Rahmfeld, ''The gs string action on $ads_5\times S^5$,'' hep-th/9808038 (1998).

[8] R. Kallosh and A. A. Tseytlin, J. High Energy Phys. **10**, 016 (1998).

[9] A. Giveon, D. Kutasov, and N. Seiberg, Adv. Theor. Math. Phys. **2**, 733 (1998).

[10] M. B. Green and J. H. Schwarz, Phys. Lett. **136B**, 367 (1984).

[11] J. Maldacena and A. Strominger, J. High Energy Phys. **12**, 605 (1998).

[12] K. Behrndt, I. Brunner, and I. Gaida, Phys. Lett. B **432**, 310 (1998).

[13] S. Deger, A. Kaya, E. Sezgin, and P. Sundell, Nucl. Phys. **B536**, 110 (1998).

[14] J. de Boer, ''Six-dimensional supergravity on $S^3\times ads_3$ and 2-d conformal field theory,'' hep-th/9806104 (1998).

[15] M. J. Duff, H. Lu, and C. N. Pope, Nucl. Phys. **B544**, 145 (1999).

[16] L. J. Romans, Nucl. Phys. **B276**, 71 (1986).

[17] R. Kallosh and A. Rajaraman, Phys. Rev. D **58**, 125003 (1998).

[18] P. Claus, R. Kallosh, J. Kumar, P. Townsend, and A. v. Proeyen, J. High Energy Phys. **06**, 004 (1998).

[19] R. Kallosh, Phys. Rev. D **57**, 3214 (1998).

[20] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory*

(Cambridge University Press, Cambridge, England, 1987), Vol. 1, Introduction.

[21] E. Bergshoeff, E. Sezgin, and P. K. Townsend, Phys. Lett. **169B**, 191 (1986).

[22] M. Awada, P. K. Townsend, and G. Sierra, Class. Quantum Grav. **2**, L85 (1985).

[23] M. J. Duff, J. T. Liu, and J. Rahmfeld, Nucl. Phys. **B459**, 125 (1996).

[24] R. R. Metsaev and A. A. Tseytlin, Phys. Lett. B **436**, 281 (1998).

[25] H. Lu, C. N. Pope, and J. Rahmfeld, ''A construction of killing spinors on S^n,'' hep-th/9805151 (1998).

[26] P. Claus, R. Kallosh, and A. v. Proeyen (unpublished).

[27] H. Pesando, J. High Energy Phys. **02**, 007 (1999).