Supersymmetry and primordial black hole abundance constraints

Anne M. Green
Astronomy Unit, School of Mathematical Sciences, Queen Mary and Westfield College, Mile End Road, London E1 4NS, United Kingdom

(Received 1 April 1999; published 24 August 1999)

We study the consequences of supersymmetry for primordial black hole (PBH) abundance constraints. PBHs will emit supersymmetric particles during their evaporation when their mass is less than about 10^{11} g. In most models of supersymmetry the lightest of these particles, the lightest supersymmetric particle (LSP), is stable and will hence survive to the present day. We calculate the limit on the initial abundance of PBHs from the requirement that the present day LSP density be less than the critical density. We apply this limit, along with those previously obtained from the effects of PBH evaporation on nucleosynthesis and the present day density of PBHs, to PBHs formed from the collapse of inflationary density perturbations in the context of supersymmetric inflation models. If the reheat temperature after inflation is low, so as to avoid the overproduction of gravitinos and moduli, then the lightest PBHs which are produced in significant numbers will be evaporating around the present day and there are therefore no constraints from the effects of the evaporation products on nucleosynthesis or from the production of LSPs. We then examine models with a high reheat temperature and a subsequent period of thermal inflation. In these models avoiding the overproduction of LSPs limits the abundance of low mass PBHs which were previously unconstrained. Throughout we incorporate the production, at fixed time, of PBHs with a range of masses, which occurs when critical collapse is taken into account. [S0556-2821(99)01318-1]

PACS number(s): 98.80.Cq, 14.80.Ly

I. INTRODUCTION

Primordial black holes (PBHs) may form in the early universe via a number of mechanisms [1,2], the simplest of which is the collapse of large density perturbations [1]. Because of quantum effects, PBHs evaporate, mimicking the emission from a blackbody with finite size and temperature $T_{\rm BH}$ where

$$T_{\rm BH}=\frac{\hbar c^3}{8\pi G M_{\rm BH}}=1.06\left(\frac{10^{13}}{M_{\rm BH}}\right)\ \ {\rm GeV}, \tag{1}$$

and $M_{\rm BH}$ is the PBH mass in grams. The standard picture of PBH evaporation is that all particles which appear elementary at the energy scale of the PBH and have rest mass less than $T_{\rm BH}$ are emitted directly [3]. For instance PBHs with $T_{\rm BH}$ above the QCD quark-hadron transition scale $\Lambda_{\rm QH}$ $\approx$250–300 MeV, emit relativistic quark and gluon jets which then fragment into photons, leptons and hadrons as in high energy accelerator collisions.

The mass loss, in grams per second, is given by

$$\frac{{\rm d}M_{\rm BH}}{{\rm d}t}=-5.34\times10^{25}f(M_{\rm BH})M_{\rm BH}^{-2}, \tag{2}$$

where $f(M_{\rm BH})$ is a function of the number of species emitted and is normalized to 1 for PBHs with $M_{\rm BH}\gg10^{17}$ g which emit only massless particles (see Ref. [3] and Sec. II for more details). The effects of the evaporation products allow constraints to be put on the initial abundance of PBHs over a range of masses. PBHs with mass in the range 10^9 g $<M_{\rm BH}<10^{13}$ g would have evaporated after nucleosynthesis and could have a number of effects on the successful predictions of nucleosynthesis [4] while PBHs with $M_{\rm BH}$ $\sim5\times10^{14}$ g are evaporating today and the number density of photons produced must not exceed the observed γ-ray background [5]. In the case of lighter PBHs with $T_{\rm BH}>100$ GeV the fundamental particles emitted, and hence $f(M_{\rm BH})$, depend upon the particle physics model assumed.

Currently the widely accepted extension of the standard model is supersymmetry [6], where each standard model particle has a supersymmetric partner known as a sparticle. Motivated mainly as an attempt to understand why the weak scale is much smaller than the Planck scale (known as the gauge hierarchy problem [7]), supersymmetry also leads to the unification of gauge couplings at an energy of about 10^{16} GeV [8]. The phenomenology of the sparticles is complicated, being governed by up to 105 independent and unknown parameters. In the simplest models the sparticles have masses of order 100 GeV and there is a multiplicatively conserved quantum number known as R parity, where standard model particles have $R=+1$ and sparticles have $R=-1$. Consequentially heavier sparticles decay into lighter sparticles and the lightest supersymmetric particle (LSP) is stable, since it has no allowed decay mode. In most models the LSP is nonrelativistic at freeze-out and is therefore a candidate for the cold dark matter (CDM).[1]

The LSP must be neutral and weakly interacting [10] as otherwise it would have condensed, along with the baryonic matter, into astrophysical structures and the resultant abundance of anomalous heavy isotopes would exceed observational limits [11]. The LSP may therefore be a sneutrino or

[1]In gauge-mediated models of supersymmetry breaking the LSP could be a gravitino with mass ~1 keV which would constitute warm dark matter [9].

the gravitino but in most supersymmetric theories it is the lightest neutralino χ, which is a mix of the supersymmetric partners of the photon, the Z boson and the neutral Higgs boson. Throughout this paper we therefore use χ to denote the LSP.

Purely experimental searches at the CERN e^+e^- collider LEP have led to the limit $m_\chi \geqslant 30$ GeV [12]. This limit can be tightened to $m_\chi > 42$ GeV by making various theoretical assumptions and requiring that the present day LSP density lie in the interesting range for CDM: $0.1 < \Omega_\chi h^2 < 0.3$ [13]. The LSP comoving number density has remained constant since annihilations ceased at the freeze-out temperature T_f. This leads to a simple estimate (see Ref. [14] for a review of this and more detailed calculations) of the current LSP density:

$$\Omega_\chi h^2 = \frac{10^{-3}}{\langle \sigma_{\mathrm{ann}}(\chi\chi) v_\chi \rangle T_0 m_{\mathrm{Pl}}}, \tag{3}$$

where $\langle \sigma_{\mathrm{ann}}(\chi\chi) v_\chi \rangle$ is the thermally averaged annihilation cross section. As m_χ increases $\langle \sigma_{\mathrm{ann}}(\chi\chi) v_\chi \rangle$ decreases so that $\Omega_\chi h^2$ increases. This leads to an upper limit of $m_\chi \leqslant 300$ GeV [15], or $m_\chi \leqslant 600$ GeV if co-annihilations with the stau slepton, which are important in some regions of parameter space, are taken into account [16].

Supersymmetric particles which are produced by the evaporation of PBHs after the temperature of the universe has fallen below T_f will not be able to equilibrate due to the inefficiency of annihilations. In Sec. II we calculate the mass range of PBHs which are heavy enough to evaporate after LSP freeze-out but light enough to produce sparticles. The LSPs produced by the decay of these sparticles, along with the LSPs evaporated directly, will therefore provide an additional contribution to the present day density of LSPs. While it is possible that the number density of LSPs produced via PBH evaporation may be comparable to the freeze-out number density, reducing the upper limit on Ω_χ and hence m_χ, this would require extreme fine-tuning of the initial abundance of PBHs. We therefore use the conservative requirement that the present day density of LSPs produced via PBH evaporation after T_f is less than the critical density, to constrain the initial abundance of LSP producing PBHs.

In Sec. III we outline the resultant constraints on the mass fraction of the universe in PBHs formed from the collapse of inflationary density perturbations, taking into account the formation, due to critical collapse [17], of PBHs with a range of masses at fixed horizon mass. Since we are assuming that supersymmetry is the correct model of particle physics above ~ 100 GeV we must apply these constraints in the context of supersymmetric inflation models. In Sec. IV we discuss the constraints on successful supersymmetric inflation models and calculate and review PBH abundance constraints for two classes of inflation model: first those where the reheat temperature after inflation is low and then those with a high reheat temperature and a subsequent period of thermal inflation.

II. ABUNDANCE OF LSPs EMITTED BY PBHs

A PBH will emit LSPs if T_{BH} is greater than m_χ so that, using Eq. (1), if $M_{\mathrm{BH}} < M_1$ where

$$M_1 = \frac{1.06 \times 10^{13}\ \mathrm{GeV}}{m_\chi}, \tag{4}$$

then the PBH will emit LSPs throughout its evaporation. A PBH with mass greater than M_1 will emit LSPs during the later stages of its evaporation once its mass has fallen below M_1. The fraction of the PBH energy density eventually in LSPs will be reduced by a factor of M_1/M_{BH} relative to that for PBHs with $M_{\mathrm{BH}} < M_1$. In this paper we will focus on the case of PBHs formed from the collapse of large inflationary density perturbations. If the horizon mass at the time the PBHs form is M_{H}, then if $M_{\mathrm{H}} < M_1$, the LSP emission will be dominated by PBHs with $M_{\mathrm{BH}} < M_1$. On extrapolating the constraint on the initial mass fraction in PBHs which is found in Sec. IV B (and incorporating the additional weakening factor of M_{BH}/M_1) we can see that for $M_{\mathrm{H}} > M_1$ the constraint from LSP emission will be weaker than those from the effects of PBH evaporation on nucleosynthesis. Therefore we will only calculate the constraints from LSP emission by PBHs with $M_{\mathrm{BH}} < M_1$.

The PBH lifetime, in grams per second, is [3]

$$\tau(M_{\mathrm{BH}}) = \frac{6.24 \times 10^{-27} M_{\mathrm{BH}}^3}{f(M_{\mathrm{BH}})}, \tag{5}$$

so that the temperature at evaporation, $T_{\mathrm{evap}} = T_{\mathrm{Pl}}[t_{\mathrm{Pl}}/\tau(M_{\mathrm{BH}})]^{1/2}$, is given by

$$T_{\mathrm{evap}} = \left(\frac{1.24 \times 10^{21} f(M_1)}{M_{\mathrm{BH}}^3} \right)^{1/2} \mathrm{GeV}. \tag{6}$$

For evaporation to occur after LSP freeze-out ($T_{\mathrm{evap}} < T_\mathrm{f}$), M_{BH} must be greater than M_2 where, using the fact that $T_\mathrm{f} \sim m_\chi/25$,

$$M_2 = \left(\frac{7.75 \times 10^{23} f(M_2)}{m_\chi^2} \right)^{1/3}. \tag{7}$$

A PBH will therefore evaporate after LSP freeze-out and also emit LSPs throughout its evaporation if its mass is in the range $M_2 < M_{\mathrm{BH}} < M_1$.

If at some initial time t_i the fraction of the total energy density of the universe in PBHs which evaporate after freeze-out producing LSPs is $\beta_{\chi,\mathrm{i}} = \rho^\chi_{\mathrm{pbh,i}}/\rho_{\mathrm{tot,i}}$, then immediately before the PBHs evaporate

$$\left(\frac{\rho_{\mathrm{pbh}}}{\rho_{\mathrm{rad}}} \right)_{\mathrm{evap}} = \frac{\beta_{\chi,\mathrm{i}}}{1 - \beta_{\chi,\mathrm{i}}} \frac{T_\mathrm{i}}{T_{\mathrm{evap}}}. \tag{8}$$

If the PBHs dominate the energy density of the universe before they evaporate, then, since the radiation emitted by the PBH dominates the background radiation energy density,

soon after evaporation the LSPs will come to dominate the energy density of the universe soon afterwards. To avoid this we require

$$\frac{\beta_{\rm i}}{1-\beta_{\rm i}} < \frac{T_{\rm evap}}{T_{\rm i}}. \tag{9}$$

The LSPs emitted will initially be relativistic with mean energy per particle $3T_{\rm BH}$. The ratio of the energy density in LSPs to that in radiation is therefore constant,

$$\left(\frac{\rho_\chi}{\rho_{\rm rad}}\right) = \epsilon_\chi, \tag{10}$$

where ϵ_χ is the fraction of the PBH mass energy evaporated into LSPs, until the LSPs become non-relativistic at $T_{\rm nr}$ where, since $\rho \propto T^4$ for relativistic fluids,

$$T_{\rm nr} = T_{\rm evap}\left(\frac{m_\chi}{3T_{\rm BH}}\right)^{1/4}. \tag{11}$$

At any subsequent epoch, with temperature T, before matter-radiation equality

$$\left(\frac{\rho_\chi}{\rho_{\rm rad}}\right) = \left(\frac{\rho_\chi}{\rho_{\rm rad}}\right)_{\rm nr} \frac{T_{\rm nr}}{T} = \left(\frac{\rho_\chi}{\rho_{\rm rad}}\right)_{\rm evap} \frac{T_{\rm evap}}{T}\left(\frac{m_\chi}{3T_{\rm BH}}\right)^{1/4}. \tag{12}$$

The fraction of the energy density of the universe in LSPs at the present day is therefore given by

$$\begin{aligned}\Omega_{\chi,0} = \Omega_{\chi,{\rm eq}} &= 2\frac{T_{\rm evap}}{T_{\rm eq}}\left(\frac{m_\chi}{3T_{\rm BH}}\right)^{1/4}\left(\frac{\rho_\chi}{\rho_{\rm rad}}\right)_{\rm evap} \\ &= 2\epsilon_\chi \frac{\beta_{\chi,{\rm i}}}{1-\beta_{\chi,{\rm i}}}\frac{T_{\rm i}}{T_{\rm eq}}\left(\frac{m_\chi}{3T_{\rm BH}}\right)^{1/4},\end{aligned} \tag{13}$$

where ''eq'' denotes the epoch of matter-radiation equality. This relation can be inverted simply to obtain the constraint on $\beta_{\chi,{\rm i}}$ from the requirement $\Omega_{\chi,0} < 1$, as a function of $T_{\rm i}$. This constraint is independent of the mechanism of PBH formation and while we have assumed that all PBHs form at the same time $t_{\rm i}$ it would be simple to recalculate the constraint allowing for PBH formation at a range of times.

The relevant mass range and the fraction of the mass of the PBH which is evaporated into LSPs both depend on $f(M_{\rm BH})$. To fully calculate $f(M_{\rm BH})$ as a function of $M_{\rm BH}$ we would need to know the full mass spectrum of the sparticles. We can however calculate $f(M_{\rm BH})$ in two limiting cases. If $T_{\rm BH} \sim m_\chi$, then the LSP will be the only sparticle emitted (along with photons, gravitons, gluons, the three lepton families and six quark flavors), while if $T_{\rm BH} \gg m_\chi$, then all the standard model particles (including the Higgs, Z and W bosons) and their supersymmetric partners will be emitted. The relativistic contributions to $f(M_{\rm BH})$ per particle degree of freedom are [3]

$$f_{s=0} = 0.267, \quad f_{s=1} = 0.060,$$

$$f_{s=3/2} = 0.020, \quad f_{s=2} = 0.007,$$

$$f_{s=1/2} = 0.147 \ \text{uncharged},$$

$$f_{s=1/2} = 0.142 \ \text{electrically charged}. \tag{14}$$

A. $T_{\rm BH} \sim m_\chi$

In this regime a PBH can emit the 3 lepton families, 6 quark flavors,[2] the graviton, the gluon and the LSP. The photon has 2 possible polarization states and the gluon has 48 degrees of freedom, giving a total of 50 $s=1$ degrees of freedom. Each type of neutrino has 2 degrees of freedom, resulting in 6 neutral $s=1/2$ degrees of freedom. The electron, muon and tau leptons each have 4 degrees of freedom, while each quark has 12 degrees of freedom, resulting in a total of 84 charged $s=1/2$ states. Finally the graviton has $s=2$ and 4 degrees of freedom. The total contribution of standard model states to $f(M_{\rm BH})$ is therefore $(50\times 0.06)+(6\times 0.147)+(84\times 0.142)+(4\times 0.007)=15.84$. Finally, as outlined in the Introduction, the LSP is most likely to be the lightest neutralino which is uncharged and has $s=1/2$, giving a final value of $f(M_{\rm BH})=15.84+(2\times 0.147)=16.13$. The fraction of the total mass which is evaporated into the LSP is $\epsilon_\chi = 0.294/16.13 = 0.018$.

B. $T_{\rm BH} \gg m_\chi$

If $T_{\rm BH}$ is much larger than m_χ, all the standard model particles and their supersymmetric partners will be emitted. In the minimal supersymmetric standard model there are 2 Higgs doublet fields which give rise to 5 physical states: the charged Higgs scalar, the light Higgs scalar, the heavy Higgs scalar and the Higgs pseudoscalar (see, e.g., Ref. [14] for details). Along with the W and Z bosons these states lead to 9 $s=1$ and 5 $s=0$ degrees of freedom in addition to the standard model degrees of freedom considered in Sec. II A, so that the total contribution of standard model states to $f(M_{\rm BH})$ is 17.10.

Each standard model (SM) degree of freedom had a supersymmetric (SUSY) degree of freedom with $s=1/2$, 0, 1/2 and 3/2 for standard model states with $s=0,1/2,1$ and 2 respectively. This leads to a grand total of 95 $s=0$ (5 SM, 90 SUSY), 92 charged $s=1/2$ (84 SM, 8 SUSY), 30 uncharged $s=1/2$ (6 SM, 56 SUSY), 59 $s=1$ (all SM), 4 $s=3/2$ (all SUSY) and 4 $s=2$ (all SM) degrees of freedom, giving $f(M_{\rm BH})=46.79$. The fraction of the total mass evaporated into supersymmetric particles is $\epsilon_{\rm SUSY} = 29.69/46.79 = 0.63$, with $\epsilon_\chi = 0.294/46.79 = 0.006$ of the total mass evaporated directly into LSPs. While the fraction of the PBH mass evaporated directly into LSPs is small all the other supersymmetric particles emitted will decay rapidly, producing at least one additional LSPs each so that we can set $\epsilon_\chi = \epsilon_{\rm SUSY}$ when we estimate the fraction of the PBH mass ending up in the form of LSPs.

[2]The LSP mass may be smaller than that of the top quark in which case there may be a narrow range of PBH masses where the LSP is emitted but the top quark is not, which would decrease the value of $f(M_{\rm BH})$ calculated here by about 1.704.

III. PBH FORMATION FROM DENSITY PERTURBATIONS

In order for a PBH to be formed, a collapsing region must be large enough to overcome the pressure force resisting its collapse as it falls within its Schwarzschild radius. This occurs if the perturbation is bigger than a critical size δ_c at horizon crossing. There also is an upper limit of $\delta<1$ since a perturbation which exceeded this value would correspond initially to a separate closed universe [18]. Analytic calculations [1] find $\delta_c\sim 1/3$ and assume that all PBHs have mass roughly equal to the horizon mass at the time they form, independent of the size of the perturbation. Recent studies [17] of the evolution of density perturbations have found that the mass of the PBH formed in fact depends on the size of the perturbation:

$$M_{\rm BH}=kM_{\rm H}(\delta-\delta_c)^{\gamma}, \tag{15}$$

where $\gamma\approx 0.37$ and k and δ_c are constant for a given perturbation shape (for Mexican hat shaped fluctuations $k=2.85$ and $\delta_c=0.67$).

In order to determine the number of PBHs formed on a given scale we must smooth the density distribution using a window function, $W(kR)$. For Gaussian distributed fluctuations the probability distribution of the smoothed density field $p(\delta(M_{\rm H}))$ is given by

$$p(\delta(M_{\rm H}))\mathrm{d}\delta(M_{\rm H})=\frac{1}{\sqrt{2\pi}\sigma(M_{\rm H})}\times\exp\left(-\frac{\delta^2(M_{\rm H})}{2\sigma^2(M_{\rm H})}\right)\mathrm{d}\delta(M_{\rm H}), \tag{16}$$

where $\sigma(M_{\rm H})$ is the mass variance evaluated at horizon crossing. For power law spectra, $P(k)\propto k^n$, where $P(k)=\langle|\delta_{\mathbf{k}}|^2\rangle$ and n is the spectral index, $\sigma^2(M_{\rm H})\propto M_{\rm H}^{(1-n)/4}$ during radiation domination [19]. The formation of PBHs on a range of scales has recently been studied [20] for power law power spectra and for flat spectra with a spike on a given scale. In both cases it was found that, in the limit where the number of PBHs formed is small enough to satisfy the observational constraints on their abundance at evaporation and at the present day, it can be assumed that all the PBHs form at a single horizon mass. In particular, if the power spectrum is a power law with $n>1$, as is the case in tree-level hybrid inflation models, all PBHs form at the smallest horizon scale immediately after reheating.

The initial mass fraction of the universe in PBHs with masses in the range $M_2<M_{\rm BH}<M_1$, which evaporate after LSP freeze out and produce LSPs throughout their evaporation, is given by

$$\beta_{\chi{\rm i}}=\int_{\delta_2}^{\delta_1}\frac{M_{\rm BH}}{M_{\rm H}}p(\delta(M_{\rm H}))\mathrm{d}\delta(M_{\rm H})=\int_{\delta_2}^{\delta_1}\frac{k(\delta-\delta_c)^{\gamma}}{\sqrt{2\pi}\sigma(M_{\rm H})}\exp\left(-\frac{\delta^2}{2\sigma^2(M_{\rm H})}\right)\mathrm{d}\delta(M_{\rm H}), \tag{17}$$

where, using Eq. (15),

$$\delta_{\rm j}=\delta_c+\left(\frac{M_j}{kM_{\rm H}}\right)^{\gamma}:\quad j=1,2. \tag{18}$$

Equation (17) can be used to translate the constraints on $\beta_{\chi,{\rm i}}$ into constraints on $\sigma(M_{\rm H})$, which in turn can be used to constrain n [21,22,19]. We can also find the maximum fraction of the universe in PBHs of all mass, $\beta_{\rm i}$:

$$\beta_{\rm i}=\int_{\delta_c}^{1}\frac{k(\delta-\delta_c)^{\gamma}}{\sqrt{2\pi}\sigma(M_{\rm H})}\exp\left(-\frac{\delta^2}{2\sigma^2(M_{\rm H})}\right)\mathrm{d}\delta(M_{\rm H}). \tag{19}$$

IV. CONSTRAINTS ON SUPERSYMMETRIC INFLATION MODELS

Since we are assuming that supersymmetry is the correct model of particle physics above $\sim$100 GeV, then we must apply the constraints on the abundance of PBHs formed from the collapse of large inflationary density perturbation to models of inflation constructed in the context of supersymmetry. Avoiding the overproduction of various relic particles, which would alter the subsequent evolution of the universe and wreck the successful predictions of the standard hot big bang model, leads to constraints on supersymmetric inflation models.

The gravitino (the supersymmetric partner of the graviton) has only gravitational interactions and mass of order 100 GeV, and will decay after nucleosynthesis. The requirement that its decay products do not destroy the successful predictions of big-bang nucleosynthesis places limits on its abundance. Since the number density of gravitinos is proportional to the reheat temperature after inflation, $T_{\rm RH}$, this leads to an upper limit on $T_{\rm RH}$ [23]. The exact limit depends on the gravitino mass, the available decay channels and the baryon-to-photon ratio before the gravitino decays [24] but, very conservatively, $T_{\rm RH}$ must be less than 10^9 GeV. Similarly in almost all theories in which supersymmetry is broken at an intermediate scale there are scalar fields, known as moduli,[3] which typically have the same mass and lifetime as the gravitino [25]. Avoiding their production after inflation requires $T_{\rm RH}$ to be less than 10^{12} GeV.

There are several ways to avoid the gravitino and moduli problems. First, inflation models can be constructed where inflation occurs at a low energy scale [26] so that the reheat temperature is automatically low enough to avoid these problems. However, these models in general require fine-tuning [27]. Second, the reheat temperature can be sufficiently low if the inflaton is long lived. For instance a model has been constructed [28], using a singlet field in a hidden sector, where inflation occurs at the scale of the spontaneous breaking of the gauge symmetry which is of order 10^{14} GeV. In this case, since the inflaton has only gravitational strength

[3]Specific examples are the dilaton of string theory and the massless gauge singlets of string compactifications or, in general, any gauge singlet field responsible for SUSY breaking.

couplings, the reheat temperature is only of order 10^5 GeV.

There is however a generic solution to these problems in high energy scale inflation models which relies on the properties of flaton fields which also arise naturally in supersymmetric theories [29]. Flaton fields, have vacuum expectation values $M \gg 10^3$ GeV, even though their mass m is only of the order of the supersymmetry scale, so that their potential is almost flat. In the early universe these fields are held at zero by finite temperature effects, with false vacuum energy density $V_0 \sim m^2 M^2$. Once the temperature falls below $V_0^{1/4}$, the false vacuum energy density dominates the thermal energy density of the universe and begins to drive a period of inflation known as thermal inflation. This inflation continues until the temperature drops to $T \sim m$, at which point thermal effects are no longer strong enough to anchor the flaton in the false vacuum. Taking $M \sim 10^{12}$ GeV gives $V_0^{1/4} \sim 10^7$ GeV so that around $\ln(10^7/10^3) \sim 10$ e-foldings of thermal inflation occur, sufficient to dilute the moduli and gravitinos existing before thermal inflation but small enough to not affect the density perturbations generated during the first period of inflation.

We examine two classes of inflation model: those with a low reheat temperature and those with a high reheat temperature and a subsequent period of thermal inflation.

A. Low reheat temperature

If $T_{\rm RH} = 10^9$ GeV, so as to just satisfy the gravitino constraint, then, using the relationship between horizon mass and temperature in a radiation dominated universe,

$$M_{\rm H} \simeq 10^{18}\ {\rm g} \left(\frac{10^7\ {\rm GeV}}{T} \right)^2 , \qquad (20)$$

the maximum horizon mass is 1×10^{14} g. Although when critical collapse is taken into account PBHs with a range of masses are formed at fixed horizon mass, the vast majority of PBHs have mass within an order of magnitude of $M_{\rm H}$. From Eq. (13), $\beta_{\chi,{\rm i}}$ must be less than 5×10^{-17}. However, for $m_\chi = 30$ GeV, only PBHs with $M_{\rm BH} < M_1 = 3.5 \times 10^{11}$ g $\sim 0.0035 M_{\rm H}$ are hot enough to emit LSPs. The fraction of PBHs which have mass this much smaller than the horizon mass is negligible so that there is no resultant constraint on $\beta_{\rm i}$. Similarly the constraints from the effect of the products of PBH evaporation on nucleosynthesis only hold for $M_{\rm BH} < 10^{13}$ g. Therefore in a low reheat temperature inflation model only the constraint from the present day density of PBHs and, if $T_{\rm RH} > 5 \times 10^8$ GeV, that on the abundance of PBHs evaporating today hold. The constraints on $\beta_{\rm i}$ are shown in Fig. 1. The U-shaped dip arises from the limit on the abundance of PBHs with mass $M_{\rm BH} \sim 5 \times 10^{14}$ g, which are evaporating at the present day (as calculated by Yokoyama [30]) and the straight line from the limit on the present day density of PBHs which have not evaporated.

The constraints on $\beta_{\rm i}$ can be translated into limits on the spectral index of the density perturbations [21,22,19] using Eq. (19) and the scale dependence of $\sigma(M_{\rm H})$:

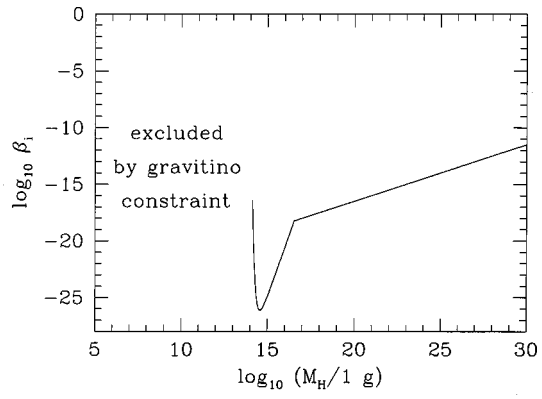

FIG. 1. The constraints on the initial mass fraction of PBHs, $\beta_{\rm i}$, in inflation models with $T_{\rm RH} < 10^9$ GeV.

$$\sigma(M_{\rm H}) = \sigma(M_0) \left(\frac{M_{\rm H}}{M_{\rm eq}} \right)^{(1-n)/4} \left(\frac{M_{\rm eq}}{M_0} \right)^{(1-n)/6} , \qquad (21)$$

where M_0 and $M_{\rm eq}$ are the horizon masses at the present day and at matter radiation equality and, using the Cosmic Background Explorer (COBE) normalization, $\sigma(M_0 = 10^{56}\ {\rm g}) = 9.5 \times 10^{-5}$. In models with $T_{\rm RH} < 10^9$ GeV, the tightest limit on n (~ 1.28) arises from the constraint on PBHs which are evaporating today. This is weaker than the limits found in Ref. [19] first because the tightest limits arise from the constraints on PBHs of mass $M_{\rm BH} \sim 10^{10-11}$ g which are not formed in low reheat temperature inflation models. Also the analytically derived value of $\delta_{\rm c}$ ($\sim 1/3$) used in those calculations is roughly half the new value $\delta_{\rm c} = 0.67$, determined from numerical simulations [17], which is used in this paper.

B. High reheat temperature with thermal inflation occurring

The effects of a period of thermal inflation on the constraints on PBH abundance were studied in Ref. [19]. It is assumed that thermal inflation commences at $T = 10^7$ GeV and continues until $T = 10^3$ GeV, when the flaton field rolls to its true vacuum state. Assuming for simplicity that reheating is efficient, then the universe is reheated to $T = 10^7$ GeV with the subsequent evolution of the universe having its standard form. The duration of thermal inflation is negligible compared to the PBH lifetime so that its main effect on the PBHs is to dilute their density by a factor $\rho_{\rm i}/\rho_{\rm f} = (a_{\rm f}/a_{\rm i})^3 \sim (10^4)^3$ so that the constraints on $\beta_{\rm i}$ are weakened by a factor of $\sim 10^{12}$.

If thermal inflation occurs, then the right hand sides of Eqs. (8)–(13) are each multiplied by a factor of 10^{-12} due to the dilution of the PBHs during thermal inflation. Using the relation between temperature and horizon mass, $T_{\rm i} = T_{\rm Pl}(m_{\rm Pl}/M_{\rm H})^{1/2}$, the requirement that the PBHs do not dominate the universe at evaporation becomes

$$\frac{\beta_{\chi,{\rm i}}}{1 - \beta_{\chi,{\rm i}}} < 6.7 \times 10^6 \left(\frac{M_{\rm H}}{M_{\rm BH}^3} \right)^{1/2} , \qquad (22)$$

and the limit from the present day density of LSPs becomes

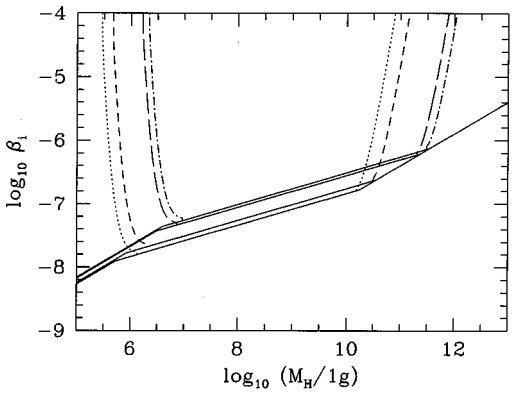

FIG. 2. The constraints on the initial mass fraction of PBHs from the present day density of LSPs, in supersymmetric inflation models with a high reheat temperature and a subsequent period of thermal inflation for $\epsilon_\chi = 0.018$. The solid lines show the constraints on $\beta_{\chi,\mathrm{i}}$, the fraction of the universe in PBHs which evaporate after freeze-out and produce LSPs throughout their evaporation, for (from bottom to top) $m_\chi = 600$, 300, 45 and 30 GeV. The dotted, short dashed, long dashed and dot-dashed lines show the maximum allowed fraction of the universe in PBHs of all masses, β_i, for $m_\chi = 600$, 300, 45 and 30 GeV, respectively.

$$\frac{\beta_{\chi,\mathrm{i}}}{1-\beta_{\chi,\mathrm{i}}} < \frac{5\times 10^{11}}{\epsilon_\chi}\frac{T_\mathrm{eq}}{T_\mathrm{Pl}}\left(\frac{M_\mathrm{H}}{m_\mathrm{Pl}}\right)^{1/2}\left(\frac{3\times 10^{13}}{m_\chi M_\mathrm{BH}}\right)^{1/4}. \qquad (23)$$

To evaluate these constraints on $\beta_{\chi,\mathrm{i}}$ we neglect the spread in PBHs masses, since the vast majority of PBHs will have the same mass to within a factor of a few. If M_H is between M_2 and M_1, then most of the PBHs will have $M_\mathrm{BH} \sim M_\mathrm{H}$, while if $M_\mathrm{H} < M_2$, the vast majority of the PBHs which evaporate after freeze-out will have $M_\mathrm{BH} \sim M_2$ and similarly if $M_\mathrm{H} > M_1$ most of the LSP producing PBHs will have $M_\mathrm{BH} \sim M_1$. The tightest constraint arises from the present day abundance of LSPs for $M_\mathrm{H} < M_1$ and also for $M_\mathrm{H} > M_1$ if $M_1 < 2\times 10^{12}$ g (which is the case for $m_\chi < 30$ GeV as found experimentally). We calculated the resulting constraints on $\sigma(M_\mathrm{H})$, and hence β_i, for 4 sample values of the LSP mass: $m_\chi = 30$, 45, 300 and 600 GeV, using $\epsilon_\chi \sim 0.018$. As the horizon mass decreases the fraction of the total number of PBHs formed which are heavy enough to evaporate after freeze-out decreases. Similarly as the horizon masses increases the fraction of PBHs which are light enough to emit LSPs decreases. Emission of LSPs after freeze-out constrains β_i for 10^6 g $\leqslant M_\mathrm{H} \leqslant 10^{10}$ g. The constraints on $\beta_{\chi,\mathrm{i}}$ and β_i are shown in Fig. 2. In Fig. 3 the constraints on β_i for $m_\chi = 45$ GeV are shown, along with those from the effects of PBH evaporation on the products of nucleosynthesis,[4] the abundance of PBHs evaporating at present and the present day density of PBHs as calculated in Refs. [19] and [30]. The missing mass range corresponds to comoving scales which

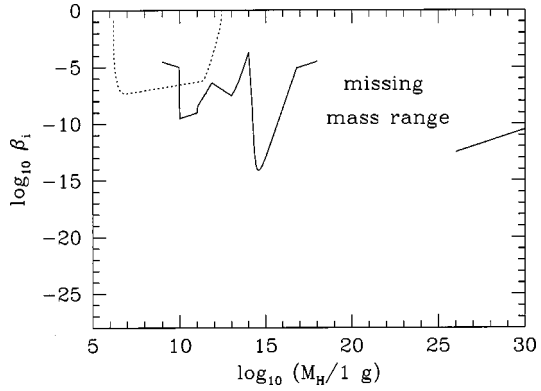

FIG. 3. A compilation of the constraints on the initial mass fraction of PBHs in supersymmetric inflation models with a high reheat temperature and a period of thermal inflation. The dotted line shows the constraints from the present day density of LSPs. From left to right the solid lines show the constraints from the effects of PBH evaporation after nucleosynthesis, the abundance of PBHs evaporating at present and the present day density of PBHs formed before and (on the far right) after thermal inflation.

enter the horizon before thermal inflation, and are then pulled back outside again during thermal inflation. Any new density perturbations are expected to be small, since the energy scale of thermal inflation is much lower than the original inflationary period, and hence unable to form black holes when they re-enter the horizon again after thermal inflation.

If thermal inflation occurs, then the temperature at which a given comoving scale crosses the Hubble radius is changed, while the relation between horizon mass and temperature remains the same so that the scale dependence of $\sigma(M_\mathrm{H})$ becomes [19]

$$\sigma(M_\mathrm{H}) = \sigma(M_0)\left[\left(\frac{10^7}{10^3}\right)^2\frac{M_\mathrm{H}}{M_\mathrm{eq}}\right]^{(1-n)/4}\left(\frac{M_\mathrm{eq}}{M_0}\right)^{(1-n)/6}. \qquad (24)$$

The limits on n from the constraints on β_i due to LSP emission range from $n < 1.36$ at $M_\mathrm{H} \sim 10^{11}$ g to $n < 1.33$ at $M_\mathrm{H} \sim 5\times 10^5$ g. These limits are slightly tighter than those from the nucleosynthesis constraints ($n \sim 1.34$–1.37 if the accurate value of $\delta_\mathrm{c} = 0.67$ is used).

V. CONCLUSIONS

We have examined the consequences of supersymmetry for PBH abundance constraints. PBHs with mass in the range 10^5 g $< M_\mathrm{BH} < 10^{11}$ g evaporate after LSP freeze-out and produce LSPs (and other supersymmetric particles) throughout their evaporation. In most models of supersymmetry the LSP is stable and the requirement that the present day density of LSPs not exceed the critical density places a limit on the initial abundance of PBHs in this mass range.

We have studied the constraints on PBH abundance for two classes of supersymmetric inflation model: those with a low reheat temperature and those with a high reheat temperature and a subsequent period of thermal inflation. If the reheat temperature is low, the lightest PBHs which can be produced will be evaporating at the present day and the con-

[4]The sharply rising line from $M_\mathrm{H} \sim 10^{13}$ g arises due to the constraint from entropy production after nucleosynthesis on PBHs with mass $M_\mathrm{BH} < 10^{13}$ g.

straints from the present day density of LSPs and the effects of evaporation on the products of nucleosynthesis, which provide the tightest limits on n, do not apply. For models with a high reheat temperature and a subsequent period of thermal inflation the constraints from the present day density of LSPs provide the tightest limit on n.

ACKNOWLEDGMENTS

A.M.G. was supported by PPARC and acknowledges use of the Starlink computer system at QMW. A.M.G. thanks Lev Kofman, Andrew Liddle and David Lyth for useful discussions.

[1] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. **168**, 399 (1974).

[2] S. W. Hawking, I. Moss, and J. Stewart, Phys. Rev. D **26**, 2681 (1981); M. Crawford and D. N. Schramm, Nature (London) **298**, 538 (1982); A. G. Polnarev and R. Zembowicz, Phys. Rev. D **43**, 1106 (1988); D. La and P. J. Steinhardt, Phys. Lett. B **220**, 375 (1989); S. W. Hawking, *ibid.* **231**, 237 (1989); S. D. H. Hsu, *ibid.* **251**, 343 (1990).

[3] J. H. MacGibbon and B. R. Weber, Phys. Rev. D **41**, 3052 (1990); J. H. MacGibbon, *ibid.* **44**, 376 (1991).

[4] Ya. B. Zel'dovich, A. A. Starobinsky, M. Y. Khlopov, and V. M. Chechetkin, Pis'ma Astron. Zh. **3**, 308 (1977) [Sov. Astron. Lett. **22**, 110 (1977)]; B. V. Vainer, D. V. Dryzhakova, and P. D. Nasselskii, *ibid.* **4**, 344 (1978) [**4**, 185 (1978)]; S. Mujana and K. Sato, Prog. Theor. Phys. **59**, 1012 (1978); B. V. Vainer and P. D. Nasselskii, Astron. Zh. **55**, 231 (1978) [Sov. Astron. **22**, 138 (1978)]; D. Lindley, Mon. Not. R. Astron. Soc. **193**, 593 (1980).

[5] J. H. MacGibbon, Nature (London) **320**, 308 (1987); J. H. MacGibbon and B. Carr, Astrophys. J. **371**, 447 (1991); H. I. Kim, C. H. Lee, and J. H. MacGibbon, Phys. Rev. D **59**, 063004 (1999).

[6] See, e.g., J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, 1983).

[7] L. Susskind, Phys. Rev. D **20**, 2619 (1979).

[8] U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B **260**, 447 (1991); C. Giunti, C. W. Kim, and U. W. Lee, Mod. Phys. Lett. A **6**, 1745 (1991); J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); **260**, 131 (1991); P. Langacker and M.-X. Luc, Phys. Rev. D **44**, 817 (1991).

[9] G. Giuduce and R. Rattazzi, hep-ph/9801271.

[10] J. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Nucl. Phys. **B238**, 453 (1984).

[11] J. Rich, M. Spiro, and J. Lloyd-Owen, Phys. Rep. **151**, 239 (1987); P. F. Smith, Contemp. Phys. **29**, 159 (1998); T. K. Hemmick *et al.*, Phys. Rev. D **41**, 2074 (1990).

[12] LEP Experiments Committee meeting, 12 November 1998, http://www.cern.ch/Committees/LEPC/minutes/LEPC50.html

[13] J. Ellis, T. Falk, K. Olive, and M. Schmitt, Phys. Lett. B **413**, 355 (1997).

[14] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996).

[15] K. A. Olive and M. Srednicki, Phys. Lett. B **230**, 78 (1989); Nucl. Phys. **B355**, 208 (1991); K. Griest, M. Kamionkowski, and M. S. Turner, Phys. Rev. D **41**, 3565 (1990).

[16] J. Ellis, T. Falk, and K. A. Olive, Phys. Lett. B **444**, 367 (1998).

[17] J. C. Niemeyer and K. Jedamzik, Phys. Rev. Lett. **80**, 5481 (1998); J. C. Niemeyer and K. Jedamzik, Phys. Rev. D **59**, 124013 (1999).

[18] E. R. Harrison, Phys. Rev. D **1**, 2726 (1970).

[19] A. M. Green and A. R. Liddle, Phys. Rev. D **54**, 6166 (1997).

[20] A. M. Green and A. R. Liddle, Phys. Rev. D **60**, 063509 (1999).

[21] B. J. Carr, J. H. Gilbert, and J. E. Lidsey, Phys. Rev. D **50**, 4853 (1994).

[22] H. I. Kim and C. H. Lee, Phys. Rev. D **54**, 6001 (1996).

[23] J. Ellis, D. Nanopoulos, and M. Quiros, Phys. Lett. B **174**, 176 (1986).

[24] T. Moroi, Ph.D. thesis, Tohoku, 1995, hep-ph/9503210.

[25] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby, and G. G. Ross, Phys. Lett. **131B**, 59 (1983); J. Ellis, D. V. Nanopoulos, and M. Quiros, Phys. Lett. B **174**, 176 (1986); B. de Carlos, J. A. Casas, F. Quevedo, and E. Roulet, *ibid.* **318**, 447 (1993); T. Banks, D. B. Kaplan, and A. E. Nelson, Phys. Rev. D **49**, 779 (1994); L. Randall and S. Thomas, Nucl. Phys. **B449**, 229 (1995); T. Banks, M. Berkooz, and P. J. Steinhardt, Phys. Rev. D **52**, 705 (1995); M. Dine, L. Randall, and S. Thomas, Phys. Rev. Lett. **75**, 398 (1995).

[26] L. Randall, M. Soljačić, and A. Guth, Nucl. Phys. **B472**, 377 (1996).

[27] L. Randall, in *Perspectives on Supersymmetry*, edited by G. L. Kane (World Scientific, Singapore, 1998).

[28] G. G. Ross and S. Sarkar, Nucl. Phys. **B461**, 597 (1996).

[29] D. H. Lyth and E. D. Stewart, Phys. Rev. Lett. **75**, 201 (1995); Phys. Rev. D **53**, 1784 (1996).

[30] J. Yokoyama, Phys. Rev. D **58**, 107502 (1998).