Tunneling in Λ decaying cosmologies and the cosmological constant problem

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The tunneling rate, with an exact prefactor, is calculated to first order in $\hbar$ for an empty closed Friedmann-Robertson-Walker universe with a decaying cosmological term $\Lambda \sim R^{-m}$ (R is the scale factor and m is a parameter $0 \leqslant m \leqslant 2$). This model is equivalent to a cosmology with the equation of state $p_\chi = (m/3 - 1)\rho_\chi$. The calculations are performed by applying the dilute-instanton approximation on the corresponding Duru-Kleinert path integral. It is shown that the highest tunneling rate occurs for $m=2$ corresponding to the cosmic string matter universe. The most probable cosmological term obtained, such as the one obtained by Strominger, accounts for a possible solution to the cosmological constant problem. [S0556-2821(99)03016-7]

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I. INTRODUCTION

The most accurately measured value of the cosmological constant Λ provided by measurements of the Hubble constant places an upper bound on its present value [1]:

$$|\Lambda|/8\pi G \leqslant 10^{-29} \ \ \mathrm{g/cm^3}.$$

According to modern quantum field theory, the structure of a vacuum turns out to be interrelated with some spontaneous symmetry-breaking effects through the condensation of quantum (scalar) fields. This phenomenon gives rise to a nonvanishing vacuum energy density $\rho_{\mathrm{vac}} \sim M_p^4$ (M_p is the Planck mass). The appearance of this characteristic mass scale may have an important effect on the cosmological constant because it receives potential contributions from this mass scale due to the mass spectrum of the corresponding physical fields in quantum field theory. By taking into account this contribution, an effective (observed) cosmological constant is defined as the sum of the bare cosmological constant λ and $8\pi G\rho_{\mathrm{vac}}$ [2,3]. This type of contribution, however, gives rise to an immediate difficulty called the cosmological constant problem. The essence of this problem is that it is very difficult to believe that the bare cosmological constant λ is fine tuned such that the effective cosmological constant Λ satisfies the above inequality after all symmetry breakings. There are some possible solutions to this problem rendering Λ exactly or almost exactly vanishing; some outstanding ones are as follows: (1) Trying to find some relaxation mechanisms by which Λ could relax to zero or its present small value [4]; (2) the Baum-Coleman-Hawking mechanism that wormhole solutions can lead the cosmological constant Λ to become a dynamical variable giving rise to distribution functions $P(\Lambda) \sim \exp(3M_p^2/8\Lambda)$ and $P(\Lambda) \sim \exp[\exp(3M_p^2/8\Lambda)]$ peaking at $\Lambda = 0$ [5]; (3) a Lorentzian analysis of quantum cosmological tunneling leading to the most probable value $\Lambda \sim 9M_p^2/16R^2$ at a given scale factor R [6]. One indirect solution to the cosmological constant problem is also suggested: (4) To assume that Λ is dynamically evolving and not constant, that is evolving from very large value to its present small value [2,3]. This last case, although not well stablished, is interesting to the present work. There are strong observational motivations for considering models in which Λ decreases as $\Lambda \sim R^{-m}$ (m is a parameter).

For $0 \leqslant m < 3$ [3], the effect of the decaying cosmological constant on the cosmic microwave background anisotropy is studied and the angular power spectrum for different values of m and density parameter Ω_{m0} is computed. Models with $\Omega_{m0} \geqslant 0.2$ and $m \geqslant 1.6$ are shown to be in good agreement with data.

For $m=2$ [2], it is shown that in the early universe Λ could be several tens of orders bigger than its present value, but not big enough to disturb the physics in the radiation-

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dominant epoch in the standard cosmology. In the matter-dominant epoch such a varying Λ shifts the three space curvature parameter k by a constant which changes the standard cosmology predictions reconciling observations with the inflationary scenario. Such a vanishing cosmological constant also leads to present creation of matter with a rate comparable to that in the steady-state cosmology. Although the ansatz $\Lambda \sim R^{-2}$ does not directly solve the cosmological constant problem but reduces it and the age problem to one and the same ''reduced problem'': How can our universe be so old having a radius R much larger than the Planck scale? [2].

In the present work we advocate the possibility that Λ varies as $R^{-m}(t)$ in favor of the forth solution to the cosmological constant problem. We approach this problem indirectly in the sense that we try to find a possible solution to the reduced problem instead of cosmological constant problem. We shall study the quantum tunneling for an empty closed Friedmann-Robertson-Walker (FRW) cosmology with $\Lambda \sim R^{-m}(t)$ $(0 \leqslant m \leqslant 2)$ as effectively being a cosmological model with an exotic χ fluid with the equation of state $p_\chi = (m/3-1)\rho_\chi$. Then we calculate the tunneling rate for this cosmology and show that the maximum tunneling rate corresponds to $\Lambda \sim R^{-2}$ $(m=2)$ as the most probable cosmological term as obtained by Strominger [6]. Therefore, we obtain a reasonable answer to the reduced problem insisting on the birth from ''nothing'' of our universe through the tunneling effect consistent with $\Lambda \sim R^{-2}$ simply because it can lead, through various symmetry breakings [2], to an old universe after tunneling. It is worth emphasizing that some connections between quantum tunneling and inflation [7] have already been discussed whose relation to this work may deserve further investigations.

We shall calculate the tunneling rate by applying the dilute-instanton approximation to first order in $\hbar$ [8], on the corresponding Duru-Kleinert path integral [9]. Its prefactor is calculated by the heat kernel method [10], using the shape invariance symmetry [11].

This paper is organized as follows. In Sec. II, the Duru-Kleinert path integral formula and Duru-Kleinert equivalence of corresponding actions is briefly reviewed. In Sec. III, we introduce the cosmological model of a closed FRW universe filled with an exotic fluid matter. This is effectively an empty closed FRW universe with an R varying cosmological term. Finally in Sec. IV, the tunneling rate for this model is fully calculated to first order in $\hbar$ by applying the dilute-instanton approximation on the corresponding Duru-Kleinert path integral. The paper is ended with a conclusion.

II. DURU-KLEINERT PATH INTEGRAL

In this section we briefly review the Duru-Kleinert path integral [12]. The fundamental object of path integration is the time displacement amplitude or propagator of a system $(X_b t_b | X_a t_a)$. For a system with a time independent Hamiltonian, the object $(X_b t_b | X_a t_a)$ supplied by a path integral is the causal propagator

$$(X_b t_b | X_a t_a) = \theta(t_a - t_b)\langle X_b | \exp[-i\hat{H}(t_b - t_a)/\hbar] | X_a \rangle. \tag{1}$$

Fourier transforming the causal propagator in the time variable, we obtain the fixed energy amplitude

$$(X_b | X_a)_E = \int_{t_a}^{\infty} dt_b e^{iE(t_b - t_a)/\hbar} (X_b t_b | X_a t_a). \tag{2}$$

This amplitude contains as much information on the system as the propagator $(X_b t_b | X_a t_a)$, and its path integral form is as follows:

$$(X_b | X_a)_E = \int_{t_a}^{\infty} dt_b \int \mathcal{D}x(t) e^{i\mathcal{A}_E/\hbar} \tag{3}$$

with the action

$$\mathcal{A}_E = \int_{t_a}^{t_b} dt \left[\frac{M}{2}\dot{x}^2(t) - V[x(t)] + E \right], \tag{4}$$

where $\dot{x}$ denotes the derivatives with respect to t . In Ref. [12] it has been shown that fixed energy amplitude (3) is equivalent to the fixed energy amplitude

$$(X_b | X_a)_E = \int_0^{\infty} dS \left[f_r(x_b) f_l(x_a) \int \mathcal{D}x(s) e^{i\mathcal{A}_E^f/\hbar} \right] \tag{5}$$

with the action

$$\mathcal{A}_E^f = \int_0^S ds \left\{ \frac{M}{2f[x(s)]} x'^2(s) - f[x(s)]\{V[x(s)] - E\} \right\}, \tag{6}$$

where f_r and f_l are arbitrary regulating functions such that $f = f_l f_r$ and x' denotes the derivatives with respect to time s. The actions $\mathcal{A}_E$ and $\mathcal{A}_E^f$, both of which lead to the same fixed-energy amplitude $(X_b | X_a)_E$ are called Duru-Kleinert equivalent.[1]

In the following section we shall use this equivalence to calculate the quantum tunneling rate. For a quantum-mechanical decay of the ground state, the standard instanton calculation [8] yields the transition amplitude

$$\langle f | i \rangle \equiv \int \mathcal{D}q \exp\left\{ -i \int_0^T \left[\frac{1}{2}\dot{x}^2 - V \right] dt \right\} \simeq e^{-\Gamma T}, \tag{7}$$

where Γ is the tunneling rate. The essential feature of Eq. (7) is that the ground state energy of the corresponding Hamiltonian picks up a small imaginary part Γ signaling the instability. In the instanton calculation this is taken care by the negative mode in the bounce solution. Note that the basic object in these calculations is the transition amplitude which plays a key role in the Duru-Kleinert equivalence. It is well known that for a quantum-cosmological tunneling we should impose the ''zero energy'' condition on the corresponding transition amplitude. Thus we rewrite the action $\mathcal{A}_E^f$ in a suitable form such that it describes a system with zero en-

[1] Of course a third action $\mathcal{A}_{E,\varepsilon}^{\mathrm{DK}}$ is also Duru-Kleinert equivalent of $\mathcal{A}_E$ and $\mathcal{A}_E^f$ but we do not consider it here [12].

ergy; as only in this sense can we describe a quantum-cosmological model with zero energy. Imposing $E=0$ in Eq. (6), with a simple manipulation, gives

$$\mathcal{A}_E^f=\int_0^1 ds' Sf[X(s')] \times\left\{\frac{M}{2\{Sf[X(s')]\}^2}\dot{X}^2(s')-V[X(s')]\right\}, \quad (8)$$

where $\dot{X}$ denotes the derivative with respect to new parameter s' defined by

$$s'=S^{-1}s \quad (9)$$

with S as a dimensionless scale parameter.

After a Wick rotation $s'=-i\tau$, we get the required Euclidean action and the path integral

$$I_0^f=\int_0^1 d\tau Sf[X(\tau)]\left\{\frac{M}{2\{Sf[X(\tau)]\}^2}\dot{X}^2(\tau)+V[X(\tau)]\right\}, \quad (10)$$

$$(X_b|X_a)=\int_0^\infty dS\left[f_r(X_b)f_l(X_a)\int \mathcal{D}X(\tau)e^{-I_0^f/\hbar}\right], \quad (11)$$

where τ is the Euclidean time. The action (10) is the Duru-Kleinert equivalent of

$$I_0=\int_{\tau_a}^{\tau_b} d\tau\left[\frac{M}{2}\dot{X}^2(\tau)+V[X(\tau)]\right], \quad (12)$$

where τ_a and τ_b correspond to t_a and t_b, respectively, and $\dot{X}$ denotes the derivative with respect to Euclidean time τ.

III. MODEL

We shall consider a closed FRW universe filled with an exotic fluid having the equation of state $p_\chi=(m/3-1)\rho_\chi$ with the parameter m restricted to the range $0\leqslant m\leqslant 2$. Such fractional equation of state is possible since the exotic matter may have an effective equation of state anywhere in a range between well established values. For instance, for cosmic strings $\frac{2}{3}\leqslant m/3\leqslant\frac{4}{3}$, and for domain walls $\frac{1}{3}\leqslant m/3\leqslant\frac{4}{3}$, depending on their velocities [13].

The system has only one collective coordinate, namely, the scale factor R. Using the usual Robertson-Walker metric we obtain the scalar curvature

$$\mathcal{R}=6\left[\frac{\ddot{R}}{R}+\frac{1+\dot{R}^2}{R^2}\right]. \quad (13)$$

Substituting Eq. (13) into the Einstein-Hilbert action plus a matter term indicating an exotic χ fluid with the equation of state $p_\chi=(m/3-1)\rho_\chi$ leads to the action[2]

[2]In what follows we shall take units such that $8\pi G=1$.

$$I=\int_0^1 dt\left[-\frac{1}{2}R\dot{R}^2+\frac{1}{2}R\left(1-\frac{\rho_\chi}{3}R^2\right)\right] \quad (14)$$

with the constraint of Einstein equation

$$\dot{R}^2+\left[1-\frac{\rho_\chi}{3}R^2\right]=0. \quad (15)$$

It is easy to show that the equation of state $p_\chi=(m/3-1)\rho_\chi$ upon substitution into the continuity equation $d\rho_\chi/dR=-(3/R)(\rho_\chi+p_\chi)$ leads to the following behavior of the energy density in a closed FRW universe [14]:

$$\rho_\chi(R)=\rho_\chi(R_0)\left(\frac{R_0}{R}\right)^m. \quad (16)$$

Now, we may define the cosmological term

$$\Lambda\equiv\rho_\chi(R)$$

which leads to

$$I=\int_0^1 dt\left[-\frac{1}{2}R\dot{R}^2+\frac{1}{2}R\left(1-\frac{\Lambda}{3}R^2\right)\right] \quad (17)$$

and

$$\dot{R}^2+\left[1-\frac{\Lambda}{3}R^2\right]=0. \quad (18)$$

Using $\Lambda\equiv\rho_\chi$, we may have equivalently

$$\Lambda(R)=\Lambda(R_0)\left(\frac{R_0}{R}\right)^m, \quad (19)$$

where R_0 is the value of the scale factor at an arbitrary reference time. It is worth emphasizing that one possible explanation for a small Λ term is to assume that it is dynamically evolving and not constant, that is, as the universe evolves from an earlier hotter and denser epoch, the effective cosmological term also evolves and decreases to its present value [15]. There are also strong observational motivations for considering cosmological models with a decaying Λ term instead of a constant one [3].

Chen and Wu [2] have given some interesting arguments in favor of a cosmological term $\Lambda\sim R^{-2}$ which was phenomenological and did not come from particle physics first principles. This behavior could be obtained under some simple and general assumptions conforming quantum cosmology. From dimensional considerations one can always write Λ as M_{pl}^4 times a dimensionless product of quantities. Supposing that no other parameters are relevant except the scale factor R, the natural ansatz is that Λ varies according to a power law in R as [2]

$$\Lambda(R)\sim M_{pl}^4\left(\frac{R_{pl}}{R}\right)^m \quad (\text{with } \hbar=c=1),$$

where M_{pl} and R_{pl} are the Planck mass and the Planck length, respectively.

Silveira and Wega [3] have also suggested a class of models in which Λ decreases as a power-law dependence on the scale factor $\Lambda \sim R^{-m}$, where m is a constant ($0 \leqslant m \leqslant 3$). Recently they investigated some properties of flat cosmologies with a cosmological term as [3]

$$\Lambda = 8\pi G \rho_{\text{vac}} = 3\alpha R^{-m}$$

with $\alpha \geqslant 0$ and $0 \leqslant m < 3$. These models are equivalent to standard cosmology with matter and radiation plus an exotic fluid with the equation of state $p_\chi = (m/3-1)\rho_\chi$. They studied the effect of the decaying Λ term on the cosmic microwave background anisotropy and computed the angular power spectrum for different values of m and density parameter Ω_{m0}.

It is to be noted that regarding the equation of state $p_\chi = (m/3-1)\rho_\chi$ with $0 \leqslant m \leqslant 2$, our model resembles a negative pressure matter universe violating the strong energy condition. Dabrowski [16] has already considered similar problem for oscillating closed Friedmann models with the matter source being domain walls (which scale as R^{-1}) and a negative cosmological constant. Domain walls are, of course, an example of the matter violating strong energy condition since for them $m=1$. Cosmic strings on the other hand have $m=2$. Thus, we can reinterpret them as decaying cosmological terms.[3]

Based on these Λ decaying models, we were motivated to take the present model in which a time dependent Λ term with a power-law dependence on the scale factor R is considered. By introducing a new parameter α restricted to the range $1 \leqslant \alpha < \infty$ we may rewrite Eq. (19) as

$$\Lambda(R) = \Lambda(R_0)\left(\frac{R_0}{R}\right)^{2-2/\alpha}. \tag{20}$$

The case $m=2$, having some interesting implications in reconciling observations with the inflationary models [2], may be obtained as $\alpha \to \infty$. Also, this value for m accounts for an exotic fluid matter source, namely the cosmic string. Substituting Eq. (20) into the action (17) and Eq. (18) leads to

$$I = \int_0^1 dt \left\{ -\frac{1}{2}R\dot{R}^2 + \frac{1}{2}R\left[1-\left(\frac{R}{R_0}\right)^{2/\alpha}\right]\right\}, \tag{21}$$

$$\dot{R}^2 + \left[1-\left(\frac{R}{R_0}\right)^{2/\alpha}\right] = 0, \tag{22}$$

where $\Lambda(R_0) = 3/R_0^2$. The issue of quantum tunneling for this Λ decaying model may be investigated in two ways: the WKB approximation and dilute-instanton approximation techniques. In the first one, we may solve the corresponding Wheeler-DeWitt equation obtaining the tunneling wave functions to calculate the tunneling probability Γ, while in the second one we may solve the Euclidean field equations obtaining instanton solutions to calculate Γ. Here, in order to calculate Γ we shall follow the second approach.

IV. TUNNELING RATE

The Euclidean form of the action (21) is not suitable to be used in instanton calculation techniques. The reason is that the kinetic term is not in its standard quadratic form. It has been recently shown [9] that in such a cosmological model one may use the Duru-Kleinert equivalence to work with the standard form of the action. Using the same procedure, we find the Duru-Kleinert equivalent action in the cosmological model here as follows:

$$I_0 = \int_{\tau_a}^{\tau_b} dt \left\{ \frac{1}{2}\dot{R}(\tau)^2 + \frac{1}{2}R^2\left[1-\left(\frac{R}{R_0}\right)^{2/\alpha}\right]\right\}. \tag{23}$$

Now, the Euclidean action (23) has the right kinetic term to be used in instanton calculations. The Euclidean-type Hamiltonian corresponding to the action (23) is given by

$$H_E = \frac{\dot{R}^2}{2} - \frac{1}{2}R^2\left[1-\left(\frac{R}{R_0}\right)^{2/\alpha}\right] \tag{24}$$

whose vanishing constraint[4] $H_E = 0$ gives a nontrivial instanton solution

$$R(\tau) = \frac{R_0}{[\cosh(\tau/\alpha)]^\alpha} \tag{25}$$

corresponding to the potential

$$V(R) = \frac{1}{2}R^2\left[1-\left(\frac{R}{R_0}\right)^{2/\alpha}\right] \quad \text{for} \quad R \geqslant 0. \tag{26}$$

Each solution with $\alpha > 0$ describes a particle rolling down from the top of the potential $-V(R)$ at $\tau \to -\infty$ and $R=0$, bouncing back at $\tau = 0$ and $R = R_0$ and finally reaching the top of the potential at $\tau \to +\infty$ and $R=0$. The region of the barrier $0 < R < R_0$ is classically forbidden for the zero energy particle, but quantum mechanically it can tunnel through it with a tunneling probability which is calculated using the instanton solution (25). The quantized FRW universe is mathematically equivalent to this particle, such that the particle at $R=0$ and $R=R_0$ represents ''nothing'' and ''FRW'' universes, respectively. Therefore one can find the probability

$$|\langle \text{FRW}(R_0)|\text{nothing}\rangle|^2.$$

The rate of tunneling Γ is calculated through the dilute instanton approximation to first order in $\hbar$ as [8]

$$\Gamma = \left[\frac{\det'[-\partial_\tau^2 + V''(R)]}{\det(-\partial_\tau^2 + \omega^2)}\right]^{-1/2} e^{-I_0(R)/\hbar}\left[\frac{I_0(R)}{2\pi\hbar}\right]^{1/2}, \tag{27}$$

where $\det'$ is the determinant without the zero eigenvalue, $V''(R)$ is the second derivative of the potential at the instan-

[3]Private communication with M. P. Dabrowski.

[4]The constraint $H_E = 0$ corresponds to Euclidean form of the Einstein equation.

ton solution (25), $\omega^2 = V''(R)|_{R=0}$ with $\omega^2=1$ for the potential (26), and $I_0(R)$ is the corresponding Euclidean action evaluated at the instanton solution (25). The determinant in the numerator is defined as

$$\det{}'[-\partial_\tau^2 + V''(R)] \equiv \prod_{n=1}^{\infty} |\lambda_n|, \tag{28}$$

where λ_n are the nonzero eigenvalues of the operator $-\partial_\tau^2 + V''(R)$.

The explicit form of this operator is obtained as

$$O \equiv \alpha^{-2}\left[-\frac{d^2}{dx^2} - \frac{(\alpha+1)(\alpha+2)}{\cosh^2 x} + \alpha^2\right], \tag{29}$$

where we have used Eqs. (25) and (26) with a change of variable $x=\tau/\alpha$. Now, in order to find the eigenvalues and eigenfunctions of the operator (29) exactly we assume α to be a positive integer. By relabeling $l=\alpha+1$, the eigenvalue equation of the operator (29) can be written as

$$\Delta_l \psi_l(x) = (E_l - 2l + 1)\psi_l(x) \tag{30}$$

with

$$\Delta_l := -\frac{d^2}{dx^2} - \frac{l(l+1)}{\cosh^2 x} + l^2, \tag{31}$$

where the factor α^{-2} is ignored for the moment. Equation (30) is a time-independent Schrödinger equation. Now, by ignoring the constant shift of energy $2l-1$ and by introducing the following first order differential operators:

$$\begin{cases} B_l(x) := \dfrac{d}{dx} + l\tanh x, \\ B_l^\dagger(x) := -\dfrac{d}{dx} + l\tanh x, \end{cases} \tag{32}$$

the operator Δ_l can be factorized and using the shape invariance symmetry we have [11]

$$\psi_l(x) = \frac{1}{\sqrt{E_l}} B_l^\dagger(x)\psi_{l-1}(x), \tag{33}$$

$$\psi_{l-1}(x) = \frac{1}{\sqrt{E_l}} B_l(x)\psi_l(x). \tag{34}$$

Therefore, for a given l, its first (bounded) excited state can be obtained from the ground state of $l-1$. Consequently, the excited state m of a given l, that is $\psi_{l,m}$, can be written as

$$\psi_{l,m}(x) = \sqrt{\frac{2(2m-1)!}{\Pi_{j=1}^m j(2l-j)}\frac{1}{2^m(m-1)!}} \times B_l^\dagger(x) B_{l-1}^\dagger(x) \cdots B_{m+1}^\dagger(x) \frac{1}{\cosh^m x}, \tag{35}$$

with eigenvalues $E_{l,m} = l^2 - m^2$. Also its continuous spectrum consists of

$$\psi_{l,k} = \frac{B_l^\dagger(x)}{\sqrt{k^2+l^2}} \frac{B_{l-1}^\dagger(x)}{\sqrt{k^2+(l-1)^2}} \cdots \frac{B_1^\dagger(x)}{\sqrt{k^2+1^2}} \frac{e^{ikx}}{\sqrt{2\pi}}, \tag{36}$$

with eigenvalues $E_{l,k} = l^2 + k^2$ where $\int_{-\infty}^{+\infty} \psi_{l,k}^*(x)\psi_{l,k'}(x)dx = \delta(k-k')$. Now, we can calculate the ratio of the determinants as follows. First we explain very briefly how one can calculate the determinant of an operator by the heat kernel method [10]. We introduce the generalized Riemann zeta function of the operator A by

$$\zeta_A(s) = \sum_m \frac{1}{|\lambda_m|^s}, \tag{37}$$

where λ_m are eigenvalues of the operator A, and the determinant of the operator A is given by

$$\det A = e^{-\zeta_A'(0)}. \tag{38}$$

It is obvious from Eqs. (37) and (38) that for an arbitrary constant c

$$\det(cA) = c^{\zeta_A(0)} \det A. \tag{39}$$

On the other hand $\zeta_A(s)$ is the Mellin transformation of the heat kernel[5] $G(x,y,\tau)$ which satisfies the following heat diffusion equation:

$$AG(x,y,\tau) = -\frac{\partial G(x,y,\tau)}{\partial \tau}, \tag{40}$$

with an initial condition $G(x,y,0) = \delta(x-y)$. Note that $G(x,y,\tau)$ can be written in terms of its spectrum

$$G(x,y,\tau) = \sum_m e^{-\lambda_m \tau}\psi_m^*(x)\psi_m(y). \tag{41}$$

An integral is written for the sum if the spectrum is continuous. From relations (38) and (40) it is clear that

$$\zeta_A(s) = \frac{1}{\Gamma(s)}\int_0^\infty d\tau \tau^{s-1}\int_{-\infty}^{+\infty} dx G(x,x,\tau). \tag{42}$$

Now, in order to calculate the ratio of the determinants in Eq. (27), called a prefactor, we need to find the difference of the functions $G(x,y,\tau)$ for two operators $\Delta_l, \Delta_l(0)$, where

$$\Delta_l(0) := -\frac{d^2}{dx^2} + l^2. \tag{43}$$

Considering the fact that $\Delta_l + 1 - 2l$ [or $\Delta_l(0) + 1 - 2l$] has the same eigenspaces as Δ_l [or $\Delta_l(0)$] and the eigenspectrum is shifted by $1-2l$, we have

[5]Here τ is a typical time parameter.

$$G_{\Delta_l(0)+1-2l}(x,y,\tau)=\frac{e^{-(l-1)^2\tau}}{2\sqrt{\pi\tau}}e^{-(x-y)^2/4\tau}, \tag{44}$$

$$G_{\Delta_l+1-2l}(x,y,\tau)=\sum_{m=0,m\neq 1}^{l-1}\psi^*_{l,m}(x)\psi_{l,m}(y)\times e^{-|(m-1)[2l-(m+1)]|\tau}+\int_{-\infty}^{+\infty}dk e^{-[(l-1)^2+k^2]\tau}\psi^*_{l,k}(x)\psi_{l,k}(y). \tag{45}$$

In order to calculate the function ζ_{Δ_l+1-2l}, according to the relation (42) we have to take the trace of heat kernel $G_{\Delta_l+1-2l}(x,y,\tau)$ where we need to integrate over $|\psi_{l,k}|^2$. Using the relation $(B_l/\sqrt{E_{l,k}})\psi_{l,k}(x)=\psi_{l-1,k}(x)$ we have

$$\int_{-\infty}^{+\infty}dx\psi^*_{l,k}(x)\psi_{l,k}(x)=-\lim_{x\to\infty}\frac{1}{\sqrt{(E_{l,k}}}\psi^*_{l,k}(x)\psi_{l-1,k}(x)+\lim_{x\to-\infty}\frac{1}{\sqrt{(E_{l,k}}}\psi^*_{l,k}(x)\psi_{l-1,k}(x)+\int_{-\infty}^{+\infty}dx\psi^*_{l-1,k}(x)\psi_{l-1,k}(x). \tag{46}$$

The first and the second terms appearing on the right hand side of the recursion relation (46) are proportional to the asymptotic value of the wave functions at ∞ and $-\infty$, respectively, where the latter is calculated as

$$\lim_{x\to\pm\infty}\psi_{m,k}(x)=\frac{-ik\pm m}{\sqrt{k^2+m^2}}\frac{-ik\pm(m-1)}{\sqrt{k^2+(m-1)^2}}\cdots\times\frac{-ik\pm 1}{\sqrt{k^2+1}}\frac{\exp(ikx)}{\sqrt{2\pi}}=\frac{1}{\sqrt{2\pi}}\prod_{j=1}^{m}\left(\frac{-ik\pm j}{\sqrt{k^2+j^2}}\right)\exp(ikx). \tag{47}$$

Substituting these asymptotic behaviors in the recursion relations between the norms of the wave functions $\psi_{m,k}$ associated with the continuous spectrum (36), then using the obtained recursion relations together with the orthonormality of discrete spectrum we get the following result for the difference of traces of heat kernels:

$$\int_{-\infty}^{+\infty}dx[G_{\Delta_l+1-2l}(x,x,\tau)-G_{\Delta_l(0)+1-2l}(x,x,\tau)]=\sum_{m=0,m\neq 1}^{l-1}\exp\{-|(m-1)[2l-(m+1)]|\tau\}-\frac{1}{\pi}\sum_{m=1}^{l}m\left(\int_{-\infty}^{+\infty}dk\frac{\exp\{-[(l-1)^2+k^2]\tau\}}{[k^2+(l-1)^2]}+[(l-1)^2-m^2]\int_{-\infty}^{+\infty}dk\frac{\exp\{-[(l-1)^2+k^2]}{(k^2+m^2)[k^2+(l-1)^2]}\right). \tag{48}$$

Hence, using the Mellin transformation (42) and the well-known Feynman integral (A1) we finally get

$$\zeta_{\Delta_l+1-2l}(s)-\zeta_{\Delta(0)+1-2l}(s)=\sum_{m=0,m\neq 1}^{l-1}\{|(m-1)[2l-(m+1)]|\}^{-s}-\frac{1}{2\pi}l(l+1)(l-1)^{-(2s+1)}\beta\left(s+\frac{1}{2},\frac{1}{2}\right)-\frac{1}{\sqrt{\pi}}\frac{\Gamma(s+3/2)}{\Gamma(s+2)}\sum_{m=1}^{l-1}m(l-1)^{-(2s+3)}[(l-1)^2-m^2]{}_2F_1\left(s+\frac{3}{2},1,s+2,1-\frac{m^2}{(l-1)^2}\right)-\frac{1}{\sqrt{\pi}}\frac{\Gamma(s+3/2)}{\Gamma(s+2)}l_2^{-2(s+1)(1-2l)}F_1\left(s+\frac{3}{2},s+1,s+2,1-\frac{(l-1)^2}{l^2}\right), \tag{49}$$

where β is the beta function.

For $s=0$, we obtain

$$\zeta_{\Delta_l+1-2l}(s)-\zeta_{\Delta_l(0)+1-2l}(s)|_{s=0}=-1. \tag{50}$$

This means that the operators Δ_l+1-2l and $\Delta_l(0)+1-2l$ have the same number of eigenspaces (even though for both of them this number is infinite) since from the definition of Riemann's zeta function, it is obvious that its value at $s=0$ can be interpreted as the number of eigenspaces of the corresponding operator. The appearance of -1 on the right hand side of the relation is due to the ignorance of the eigenfunctions associated with the zero eigenvalue of the operator Δ_l+1-2l. Therefore, its number of eigenstates is the same as that of the operator $\Delta_l(0)+1-2l$. In order to calculate the ratio of the determinant of the operators Δ_l+1-2l and $\Delta_l(0)+1-2l$ we need to know the derivative of their associated zeta functions at $s=0$. Hence differentiating both sides of the relation (50) with respect to s and evaluating such integrals as Eqs. (A2),(A3) we get

$$\zeta'_{\Delta_l+1-2l}(s)-\zeta'_{\Delta_l(0)+1-2l}(s)|_{s=0}=\log\left(\frac{2(2l-1)!}{[(l-2)!]^2}\right). \tag{51}$$

Therefore, according to the relations (27), (38), (39), (50), and (51) the prefactor associated with the potential (26) is calculated as

$$\frac{\det\{[1/(l-1)^2][-d^2/dx^2-l(l+1)/\cosh^2 x+(l-1)^2]\}}{\det\{[1/(l-1)^2][-d^2/dx^2+(l-1)^2]\}} =\frac{[(l-1)!]^2}{2(2l-1)!}. \tag{52}$$

Finally, the decay rate of metastable state of this potential is calculated as

$$\Gamma=\frac{1}{\sqrt{\pi\hbar}}R_0 2^{l-1}\exp\left(-\frac{(l-1)R_0^2\beta(3/2,l-1)}{\hbar}\right)+O(\hbar), \tag{53}$$

where we have used the value of I_0 at the instanton solution (25):

$$I_0=\int_{-\infty}^{\infty}R_0^2\frac{[\sinh(\tau/\alpha)]^2}{[\cosh(\tau/\alpha)]^{2(1+\alpha)}}d\tau=\alpha R_0^2\beta\left(\frac{3}{2},\alpha\right). \tag{54}$$

V. CONCLUSION

In this paper we have calculated the tunneling rate, with exact prefactor, to first order in $\hbar$ from ''nothing'' to a closed FRW universe with decaying Λ. The tunneling rate (53) increases for higher values of the positive integer α (or l) such that for $\alpha\to\infty$ the most tunneling rate corresponds to the most probable cosmological term as $\Lambda\sim R^{-2}$. This decaying cosmological term may have its origin in the cosmic string as the exotic matter with the effective equation of state $p_\chi=-\frac{1}{3}\rho_\chi$. It is probable that the universe could have tunneled with highest probability from nothing to an empty closed FRW cosmology with a typical Planck size R_p and a large cosmological term $\Lambda\sim R_p^{-2}$. Then, after tunneling, Λ may have evolved and decreased to its present small value as the universe has expanded classically. As is discussed in Ref. [2] it does not directly solve the cosmological constant problem but reduces it with the age problem to one and the same problem: How could our universe have escaped the death at the Planck size?

One possible solution to this problem is that the value of Λ after tunneling might be large enough to derive various symmetry breakings necessary to the appearance of a universe which has evolved to the present universe with a small cosmological constant. This may be consistent with an inflationary model in which an extraordinarily brief period of rapid expansion occurs where the universe is about the Planck size after quantum tunneling. We remark that this Λ decaying model has some advantages in that alleviates some problems in reconciling observations with the inflationary scenario; in particular it leads to creation of matter [2]. It is interesting to note that the tunneling rate in this model is the same one obtained for a closed FRW cosmology with perfect fluid violating the strong energy condition with the equation of state $p=(m/3-1)\rho$ [17] such that the most probable cosmological term corresponding to $m=2$ is equivalent to the least violation of the strong energy condition. This may account for another possible solution to the reduced problem from the point of view of *energy conditions*. In other words, one may think that our universe could have escaped the death at the Planck size because the violation of energy conditions is minimized right after quantum tunneling.

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APPENDIX

$$\frac{1}{D_1^{a_1}D_2^{a_2}\cdots D_n^{a_n}} =\frac{\Gamma(a_1+a_2+\cdots+a_n)}{\alpha(a_1)\Gamma(a_2)\cdots\Gamma(a_n)}\int dt_1dt_2\cdots dt_n \times\frac{\delta(1-t_1-t_2\cdots-t_n)t_1^{a_1-1}t_2^{a_2-1}\cdots t_n^{a_n-1}}{(t_1D_1+t_2D_2+\cdots+t_nD_n)^{a_1+a_2+\cdots+a_n}}. \tag{A1}$$

$$\int_0^1 dt\{[(l-1)^2-m^2]t+m^2\}^{-3/2}\log t =\frac{4}{m[(l-1)^2-m^2]}[\log 2+\log m-2\log(l-1+m)], \tag{A2}$$

$$\int_0^1 dt\{[(l-1)^2-m^2]t+m^2\}^{-3/2} \times\log\{[(l-1)^2-m^2]t+m^2\}\log t =\frac{4}{m[(l-1)^2-m^2]}\left(\log m\frac{m}{l-1}\log(l-1)+\frac{(l-1-m)}{l-1}\right)m -2\log(l-1+m). \tag{A3}$$

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