Baryogenesis from primordial black holes after the electroweak phase transition

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Incorporating a realistic model for accretion of ultrarelativistic particles by primordial black holes (PBHs), we study the evolution of an Einstein–de Sitter universe consisting of PBHs embedded in a thermal bath from the epoch $\sim 10^{-33}$ sec to $\sim 5\times10^{-9}$ sec. In this paper we use the ansatz of Barrow *et al.* to model black hole evaporation in which the modified Hawking temperature goes to zero in the limit of the black hole attaining a relic state with a mass $\sim m_{\rm Pl}$. Both the single mass PBH case as well as the case in which black hole masses are distributed in the range 8×10^{2}–3×10^{5} g have been considered in our analysis. Black holes with a mass larger than $\sim 10^{5}$ g appear to survive beyond the electroweak phase transition and, therefore, successfully manage to create baryon excess via X-$\bar{X}$ emissions, averting the baryon number washout due to sphalerons. In this scenario, we find that the contribution to the baryon-to-entropy ratio by PBHs of initial mass m is given by $\sim \epsilon\zeta(m/1\,{\rm g})^{-1}$, where ϵ and ζ are the CP-violating parameter and the initial mass fraction of the PBHs, respectively. For ϵ larger than $\sim 10^{-4}$, the observed matter-antimatter asymmetry in the universe can be attributed to the evaporation of PBHs. [S0556-2821(99)05416-8]

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I. INTRODUCTION

That the Milky Way is essentially made of matter is evident not only from the landings of space probes on the Moon and other planets without any disastrous consequences, but also from the absence of antinuclei in the observed cosmic rays, and from the observations of Faraday rotation [1]. Observational support for the absence of a significant quantity of antimatter beyond our Galaxy exists, but it is of an indirect nature [1,2]. Since visible mass in the universe is chiefly in the form of baryonic matter, the inferred matter-antimatter asymmetry essentially boils down to the problem of the origin of baryon asymmetry.[1] The baryon asymmetry is characterized by the baryon-to-photon ratio $\eta = n_B/n_\gamma$, with n_B and n_γ being the number densities of net baryons and photons, respectively. According to standard big-bang nucleosynthesis calculations, the predicted abundances of light elements depend only on the free parameter η and are in apparent agreement with the observed abundances provided η lies in the range $(2.8–4.5)\times10^{-10}$ [3]. Recently, Tytler *et al.* [4] have estimated the baryon-to-photon ratio from the observations of deuterium abundance in a high redshift quasar absorption system and according to their measurements, $\log \eta = -9.18 \pm 0.4 \pm 0.4 \pm 0.2$.

The esthetically appealing scenario of the universe consisting of equal amount of baryons and antibaryons at the instant of creation is still compatible with a nonzero η if one invokes Sakharov conditions, namely, of having B, C, and CP violating interactions in out-of-thermodynamic equilibrium condition sometime in the early history of the universe [5]. The grand unified theories (GUTs) of fundamental forces incorporate baryon number violating interactions naturally while CP violation can be introduced in such theories in many different ways (it is to be noted that CP violation added theoretically in GUTs, in general, is not related to the observed CP violation in the $K^\circ - \bar{K}^\circ$ system [6]) and therefore it is not surprising that GUTs provide a natural framework for the generation of baryon asymmetry through decay of X-$\bar{X}$ bosons [7]. However, through the work of Kuzmin, Rubakov, and Shaposhnikov [8] it came to be appreciated that sphalerons could induce transitions between degenerate vacua differing in baryon number and that such processes could erase baryon-asymmetry generated prior to the electroweak phase transition (EWPT) era.

Use of B violation in electroweak theories to produce excess baryons has also been made in the literature [9] but one of the major obstacles in this scenario is the requirement of low Higgs mass which is in direct conflict with the experimental lower limit of $m_H > 88$ GeV [10]. It appears that in the minimal version of electroweak theory, generating baryon asymmetry may not be possible at all [11] and especially with the discovery of the top-quark with a mass around 175 GeV [12] there is hardly any region left in the parameter space of the standard model to produce observed baryon-to-photon ratio [13].

The other major scenario of generating baryon asymmetry is to invoke Hawking evaporation of black holes. The early sketchy ideas of Hawking and Zeldovich took proper shape with the advent of GUTs, giving rise to a picture of black holes of small mass emitting X and $\bar{X}$ bosons thermally which subsequently decay and in the process violate B, C, and CP, leading to a production of baryon excess [14]. At the fundamental level, this scenario has an attractive feature in that it combines ideas of black-hole thermodynamics [15,16] on one hand and GUT on the other, to explain the observed matter-antimatter asymmetry in the universe. One

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[1] If neutrinos are massive then the gravitating mass may as well be dominated by leptons. However, there is hardly any direct measure of the lepton number of the universe.

of the important ingredients of this picture is the occurrence of mini black holes having a mass less than 10^{14} g. It is obvious that such black holes cannot emerge as end products of stellar evolution. However, Zeldovich and Novikov [17] and Hawking [18] argued that primordial black holes (PBHs) of small mass can be generated from the space-time curvature, and subsequently, Carr [19] showed the possibility of creating PBHs from density fluctuations in the early universe. In the context of inflation, several authors have discussed mechanisms to produce PBHs using the general idea that bubble wall collisions may trap pockets of false vacuum region that subsequently collapse to form black holes [20]. In a recent work, Nagatani [21] has proposed an interesting black-hole–electroweak mechanism of baryogenesis that requires the presence of a black hole to create a domain wall around it, leading to genesis of baryon excess without the need of a first order electroweak phase transition.

Previous paragraphs of this section indicate that although GUTs can naturally generate baryon asymmetry, any baryon excess generated prior to electroweak era is erased due to sphaleron transitions, while at the same time, creation of baryon asymmetry solely due to electroweak processes is fraught with uncertainties as well as the requirement of low Higgs mass, contrary to the experimental situation. Under the present circumstances, it is therefore natural to explore alternate means to explain matter-antimatter asymmetry. Since the existence of PBHs in the early universe is rather generic, one ought to carefully reexamine the mechanism of generating baryon asymmetry through black-hole evaporation. In such a scenario, the crucial point to investigate is whether PBHs survive after the EWPT has taken place, so that the baryon asymmetry created due to their subsequent Hawking evaporation survives, leaving an imprint till the present epoch.

The present paper is an attempt to critically examine the evolution of the masses of a collection of PBHs created after the end of inflation, taking into account both the accretion of background matter by the black holes as well as the mass loss due to Hawking emission. The paper has been organized in the following manner. In Sec. II we discuss processes responsible for the change in a black hole's mass, and thereafter, we develop a formalism to describe accretion of relativistic matter by mini black holes. The subject of the black-hole mass spectrum and its evolution is tackled next, in Sec. III, along with a discussion on the cosmological evolution of a mixture of PBHs and relativistic matter. Section IV deals with the study of evolution equations numerically as well as a detailed analysis of the numerical solutions pertaining to the survival of PBHs past the EWPT. In Sec. V, we calculate baryon excess resulting from the decay of X-$\bar{X}$ bosons emitted by the PBHs during their final stages of Hawking evaporation, and then discuss the implications of these results to the question of matter-antimatter asymmetry. Finally, we end with a brief discussion of the above scenario in Sec. VI.

II. EVOLUTION OF THE MASS OF A BLACK HOLE

A. Mass loss due to evaporation

Bekenstein's conjecture [15] that the area of the event horizon of a black hole being a measure of its entropy was vindicated by the classic work of Hawking in the early 1970's who showed that when quantum effects around a black hole are included, the black hole emits particles with a thermal distribution corresponding to a temperature $T_{\rm BH}$ that is proportional to the surface gravity at the event horizon [15,16], and is given by the relation

$$T_{\rm BH}=\frac{m_{\rm Pl}^2}{8\pi m}\,\frac{c^2}{k}, \tag{1}$$

where m and $m_{\rm Pl}$ are the mass of the black hole and the Planck mass, respectively. According to Eq. (1), PBHs created in the early universe with a mass $\approx 10^{14}$ g would be decaying today in a burst of high energy radiation, and there exists in the literature, upper bounds on the abundance of such PBHs from the observed level of cosmic γ-ray flux [22]. As pointed out by Zeldovich and others, the expression in Eq. (1) for the Hawking temperature can only be an approximation and is amenable to modifications at Planck scale because of the effects of quantum gravity. In fact, particle physicists have shown from various angles that Hawking evaporation may cease when the black hole reaches the Planck mass scale leading to a massive relic. In this context, an interesting toy model inspired by superstring theories has been considered by Barrow *et al.* [23] in which the expression for the black-hole temperature has been modified by including correction terms that contain powers of black-hole mass in units of Planck mass. Following the ansatz of Barrow *et al.*, one can therefore express the black-hole temperature as

$$T_{\rm BH}=\frac{m_{\rm Pl}}{8\pi}\left[\frac{m_{\rm Pl}}{m}-\kappa\left(\frac{m_{\rm Pl}}{m}\right)^n\right]\frac{c^2}{k}, \tag{2}$$

where κ is a non-negative constant. For $n>2$ and $\kappa \approx O(1)$, it is clear that for holes of mass $m \gg m_{\rm Pl}$, Eq. (1) is a limiting case of Eq. (2). According to Eq. (2), as the hole mass decreases due to evaporation, initially there is a rise in the hole's temperature but as m approaches $m_{\rm Pl}$ the temperature starts falling and becomes zero when the mass of the hole reaches the value $m_{\rm rel}=\kappa^{1/(n-1)}m_{\rm Pl}$. Therefore, the ansatz of Barrow *et al.* implies stable black-hole relics of mass $m_{\rm rel}\approx m_{\rm Pl}$. To estimate the rate of mass loss from Eq. (2), we may work in the framework of radiative transfer, assuming that the hole's event horizon acts as a perfect black-body surface. In such a case, it is easy to show that the energy flux F is related to the energy density ε at the surface of interest in the following manner [24]:

$$F=\frac{c}{4}\,\varepsilon. \tag{3}$$

The effective energy density of ultrarelativistic particles due to Hawking evaporation in the vicinity of the event horizon is related to the temperature of the black hole by

$$\varepsilon=\frac{\pi^2 g_\star^{\rm BH}}{30}\,\frac{k^4}{(\hbar c)^3}\,T_{\rm BH}^4, \tag{4}$$

where $g_\star^{\rm BH}=g_b^{\rm BH}+(7/8)g_f^{\rm BH}$ is the effective number of degrees of freedom at the temperature $T_{\rm BH}$, and $g_b^{\rm BH}$ and $g_f^{\rm BH}$ are the corresponding degrees of freedom for bosons and fermions, respectively. Therefore, the rate of mass loss from the event horizon is given by

$$\frac{dm}{dt}=-\frac{1}{c^2}F4\pi R_S^2 \tag{5}$$

$$=-\alpha_2 m^2\left[\frac{m_{\rm Pl}}{m}-\kappa\left(\frac{m_{\rm Pl}}{m}\right)^n\right]^4, \tag{6}$$

where $\alpha_2=g_\star^{\rm BH}c^2/(30720\pi\hbar)$ and $\kappa\approx 1$. In arriving at Eq. (6), we have made use of Eqs. (2)–(4) as well as the standard result for the Schwarzschild radius $R_S=2Gm/c^2$. The calculations that led to Eq. (6) were based on modeling the black hole event-horizon to be the surface of a black body of radius R_S at a thermodynamic temperature $T_{\rm BH}$. It is, therefore, interesting to compare our result with that of Page [25] which is based on rigorous numerical computations for black holes of mass $m>10^{17}$ g. According to his calculations, the mass-loss rate for such holes is

$$\frac{dm}{dt}=-2.011\times 10^{-4}\frac{\hbar c^4}{G^2m^2}. \tag{7}$$

If we assume that Eq. (7) is valid also for $10^{-2}\,{\rm g}<m<10^{17}\,{\rm g}$, then comparing Eqs. (6) and (7) one obtains $g_\star^{\rm BH}\approx 20$, which is not too unreasonable since for holes of mass 10^{-2} g one expects $g_\star^{\rm BH}$ to be as high as ≈ 100 (in most GUTs).

B. Accretion of relativistic matter by a mini black hole

The temperature of the universe is expected to be extremely high just after the end of inflation, and therefore matter during that period will be in the form of ultrarelativistic particles. For particles with de Broglie wavelength $\lambda \ll R_S$, the capture cross section corresponding to a Schwarzschild black hole is $\sim\pi r_c^2$, where $r_c=(3\sqrt{3}/2)R_S$ [26]. When the de Broglie wavelength of a particle is larger than R_s, the capture cross section is likely to be negligible as the black hole sees an incident wave rather than a point particle. For high energy particles with $\lambda \ll R_S$, we will make use of the geometric optics approximation in which any such ultrarelativistic particle hitting a fictitious sphere of radius r_c around the hole will be absorbed.

If I_ν represents the specific intensity of such particles corresponding to energy $h\nu$ and if dA is an area element on this fictitious sphere then the rate at which energy is accreted by the hole per unit range of ν per unit area is given by

$$\frac{dE_\nu}{dt\,d\nu\,dA}=\int d\Omega\cos\theta I_\nu=\pi I_\nu. \tag{8}$$

Since the effective area of capture is $4\pi r_c^2$, the rate at which energy is accreted in the frequency range $[\nu,\nu+d\nu]$ is given by

$$\frac{dE_\nu}{dt}=(2\pi r_c)^2 I_\nu d\nu. \tag{9}$$

To obtain the total rate of accretion of energy we integrate Eq. (9) over frequency keeping in mind that geometric optics approximation requires the lower limit of integration $\nu_{\rm min}$ to be a few times c/r_c. For ultrarelativistic particles, momentum is $p\approx h\nu/c$ so that the number density of particles of species A in the frequency range $(\nu,\nu+d\nu)$ takes the form

$$n_A(\nu)d\nu=\frac{4\pi g_A}{c^3}\frac{\nu^2 d\nu}{e^{h\nu/kT}\pm 1}, \tag{10}$$

where g_A is the spin-degeneracy factor for the Ath species and the $+(-)$ sign refers to fermions (bosons). In Eq. (10) T is the temperature of the universe. Therefore, the specific intensity $I_{\nu A}$ corresponding to the species A is given by [24]

$$I_{\nu A}=\frac{ch\nu n_A(\nu)}{4\pi}=\frac{g_A}{c^2}\frac{h\nu^3}{e^{h\nu/kT}\pm 1}. \tag{11}$$

Making use of Eqs. (9) and (11), we can express the net rate of energy accretion by a hole in the following manner:

$$\frac{dE}{dt}=\left(\frac{2\pi r_c}{c}\right)^2\left[g_b^{uni}\int_{\nu_{\rm min}}^{\infty}\frac{h\nu^3}{e^{h\nu/kT}-1}d\nu + g_f^{uni}\int_{\nu_{\rm min}}^{\infty}\frac{h\nu^3}{e^{h\nu/kT}+1}d\nu\right], \tag{12}$$

where $\nu_{\rm min}=\alpha_1 c/r_c$ is the lower frequency cutoff, α_1 being a number of the order of 10 (this takes care of the fact that only particles with $\lambda \ll R_s$ are considered to have been captured by the black hole). In Eq. (12), g_b^{uni} and g_f^{uni} are the total bosonic and fermionic degrees of freedom, respectively, for the cosmic soup. These are to be distinguished from $g_b^{\rm BH}$ and $g_f^{\rm BH}$ introduced in Sec. II A.

The rate at which the hole's mass grows as a result of accretion is

$$\frac{dm}{dt}=\frac{405}{\pi^3c^5}\varepsilon_R G^2m^2\left[\frac{g_b^{uni}}{g_\star^{uni}}\int_{x_{\rm min}}^{\infty}\frac{x^3}{e^x-1}dx + \frac{g_f^{uni}}{g_\star^{uni}}\int_{x_{\rm min}}^{\infty}\frac{x^3}{e^x+1}dx\right], \tag{13}$$

where $x_{\rm min}=h\nu_{\rm min}/kT$. In obtaining the above equation, we have made use of a change of variable in Eq. (12) along with $r_c=(3\sqrt{3}/2)R_S$. We note that ε_R appearing in Eq. (13) is the energy density of the background relativistic particles $\varepsilon_R=\pi^2 g_\star^{uni}(kT)^4/(30\hbar^3c^3)$, $g_\star^{uni}$ being the temperature-dependent effective spin-degeneracy factor and is equal to $g_b^{uni}+7/8g_f^{uni}$. From Eq. (13) it is evident that accretion plays an important role for massive PBHs at early epochs when the temperature of the universe is very high so that energy density ε_R of the relativistic particles is large while $x_{\rm min}$ is small. This is easy to understand from a physical point of view in the sense that only when the temperature is large that there are sufficient number of particles with de Broglie wavelength much less than the Schwarzschild radius of the PBHs, ready to be accreted. By the same token, when the hole mass reaches a size of the order of $m_{\rm Pl}$, neither accretion nor quantum evaporation is significant.

III. BLACK HOLE MASS SPECTRUM AND EVOLUTION OF THE UNIVERSE

It is evident that the mass distribution of PBHs is intimately linked to the mechanism of their production. Several authors [19,20,27,28] in the literature have discussed the black hole mass spectrum from diverse angles. Since the mass spectrum is sensitive to production mechanisms and, since so far no particular model of PBH creation has been singled out, we adopt a very general procedure in this paper to analyze the evolution of the black hole mass spectrum.

We consider a distribution function $N(m,t)$ such that $N(m,t)dm$ represents the number density of PBHs with mass in the range $(m,m+dm)$ at the cosmic epoch t. We assume that the creation of PBHs stopped after a cosmic epoch $t_{\rm PBH}$ so that at later times in a given comoving volume the number of holes remain the same while their masses change due to a combination of Hawking radiation and accretion of background matter. Note that we are working under the assumption that the ultimate state of a PBH along the course of its evolution is a stable relic of mass $\approx m_{\rm Pl}$, i.e., a hole does not disappear completely as the original Hawking radiation mechanism would demand. Also, since the mass m of a hole changes with time, the mass distribution function at time t and at time $t+dt$ are related as

$$a^3(t)N(m,t)dm = a^3(t+dt)N(m',t+dt)dm', \quad (14)$$

where m' is related to m through $m'=m+\dot{m}dt$ and $a(t)$ is the Friedmann-Robertson-Walker (FRW) scale-factor at cosmic epoch t. Making a Taylor expansion of quantities on the right-hand side of Eq. (14), and using the relation

$$dm' = dm\left(1+\frac{\partial \dot{m}}{\partial m}dt\right) \quad (15)$$

we obtain

$$\frac{\partial N}{\partial t} + 3\frac{\dot{a}}{a}N + \frac{\partial}{\partial m}(N\dot{m}) = 0. \quad (16)$$

With the help of the mass distribution function $N(m,t)$, we can also obtain an expression for the mass density associated with PBHs as

$$\rho_{\rm PH}(t) = \int_{m_{\rm rel}}^{\infty} mN(m,t)dm. \quad (17)$$

It is useful to express the black-hole mass distribution as

$$N(m,t) = N_0(t)f(m,t), \quad (18)$$

where $N_0 \propto a^{-3}(t)$ so that $\int_{m_{\rm rel}}^{\infty} f(m,t)dm$ is independent of time. With the help of Eq. (18) it can be easily shown that Eq. (16) reduces to

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial m}(\dot{m}f) = 0. \quad (19)$$

Essentially, $f(m,t)dm$ represents the number of black holes with mass in the interval $(m,m+dm)$ in a unit coordinate volume at the cosmic epoch t, while the dilution of black-hole number density due to the expansion of the universe is taken care of by the factor $N_0(t)=A/a^3(t)$. Differentiating Eq. (17) with respect to t and then making use of Eqs. (18) and (19) it can be shown that

$$\frac{d\rho_{\rm BH}}{dt} + \frac{3\dot{a}}{a}\rho_{\rm BH} = N_0\int_{m_{\rm rel}}^{\infty}\dot{m}f(m,t)dm. \quad (20)$$

Since the total energy-momentum tensor is divergence free, we also have the equation [29]

$$c^2\frac{d}{dt}[(\rho_R+\rho_{\rm BH})a^3] + 3p_Ra^2\dot{a} = 0, \quad (21)$$

where $\rho_R = \varepsilon_R/c^2$ is the mass density of radiation. Here we have assumed that the black holes possess negligible peculiar speeds so that their contribution to pressure is insignificant. Using $p_R = c^2\rho_R/3$ and Eq. (21) in Eq. (20) we obtain

$$\frac{d\rho_R}{dt} + 4\frac{\dot{a}}{a}\rho_R = -N_0\int_{m_{\rm rel}}^{\infty}\dot{m}f(m,t)dm. \quad (22)$$

Equation (22) just reflects, as is to be expected, the fact that an effective black-hole mass loss (or gain) would imply $\rho_R \propto a^{-4-\alpha}$ where $\alpha(t)$ is negative (positive) because of black holes acting as source (sink) of radiation.

In any mechanism of PBH production, the actual masses of the black holes will be distributed in a discrete fashion, and therefore without loss of generality the distribution function can be expressed as

$$f(m,t) = \sum_{i=1}^{K}\beta_i\delta[m-m_i(t)], \quad (23)$$

where β_i are constant weights corresponding to m_i, and K is the number of distinct black-hole masses. It can easily be ascertained that the distribution function in Eq. (23) indeed is a solution of Eq. (19), since

$$\frac{\partial}{\partial t}\delta[m-m_i(t)] = \frac{\dot{m}_i}{m-m_i(t)}\delta[m-m_i(t)] \quad (24)$$

and

$$\frac{\partial}{\partial m}[\dot{m}f(m,t)] = -\sum_{i=1}^{K}\beta_i\frac{\dot{m}_i}{m-m_i(t)}\delta[m-m_i(t)]. \quad (25)$$

Consequently, we have

$$\frac{d\rho_R}{dt}+4\rho_R\frac{\dot{a}}{a}=-N_0(t)\sum_{i=1}^{K}\beta_i\dot{m}_i. \tag{26}$$

The manner in which the individual mass m_i of a PBH changes with time depends on the combination of Hawking evaporation rate and the accretion of background relativistic matter as discussed in Sec. II. Therefore, making use of Eqs. (6) and (13) in the context of a PBH with mass m_i we get the following result:

$$\frac{dm_i}{dt}=405/\pi^3c^3\rho_RG^2m_i^2\left[\frac{g_b^{uni}}{g_\star^{uni}}\int_{x_{\min}}^{\infty}\frac{x^3dx}{e^x-1}+\frac{g_f^{uni}}{g_\star^{uni}}\int_{x_{\min}}^{\infty}\frac{x^3dx}{e^x+1}\right]-\alpha_2m_i^2\left[\frac{m_{\rm Pl}}{m_i}-\kappa\left(\frac{m_{\rm Pl}}{m_i}\right)^n\right]^4. \tag{27}$$

From Eqs. (17), (18), and (23), the mass density $\rho_{\rm BH}$ associated with the PBHs can be written as

$$\rho_{\rm BH}(t)=N_0(t)\sum_{i=1}^{K}\beta_im_i(t). \tag{28}$$

The evolution of the scale factor $a(t)$ then follows from the flat FRW Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\left[\rho_R+N_0(t)\sum_{i=1}^{K}\beta_im_i(t)\right]. \tag{29}$$

In writing down the above equation, we have adopted the inflationary paradigm according to which the universe in the post-inflationary phase is described essentially by a flat FRW model. In this paper, the evolution of the universe is determined by three coupled differential equations (26), (27), and (29) along with the fact that $N_0(t)\propto a^{-3}(t)$.

IV. NUMERICAL EVOLUTION

In this section, we solve $2+K$ coupled nonlinear, first order differential equations (26), (27), and (29), set up in the preceding section, numerically using Hemming's fourth-order, double precision predictor-corrector method. To begin with, we fix $N_0(t)$ by demanding that $\beta_im_i(t_0)N_0(t_0)$ represents the initial fraction ζ_i of total mass density $\rho(t_0)$ that lies in black holes having initial mass $m_i(t_0)$ so that

$$\beta_im_i(t_0)N_0(t_0)=\zeta_i\rho(t_0). \tag{30}$$

As $N_0(t)\propto a^{-3}(t)$, we have from Eq. (30)

$$N_0(t)=\frac{a^3(t_0)}{a^3(t)}\frac{\zeta_i\rho(t_0)}{\beta_im_i(t_0)}. \tag{31}$$

Since $N_0(t)$ is independent of i, we obtain the following relation between ζ_i, and β_i:

$$\zeta_i\propto\beta_im_i(t_0), \tag{32}$$

where the constant of proportionality in Eq. (32) can be determined from the identity

$$\rho_R(t_0)=\rho(t_0)-\sum_{i=1}^{K}\zeta_i\rho(t_0) \tag{33}$$

leading to the following expression:

$$\text{proportionality const}=\left(\sum_{i=1}^{K}\beta_im_i(t_0)\right)^{-1}\left(1-\frac{\rho_R(t_0)}{\rho(t_0)}\right). \tag{34}$$

Substituting Eq. (31) in Eqs. (26) and (29), we obtain

$$\frac{d\rho_R}{dt}+4\rho_R\frac{\dot{a}}{a}=-\frac{a^3(t_0)}{a^3(t)}\rho(t_0)\sum_{i=1}^{K}\zeta_i\frac{\dot{m}_i}{m_i(t_0)}, \tag{35}$$

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\left[\rho_R+\frac{a^3(t_0)}{a^3(t)}\rho(t_0)\sum_{i=1}^{K}\zeta_i\frac{m_i(t)}{m_i(t_0)}\right], \tag{36}$$

respectively.

In our original formulation (see Sec. III), the black-hole initial mass spectrum was completely specified by the set of numbers $\{\beta_i,m_i(t_0);\ i=1,...,K\}$. Equivalently, since β_i and ζ_i are related by Eq. (32), we may as well specify the spectrum by the set $\{\zeta_i,m_i(t_0);\ i=1,...,K\}$.

Before we embark on solving the set of differential equations numerically let us consider a highly simplified picture to verify the possibility of PBHs surviving beyond EWPT, without causing reheating above $\sim$100 GeV when they eventually decay. Otherwise, baryon excess generated from the black-hole evaporations get erased due to sphalerons in the reheated phase. In this simplified scenario, the black holes created at $t_0\approx10^{-33}$ s with initial mass fraction ζ and mass m_0 are assumed not to gain or lose mass till the epoch t_{eva}. At the end of t_{eva}, the PBHs evaporate instantaneously, converting their entire rest mass energy into ultrarelativistic thermal particles, and hence lead to a reheating of the universe.

At epoch t before t_{eva}, the ratio of black hole to radiation energy density is given by

$$\frac{\varepsilon_{\rm BH}}{\varepsilon_R}=\frac{a}{a_0}\frac{\zeta}{1-\zeta}, \tag{37}$$

where a and a_0 are the scale factors at times t and t_0, respectively. From Eq. (37) it is evident that after sufficient expansion, the universe gets into the black hole dominated phase (e.g., if $\zeta\sim0.01$ then this transition occurs around $\sim10^{-31}$ s). When this happens, till about t_{eva} the scale factor increases as $\sim t^{2/3}$, so that the ambient temperature drops as

$$\frac{T_0}{T}\approx2\times10^{22}t^{2/3}, \tag{38}$$

where $T_0\sim10^{14}$ GeV. If one chooses the mass of the PBHs to be $m_0\sim5\times10^5$ g then these evaporate at $t_{eva}\sim3\times10^{-10}$ s, when the temperature of the universe according to Eq. (38) is $\sim$0.01 GeV (well past the EWPT). The maximum extent of reheating due to all the particles (including X bosons) emitted from the decaying PBHs can be estimated by assuming that the entire energy locked up in the rest mass of the black holes is converted to radiation. Therefore, the ratio of radiation energy density immediately after t_{eva} to just before this epoch follows from Eq. (37) to be

$$\frac{\varepsilon_R(t_{eva}^{+})}{\varepsilon_R(t_{eva}^{-})} \approx \frac{T_0}{T(t_{eva}^{-})}\frac{\zeta}{1-\zeta}. \tag{39}$$

Since $T(t_{eva}^{-})$ is about 0.01 GeV, one finds from Eq. (39) that the reheat temperature $\propto \varepsilon_R^{1/4}(t_{eva}^{+})$ is given by

$$T_{RH} \approx 100\left(\frac{\zeta}{1-\zeta}\right)^{1/4}\ \mathrm{GeV}. \tag{40}$$

For $\zeta \sim 0.01$, one obtains $T_{RH} \sim 32$ GeV, which is below the EWPT temperature.

According to this simplified analysis, it is indeed possible for PBHs of mass $\sim 10^5$ g created with $\zeta \sim 0.01$ to be extant even after EWPT and not cause electroweak scale reheating when they evaporate. Does the numerical solution support this picture?

For the purpose of numerical evolution, it is convenient to cast Eqs. (27), (35), and (36) in terms of dimensionless quantities defined below:

$$\tau = t\sqrt{G\rho_0}, \tag{41}$$

$$\alpha(\tau) = \frac{a(t)}{a_0}, \tag{42}$$

$$R(\tau) = \frac{\rho_R(t)}{\rho_0}\alpha^4, \tag{43}$$

$$M_i(\tau) = \frac{m_i(t)}{m_i(t_0)}, \tag{44}$$

where $\rho_0 \equiv \rho(t_0)$ and $a_0 \equiv a(t_0)$. In terms of the above quantities, the system of differential equations assumes the following form:

$$\alpha' = \frac{1}{\alpha}\sqrt{\frac{8\pi}{3}\left(R + \alpha\sum_{i=1}^{K}\zeta_i M_i\right)}, \tag{45}$$

$$R' = -\alpha\sum_{i=1}^{K}\zeta_i M_i', \tag{46}$$

$$\begin{aligned} M_i' = {} & m_i(t_0)M_i^2(G\rho_0)^{-1/2} \\ & \times\left(\frac{405}{\pi^3 c^3}G^2\rho_0 R\alpha^{-4}J_i - \alpha_2\left[\left(\frac{m_{\mathrm{Pl}}}{m_i(t_0)}\frac{1}{M_i}\right)\right.\right. \\ & \left.\left. - \kappa\left(\frac{m_{\mathrm{Pl}}}{m_i(t_0)}\frac{1}{M_i}\right)^n\right]^4\right), \end{aligned} \tag{47}$$

where the prime denotes differentiation with respect to τ and for convenience we have introduced

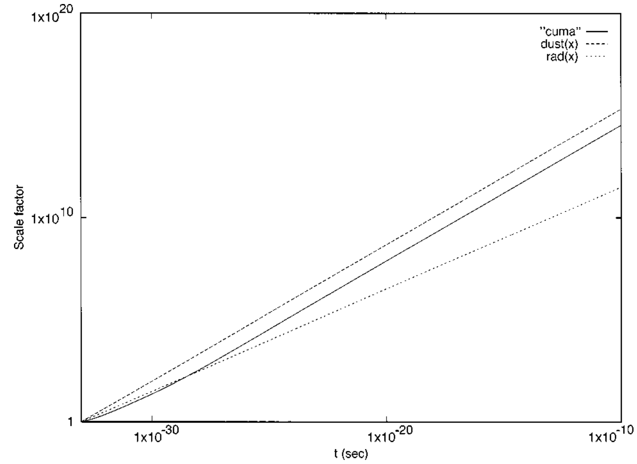

FIG. 1. The evolution of the scale factor $a(t)$ for $\zeta = 0.01$, $m_0 = 2.5\times 10^5$ g. The plots $a \sim t^{1/2}$ and $a \sim t^{2/3}$ are provided for comparison.

$$J_i \equiv J(x_{0_i}, T) = \frac{g_b^{uni}}{g_\star^{uni}}\int_{x_{0_i}}^{\infty}\frac{x^3 dx}{e^x - 1} + \frac{g_f^{uni}}{g_\star^{uni}}\int_{x_{0_i}}^{\infty}\frac{x^3 dx}{e^x + 1}$$

with

$$x_{0_i} = \frac{h}{kT}\left[\frac{\alpha_1 c}{r_{ci}}\right]$$

and

$$r_{ci} = \frac{3\sqrt{3}Gm_i(t)}{c^2}.$$

We choose t_0 to be the cosmic epoch when inflation ends $\approx 10^{-33}$ s, and set $\rho_0 = 10^{56}\,\mathrm{GeV}^4$, which is the density expected at GUT scale. In our numerical evolution program, the actual values used for the following parameters are listed below:

$$\alpha_1 = 10,$$

$$\kappa = 0.1,$$

$$n = 3,$$

$$\begin{aligned} g_b^{\mathrm{BH}} &= g_f^{\mathrm{BH}} \\ &= g_b^{uni} \\ &= g_f^{uni} \\ &= 50. \end{aligned}$$

First we consider the case when $K=1$, i.e., at the end of inflation a fraction ζ of matter lies in black holes, all with initial mass $m_0 = m_0(t_0)$. We study different models by varying ζ in the range 10^{-3} to 10^{-1} while m_0 runs through the range 10^3 to 5×10^5 g. In Fig. 1 we plot $a(t)$ for a typical choice of ζ and m_0. The plots of $a(t)$ for a radiation-dominated (RD) FRW universe ($a \sim t^{1/2}$) and a matter-

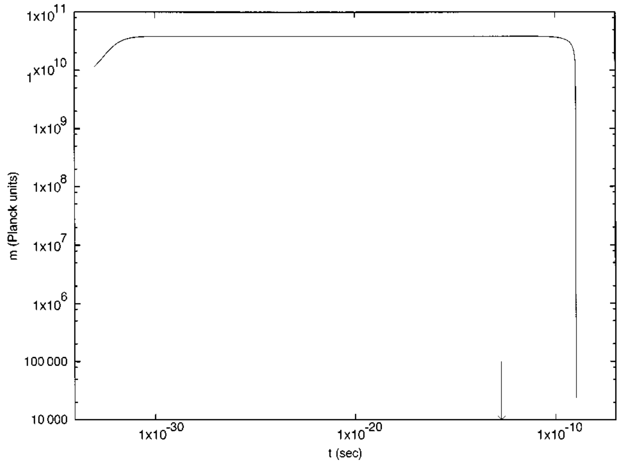

FIG. 2. The evolution of the mass $m(t)$ of the PBHs for a typical choice $\zeta=0.01$, $m_0=2.5\times10^5$ g.

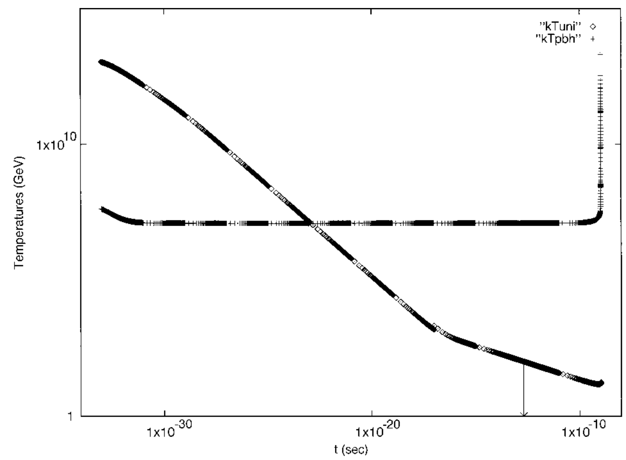

FIG. 3. The temperature T of the background thermal bath and the Hawking temperature T_{BH} for a typical choice $\zeta=0.01$ and $m_0=2.5\times10^5$ g. The instant of EWPT is marked by an arrow.

dominated (MD) FRW universe ($a\sim t^{2/3}$) are also given in the same figure. The initial behavior of the system is that of a RD universe but soon the evolution of the scale factor a becomes similar to that in an MD universe, and subsequently, as the PBHs evaporate, the dynamics becomes RD again. This is because initially energy density of relativistic matter gets depleted owing to accretion by PBHs, resulting in its decrease faster than the kinematic rate a^{-4} (see Fig. 1) so that the dominant contribution from ''dustlike'' black holes drives a faster expansion rate. We have also compared our results with approximate estimates obtained by assuming that $a(t)\sim t^{2/3}$ from $t=t_0$ (end of inflation) to $t=t_{\mathrm{EWPT}}$ (epoch of EWPT) and that $a(t)\sim t^{1/2}$ afterwards. For the range of parameters $10^3<m_0<10^5$ (g) and $0.001<\zeta<0.1$, the estimates agree with our numerical results to within an order of magnitude.

In Fig. 2 we plot a typical mass m as a function of time. It is evident from the figure that the growth of the black hole mass due to accretion takes place only in the initial period when the temperature and density of the universe is very high. This is anyway expected since the de Broglie wavelength λ of a typical particle just after the end of inflation is $\sim10^{-28}$ cm, while the R_S for a black hole of mass as low as $\sim$100 g is $\sim10^{-26}$ cm leading to a substantial accretion because of $\lambda<R_S$ criteria. At intermediate times the curve flattens out reflecting a balance between accretion and Hawking evaporation. During this phase, the dynamics is essentially MD since radiation loses out in the competition because of the expansion of the universe as well as its attenuation due to accretion by PBHs.

Towards the end, Hawking evaporation begins to dominate the evolution of PBH mass as the accretion automatically gets switched off due to the decrease in temperature and density of background radiation. In Fig. 3 we plot, for a typical choice of parameters $\zeta=0.01$ and $m_0=2.5\times10^5$, the ambient temperature of the universe T as well as the Hawking temperature T_{BH} of the black hole. The straight line portion of the curve has slope equal to $-2/3$. Thus T falls, at intermediate times, as if the dynamics of the universe was akin to that of a MD universe. At later times, when evaporation becomes the dominant process in the evolution of the holes, the universe at first starts cooling at a slower rate, but eventually reheats due to the rapid evaporation of the black holes (the reheat portion is not included in the figure). The point at which EWPT occurs is marked by an arrow in the figure (i.e., T is $\sim$100 GeV at this instant of time) corresponding to a value of $\sim2\times10^{-13}$ sec. We note that the epoch of EWPT is considerably lower than the standard value of $\sim10^{-10}$ sec obtained from the time-temperature relation in big-bang models. The reason for this is not hard to understand, as depletion of radiation by the accreting PBHs leads to a MD phase causing the T to decline faster than the usual $t^{-1/2}$ fall.

Now, the amount of reheating should be such that the temperature of the universe does not rise above the EWPT temperature ($\sim$100 GeV), because, otherwise the sphaleron processes will be reignited, leading once again to a washing out of BAU generated. This, in effect, constrains our parameters ζ and m_0. In Fig. 4 we plot the combination of ζ and

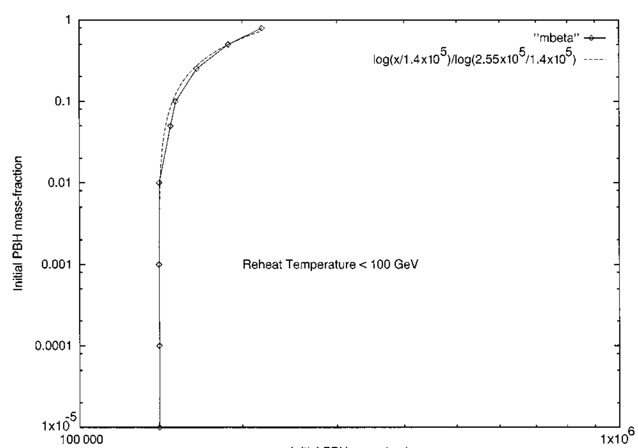

FIG. 4. The combinations ζ and m_0 for which the reheat temperature$=T_{\mathrm{EWPT}}=100$ GeV. The region with acceptable reheat temperatures $<$100 GeV is indicated in the figure. The analytical fit with dotted line is purely empirical.

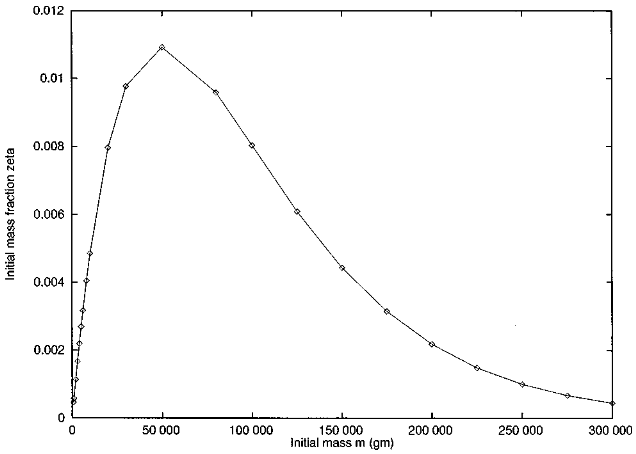

FIG. 5. Black-hole mass spectrum: plot of ζ_i against $m_i(t_0)$.

m_0 for which the reheat temperature is 100 GeV, and these points are empirically fitted with a curve. From the numerical evolution, we find that the region lying below to the right of the curve consists of those values of (ζ, m_0) for which reheat temperature remains below 100 GeV. While the region lying left of the curve consists of those combinations for which PBHs evaporate away, reaching the relic state before EWPT, and therefore are not of any use as far as baryogenesis is concerned. From Fig. 4 it is evident that for ζ lying in the interval $(10^{-5}, 0.1)$ a PBH with initial mass less than $\sim 2\times 10^5$ g converges to the relic state before EWPT, and hence does not contribute to generation of baryons. Therefore, we find that if the initial PBH mass spectrum is a delta function peaking at the mass m_0, baryogenesis through black-hole evaporation is viable only when the initial mass of the PBHs exceeds $\sim 2\times 10^5$ g for reasonably low values of ζ.

Next, we consider the case in which black-hole masses at time t_0 are distributed in a pseudo-Maxwellian manner as shown in Fig. 5. The black-hole masses fall in a range from 8×10^2 to 3×10^5 g, with 22 distinct mass values contributing to a total fraction $\Sigma_i^{22}\zeta_i \sim 0.09$ of the mass density of the universe just after the end of inflation. The chosen mass range is more or less what several PBH formation scenarios would predict (see Refs. [19,20,27]). From Fig. 6 it is apparent that black holes with larger initial mass accrete background hot matter at higher rates than those with smaller initial mass, as expected from the fact that higher mass PBHs have larger cross section for absorbing matter. We find that those PBHs with initial mass greater than 1.5×10^5 g reach X-$\bar{X}$ emitting phase after the epoch 1.9×10^{-11} s, the instant at which EWPT takes place for this spectrum of masses. Once again we find that EWPT occurs sooner than that in the standard model. There are seven such black-hole masses which finally contribute to the production of baryon excess. Because of the wide distribution of black-hole masses, the instants at which the PBHs reach the relic mass are staggered, hence no sharp reheating takes place in our analysis, rather the temperature of the universe falls at a slower rate till the largest size black hole (with initial mass $= 3\times 10^5$ g) evaporates, leaving behind a relic mass around the epoch $\sim 3\times 10^{-9}$ s, when the temperature of the universe is ~ 9 GeV. The decline of temperature with time is shown in Fig. 7.

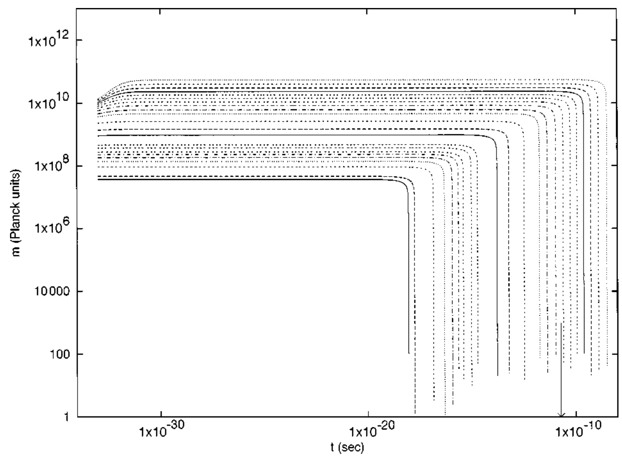

FIG. 6. The evolution of the masses $m_i(t)$ of a collection of PBH masses distributed according to the spectrum shown in Fig. 5.

Even in the case of black-hole mass distribution, K being larger than 1, in principle, one can constrain the parameter space $[\zeta_i, m_i(t_0)]$ from the requirement of reheating less than 100 GeV (as undertaken when $K=1$, see Fig. 4), however, the exercise is enormously time consuming, and is beyond the scope of the present paper.

V. BARYOGENESIS

We saw in the previous section that for $\zeta \approx 0.01$, black holes created with mass less than $\approx 2\times 10^5$ g evaporate and reach the relic state before the EWPT and hence their contribution to baryon asymmetry is doubtful due to the expected B violation induced by sphalerons. However, black holes with initial mass larger than $\approx 2.5\times 10^5$ g certainly ought to be considered as sources of baryogenesis.

There are two possible ways in which black holes that

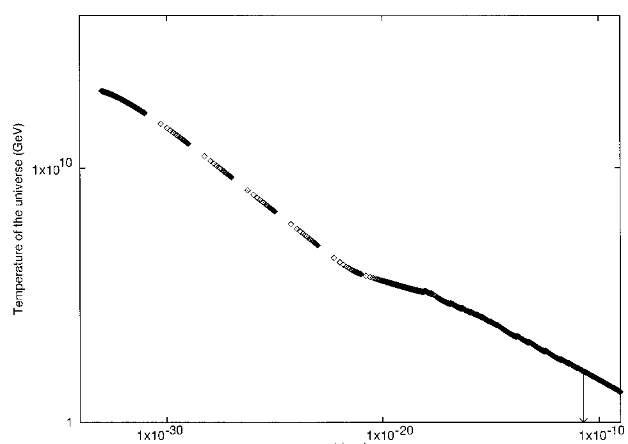

FIG. 7. The cooling of the universe for the case where PBH masses are distributed according to the spectrum displayed in Fig. 5. The epoch of EWPT is marked by an arrow, and it takes place at 1.9×10^{-11} sec.

avert spaleron washout may contribute to baryon excess: (1) Hawking emission may directly lead to a nonzero net baryon flux and (2) since such black holes reach the X-$\overline{X}$ emission phase well past the EWPT, subsequent decay of such super heavy GUT bosons can generate matter-antimatter asymmetry. It is difficult to estimate the actual amount of baryon excess that possibility (1) would give rise to, although Toussaint *et al.* [30] have demonstrated considering a toy model that if the effective Lagrangian of particle fields outside the event horizon contains a gravitational correction term that incorporates *C*, *T*, and *B* violations, then the Hawking evaporation can lead to matter-antimatter asymmetry. The difficulty lies in translating the results of the Toussaint *et al.* to realistic GUT models. In this paper, we limit ourselves to the second mechanism, and proceed to estimate the extent of baryon asymmetry resulting from decaying X-$\overline{X}$ emitted by the PBHs whose Hawking temperature reaches the GUT scale after EWPT.

Representing the specific intensity of X bosons radiated with energy $h\nu$ from a black hole by I_ν^X, we have the relation (e.g., see Ref. [24])

$$I_\nu^X = \frac{u_\nu^X(\Omega)}{v}, \tag{48}$$

where $u_\nu^X(\Omega)$ is the specific energy density and v is the speed of the emanating X bosons. With

$$v = c\left[1-\left(\frac{mc^2}{h\nu}\right)^2\right]^{1/2} \tag{49}$$

and

$$u_\nu^X(\Omega) = \frac{h\nu^3}{c^4}\,\frac{v g_X}{e^{h\nu/kT_{\rm BH}(m)}-1} \tag{50}$$

we may express I_ν^X as

$$I_\nu^X = \frac{h\nu^3}{c^2}\left[1-\left(\frac{mc^2}{h\nu}\right)^2\right]\frac{g_X}{e^{h\nu/kT_{\rm BH}(m)}-1}. \tag{51}$$

It is to be noted that g_X and $T_{\rm BH}(m)$ are the spin degeneracy factor of X bosons and Hawking temperature of a black hole of mass *m*, respectively.

The flux density of X bosons at a distance r from the black hole is given by

$$F_\nu^X = \pi I_\nu^X \frac{R_S^2}{r^2}. \tag{52}$$

Therefore, from Eqs. (51) and (52) the rate of emission of X bosons from a black hole of mass m is derived to be

$$\frac{dN_X(m)}{dt} = \int_{m_X c^2/h}^{\infty} \frac{F_\nu^X 4\pi r^2 d\nu}{h\nu} \tag{53}$$

$$= \frac{4\pi R_S^2 c^4 g_X m_X^3}{h^3} I(y_i), \tag{54}$$

where m_X is the mass of the X boson and

$$I(y_i) = \int_1^\infty \frac{y^2-1}{e^{y/y_i}-1}dy \tag{55}$$

while

$$y_i(t) \equiv \frac{kT_{\rm BH}[m_i(t)]}{m_X c^2}. \tag{56}$$

Since, at any given cosmic epoch t, the number density of black holes with a mass lying in the interval $(m, m+dm)$ is $N_0(t)f(m,t)$ [see Eq. (18)], the rate at which X and $\overline{X}$ bosons are generated in a unit proper volume is given by

$$\frac{dn_{X\overline{X}}}{dt} = 2N_0(t)\int \frac{dN_X(m)}{dt} f(m,t)dm. \tag{57}$$

In Eq. (57) the factor 2 arises because we have included production of $\overline{X}$ bosons as well. Making use of the form given in Eq. (23) we can express Eq. (57) as

$$\frac{dn_{X\overline{X}}}{dt} = 2N_0(t)\sum_{i=1}^{K} \beta_i \frac{dN_X(m_i)}{dt}. \tag{58}$$

The lifetime of a X boson $\tau_X = \Gamma_X^{-1}$ turns out to be $\approx 10^{-36}$ s when $m_X \approx 10^{14}$ GeV [31] which is negligible in comparison with the time scales over which the black-hole mass changes or the universe expands appreciably. Hence, the rate of increase of net baryon number in a unit proper volume is

$$\approx \epsilon \frac{dn_{X\overline{X}}}{dt}$$

with

$$\epsilon \equiv \frac{\Gamma(X\to ql) - \Gamma(\overline{X}\to \overline{q}\overline{l})}{\Gamma_{\rm tot}} \tag{59}$$

being the net baryon number generated by the decay of a pair of X and $\overline{X}$ [32].

If $n_B(t)$ represents net baryon number density at the cosmic epoch t then

$$\frac{d}{dt}[a^3(t)n_B(t)] = \epsilon a^3(t)\frac{dn_{X\overline{X}}}{dt}. \tag{60}$$

Employing Eqs. (31) and (58) in Eq. (60) and then integrating the latter, we obtain

$$\begin{aligned} &a^3(t)n_B(t) - a^3(t_{\rm EWPT})n_B(t_{\rm EWPT}) \\ &\quad = 2\epsilon(\rho_0 a_0^3)\sum_{i=1}^{K}\frac{\zeta_i}{m_i(t_0)}\int_{t_{\rm EWPT}}^{t}\frac{dN_X(m_i)}{dt'}dt'. \end{aligned} \tag{61}$$

Assuming that prior to black-hole baryogenesis, the net baryon number in the universe is zero [i.e., $n_B(t_{\mathrm{EWPT}})=0$] and making use of Eq. (54) in Eq. (61), we get the following expression for the net baryon number density at any time:

$$n_B(t)=\frac{\rho_0 a_0^3}{a^3(t)}\left(\frac{4G^2}{\pi\hbar^3}\right)\epsilon g_X m_X^3\sum_{i=1}^{K}\frac{\zeta_i}{m_i(t_0)}\times\int_{t_{\mathrm{EWPT}}}^{t}dt' m_i^2(t')I[y_i(t')]. \tag{62}$$

After EWPT has taken place, the evolution of PBH mass is totally dominated by, Eq. (6) since the de Broglie wavelength λ of a typical particle is larger than $\sim 10^{-15}$ cm, while the R_S corresponding to a black hole of mass as high as $\sim 10^7$ g is only $\sim 10^{-21}$ cm. Hence, using Eq. (6) we can change the variable of integration in Eq. (62) from t' to $m_i(t')$ so that

$$\int_{t_{\mathrm{EWPT}}}^{t} m_i^2(t')I[y_i(t')]dt' =\frac{2m_{\mathrm{PL}}}{\alpha_2(8\pi m_X)^4}H(m_i(\mathrm{lower}),m_i(t_{\mathrm{EWPT}})), \tag{63}$$

where H is defined to be

$$H(m_i(\mathrm{lower}),m_i(t_{\mathrm{EWPT}}))\equiv\int_{m_i(\mathrm{lower})}^{m_i(t_{\mathrm{EWPT}})}\frac{1}{y_i^2}\left[y_i\sum_{k=1}^{\infty}\frac{e^{-k/y_i}}{k^3}+\sum_{k=1}^{\infty}\frac{e^{-k/y_i}}{k^2}\right]dm_i. \tag{64}$$

In obtaining Eqs. (63) and (64) we have used the series equivalent of the integral given in Eq. (55). The value of $m_i(\mathrm{lower})$ is set by requiring y_i to be 10^{-3} since the series given in Eq. (64) is negligibly small for smaller values of y_i. This automatically takes into account the fact that only those PBHs matter for BAU that are capable of emitting X-$\overline{X}$ after EWPT. For PBH masses larger than $10^5 m_{\mathrm{Pl}}$, the value of H is 2.7×10^{-2} and becomes insensitive to the exact value of $m_i(t_{\mathrm{EWPT}})$ thereafter. Therefore, for the 7 PBHs that survive the EWPT, we have

$$\sum_{i=16}^{22}\frac{\zeta_i}{m_i(t_0)}H[m_i(\mathrm{lower}),m_i(t_{\mathrm{EWPT}})]=1.97\times 10^{-9}. \tag{65}$$

The entropy density of the universe at any epoch t is given by

$$s=\frac{2\pi^2}{45}g_\star^{uni}\frac{k^4}{(\hbar c)^3}T^3(t). \tag{66}$$

We estimate the baryon-to-entropy ratio at $t\sim 3\times 10^{-9}$ s, when all the PBHs settle on to the relic state, by making use of Eqs. (62)–(66),

$$\frac{n_B(t)}{s(t)}=7.5\times 10^{-8}\epsilon g_X\left(\frac{g_\star^{\mathrm{BH}}}{100}\right)^{-1}\left(\frac{g_\star^{uni}}{100}\right)^{-1}. \tag{67}$$

The contribution to baryon-to-entropy ratio by PBHs with initial mass m_0 and initial mass fraction ζ goes roughly as

$$\frac{n_B}{s}\approx\epsilon\zeta g_X\left(\frac{m_0}{1\ \mathrm{g}}\right)^{-1}\left(\frac{g_\star^{\mathrm{BH}}}{100}\right)^{-1}\left(\frac{g_\star^{uni}}{100}\right)^{-1}. \tag{68}$$

Hence, in the case of a delta-function mass spectrum with $\zeta\approx 0.01$ and $m_0\approx 2.5\times 10^5$ g, one obtains a baryon-to-entropy ratio of $\approx 4\times 10^{-8}\epsilon$, with $g_X=1$. Thus one may use Eq. (68) along with the value of $n_B/s\approx 10^{-11}$, that follows from observations, to put a constraint on $\epsilon\zeta/m_0$. This implies that one requires the CP-violating parameter ϵ to be around $\sim 10^{-4}$ to generate excess baryons from evaporating PBHs.

VI. DISCUSSIONS

To study the evolution of PBHs, in the early universe, that undergo accretion along with steady mass loss due to Hawking evaporation, we have laid down a formalism which can handle any black-hole mass spectrum that can be decomposed as a sum of weighted δ functions. Accretion of ambient hot matter by a black hole has been modeled in the limit of geometric approximation, so that only those particles with de Broglie wavelength less than about a tenth of Schwarzschild radius are considered for absorption by the black hole. The evolution of a flat FRW universe and the PBHs has been studied numerically to find conditions under which black holes survive past the electroweak phase transition in order that their subsequent evaporation leads to baryogenesis.

The basic picture which emerges is the following. In the case of a black-hole mass spectrum that peaks sharply at a single mass value m_0, when ζ (the initial mass fraction of PBHs) is of the order of 1%, PBHs with initial mass m_0 less than about 2.3×10^5 g evaporate *before* EWPT. Therefore, only PBHs with m_0 greater than this critical value need be considered for generation of the baryon asymmetry of the universe (BAU). Here, we wish to point out that the model of accretion which one considers can make an immense difference in the final result of the analysis. If one uses a simple spherical model of accretion in which the capture cross section is just πR_S^2 and with no de Broglie wavelength based cutoff then black holes of initial mass $m_0\approx 10^3$ g can successfully live past the EWPT, and eventually contribute to the BAU (see Majumdar *et al.* in Ref. [14]). While on using the same set of parameters with a wavelength based cutoff model of accretion, we find that PBHs of such small initial mass do not survive beyond the EWPT.

For reasonable choice of parameters, we find that in the case of PBHs with a distribution of mass ranging from 8×10^2–3×10^5 g, blackholes with initial mass larger than about $\sim 10^5$ g reach the relic state much after EWPT. Because of the presence of black holes with mass less than 10^5 g that evaporate at a faster rate, pumping in energetic particles into the surrounding medium, the ambient temperature in this case declines at a slower rate, and hence EWPT takes place later than in the case when all PBHs had the same mass of 2.5×10^5 g. As described in Secs. III and IV the evolution of mass spectrum is totally determined by the manner in which individual blackhole masses change with time, β_i or equivalently ζ_i remaining fixed for all times. As an illustration, we have shown the evolution of mass spectrum in Fig. 6 for a particular set of ζ_i.

In a previous study, Barrow *et al.* [33] had examined evolution of PBHs in an expanding universe, obtaining exact solutions for a wide range of the black-hole mass spectrum. However, accretion of ambient radiation by black holes was neglected in their work. Although we have included the possibility of black-hole mass gain in this paper, we wish to point out that accretion is important only during the initial stages, just after the end of inflation when the temperature of the universe is $\sim10^{13}$ GeV, causing an increase in the mass of a black hole by a factor of $\sim$4 in some cases. There are two factors responsible for a black hole of initial mass of $\sim10^5$ g to live after the EWPT. One is the increase in the mass due to accretion, while the other is the occurrence of EWPT sooner than that in a model in which there is no depletion of radiation due to PBHs acting as sinks. For black holes with a mass less than $\sim10^5$ g, accretion is less due to the reduction in the capture cross section because the rate of the depletion of radiation is not large leading to a delayed occurance of EWPT, after the black holes have reached the final relic state.

The ansatz of Barrow *et al.* [23] which has been used in this paper to take the expected modification of Hawking emission into account becomes important only when the mass of evaporating black holes fall below $\sim10m_{\mathrm{Pl}}$. Our numerical results are not sensitive to the exact form of modified black-hole temperature. For baryogenesis, significant quantities of X-$\overline{X}$ are emitted only during the phase when the black-hole temperature is $\sim T_{\mathrm{GUT}}$, because of which the integral H is not sensitive to the upper limit $m(t_{\mathrm{EWPT}})$ so long as the latter is larger than 10^5m_{Pl}. Therefore, the final expression for baryon-to-entropy ratio turns out to be rather simple [see Eq. (68)], implying that if at the end of inflation 1% of total matter goes into creating PBHs with initial mass 2.5×10^5 g then this scenario can successfully lead to BAU provided the CP-violating parameter ϵ is over 10^{-4}. Thus production of baryon excess through black-hole evaporation is a viable alternative to GUTs or electroweak baryogenesis, although there is no denying that because of the presence of parameters such as ζ_i and $m_i(t_0)$ whose values *a priori* are uncertain, this scenario cannot provide meaningful constraint on the value of ϵ.

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