Quantum field theory of meson mixing

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We develop a quantum-field-theoretic framework for scalar and pseudoscalar meson mixing and oscillations in time. The unitary inequivalence of the Fock space of base (unmixed) eigenstates and the physical mixed eigenstates is proven and shown to lead to a rich condensate structure. This is exploited to develop formulas for two-flavor boson oscillations in systems of arbitrary boson occupation number. The mixing and oscillation can be understood in terms of a vacuum condensate which interacts with the bare particles to induce nontrivial effects. We apply these formulas to analyze the mixing of η with η' and comment on the $K_L K_S$ system. In addition, we consider the mixing of boson coherent states, which may have future applications in the construction of meson lasers. [S0556-2821(99)00715-8]

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I. INTRODUCTION

The study of mixing transformations plays an important part in particle physics phenomenology [1]. The standard model incorporates the mixing of fermion fields through the Kobayashi-Maskawa [2] mixing of three quark flavors, a generalization of the original Cabibbo [3] mixing matrix between the *d* and *s* quarks. In addition, neutrino mixing and oscillations are the likely resolution of the famous solar neutrino puzzle [4]. In the boson sector, the mixing of K^0 with $\overline{K^0}$ via weak currents provided the first evidence of *CP* violation [5]. The $\eta\eta'$ mixing in the SU(3) flavor group provides a unique opportunity for testing QCD and the constituent quark model. Furthermore, the particle mixing relations for both the fermion and boson case are believed to be related to the condensate structure of the vacuum. The nontrivial nature of the vacuum is expected to hold the answer to many of the most salient questions regarding confinement and the symmetry-breaking mechanism.

The importance of the fermion mixing transformations has recently prompted a fundamental examination of them from a quantum-field-theoretic perspective [6,7]. To our knowledge, a similar analysis in the bosonic sector has not yet been undertaken. Moreover, the statistics of bosons and fermions are intrinsically different. Thus, the results for boson mixing are expected to be quite different from the previous analysis of fermions. That is the motivation for the present work.

We begin in Sec. II with an investigation of the vacuum structure and the related condensation, using the relation between the base eigenstate and the physical mixed eigenstate fields as our starting point. The unitary inequivalence of the associated Fock spaces is proven and an explicit formula for the condensation density is derived. In Sec. III, the ladder operators are contructed in the mixed basis. These are used to derive time-dependent oscillation formulas for one-boson states, *n*-boson states, and boson-coherent states. We also show how the ladder operators can be generated from two successive similarity transformations. Section IV is devoted studying specific cases in our formalism, such as the $\eta\eta'$ system. Finally, in Sec. V we offer some concluding remarks and explore future possibilities.

II. THE VACUUM STRUCTURE AND CONDENSATION

We take the mixing of two arbitrary flavors of bosons to be given by

$$\Phi_\alpha(x) = \cos\theta\phi_1(x) + \sin\theta\phi_2(x),$$
$$\Phi_\beta(x) = -\sin\theta\phi_1(x) + \cos\theta\phi_2(x), \tag{1}$$

where ϕ_i $(i=1,2)$ are solutions to the real Klein-Gordon equation and are given by

$$\phi_i(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{2VE_{k,i}}}[a_{k,i}e^{-ik\cdot x} + a^\dagger_{k,i}e^{ik\cdot x}]. \tag{2}$$

The commutation relations are

$$[a_{k,i}, a^\dagger_{k',j}] = \delta_{kk'}\delta_{ij} \tag{3}$$

from which it follows

$$[\phi_i(\mathbf{x}), \dot{\phi}_j(\mathbf{y})] = i\delta(\mathbf{x}-\mathbf{y})\delta_{ij}. \tag{4}$$

For calculational simplicity in the following we shall redefine $a_{k,i} \rightarrow a_{k,i}e^{-iE_{k,i}t}$. It is not difficult to see that the algebra of the annihilation and creation operators remains intact and that this redefinition does not effect any of the results we obtain.

In order to analyze the condensation density and structure of the vacuum, we must first determine the relationship between the Fock space of base eigenstates and the Fock space of physical mixed states. To this end we need the unitary generator that rotates base eigenstates into physical eigenstates:

$$\Phi_\alpha(x) = G^{-1}(\theta)\phi_1(x)G(\theta),$$
$$\Phi_\beta(x) = G^{-1}(\theta)\phi_2(x)G(\theta). \tag{5}$$

Using the Baker-Hausdorf lemma one can easily verify that

$$G(\theta) = \exp\left[-i\theta\int d^3x(\dot{\phi}_1(x)\phi_2(x) - \dot{\phi}_2(x)\phi_1(x))\right] \tag{6}$$

is the generator. The commutation relations allow us to rewrite this as $G(\theta)=e^{iS\theta}$ where

$$S=\sum_{\mathbf{k}} \frac{i}{2}[\gamma_- a_{-1}a_2+\gamma_+ a_{-1}a^\dagger_{-2}-\gamma_+ a_1^\dagger a_2-\gamma_- a_1^\dagger a^\dagger_{-2}]. \tag{7}$$

and $\gamma_\pm=\sqrt{E_1/E_2}\pm\sqrt{E_2/E_1}$. Here we have suppressed all of the **k** subscripts on the ladder operators for notational simplicity and a_{-1} stands for $a_{-k,1}$, for example. Similarly we will use a_α for $a_{k,\alpha}$.

We note that

$$_{1,2}\langle a|\phi_1(x)|b\rangle_{1,2}={}_{1,2}\langle a|G(\theta)\Phi_\alpha(x)G^{-1}(\theta)|b\rangle_{1,2} \tag{8}$$

implies $G^{-1}(\theta)|b\rangle_{1,2}\in\mathcal{H}_{\alpha,\beta}$ and $|0\rangle_{\alpha,\beta}=G^{-1}(\theta)|0\rangle_{1,2}$. Here $|\rangle_{1,2}\in\mathcal{H}_{1,2}$ and $|\rangle_{\alpha,\beta}\in\mathcal{H}_{\alpha,\beta}$, where $\mathcal{H}_{1,2}$ and $\mathcal{H}_{\alpha,\beta}$ are the Fock space of base (unmixed) eigenstates and the Fock space of physical mixed eigenstates, respectively. In this form we see that

$$_{\alpha,\beta}\langle 0|0\rangle_{1,2}=0 \tag{9}$$

trivially follows. This proves the unitary inequivalence of the Fock space of base and physical mixed eigenstates even in the finite volume regime. For fermions, Blasone and Vitiello [6] have found that the respective Fock spaces are unitarily inequivalent only in the infinite volume limit. This contrast arises because fermions have a finite number of states in a finite volume whereas bosons have an uncountable infinity of states in a finite volume. Thus, to obtain the aggregate particle behavior which manifests itself in the vacuum states, it is necessary to go to an infinite volume for fermions but not for bosons.

We define the number operator in the natural way, $N_{k,i}\equiv N_i=a_i^\dagger a_i$. The condensation density of the physical vacuum is defined as $_{\alpha,\beta}\langle 0|N_1|0\rangle_{\alpha,\beta}$. It follows that

$$_{\alpha,\beta}\langle 0|N_1|0\rangle_{\alpha,\beta}={}_{1,2}\langle 0|e^{iS\theta}a_1^\dagger e^{-iS\theta}e^{iS\theta}a_1 e^{-iS\theta}|0\rangle_{1,2}, \tag{10}$$

where

$$e^{iS\theta}a_1e^{-iS\theta}=a_1\cos\theta-\frac{\sin\theta}{2}(\gamma_+a_2+\gamma_-a^\dagger_{-2}). \tag{11}$$

From these we easily obtain

$$_{\alpha,\beta}\langle 0|N_1|0\rangle_{\alpha,\beta}={}_{\alpha,\beta}\langle 0|N_2|0\rangle_{\alpha,\beta}=\frac{\gamma_-^2}{4}\sin^2\theta. \tag{12}$$

Therefore, an admixture of base-eigenstate particles is found in the physical vacuum state. As we will see, this condensation density becomes manifest in the boson mixing relations to be derived later. Note that the converse is also true. The base vacuum state contains an admixture of physical eigenstate particles and the condensation density, given by $_{1,2}\langle 0|N_\alpha|0\rangle_{1,2}={}_{1,2}\langle 0|N_\beta|0\rangle_{1,2}$, is the same as above [Eq. (12)].

III. LADDER OPERATORS AND MIXING RELATIONS

The ladder operators in the mixed basis are given from Eq. (5) as

$$a_\alpha=G^{-1}(\theta)a_1G(\theta), \tag{13}$$

assuming equal masses in the two eigenstate representations, or after a simple redefinition of the operators. This leads to the following operators:

$$a_\alpha=a_1\cos\theta+\frac{\sin\theta}{2}(\gamma_+a_2+\gamma_-a^\dagger_{-2}),$$

$$a_\beta=a_2\cos\theta-\frac{\sin\theta}{2}(\gamma_+a_1+\gamma_-a^\dagger_{-1}). \tag{14}$$

The number operators $N_\alpha\equiv a_\alpha^\dagger a_\alpha$ and $N_\beta\equiv a_\beta^\dagger a_\beta$ are easily constructed from these.

We would like to consider the mixing of one meson states which, for arbitrary meson flavor α, are given by

$$|\alpha\rangle=a_\alpha^\dagger|0\rangle=\cos\theta|1\rangle+\frac{\gamma_+\sin\theta}{2}|2\rangle. \tag{15}$$

This gives a normalization factor of

$$\langle\alpha|\alpha\rangle=\cos^2\theta+\frac{\gamma_+^2\sin^2\theta}{4}=1+C_b, \tag{16}$$

where $\gamma_+^2=4+\gamma_-^2$ was used and C_b is the boson condensation density given in Eq. (12). The significance of the normalization will be commented upon later. From the definitions of the number operator and the meson state it is easy to see that

$$\langle 1|N_\alpha|1\rangle=\cos^2\theta+\frac{\gamma_-^2\sin^2\theta}{4},$$

$$\langle 1|N_\alpha|2\rangle=\frac{\gamma_+\cos\theta\sin\theta}{2},$$

$$\langle 2|N_\alpha|2\rangle=\frac{(\gamma_-^2+\gamma_+^2)\sin^2\theta}{4}. \tag{17}$$

From these relations we find

$$\langle\alpha|N_\alpha|\alpha\rangle_N=\cos^2\theta+\frac{(\gamma_+^2+\gamma_-^2)\sin^2\theta}{4}=(1+C_b)+C_b, \tag{18}$$

where $\langle\alpha|N_\alpha|\alpha\rangle_N=\langle\alpha|N_\alpha|\alpha\rangle/\langle\alpha|\alpha\rangle$. Similarly, we obtain

$$\langle\alpha|N_\beta|\alpha\rangle_N=\frac{\gamma_-^2\sin^2\theta}{4}=C_b. \tag{19}$$

In order to find formulas for the oscillation of flavors in time we use the time evolution operator given by $U(t)=\exp(-iH_{1,2}t)$, where $H_{1,2}|1\rangle=E_1|1\rangle$, etc. The calculation yields

$$\langle\alpha(t)|N_\alpha|\alpha(t)\rangle_N=\langle\alpha|N\alpha|\alpha\rangle_N-\frac{\gamma_+^2\cos^2\theta\sin^2\theta}{1+C_b}\sin^2\frac{\Delta Et}{2} \tag{20}$$

and

$$\langle\alpha(t)|N_\beta|\alpha(t)\rangle_N=\langle\alpha|N_\beta|\alpha\rangle_N+\frac{\gamma_+^2\cos^2\theta\sin^2\theta}{1+C_b}\sin^2\frac{\Delta Et}{2}. \tag{21}$$

We observe that the sum of the number of both species is constant in time, as expected. This suggests the interpretation that the oscillation phenomena results from particle flavors interacting with the nontrivial vacuum condensation.

Unlike fermions, multiple bosons can occupy a single quantum state. Thus, we would like to see how particle flavors mix in an identically prepared state of n scalar or pseudoscalar bosons of flavor α defined by $|n,\alpha\rangle=(a_\alpha^\dagger)^n/\sqrt{n!}|0\rangle_{1,2}$. The calculation is a straightforward generalization of the above methods and the results are

$$\langle n,\alpha|N_\alpha|n,\alpha\rangle_N=n\left(\cos^2\theta+\frac{\gamma_+^2\sin^2\theta}{4}\right)+\frac{\gamma_-^2\sin^2\theta}{4}=n(1+C_b)+C_b \tag{22}$$

and

$$\langle n,\alpha|N_\beta|n,\alpha\rangle_N=\frac{\gamma_-^2\sin^2\theta}{4}=C_b. \tag{23}$$

Here the normalization of states is given by

$$\langle n,\alpha|n,\alpha\rangle=\left(\cos^2\theta+\frac{\gamma_+^2\sin^2\theta}{4}\right)^n=(1+C_b)^n. \tag{24}$$

The fact that the states in the α, β basis are not already normalized follows from the nontrivial condensation density and the unitary inequivalence of the Fock bases. This is observed in Fig. 2, where the total number of particles in a "one"-particle state is seen to be greater than one. In general, the normalization factor grows exponentially with n. The preceding equations written in terms of C_b provide a clear and very interesting physical interpretation of the mixing. In Eq. (22) the term C_b is simply the static vacuum condensation, whereas the term $n(1+C_b)$ represents a "renormalized" number of particles. Each of the n bosons obtains a particle number slightly larger than one through its nonperturbative attraction of vacuum condensate. However, this attraction of vacuum condensate leaves no holes in the pervasive vacuum condensate, as we still have the static C_b contribution. In a sense, $1+C_b$ just redefines what we mean by one particle. This is further verified in Eq. (24) where we have the normalization equal to n factors of $1+C_b$, which can be looked at as abstract particle number "volume" in Fock space. These results are somewhat different from the naive expectation that putting n bosons in the nontrivial vacuum will yield simply a boson particle number of $n+C_b$. The above results are to be contrasted with the case for fermions [6] where the authors [Eqs. (4.13)–(4.17)] find, after translating into our notation,

$$\langle\alpha|\alpha\rangle=1-C_f,\quad \langle\alpha|N_\alpha|\alpha\rangle_N=1=(1-C_f)+C_f,$$

and

$$\langle\alpha|N_\beta|\alpha\rangle_N=C_f, \tag{25}$$

where C_f is the fermionic condensation density and is of the same form as C_b. We see that the particle number in a one particle state is just one. There is no pervasive vacuum condensate nor any attracted vacuum condensate, as expected from the exclusion principle. The α fermion excludes any α vacuum condensate, while the β contribution is entirely condensate. The exclusion of condensate can also be seen in the normalization. Time evolution introduces oscillations in both α and β proportional to n, for both the fermion and boson case, though for fermions $n=1$.

Note in Eqs. (22) and (23) that one species is linearly dependent on n while the other is n independent. This is very interesting, since it implies that the ratio of the α species to the β species grows linearly with n. Thus, states with more identically prepared mesons have less mixing "per capita." This is subject to experimental test. The relationship does not hold true when the states are allowed to evolve in time:

$$\langle n(t),\alpha|N_\alpha|n(t),\alpha\rangle_N=\langle n,\alpha|N_\alpha|n,\alpha\rangle_N-\frac{n\gamma_+^2\cos^2\theta\sin^2\theta}{1+C_b}\sin^2\frac{\Delta Et}{2},$$
$$\langle n(t),\alpha|N_\beta|n(t),\alpha\rangle_N=\langle n,\alpha|N_\beta|n,\alpha\rangle_N+\frac{n\gamma_+^2\cos^2\theta\sin^2\theta}{1+C_b}\sin^2\frac{\Delta Et}{2}. \tag{26}$$

In the static case, we noted that the mixing is related to the vacuum condensation. Dynamically, the mixed state further interacts with the vacuum to produce time-dependent effects which depend on the number of interacting particles in the mixed state.

We may also consider the mixing of meson coherent states defined by

$$|\mathcal{C},\alpha\rangle\equiv\mathcal{N}e^{\mathcal{C}a_\alpha^\dagger}|0\rangle_{1,2}, \tag{27}$$

where $\mathcal{C}$ is a complex number and the normalization is $\mathcal{N}=\exp[-|\mathcal{C}|^2/2(1+C_b)]$. Defining $c=\cos\theta$ and $s=(\gamma_+/2)\sin\theta$, and using the binomial theorem to expand $(a_\alpha^\dagger)^n$ in terms of a_1 and a_2, we find a useful expression for the coherent state:

$$|\mathcal{C},\alpha\rangle=\mathcal{N}\sum_{n=0}^{\infty}\sum_{j=0}^{n}\frac{\mathcal{C}^n c^{n-j}s^j}{\sqrt{(n-j)!j!}}|n-j,j\rangle, \tag{28}$$

where $|n-j,j\rangle$ represents the state of n-j base eigenstate one particles and j base eigenstate two particles. Using this, we quickly obtain the following intermediate results: $\langle\mathcal{C},\alpha|N_1|\mathcal{C},\alpha\rangle=|\mathcal{C}|^2c^2$, $\langle\mathcal{C},\alpha|N_2|\mathcal{C},\alpha\rangle=|\mathcal{C}|^2s^2$, $\langle\mathcal{C},\alpha|a_{-2}a_{-2}^\dagger|\mathcal{C},\alpha\rangle=1$, $\langle\mathcal{C},\alpha|a_1^\dagger a_2|\mathcal{C},\alpha\rangle=\langle\mathcal{C},\alpha|a_2^\dagger a_1|\mathcal{C},\alpha\rangle=cs|\mathcal{C}|^2$. With these one can show that

$$\begin{aligned}\langle\mathcal{C},\alpha|N_\alpha|\mathcal{C},\alpha\rangle&=(c^2+s^2)^2|\mathcal{C}|^2+\frac{\gamma_-^2}{\gamma_+^2}s^2\\&=\left(\cos^2\theta+\frac{\gamma_+^2}{4}\sin^2\theta\right)^2|\mathcal{C}|^2+\frac{\gamma_-^2}{4}\sin^2\theta\\&=(1+C_b)^2|\mathcal{C}|^2+C_b\end{aligned}\tag{29}$$

and

$$\langle\mathcal{C},\alpha|N_\beta|\mathcal{C},\alpha\rangle=\frac{\gamma_-^2}{\gamma_+^2}s^2=\frac{\gamma_-^2}{4}\sin^2\theta=C_b.\tag{30}$$

The time-dependent relations are derived in a straightforward way and are

$$\begin{aligned}\langle\mathcal{C}(t),\alpha|N_\alpha|\mathcal{C}(t),\alpha\rangle&=\left(\cos^2\theta+\frac{\gamma_+^2}{4}\sin^2\theta\right)^2|\mathcal{C}|^2+\frac{\gamma_-^2}{4}\sin^2\theta\\&\quad-|\mathcal{C}|^2\cos^2\theta\sin^2\theta\gamma_+^2\sin^2\frac{\Delta Et}{2}\\&=(1+C_b)^2|\mathcal{C}|^2+C_b\\&\quad-|\mathcal{C}|^2\cos^2\theta\sin^2\theta\gamma_+^2\sin^2\frac{\Delta Et}{2}\\\langle\mathcal{C}(t),\alpha|N_\beta|\mathcal{C}(t),\alpha\rangle&=\frac{\gamma_-^2}{4}\sin^2\theta\\&\quad+|\mathcal{C}|^2\cos^2\theta\sin^2\theta\gamma_+^2\sin^2\frac{\Delta Et}{2}\\&=C_b+|\mathcal{C}|^2\cos^2\theta\sin^2\theta\gamma_+^2\sin^2\frac{\Delta Et}{2}.\end{aligned}\tag{31}$$

Now we show how each of the ladder operators in Eq. (14) can be obtained by two similarity transformations. First the base eigenstates are rotated together. Then, through a Bogoliubov transformation [9], the particles are mixed with the antiparticles moving backward in time. We seek operators R and B such that

$$\begin{aligned}a_\alpha&=B_1^{-1}R^{-1}a_1RB_1,\\a_\beta&=B_2^{-1}R^{-1}a_2RB_2.\end{aligned}\tag{32}$$

We obtain

$$R=\exp\left[\theta\sum_k\ (a_{k,1}^\dagger a_{k,2}-a_{k,2}^\dagger a_{k,1})\right]\tag{33}$$

for both mass eigenstate rotations. For the Bogoliubov transformations we need two different operators:

$$\begin{aligned}B_1&=\exp\left[\phi\sum_k\ (a_{k,2}^\dagger a_{-k,2}^\dagger-a_{k,2}a_{-k,2})\right],\\B_2&=\exp\left[-\phi\sum_k\ (a_{k,1}^\dagger a_{-k,1}^\dagger-a_{k,1}a_{-k,1})\right],\end{aligned}\tag{34}$$

where $\cosh\phi\equiv\gamma_+/2$ and $\sinh\phi\equiv\gamma_-/2$.

One should note that the nontrivial mixing phenomena are possible only if both the mixing angle θ is nonzero and the mass difference between the two mass eigenstates does not vanish. As shown in Eq. (12), the condensation density of the physical vacuum is nonzero only if these two conditions ($\theta\neq0$ and $\gamma_-\neq0$) are satisfied. The operators R and B given by Eqs. (33) and (34) are associated with these two conditions, $\theta\neq0$ and $\gamma_-\neq0$, respectively. These conditions are required in order for the two operators to be different from the identity operator. Unless both operators are nontrivial (i.e., different from the identity operator), one cannot expect the physically observable mixing phenomena.

IV. APPLICATION TO REAL MESON STATES

To illustrate the results of the previous section we examine the $\eta\eta'$ system. The masses are taken to be 549 and 958 MeV, respectively, and of course in the particle rest frame the energies in the above expressions reduce to the masses. The phenomenologically allowed mixing angle ($\theta_{SU(3)}$) range of the $\eta\eta'$ system is given between $-10°$ and $-23°$ [8], where the mixing angle $\theta_{SU(3)}$ is defined by Eq. (36) of Ref. [9]. This angle represents the breaking of the SU(3) symmetry, the eigenstates of which are already rotated $-35.26°$ from $u\bar{u}+d\bar{d}$ and $s\bar{s}$ to $\eta=u\bar{u}+d\bar{d}-2s\bar{s}$ and $\eta'=u\bar{u}+d\bar{d}+s\bar{s}$. Thus, our mixing angle is defined by $\theta=\theta_{SU(3)}-35.26°$. Recent analysis of the $\eta\eta'$ mixing angle using a constituent quark model based on the Fock states quantized on the light-front can be found in Ref. [10] and the references therein. The optimal value found for $\theta_{SU(3)}$ was $\sim-19°$, and thus $\theta=-54°$ was used in generating Figs. 1 and 3. The $\eta\eta'$ system is interesting because it is nearly maximally mixed. In Fig. 2 we see that at $|\theta|=45°$ the time-averaged occupation numbers for both particles are equal, and are nearly equal in the range of possible θ values. Figure 1 shows how the flavor oscillations occur on a very short time scale, even compared with the lifetimes of η and η', which are 7×10^{-19} s and 3×10^{-21} s, respectively. Figure 3 gives the ratio of the quantities plotted in Fig. 1.

The same formulation has been applied to the mixing of the $K_L K_S$ system, although the CP violation appears to be too minimal to lead to any appreciable meson mixing observables, unlike the case of the η and η' system. However, this issue deserves further investigation.

V. CONCLUSIONS AND DISCUSSIONS

The nontrivial scalar and pseudoscalar meson mixing effects may be understood by the condensation of correspond-

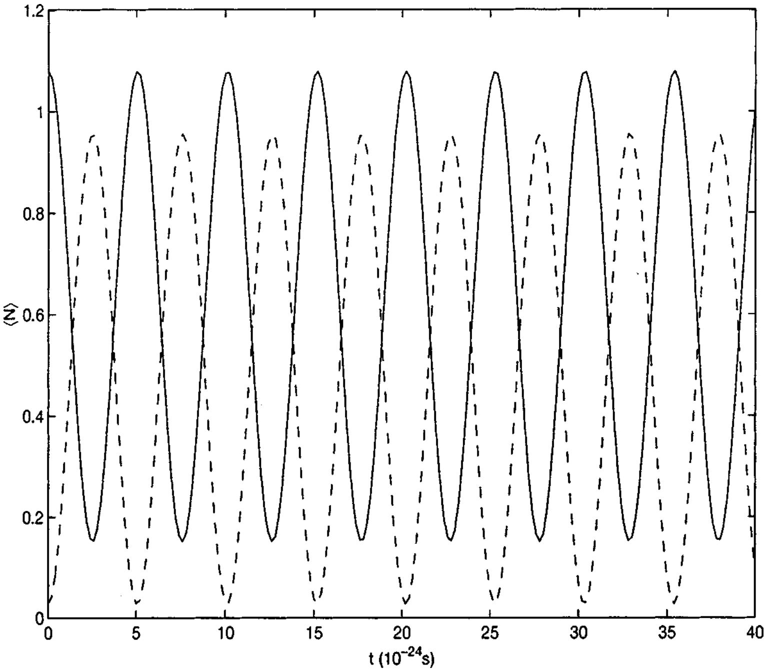

FIG. 1. The expectation value of the number operator for $\alpha=\eta$ and $\beta=\eta'$ in a $n_\eta=1$ state. The solid and dashed curves correspond to $\langle n_\eta(t)|N_\eta|n_\eta(t)\rangle_N$ and $\langle n_\eta(t)|N_{\eta'}|n_\eta(t)\rangle_N$, respectively, as given by Eq. (25). The mixing angle is taken to be $\theta=-54°$.

ing flavor states in the vacuum as presented in this work. Central to this analysis is the interplay between the base (unmixed) Fock space and the physical Fock space. Their nontrivial relationship (unitary inequivalence of the vacuum states) gives rise to the mixing and oscillation phenomena. While a similar quantum-field-theoretic formulation was presented for the fermion mixing [6], our analysis intrinsically differs from the fermion case because of the fundamental difference in statistics. As a consequence, we found that the unitary inequivalence of the base flavor states and the physical mass eigenstates holds even in the finite volume regime, in contrast to the case of fermion mixing where the unitary inequivalence holds only in the infinite volume limit [6]. An interesting physical interpretation of the results is that an n boson state can be thought of as a sum of the static vacuum condensate, a ''renormalized'' number of bosons $n(1+C_b)$, and time evolution effects. We also noted that, for both the boson and fermion cases, the nontrivial observable mixing phenomena cannot occur unless there is both a nonzero mixing angle and also a nonzero mass (energy) difference between the two physically measurable mixed states.

As a physical application, we used our formulation to analyze the $\eta\eta'$ system and found that the measured mixing angle and mass difference between η and η' can be related

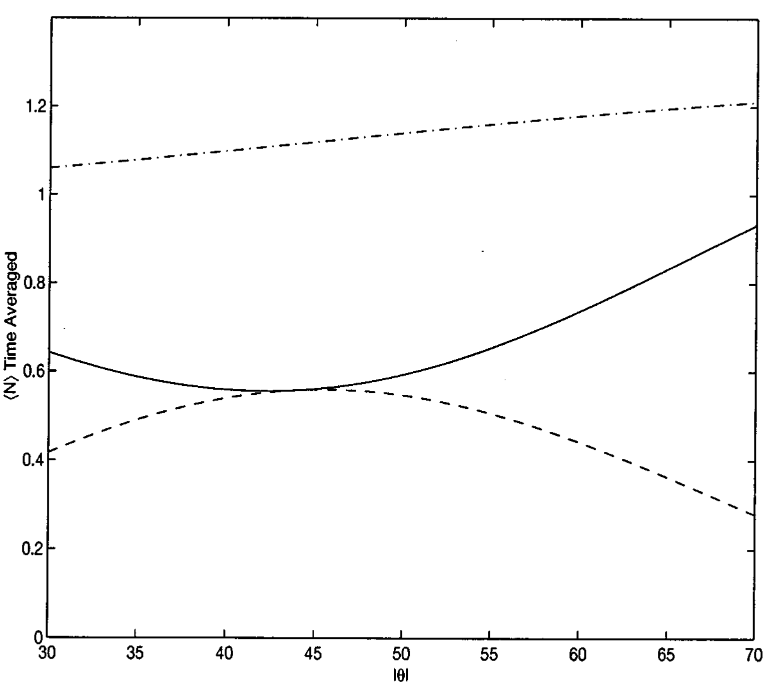

FIG. 2. The time-averaged occupation number expectation values for the $n_\eta=1$ state plotted versus $|\theta|$, the mixing angle. The solid and dashed lines represent the time-averaged values of $\langle N_\eta\rangle$ and $\langle N_{\eta'}\rangle$, respectively. The dash-dotted line is the sum of the two.

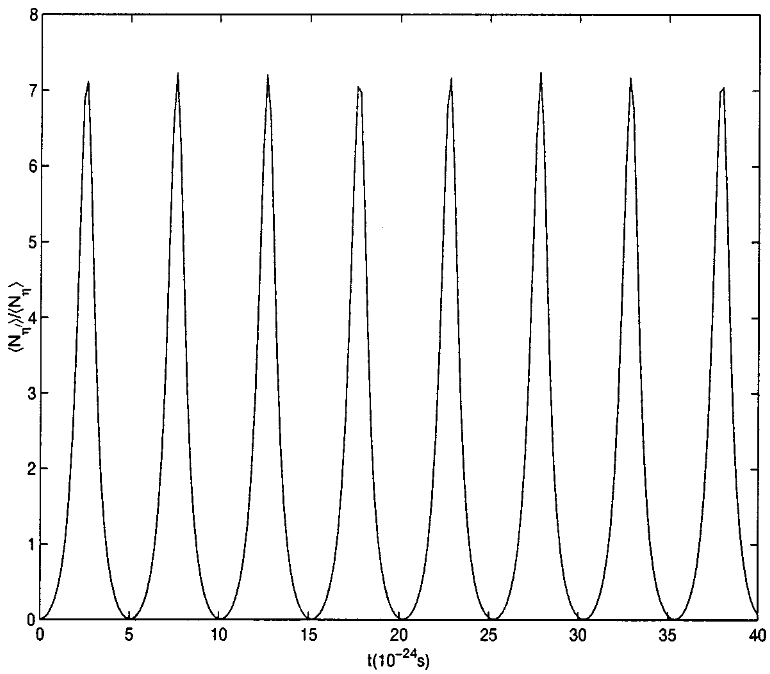

FIG. 3. The ratio of the expectation values of the number operators for η and η', as given by Eq. (25), for an arbitrary n_η state. The value n_η is unimportant since any value will yield an almost identical curve, with the $n_\eta = 1$ case only being shifted down slightly, reflecting the relative abundance of vacuum condensation.

to the nontrivial flavor condensation in the vacuum. However, more fundamental questions such as the translation of the condensation in hadronic degrees of freedom to those in quark and gluon degrees of freedom remains unanswered. The answer to this question depends on the dynamics responsible for the confinement of quark and gluon degrees of freedom and perhaps has to rely on lattice QCD and/or some phenomenological model that accommodates strongly interacting QCD. Further investigations along this line are underway. Also, it would be interesting to look at the mixing transformations between gauge vector bosons governed by the Weinberg angle in the electroweak theory as well as vector mesons such as the ρ and ω. While the statistics are the same as the scalar and pseudoscalar bosons considered here, there will be additional spin-dependent interactions which complicate the analysis.

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