

Family unification from universality

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A direct consequence of the occurrence of fermion families in the standard model is the invariance of fermion currents under certain groups of (universality) transformations. In this paper we show how these universality properties can themselves be used as a method of finding and studying “grand” family unification models. In the exact standard model limit two independent universality groups S_{wk} and S_{st} of weak and strong gauge interactions are first identified. The subgroup of any family unification group \mathcal{G} whose currents are invariant under $S_{wk}(S_{st})$ is then the centralizer $G_{wk}(G_{st})$ of $S_{wk}(S_{st})$ in \mathcal{G} . Choosing $\mathcal{G}=SU(8N)$, we find $G_{wk}=SU(2)$ and $G_{st}=U(1)\times SU(3)$; the standard model group $G=G_{wk}\times G_{st}$ is the group which respects either weak or strong universality. The fundamental representation $\mathbf{8N}$ of $SU(8N)$ decomposes under $SU(2)\times SU(3)$ as N copies of $(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{3})$; their $U(1)$ charges are the usual hypercharges. A Higgs field transforming as $\mathbf{8N}$ accomplishes the secondary symmetry reduction [from G to $U(1)_{em}\times SU(3)$] satisfactorily. The requirement that charged currents be $V-A$ forces all fermions to be left-handed in the unbroken G limit, making the model chirally invariant in a strong sense—fermions have no right-handed components to make masses with. The remedy proposed is that R -fermions are composites of L -fermions and Higgs bosons. If their binding is in one specific channel, R -fermions are shown to come in the right numbers and with the right couplings to ensure pure $VU(1)_{em}$ and $SU(3)$ currents while leaving the $SU(2)$ currents unchanged. It is finally argued that universality is most naturally understood in terms of a simple preonic structure for fermions (but not for gauge bosons), obviating the need for a primary $\mathcal{G}\rightarrow G$ Higgs mechanism. $SU(8N)$ is then best interpreted as the global “metaflavor” group of L -chiral fermions. In this picture, \mathcal{G}/G is not gauged; there are no ultraheavy gauge bosons and hence no anomaly or hierarchy problem. [S0556-2821(99)01917-7]

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I. INTRODUCTION

The problem of accommodating the existence of several replicas of a family of leptons and quarks in unified theories of all their interactions (except, of course, gravitational) has preoccupied model makers for more than two decades now. Beginning with *ad hoc* impositions of invariance under (finite [1] or Lie [2]) groups of so called horizontal symmetries, this endeavor soon moved on to the recognition that certain orthogonal groups \mathcal{G} have subgroups G and representations ρ such that the restriction of ρ to G is a direct sum of copies of a unique irreducible representation of G , making such groups plausible candidates for family unification [3]. Such models, however, have certain fundamental difficulties (primarily to do with unacceptably large right chiral interactions) which have resisted a satisfactory natural resolution. Subsequently, attention turned increasingly to supersymmetric models; in particular, very detailed work on superstring-inspired family unification models are going on apace now [4]. At the same time it has recently been shown that conventional nonsupersymmetric unification models, when combined with the topological properties of the Higgs phase of non-Abelian gauge theories—i.e., going beyond the perturbative regime—have the richness to accommodate the family structure [5,6].

The approach of the present paper to dealing with families is, by and large, orthodox, at least in its fundamentals; supersymmetry is not invoked and topological aspects of the reduction of a unifying gauge group \mathcal{G} to an observed or

effective gauge group G are ignored in the Lie algebraic considerations below. It is also a pragmatic and phenomenological approach. Instead of looking for the source of the family structure, we shall focus primarily on its most characteristic empirical signature, namely the observed universality properties of the gauge interactions of all leptons and quarks, $8N$ in number where N is the number of families. We shall seek to determine a simple group \mathcal{G} such that the subgroup of \mathcal{G} whose currents respect the demand of universality is the (unbroken) standard model group $G=SU(2)\times U(1)\times SU(3)$. In the actual implementation, as described in the next section, we proceed as follows. Ignoring fermion masses and mixings, i.e., before G is further broken to $U(1)_{em}\times SU(3)$, the G -gauge interactions of fermions are separately invariant under two types of—weak and strong—universality transformations forming two distinct subgroups S_{wk} and S_{st} of \mathcal{G} . The \mathcal{G} -gauge currents which are invariant under S_{wk} (respectively S_{st}) are easily shown to couple to the gauge bosons of a subgroup $G_{wk}(G_{st})$ of \mathcal{G} which is the centraliser of $S_{wk}(S_{st})$, namely the group of elements of \mathcal{G} which commute with all elements of $S_{wk}(S_{st})$. Moreover, the gauge group whose currents are invariant under *either* S_{wk} or S_{st} is $G_{wk}\times G_{st}$. After identifying S_{wk} and S_{st} (taking account in particular of the fact that leptons have no strong interactions), it is established that if \mathcal{G} is chosen to be $SU(8N)$, then $G_{wk}=SU(2)$ and $G_{st}=U(1)\times SU(3)$. Conversely if we want to have $G=G_{wk}\times G_{st}$ to be $SU(2)\times U(1)\times SU(3)$, then \mathcal{G} can only be $SU(8N)$. The fundamental representation $\mathbf{8N}$ decomposes under $SU(2)\times SU(3)$ as N copies of the representation $(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{3})$ i.e.,

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N families of leptons and quarks. Thus, by embedding $SU(2) \times U(1) \times SU(3)$ in $SU(8N)$ via the imposition of weak and strong universalities, $SU(8N)$ is made to play the role of a “grand” family unification group (no other simple group can serve this purpose) [7]. In particular, independently of the mechanism for the reduction of the gauge group from \mathcal{G} to G (more fundamentally, even in ignorance of the true source of universality), the relative coupling constants of $SU(2)$, $SU(3)$ and $U(1)$ get fixed.

It is noteworthy that the (“weak”) hypercharge $U(1)$ reflects the absence of strong interactions for the leptons (as it perhaps should since, empirically, it does distinguish between leptons and quarks). Other indications that the model is somewhat off the beaten track also become apparent at this point. For instance, the $SU(2)$ coupling is smaller, in relation to $SU(3)$ and $U(1)$, by a factor $\sqrt{2}$ than in most conventional unification models, resulting in a (unification) value of $\sin^2\theta$ of $3/4$. Moreover, since $SU(2)$ currents must be left-chiral, all fermions are forced to be left-handed (on the scale of the G gauge theory), making the $U(1)$ and $SU(3)$ currents also left-chiral. Thus, in the G -gauge invariant limit, all fermions are massless and chiral symmetry is exact in the strongest possible form: there are no right-handed (R -) fermions to couple to L -fermions so as to generate masses. This circumstance suggests strongly that the mechanisms responsible for the restoration of parity invariance of electromagnetic and color interactions and for the appearance of fermion masses are one and the same, namely a suitable dynamical breaking of chiral symmetry, and that this mechanism is closely linked to the spontaneous breaking of $SU(2) \times U(1) \times SU(3)$ to $U(1)_{em} \times SU(3)$.

As a first step towards justifying such a linkage, we fall back on the standard Higgs mechanism for breaking $SU(2) \times U(1)$ to $U(1)_{em}$. Accordingly, in the \mathcal{G} theory, we take the Higgs boson to constitute one $8N$ representation, decomposing as N families of leptonic Higgs bosons [$\sim(2,1)$ under $SU(2) \times SU(3)$] and N of colored Higgs bosons ($\sim(2,3)$). On assigning vacuum values to all N leptonic Higgs bosons in, say, the up flavor, the model becomes indistinguishable from the standard model except, of course, for the exact (L -) chiral invariance. But once elementary scalars are admitted, we have the possibility of generating R -fermions as composites of L -fermions and Higgs boson [8] and, thence, of producing masses by Yukawa couplings. Details of the dynamics of the proposal are beyond the essentially group-theoretic scope of this paper. It is quite easy to show however that there is a “channel” in the L -fermion Higgs system which, if assumed attractive, can produce just the right number of R -fermions with the right transformation properties under $SU(2)$ and $SU(3)$: there are $8NR$ -fermions, all of them transforming trivially under $SU(2)$, breaking up as $2N$ multiplets each transforming as $1 \oplus 3$ under $SU(3)$. Consequently, no R -fermion couples to $SU(2)$, the R -leptons do not couple to $SU(3)$ while the R -quarks do, with strength equal to that of L -quarks. In other words, $SU(2)$ currents are $V-A$ and the $SU(3)$ (quark) currents pure V . Finally, the $U(1)_{em}$ coupling strengths of R -fermions are also easily computed; they match precisely

those of the corresponding L -fermions, making the electromagnetic current also pure V .

In the last section, we turn to the problem of the physics underlying the reduction of the primitive group \mathcal{G} to the standard model group G . After exhibiting a simple Higgs representation that will achieve this, a more directly physical explanation for the manifestation of universality is sought in the very natural idea that the (L -) fermions are themselves composites of a set of preons or metafermions. The simplest explicit preonic model with built-in universality then requires $6+N$ preons (2 for flavor, 4 for color including leptonic color and N for family) [9]. All of them have meta-interactions binding them into massless, L -chiral, leptons and quarks, but 6 of them have, additionally, $SU(2) \times U(1) \times SU(3)$ gauge interactions—the photon, gluons, W and Z are thus elementary. In this perspective, our unifying $SU(8N)$ is just ‘t Hooft’s metaflavor group [10] unbroken as long as fermions are massless, but broken softly by the spontaneous generation of R -fermions and the concomitant masses—the gauging of only a subgroup of the metaflavor group is no more than a reflection of the fact that some of the preons already have such gauge interactions.

Obviously, these speculative ideas, touched upon only briefly in the concluding section, need to be worked out in detail. Another area for further work is the dynamics of chiral symmetry breaking. Particularly worthy of attention is the possibility that the Higgs fields themselves have a dynamical origin, i.e., that the spontaneous G -symmetry breaking and the chiral symmetry breaking both arise from some sort of Nambu–Jona-Lasinio mechanism involving the L -fermions. It is nevertheless encouraging that the purely group-theoretic and kinematic foundation of such an enterprise, as described in this paper, has proved to have no serious drawbacks.

II. UNIVERSALITY PROPERTIES OF FERMIONS

A. Implementing universality

Considering all leptons and quarks together, a fermion is labeled by a family index $i=1, \dots, N$, N assumed arbitrary, a flavor index $f=1,2$ (1 is “up” and 2 is “down,” say) and a generalized color index $\alpha=0,1,2,3$, $\alpha=0$ referring to leptons and $\alpha=1,2,3$ (collectively denoted by c where necessary) to conventional quark color. The $8N$ dimensional complex vector space V spanned by the orthonormal basis $\{|i,f,\alpha\rangle\}$ is the space of 1-fermion states (ignoring momenta and helicities). In any gauge model for the interactions of all the fermions, V will carry a unitary, not necessarily irreducible, representation of the global gauge group. The chirality assigned to the vectors of V (e.g., whether Dirac or Weyl spinors) will determine the parity properties of the various currents and is left open for the moment. It will turn out that the only viable choice is in favor of left-handed Weyl spinors; in fact it is a unique and nontrivial feature of the model that right-handed fermions cannot exist in the gauge-invariant limit of the model; they will be generated spontaneously (along with masses) in a very natural way.

The basic strategy pursued in this paper is to postulate first that underlying the standard $G=SU(2) \times U(1) \times SU(3)$ gauge model there is a unifying or embedding

gauge theory with group \mathcal{G} , treating all fermions on an equal footing and, then, to determine \mathcal{G} as the group having the property that its subgroup respecting all observed universality properties is G . In the actual execution, it is simpler to proceed by choosing \mathcal{G} as the smallest simple group that will suffice, namely $\mathcal{G}=SU(8N)$, and then to verify that its subgroup satisfying weak and strong universality is indeed G . The universality properties referred to are those valid at the level of $SU(2)\times U(1)\times SU(3)$, before it is spontaneously broken to the $SU(3)\times U(1)$ model incorporating fermion mass differences and family mixings.

We denote a basis for the Lie algebra of $\mathcal{G}=SU(8N)$ by $\{t_A, A=1, \dots, 64N^2-1\}$ and write the fermion current coupling to the \mathcal{G} -gauge boson X_A as

$$J_A = \bar{\psi} t_A \psi$$

where $\psi \in V$ and the space-time structure has been suppressed. Let us choose the index A to be compatible with the family, flavor and color labels as they occur in the G gauge theory: A is then a pair of sets of indices (i, f, α) and (i', f', α') . A typical universality property is most simply formulated as the statement that the current J which couples to a particular gauge boson X of the G gauge theory is unchanged by a particular set of permutations of the labels (i, f, α) of a fermion. Weak universality of leptonic currents, for instance, is the generalization of the old $e-\mu$ universality to the statement that the charged weak currents are invariant under a permutation of e, μ and τ and the *same* permutation, simultaneously, of ν_e, ν_μ and ν_τ . Postponing a more precise formulation of universality to the next two subsections, we note first that a universality transformation is a unitary matrix on V and that the set of universality transformations leaving invariant a given subset of the currents is a group.

Given a group S of unitary operators on V , a \mathcal{G} -gauge fermion current $J = \bar{\psi} t \psi$, where t is a real linear combination of $\{t_A\}$, is invariant under S ("universal with respect to S ") if

$$\bar{\psi} s^* t s \psi \equiv \bar{\psi} s^{-1} t s \psi = \bar{\psi} t \psi$$

for all $s \in S$. The set of t which commute with every $s \in S$ forms a Lie subalgebra of the Lie algebra of \mathcal{G} and the corresponding Lie group is a subgroup of \mathcal{G} , the *centralizer* of S in \mathcal{G} , denoted $C(S)$. The gauge bosons of the $C(S)$ gauge theory couple to all currents universal with respect to S and only to them, and we obtain in this way an embedding of the universal gauge theory in the \mathcal{G} gauge theory. The ratios of the gauge couplings of each simple factor group of $C(S)$, as well as the couplings of various fermion currents to any Abelian factor group of $C(S)$, are thereby fixed. Thus one of the prime motivations for unification is fulfilled by appealing to the observed pattern of families rather than by invoking complicated Higgs multiplets. Indeed, though the Higgs mechanism for reducing \mathcal{G} to G certainly remains a viable option, we are now free to explore other, physically more compel-

ling, reasons for the manifestation of universality and family structure. One such speculative possibility will be suggested at the end.

B. Weak universality

As already stated above, we follow conventional wisdom and begin by supposing that the weak interactions of leptons in the unbroken standard G gauge theory are unchanged by an arbitrary permutation π of the family index of the up leptons and the same permutation of the down family index

$$|i, u, \alpha=0\rangle \rightarrow |\pi(i), u, \alpha=0\rangle,$$

$$|i, d, \alpha=0\rangle \rightarrow |\pi(i), d, \alpha=0\rangle.$$

The physical leptons may violate strict universality through mass differences and possible family mixings; both of these are generally taken to be manifestations of the breaking of G further to the final, low energy, exact gauge group $U(1)_{em} \times SU(3)$. Under the same assumption, the weak interactions of quarks in the G -gauge theory are also invariant under similar simultaneous permutations of the up and down family indices (despite the family mixings present in low energy currents). Hence the *conventional* universality assumption can be stated as the invariance of all weak interactions under

$$|i, u, \alpha\rangle \rightarrow |\pi(i), u, \alpha\rangle, \quad |i, d, \alpha\rangle \rightarrow |\pi(i), d, \alpha\rangle \quad (1)$$

simultaneously for each fixed $\alpha=0,1,2,3$.

However, this formulation of weak universality is incomplete. First, if we ignore, once again, family mixings (i.e., at the G -invariant stage), the observed weak currents of a definite color are of the general forms $\sum_i \bar{u}_{i\alpha} \Gamma d_{i\alpha}$ (and its conjugate), $\sum_i \bar{u}_{i\alpha} \Gamma u_{i\alpha}$ and $\sum_i \bar{d}_{i\alpha} \Gamma d_{i\alpha}$ where Γ are (different) matrices which *do not* operate on i . So a unitary transformation of the family index i will commute with Γ ; the currents are invariant not just under the discrete permutations π but the more general

$$|i, u, \alpha\rangle \rightarrow U_{ij} |j, u, \alpha\rangle, \quad |i, d, \alpha\rangle \rightarrow U_{ij} |j, d, \alpha\rangle \quad (2)$$

where U in both transformations is the same unitary matrix [11]. Next, as far as the weak currents are concerned, quark color is no different from the family label; the coupling strengths of the currents do not depend on the quark color and the matrices Γ do not operate on them. We may therefore immediately extend weak universality to encompass unitary transformations on the pair of indices $(i, c=1,2,3)$. Moreover the only empirical reason for not extending it to include leptons also would appear to be the lack of equality of the coupling strengths of leptons and quarks to the photon and the Z boson. The couplings of the physical γ and Z to fermions are however fixed only after $SU(2)\times U(1) \subset G$ invariance is broken down to $U(1)_{em}$ invariance and the neutral bosons mixed and hence this apparent reason is not so compelling. This leaves the theoretical objection that the so called weak hypercharges in the standard $SU(2)\times U(1)$ model does distinguish between quarks and leptons. The remarkable fact is, as will become clear in the next subsection,

that in our unification scheme the hypercharge is not an attribute of weak but of strong universality and is a measure of the absence of strong interactions for the leptons.

In the light of the above considerations, we take the maximal weak universality group S_{wk} as the group consisting of unitary transformations with matrix elements $U_{i\alpha,j\beta}$, $i, j = 1, \dots, N$, $\alpha, \beta = 0, 1, 2, 3$, applied to the fermion basis $|i, f, \alpha\rangle$ leaving the flavor f unchanged:

$$\begin{aligned} |i, u, \alpha\rangle &\rightarrow U_{i\alpha,j\beta} |j, u, \beta\rangle, \\ |i, d, \alpha\rangle &\rightarrow U_{i\alpha,j\beta} |j, d, \beta\rangle. \end{aligned} \quad (3)$$

Abstractly, S_{wk} is the group $U(4N)$. If we write V as a tensor product of spaces spanned by family, flavor and color basis vectors, $V = V_{fam} \otimes V_{fl} \otimes V_{col}$, S_{wk} is the unitary group of $V_{fam} \otimes V_{col}$. In this abstract sense, weak universality is just the statement that, as far as weak interactions are concerned, family and color directions in V can be chosen as an arbitrary orthonormal basis for $V_{fam} \otimes V_{col}$.

In accordance with the general considerations of Sec. II A, the subgroup of $\mathcal{G} = SU(8N)$ whose currents respect weak universality is the centralizer $C(S_{wk})$. As a $(8N \times 8N)$ matrix group on V , S_{wk} consists of pairs of identical $U(4N)$ matrices s_{wk} acting on $V_{fam} \otimes V_{col}$:

$$s_{wk} = \begin{pmatrix} x_{wk} & 0 \\ 0 & x_{wk} \end{pmatrix}; \quad (4)$$

i.e., S_{wk} is the diagonal subgroup $\{(x_{wk}, x_{wk})\}$ of the direct product $U(4N) \times U(4N)$. We write this as

$$S_{wk} = D^2 U(4N), \quad (5)$$

so that

$$G_{wk} = C(D^2 U(4N)). \quad (6)$$

It is very easy to determine the group G_{wk} . In the basis used in Eq. (4) (i.e., a fixed flavor labelling the first $4N$ rows and columns), write a general element $g \in SU(8N)$ as the matrix

$$g = \begin{pmatrix} g_{uu} & g_{ud} \\ g_{du} & g_{dd} \end{pmatrix},$$

with each $g_{ff'}$ a $4N \times 4N$ matrix. For g to commute with s_{wk} , each submatrix $g_{ff'}$ must commute with x_{wk} and since x_{wk} is an arbitrary matrix, each $g_{ff'}$ is trivial:

$$g_{ff'} = a_{ff'} 1_{4N},$$

where $a_{ff'}$ are complex numbers (the subscript on 1 indicating dimension will prove its usefulness soon). Hence a general element of G_{wk} is of the form

$$g_{wk} = \begin{pmatrix} a_{uu} 1_{4N} & a_{ud} 1_{4N} \\ a_{du} 1_{4N} & a_{dd} 1_{4N} \end{pmatrix}, \quad (7)$$

$a_{ff'}$ being arbitrary complex numbers such that g_{wk} is unitary and has determinant 1. A reordering of the basis vectors of V [interchange the $(n+1)$ th row (column) with the $(4N$

$+n)$ th row (column) of the matrix] allows us to write a typical element of G_{wk} as the $8N \times 8N$ matrix having the unitary matrix

$$a = \begin{pmatrix} a_{uu} & a_{ud} \\ a_{du} & a_{dd} \end{pmatrix}$$

repeated $4N$ times along the principal diagonal. Thus G_{wk} is the unit determinant diagonal subgroup of $U(2)^{4N}$:

$$G_{wk} = SD^{4N} U(2). \quad (8)$$

The condition $\det g_{wk} = (\det a)^{4N} = 1$ allows us to rewrite this as the diagonal group $D^{4N} U(2)_{4N}$, isomorphic to $U(2)_{4N}$, the group of unitary matrices having determinant equal to any $4N$ th root of unity. This is obviously not a connected Lie group; its connected component is $SU(2)$.

Since the currents in a gauge theory are completely specified in terms of the Lie algebra of the gauge group, possible lack of connectedness (and simple-connectedness) of the group is immaterial in the (perturbative) calculation of any process. Nonperturbative results may depend on the topology of the group, but such considerations are not pursued in this paper. With this qualification, we have thus shown that

$$G_{wk} = SU(2). \quad (9)$$

It is immediately evident that the $8N$ basis vectors $\{|i, f, \alpha\rangle\}$ of the fundamental fermion representation of $SU(8N)$ form $4N$ copies of the fundamental representation of $SU(2)$, the $SU(2)$ acting on flavor and the copies labeled by family and color. The relationship between weak universality and family structure has thus been made precise and concrete by the use of $SU(8N)$ as an embedding or unifying group.

C. Strong universality

The formulation of a strong universality principle for quarks proceeds in a manner very similar to that of weak universality: the color currents of quarks responsible for strong interactions are universal in the sense that they are sums over family and flavor, with a common coupling constant. This feature is usually recognized as being the reason for the phenomenon loosely called flavor invariance (approximate because of quark masses). Exactly as above, the strong universality group is then $D^3 U(2N)$. Ignoring the existence of leptons for a moment and taking the primitive gauge group to be $SU(6N)$, the resulting strong gauge group would then be $S(D^{2N} U(3))$.

But leptons do exist and the fact that they have no strong interaction—i.e., no (low energy) current changes leptonic color to any other color—introduces a fundamental new feature. In the context of universality this means that the strong interactions are impervious to any arbitrary choice of basis of the $2N$ dimensional leptonic subspace of V spanned by $\{|i, f, \alpha = 0\rangle\}$, completely independent of the universality transformations of the quark subspace. The most general strong universality transformation, as a matrix on V , is therefore of the form

$$s_{st} = \begin{pmatrix} x'_{st} & & & \\ & x_{st} & & \\ & & x_{st} & \\ & & & x_{st} \end{pmatrix}, \quad (10)$$

with $x'_{st} \in U(2N)$ and $x_{st} \in U(2N)$, all blank entries being zero. (The basis used here is, of course, one in which the first $2N$ rows and columns correspond to leptons of all family and flavor.) The strong universality group is thus

$$S_{st} = U(2N) \times D^3 U(2N). \quad (11)$$

As before, the strong gauge group G_{st} is the centralizer of S_{st} in $SU(8N)$. To compute it, write $g \in SU(8N)$ as the matrix [in the basis used in Eq. (10)]:

$$g = \begin{pmatrix} g_{00} & \cdots & g_{03} \\ \vdots & & \\ g_{30} & \cdots & g_{33} \end{pmatrix} \equiv (g_{\alpha\alpha'})$$

each $g_{\alpha\alpha'}$ being a matrix on $V_{fam} \otimes V_{fl}$. For g to commute with s_{st} , we must have

$$\begin{aligned} g_{00}x'_{st} &= x'_{st}g_{00}, & g_{cc}x_{st} &= x_{st}g_{cc}, \\ g_{c0}x'_{st} &= x_{st}g_{c0}, & g_{oc}x_{st} &= x'_{st}g_{oc}, \\ g_{cc'}x_{st} &= x_{st}g_{cc'} & \text{for } c \neq c' \end{aligned}$$

for all $x_{st}, x'_{st} \in U(2N)$ and $c, c' = 1, 2, 3$. These conditions are solved by

$$g_{00} = b_{00}1_{2N}, \quad g_{c0} = g_{0c} = 0, \quad g_{cc'} = b_{cc'}1_{2N} \text{ for all } c, c',$$

for arbitrary complex numbers b_{00} and $\{b_{cc'}\}$. Reordering rows and columns, a typical element of G_{st} can therefore be written as the $8N \times 8N$ unitary matrix having the 4×4 matrix

$$b = \begin{pmatrix} b_{(l)} & 0 \\ 0 & b_{(q)} \end{pmatrix},$$

where $b_{(l)}$ is an arbitrary complex number with $|b_{(l)}| = 1$ and $b_{(q)}$ is a unitary 3×3 matrix, repeated $2N$ times along the diagonal. The matrix b operates on V_{col} whose leptonic and quark subspaces are distinguished by the subscripts.

Thus, as a $(8N \times 8N)$ matrix group, the strong gauge group is

$$G_{st} = SD^{2N}(U(1) \times U(3)). \quad (12)$$

The unit determinant condition says that $b_{(l)}^{2N}(\det b_{(q)})^{2N} = 1$; this puts no restriction on $\det b_{(q)}$ but merely says that its value is fixed in terms of the arbitrary phase $b_{(l)}$. Hence G_{st} is connected and is isomorphic to $U(3)$. Restricting ourselves to the Lie algebra and ignoring topological niceties once again, we therefore write

$$G_{st} = U(1) \times SU(3). \quad (13)$$

It is clear that the $SU(3)$ group operates trivially on the subspace $\alpha=0$ (leptons) and as the fundamental $\mathbf{3}$ representation on the quark subspace of each family and flavor; the total fermion space V breaks up as $2N$ copies of the color triplet and $2N$ copies of the singlet representation. The $U(1)$ subgroup is not just a group acting on leptons, as it arises from solving the unit determinant condition. Its physical interpretation will become clear in the next section.

III. THE STANDARD MODEL

A. Combining G_{wk} and G_{st}

Physically, the picture we would like to have is this: the effective gauge theory arrived at by appealing to universality should accommodate all currents satisfying *either* weak universality *or* strong universality and no others. This will be ensured if we can conclude that the effective gauge group G is just the direct product of G_{wk} and G_{st} and that, in turn, requires that G_{wk} and G_{st} are disjoint as subgroups of \mathcal{G} and that they are mutually commutative. These two properties are immediately obvious for the Lie algebras of G_{wk} and G_{st} (e.g., color generators and flavor generators commute) and hence for the groups themselves if they are connected and simply connected. Since we have already chosen to confine attention to the Lie algebra we may assert that the subgroup of $\mathcal{G} = SU(8N)$ describing the effective gauge theory which respects *either* weak *or* strong universality is indeed

$$G = G_{wk} \times G_{st} = SU(2) \times U(1) \times SU(3). \quad (14)$$

Under the $SU(2) \times SU(3)$ subgroup of G , leptons of each family transform as the representation $(\mathbf{2}, \mathbf{1})$ and quarks of each family as the representation $(\mathbf{2}, \mathbf{3})$. [m and n in the notation (\mathbf{m}, \mathbf{n}) are the dimensions of $SU(2)$ and $SU(3)$ representation.] The $U(1)$ transformation properties of [i.e., the $U(1)$ coupling constants to] the various fermions will be determined in the next subsection—it will turn out that $U(1)$ is in fact the group of what is conventionally called the weak hypercharge.

At one level one may think of the results of this section as just a systematization of the relationship between observed universality and family structure. Some striking insights have nevertheless emerged in the process:

(1) If, as we have hypothesised, the family structure (the N -fold replication of an irreducible representation) of fermions in the standard model is the result of imposing universality restrictions on the currents of a unifying gauge group \mathcal{G} , the standard gauge group $G = SU(2) \times U(1) \times SU(3)$ fixes \mathcal{G} *uniquely* to be $SU(8N)$. The number of families plays no role in the reduction of \mathcal{G} to G (except in so far as the topological properties of G are concerned).

(2) The hypercharge $U(1)$ actually arises from strong universality and reflects the absence of strong interactions among the leptons.

(3) The fact that weak $SU(2)$ currents are left-chiral forces all fermions to be left-handed Weyl spinors in the primitive \mathcal{G} gauge theory. Obtaining a G gauge theory in which the $SU(2)$ currents are $V-A$ while the $SU(3)$ currents are pure V then poses a problem.

The usual ways of introducing R -fermions in a ‘‘grand’’ unified model broadly fall into two distinct strategies: (i) They belong to a different representation of a simple unifying group \mathcal{G} chosen carefully so that when \mathcal{G} is broken to G , the parity properties of currents come out right. The prototype of this method is of course the $SU(5)$ model [12] in which this is done for each family separately. (ii) They transform trivially under \mathcal{G} but nontrivially under another subgroup \mathcal{G}' of a full unifying (nonsimple) group $\mathcal{G} \times \mathcal{G}' \equiv \mathcal{G}_L \times \mathcal{G}_R$. The gauge bosons of the standard model then result from a mixing of the bosons of \mathcal{G} and \mathcal{G}' , induced by spontaneous symmetry breaking. The prototypes here are left-right symmetric models [13]. Such strategies are not natural for us. For the first option to work, the representation of $SU(8N)$ to which R -fermions are to be assigned must evidently have dimension less than $8N$ and there are no such. The second option is excluded by universality itself. Observed universality does not discriminate between $V-A$ (charged weak) and V (electromagnetic and color) currents: if we apply universality to, say, $\mathcal{G}_L \times \mathcal{G}_R$ as the unifying group, we shall end up with $G_L \times G_R$ as the effective group, requiring further (Higgs?) gymnastics to end up with the correct $V-A$ and V currents.

Thus, accepting universality as the sole guide in the choice of the unifying group forces us to look for novel ways of understanding the parity properties of currents. A possible solution of this problem is described in Sec. IV B.

B. Coupling constants

The ratios of the squares of the coupling constants of $SU(2)$ and $SU(3)$ currents and of each individual fermion $U(1)$ current are determined by the way G is embedded in \mathcal{G} . (It is useful to remember that the physical mechanism of symmetry reduction is immaterial for this purpose.) Fix a normalization

$$\text{tr } t_A^2 = \delta \quad (15)$$

for the generators of $SU(8N)$. The generators of the $SU(2)$ subgroup can then be written, as $SU(8N)$ generators, as the matrices

$$T_a = \frac{1}{2} g \text{diag}(\tau_a, \tau_a, \dots, \tau_a), \quad a=1,2,3,$$

in the flavor basis, where τ_a are the Pauli matrices ($\text{tr } \tau_a^2 = 2$), i.e., as the matrix having τ_a repeated $4N$ times along the diagonal and all other entries zero. The normalization (15) fixes the value of g^2 :

$$\delta = \text{tr } T_a^2 = \frac{1}{4} g^2 4N \text{tr } \tau_a^2 = 2Ng^2,$$

i.e.,

$$g^2 = \frac{\delta}{2N}. \quad (16)$$

Similarly we write the generators of the $SU(3)$ subgroup as

$$L_n = \frac{1}{2} f \text{diag}(0, \lambda_n, 0, \lambda_n, \dots, 0, \lambda_n), \quad n=1, \dots, 8,$$

in the color basis, where λ_n are the Gell-Mann matrices ($\text{tr } \lambda_n^2 = 2$), with $(0, \lambda_n)$ repeated $2N$ times along the diagonal. So

$$f^2 = \frac{\delta}{N} = 2g^2. \quad (17)$$

The $U(1)$ group commutes with $SU(2)$ and $SU(3)$ and hence the hypercharge Y' as a generator of $SU(8N)$ commutes with $\{T_a\}$ and $\{L_n\}$. Therefore

$$Y' = \frac{1}{2} \text{diag}(g' 1_{2N}, g'' 1_{6N})$$

(for real numbers g' and g'') in the flavor basis. Equivalently, in the color basis,

$$Y' = \frac{1}{2} \text{diag}(g', g'' 1_3, \dots, g', g'' 1_3)$$

with $2N$ repetitions. Since Y' is traceless,

$$g'' = -\frac{1}{3} g' \quad (18)$$

and we write, to conform to usual practice,

$$Y' = \frac{1}{2} g' Y, \quad Y = \text{diag}\left(-1_{2N}, \frac{1}{3} 1_{6N}\right)$$

in the flavor basis. The correct ratio of the lepton and quark hypercharges follows strictly from the simplicity of $SU(8N)$ and the fact that there are three colors of quarks and one of leptons (it is not directly related to baryon and lepton numbers). The common coupling constant g' is fixed by the normalization $\delta = \text{tr } Y'^2$ to be

$$g'^2 = \frac{3\delta}{2N} = 3g^2. \quad (19)$$

The ratios of the coupling constants are different from the almost canonical values expected [14] in a sequential embedding of the standard model in any simple unifying group with fermions in the $(\mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{3})$ representation of $SU(2) \times SU(3)$ because there are no R -fermions in our model. [Indeed, since these ratios are fixed by the way G sits inside $SU(8N)$, this is yet another indication of the impossibility of incorporating R -fermions in any representation of $SU(8N)$ without destroying the standard model.] Consequently, the values of the mixing angle and the strong coupling constant are both twice their canonical values:

$$\sin^2 \theta = \frac{3}{4}, \quad \frac{\alpha_S}{\alpha_W} = \frac{f^2}{g^2} = 2. \quad (20)$$

The first of these numbers, in particular, shows that the unification regime of the model has to be at substantially higher energy than we have been used to (see later).

IV. BREAKING G INVARIANCE

A. The choice of Higgs fields

So far we have avoided specifying the mechanism that is responsible for reducing the gauge symmetry from the primitive (“would-be”) gauge group \mathcal{G} to the effective one of the standard model. In our model this mechanism is no different from that responsible for the universality properties of the G -gauge theory; hence, it is possible to think that looking for the underlying physical causes of universality might offer options other than the search for an astute Higgs representation of \mathcal{G} . We postpone this question to the concluding section.

In contrast, the breaking of G to $SU(3) \times U(1)_{em}$ is attributed in this paper to a conventional Higgs mechanism. The main reason is that our real concern is how to understand the (pre-Higgs) standard model and its observed systematics in terms of a unified model, rather than in the details of how it is further broken. In any case, the standard Higgs mechanism is extraordinarily successful compared to the many efforts to supplant it or to find a more fundamental explanation for it. At the same time, it is very simple—compare the complicated Higgs multiplets required in most (one-family) grand unified models. An additional reason is the central role the Higgs fields play in our approach to chiral symmetry breaking and in generating R -fermions. (See, however, the remark at the end of the Introduction.)

Accordingly, we assume that the \mathcal{G} gauge theory has one Higgs field Φ transforming as the defining (fundamental) representation of $SU(8N)$. With respect to $G \subset \mathcal{G}$, Φ decomposes exactly as the fermion representation: there are N families ϕ_i , each ϕ_i transforming under $SU(2) \times SU(3)$ as $(\mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{3})$ with $Y = -1$ and $\frac{1}{3}$ respectively (lepton-like and quark-like Higgs bosons). We assume further that only the up flavors of lepton-like Higgs bosons of all families condense in the vacuum, with vacuum expectation values η_i . Except for family replication, this is the standard Higgs picture as far as $SU(2) \times U(1)$ is concerned. But the tree level algebraic properties of the standard model—the relations involving various observables like the values of the photon and Z coupling constants and the mixing angle θ , the masses of W and Z , etc.—are insensitive to the number of Higgs doublets (having vacuum expectation values all in the same flavor). Thus the fully broken $SU(2) \times U(1)$ theory is indistinguishable from the standard model except in higher orders involving Higgs propagators (and also of course when external Higgs particles are involved). We note in particular that the electric charges of all the fermions (so far, only left-handed) have the conventional values.

In addition, we have the color triplets of quark-like Higgs bosons (which may or may not be massive). They are presumably confined and cannot have $SU(3)$ invariant couplings to quarks—indeed, Φ has no $SU(8N)$ invariant coupling to the fermion $\mathbf{8N}$. They can interact with gluons however and so will form part of the “sea” in hadrons.

B. Right-handed fermions

We have already noted (Sec. III A) the difficulty of introducing R -fermions in the \mathcal{G} gauge model in such a way as to ensure, in the effective G gauge theory, that $SU(2)$ currents are $V-A$ while the $SU(3)$ currents are V . As a way out of this difficulty, the proposal put forward here is that, while R -fermions are absent in the \mathcal{G} gauge model, they are generated dynamically, simultaneously with the spontaneous breaking of G to $SU(3) \times U(1)_{em}$. In support of the general idea that R -fermions might really be the result of the final stage of symmetry breaking (of G), we point to two very suggestive circumstances: (i) They are essential in restoring the parity invariance of strong and electromagnetic interactions, precisely those interactions whose gauge invariance survives all symmetry breaking; in other words, those and only those currents which couple to massless gauge bosons are pure V . (ii) They are necessary to generate fermion masses which are also a manifestation of symmetry breaking. The only phenomena which require R -fermions are thus closely linked to G -symmetry breaking.

Explicitly, our proposal is that R -fermions are composites of L -fermions and Higgs bosons in certain specific channels. First, the binding must be orbitally excited (to obtain the chirality flip) and must be, in a first approximation [15], within the same family (to preserve the universality properties of realistic Dirac fermions). Dropping the family index and writing ψ_L for left handed Weyl spinor fields of a given $SU(2) \times SU(3)$ multiplet, the composite field $\psi_L \partial_\mu \phi$ transforms under the Lorentz group as $(\frac{1}{2}, 0) \otimes (\frac{1}{2}, \frac{1}{2}) = (1, \frac{1}{2}) \oplus (0, \frac{1}{2})$ of which the right-handed $(0, \frac{1}{2})$ component can be projected out by taking

$$\psi_R \simeq \sigma_\mu \psi_L \partial_\mu \phi \quad (21)$$

where $\sigma_0 = 1$ and σ_i are the usual spin matrices. Evidently this equation can have, at this stage, only a schematic meaning, as a means of keeping track of quantum numbers. At the very least, we have ignored the need for proper regularizations of quantum composite fields. The dynamical problems which will have to be solved in implementing this idea cannot obviously be broached here. Nevertheless, Eq. (21) can already be seen to have an intriguing consequence. On writing

$$\psi_R \simeq \partial_\mu (\sigma_\mu \psi_L \phi) - (\partial_\mu \sigma_\mu \psi_L) \phi,$$

it is clear that if both ψ_L and $\psi_L \phi$ are (L -projections of) massless Dirac fields, ψ_R vanishes; right handed components can be generated dynamically in this way only if mass is also simultaneously generated. In the rest of this section, we shall disregard all dynamical problems concentrating only on the quantum number aspects.

In general $\psi_L \phi$ composites can transform under $SU(2) \times SU(3)$ as any of the irreducible representations occurring in the decomposition of $(\mathbf{2}, \mathbf{3} \oplus \mathbf{1}) \otimes (\mathbf{2}, \mathbf{3} \oplus \mathbf{1})$. To delimit the number of possible irreducible fields, let us assume that $\psi_L \phi$ binding takes place for exactly the same irreducible representation (with respect to G) of ϕ which also condenses in the vacuum, namely the lepton-like (colorless) representation

$\phi \sim (\mathbf{2}, \mathbf{1})$, and, furthermore, in the antisymmetric tensor product \wedge of $SU(2)$ representations. Then

$$\begin{aligned} \psi_L \phi \sim (\mathbf{2} \wedge \mathbf{2}, (\mathbf{3} \oplus \mathbf{1}) \otimes \mathbf{1}) \\ = (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{1}). \end{aligned}$$

We now identify the two irreducible representations on the right as the down components of R -quarks and R -leptons respectively. Explicitly, in a general gauge,

$$\psi_{Rd}(\mathbf{3}) \approx \psi_L(u, \mathbf{3}) \phi(d, \mathbf{1}) - \psi_L(d, \mathbf{3}) \phi(u, \mathbf{1}) \quad (22)$$

and

$$\psi_{Rd}(\mathbf{1}) \approx \psi_L(u, \mathbf{1}) \phi(d, \mathbf{1}) - \psi_L(d, \mathbf{1}) \phi(u, \mathbf{1}). \quad (23)$$

The up R -fermions are correspondingly obtained by replacing ϕ by its conjugate, ϕ^c :

$$\psi_{Ru}(\mathbf{3}) \approx \psi_L(u, \mathbf{3}) \phi^c(d, \mathbf{1}) - \psi_L(d, \mathbf{3}) \phi^c(u, \mathbf{1}), \quad (24)$$

$$\psi_{Ru}(\mathbf{1}) \approx \psi_L(u, \mathbf{1}) \phi^c(d, \mathbf{1}) - \psi_L(d, \mathbf{1}) \phi^c(u, \mathbf{1}). \quad (25)$$

The subscripts u and d on R -fermions (as distinct from L -fermions for which they are written as arguments) are meant only to indicate in advance which composite will turn out to be the R -partner of which L -fermion in the currents coupling to $U(1) \times SU(3)$ —they are all invariant under $SU(2)$. The totality of R -fermions falls into two sets of quarks $\psi_{Ru}(\mathbf{3})$ and $\psi_{Rd}(\mathbf{3})$ and two sets of leptons $\psi_{Ru}(\mathbf{1})$ and $\psi_{Rd}(\mathbf{1})$ in each family. This is exactly the pattern required by the standard model.

Finally, to fix the parity properties of the total fermionic currents of $U(1)$ and $SU(3)$, we need to know the corresponding coupling constants of the R -fermions. The transformation properties of ψ_R under $SU(2) \times SU(3)$ immediately imply that no R -currents couple to $SU(2)$ and no leptonic R currents couple to $SU(3)$. The R -quark coupling to $SU(3)$ is determined, just by $SU(3)$ gauge invariance, to be the same as the L -quark coupling (=the intrinsic gauge coupling constant of $SU(3)$). Hence all $SU(3)$ currents are pure V . The fermion couplings to Abelian groups such as the hypercharge $U(1)$ or (equivalently) the electromagnetic $U(1)$ are not fixed in this way as there is no intrinsic gauge coupling defined by the gauge field; they have to be determined for each fermion individually. But being Abelian charges, they are additive in each composite and we may compute them directly and easily. For the electric charge, we get

$$\begin{aligned} Q(\psi_{Rd}(\mathbf{3})) &= \left\{ \begin{array}{l} Q(\psi_L(u, \mathbf{3})) + Q(\phi(d, \mathbf{1})) = \frac{2}{3} - 1 \\ Q(\psi_L(d, \mathbf{3})) + Q(\phi(u, \mathbf{1})) = -\frac{1}{3} + 0 \end{array} \right\} \\ &= -\frac{1}{3} \end{aligned}$$

and, similarly,

$$Q(\psi_{Rd}(\mathbf{1})) = -1,$$

$$Q(\psi_{Ru}(\mathbf{3})) = \frac{2}{3},$$

$$Q(\psi_{Ru}(\mathbf{1})) = 0.$$

Hence the electromagnetic current is also pure V . It is pertinent to stress that the electric charges of ψ_L and ϕ are themselves fixed completely by the embedding of the hypercharge in $SU(8N)$ and by the Higgs structure of the model; they are not assigned *a priori*.

Thus the assumption that there exists an attractive force between L -fermions and Higgs bosons in a specific ‘‘channel’’ corresponding to unique angular momentum, color and flavor selection rules, strong enough to bind, leads to exactly the right global quantum numbers for the R -fermions. To make the picture complete, this is of course not enough. One needs to be able to construct local field operators for ψ_R such that they transform correctly also under *local* gauge transformations, in other words, deal with the dynamical problems alluded to earlier. In conventional (unconstrained) quantum field theories, the way to construct local field operators for composites has been known since long [16], but its generalization to gauge theories remains an open problem.

V. OPEN QUESTIONS

Of the questions we have so far left unaddressed, the most pressing is that of the physical mechanism which reduces the primitive gauge group $SU(8N)$ to $SU(2) \times U(1) \times SU(3)$ and, conversely, the significance and regime of validity of full $SU(8N)$ gauge invariance. It may be reassuring to note at the outset that it is possible to find a Higgs representation which will serve the propose. It is shown in the Appendix that when a gauge group \mathcal{G} is spontaneously broken to a subgroup G which is the centralizer of a group $S \subset \mathcal{G}$, we can always find a set of Higgs fields belonging to the adjoint representation of S and a set of non-zero vacuum values for them such that their little group (stabilizer) is precisely G ; it is also shown there that two copies of the adjoint representation of the family $SU(N)$ group, suitably embedded in the adjoint representation of $SU(8N)$, are sufficient to break $SU(8N)$ down to $SU(2) \times U(1) \times SU(3)$.

If the Higgs option is chosen, then conventional wisdom dictates that the vacuum values of the adjoint Higgs boson will determine the energy scale at which unification will hold and all $64N^2 - 1$ currents of $SU(8N)$ will be operative. The relatively large values of $\sin^2 \theta$ and of f^2/g^2 that we have found in Sec. III B indicate that this unification energy [and hence the masses of the exotic (non-universal) gauge bosons of $SU(8N)$] will be many orders of magnitude larger than the 10^{15} GeV or so that we are so used to and, even, the Planck mass. Conceptually, this is unknown territory; in any case no reasonable physical sense can be attached to a procedure of evolving low energy parameters beyond the Planck mass while ignoring gravitational effects in the renormalization group equations.

The other question touching on the magnitude of the uni-

fication energy scale is that of anomalies. Though the $SU(8N)$ gauge theory is not anomaly-free, our low energy “universal” model, being indistinguishable from the standard model, is: the dynamically generated R -fermions cancel precisely the triangle anomalies arising from the L -fermions as long as no non-standard gauge boson internal lines occur in loops. Such loops will technically be non-renormalizable; however, their contribution will be negligibly small in any regularized computation, as long as the cut-off is much smaller than the unification scale. This is just a loose paraphrase of the set of results collectively known as decoupling theorems. Since gravitational effects effectively preclude a cut-off above the Planck mass, we need not take the demand of technical renormalizability too seriously at this stage. In any case, these points are only of academic interest if, as we suggest below, the reduction $\mathcal{G} \rightarrow G$ is caused by a mechanism other than that of Higgs fields.

The option that we favor is to take leptons and quarks to be composites of a set of preons (or metaparticles) and to attribute the distinct gauge interactions of each fermion to the preons which themselves have the same gauge interactions. The model we are thus led to is one of the earliest (and conceptually simplest) preonic models proposed [9]: a fermion is assumed to be a composite of three types of spin $\frac{1}{2}$ preons, $|i, f, \alpha\rangle = |i\rangle \otimes |f\rangle \otimes |\alpha\rangle$. All of them are subject to a meta-interaction which is responsible for the binding (and about which it is futile to speculate at this stage). In addition, the flavor preons $|f\rangle$ have an $SU(2)$ gauge interaction and the quark-like color preons $|\alpha=c\rangle$, $c=1,2,3$ have an $SU(3)$ gauge interaction. If the preons are all L -chiral and the binding is assumed to be without “orbital excitation,” the composites necessarily belong to one of the irreducibles in the representation

$$\left(\frac{1}{2}, 0\right) \otimes \left(\frac{1}{2}, 0\right) \otimes \left(\frac{1}{2}, 0\right) = \left(\frac{3}{2}, 0\right) \oplus \left(\frac{1}{2}, 0\right) \oplus \left(\frac{1}{2}, 0\right)$$

of the Lorentz group. Since there are good reasons why massless particles of spin $\frac{3}{2}$ are unlikely to exist [17], we end up with the right number of L -chiral spin $\frac{1}{2}$ fermions if we assume that only one of the two $(\frac{1}{2}, 0)$ representations binds. In any case, chiral symmetry is valid strongly and the composite fermions are strictly massless. We have then a “metaflavor” group $U(8N)$ which, when the effects of instantons are taken into account, leaves $SU(8N)$ as the group of exact symmetries of all interactions of the composite fermions [10]. Thus a preonic picture provides a natural reason why the unifying group \mathcal{G} is indeed $SU(8N)$.

It is nevertheless important to highlight the one significant difference between the picture that emerges here and the one visualized by 't Hooft [10]. Once it is accepted that the universality properties of G gauge theory reflect the existence of flavor and color preons $|f\rangle$ and $|\alpha\rangle$ we are obliged to ascribe the (low energy) G -gauge interactions to the preons themselves. They are not some residual interactions left over from the meta-interactions binding them, even less the manifestation of “spectator” gauge bosons. (An instructive down-to-earth parallel is provided by a set of nuclei having the same mass number but differing atomic numbers—their electro-

magnetic interactions at energies low enough for nuclear structure effects to be ignored arise from and are fully determined by the electromagnetic interactions of the proton and the neutron.) In particular, the gauge bosons of G are themselves elementary.

We conclude by noting that there is no conceptual problem in having a “grand” unifying group \mathcal{G} of which only a subgroup G is actually a gauge group with gauge bosons associated to its generators. The determination of the gauge coupling constants of G by embedding it in \mathcal{G} is independent of whether all of \mathcal{G} is gauged. The matrix elements of universality-violating currents are suppressed, not because they couple to superheavy gauge bosons, but because the states between which the matrix elements are taken respect universality. Some of the problems occurring in traditional “grand” unification models like the naturalness and hierarchy problems are then no longer relevant. Also, we need no longer worry about anomalies connected with \mathcal{G} as long as the G theory is anomaly-free, which it is.

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APPENDIX

We describe here some elementary group theory used in the paper. Let \mathcal{G} be a group, S a subset of \mathcal{G} , not necessarily a subgroup. Then the centralizer of S in \mathcal{G} is

$$C(S) = \{g \in \mathcal{G} | gs = sg, \quad \forall s \in S\}.$$

$C(S)$ is a subgroup of \mathcal{G} containing the center of \mathcal{G} . If S_1 and S_2 are subsets of \mathcal{G} and $S_1 \subset S_2$, then obviously, $C(S_2) \subset C(S_1)$. Consider the centralizer of $C(S)$:

$$C(C(S)) = \{g \in \mathcal{G} | gx = xg, \quad \forall x \in C(S)\}.$$

$C(C(S))$ has S as a subset. Moreover, since every $x \in C(S)$ commutes with every $g \in C(C(S))$, $C(S)$ centralizes $C(C(S))$ and $C(C(S))$ is the maximal subgroup of \mathcal{G} containing S and centralized by $C(S)$. This observation is useful if we wish to find a subgroup S such that $C(S)$ is a given subgroup, in our case $G_{wk} \times G_{st} = C(S_{wk}) \times C(S_{st})$. We have $C(C(S)) = \{g \in \mathcal{G} | gx = xg, \quad \forall x \in G_{wk} \text{ and } gy = yg, \quad \forall y \in G_{st}\} = C(G_{wk}) \cap C(G_{st}) = C(C(S_{wk})) \cap C(C(S_{st}))$. It follows that $S = S_{wk} \cap S_{st}$ is the maximal subgroup centralised by $G_{wk} \times G_{st}$.

Writing the defining representation space of $SU(8N)$ as

$$V = V_{fam} \otimes V_{fl} \otimes (V_l \oplus V_q)$$

we may characterize S_{wk} and S_{st} as

$$S_{wk} = D_{fl}^2 U(V_{fam} \otimes (V_l \oplus V_q)),$$

$$S_{st} = U(V_{fam} \otimes V_{fl}) \times D_q^3 U(V_{fam} \otimes V_{fl}).$$

As the notation makes clear, S_{wk} is the diagonal product with respect to flavor of the unitary group of $V_{fam} \otimes V_{cal}$ and similarly for S_{st} . The advantage of this explicit notation is that we can read off the intersection of S_{wk} and S_{st} as the group

$$S = S_{wk} \cap S_{st} = D^6 U(V_{fam}) \times D^2 U(V_{fam}),$$

where, in the two factors on the right, the diagonal products are over $V_{fl} \otimes V_q$ and $V_{fl} \otimes V_l$ respectively. Therefore S is isomorphic to $U(N) \times U(N)$.

In the argument given in Sec. II for finding G given S_{wk} and S_{st} , these general considerations were unnecessary and were dispensed with. They become very useful however in finding a minimal Higgs scheme for reducing \mathcal{G} to G . For this purpose, we begin by noting the standard identification of the Lie algebra of \mathcal{G} with the vector space of the adjoint representation ρ_{ad} of \mathcal{G} . If $\{t_A\}$ is a basis of matrices for Lie \mathcal{G} and $\xi \in V_{ad}$ is a vector in the adjoint representation, with components $\{\xi_A\}$, this identification is

$$\xi \leftrightarrow \sum_A \xi_A t_A = X.$$

On the matrix X , the adjoint action of \mathcal{G} corresponds to the usual one of conjugation by $g \in \mathcal{G}$:

$$\rho_{ad}(g) \xi \leftrightarrow g X g^{-1}.$$

Under this identification, therefore, the centralizer of $\exp(iX) \in \mathcal{G}$ corresponds to the set $\{g \in \mathcal{G} | \rho_{ad}(g) \xi = \xi\}$, in other words, the little group of ξ .

On the other hand, we have the general characterization of the non-zero vacuum values of the Higgs fields as a numerical vector η of the representation (in general reducible) of \mathcal{G} to which the Higgs field belongs, such that the little group of η in \mathcal{G} is the unbroken subgroup G . Hence when G is given as the centralizer of some subgroup S , we may conclude that (i) the Higgs fields can be assigned to (sums of copies of) the adjoint representation of \mathcal{G} , and (ii) their nonvanishing vacuum values can be chosen to be a set of matrices spanning the Lie algebra of S . It follows that the Higgs vacuum values which will break $\mathcal{G} = SU(8N)$ to $G = SU(2) \times U(1) \times SU(3)$ form two sets of matrices each spanning the Lie algebra of $SU(N)$ [the centralizer of $SU(N)$ automatically centralises $U(N)$]; each set consists of $N^2 - 1$ linearly independent Hermitian $N \times N$ matrices.

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