Penguin operators in nonresonant $B^- \to M\bar{M}\pi^-$ ($M=\pi^-,K^-,K^0$) decays

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We investigate the contributions coming from the penguin operators in the nonresonant $B^- \to M\bar{M}\pi^-$ ($M=\pi^-,K^-,K^0$) decays. The effective Wilson coefficients of the strong penguin operators O_4 and O_6 are found to be relatively larger than those for other penguin operators. We calculate the contributions arising from the O_4 and O_6 operators in the nonresonant decays $B^- \to M\bar{M}\pi^-$ ($M=\pi^-,K^-,\bar{K}^0$) using a model combining heavy quark symmetry and the chiral symmetry, developed previously. We find that the CKM-forbidden nonresonant $B^- \to K^0\bar{K}^0\pi^-$ decay occurs through the strong penguin operators. These penguin contributions affect the branching ratios for $B^- \to M\bar{M}\pi^-$ ($M=\pi^-,K^-$) by only a few percent. The branching ratio for $B^- \to K^0\bar{K}^0\pi^-$ is estimated to be of the order 10^{-6}. [S0556-2821(99)07613-4]

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There is considerable interest in understanding the decay mechanism of the nonleptonic charmless three-body decays of B mesons [1–4]. The importance of penguin operators in three-body decays of charged B mesons has recently been questioned [1]. In the analysis of the Dalitz plot for $B^- \to \pi^+\pi^-\pi^-$ the authors of [1] have assumed that the nonresonant decay amplitude is flat, having no dependence on the Dalitz variables. They also assumed that the contributions of the penguin operators can amount to as much as 20% of the dominant decay amplitude. Others have made predictions for the branching ratios of decays [2–4] motivated in part by the CLEO limits on some of the nonresonant decays of the type $B^+ \to h^+h^+h^-$ [6]. CLEO found the upper limits on the branching ratios $\mathrm{BR}(B^+ \to \pi^+\pi^-\pi^+) \leqslant 4.1\times 10^{-5}$ and $\mathrm{BR}(B^+ \to K^+K^-\pi^+) \leqslant 7.5\times 10^{-5}$. In addition there is hope that the *CP*-violating phase γ can be measured from the asymmetry in charged B meson charmless three-body decays [2,4,5].

Motivated by the need to understand whether the nonresonant decay amplitudes for $B^- \to M\bar{M}\pi^-$ ($M=\pi^+,K^+$) involve significant effects due to the penguin operators we have investigated the contributions coming from the penguin operators [7–13] in these nonresonant decay amplitudes. The decay $B^- \to K^0\bar{K}^0\pi^-$ is forbidden [14,15] at the tree level, but can occur through penguin operators. A measurement of this rate would allow one to estimate the strength of the penguin interactions which are needed in the analysis of *CP* violation effects in $B \to \pi\pi$ and $B \to K\pi$ decays.

In our analysis we will use the factorization approximation in which the main contribution to the nonresonant $B^- \to M\bar{M}\pi^-$, ($M=\pi^+,K^+$) amplitudes comes from the product $\langle M\bar{M}|(\bar{u}b)_{V-A}|B^-\rangle\langle\pi^-|(\bar{d}u)_{V-A}|0\rangle$, where $(\bar{q}_1 q_2)_{V-A}$ denotes $\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. For the calculation of the matrix element $\langle M\bar{M}|(\bar{u}b)_{V-A}|B^-\rangle$ we extend the results obtained in [16], where the nonresonant $D^+ \to K^-\pi^+ l\nu$ decay was analyzed. The experimental result for the branching ratio of the nonresonant $D^+ \to K^-\pi^+ l\nu$ decay was successfully reproduced within a hybrid framework [16] which combines the heavy quark effective theory (HQET) and the chiral Lagrangian (CHPT) approach. The combination of heavy quark symmetry and chiral symmetry has also been quite successful in other analyses of D meson semileptonic decays [17–23].

Heavy quark symmetry is expected to be even better for the heavier B mesons [20,21]. However, CHPT might be less reliable in B decays due to the large energies of light mesons in the final state. It is really only known that the combination of HQET and CHPT is valid at small recoil momentum. To take into account the larger recoil energies of the light mesons in our previous work [16,22], we modified the hybrid model of [17–21] to describe the semileptonic decays of D mesons into one light vector or pseudoscalar meson. Our modification is quite straightforward: we retain the usual HQET Feynman rules for the *vertices* near and outside the zero-recoil region, *but we include the complete propagators instead of using the usual HQET propagator*. This quite reasonable modification of the hybrid HQET and CHPT model enabled us to use it successfully over the entire kinematic region of the D meson weak decays [16,22,23].

In the following we systematically use this model to calculate the contributions of the penguin operators to the nonresonant $B^- \to M\bar{M}\pi^-$ ($M=\pi^-,K^-,K^0$) decay amplitudes. We first analyze the contributions coming from the $O_{4,6}$ penguin operators [8,13], since their effective Wilson coefficients are the largest. We then determine the dependence on the Dalitz plot variables. The operator O_4, as defined in [8,13], has the same dependence on the Dalitz plot variables as the tree-level operator O_1, while O_6 exhibits different energy dependence. Finally, we discuss the influence of these operators on the branching ratios for $B^- \to M\bar{M}\pi^-$ $(M=\pi^+,K^+)$ and estimate the branching rate for $B^- \to K^0\bar{K}^0\pi^-$.

The effective weak Hamiltonian for the nonleptonic Cabibbo-suppressed B meson decays is given by [8–11,13]

$$\mathcal{H}_{eff}=\frac{G_F}{\sqrt{2}}[V_{ud}^*V_{ub}(c_1O_{1u}+c_2O_{2u})$$
$$+V_{cd}^*V_{cb}(c_1O_{1c}+c_2O_{2c})]$$
$$-\sum_{i=3}^{10}([V_{ub}V_{ud}^*c_i^u+V_{cb}V_{cd}^*c_i^c+V_{tb}V_{td}^*c_i^t)O_i]+\text{H.c.} \tag{1}$$

where the superscripts u, c, t denote the internal quark. The operators O_i are defined in [8,9,11,13]. We rewrite O_3-O_6, using the Fierz transformations, as follows:

$$O_3=\sum_{q=u,d,s,c,b}\bar{d}\gamma_\mu(1-\gamma_5)b\bar{q}\gamma^\mu(1-\gamma_5)q, \tag{2}$$

$$O_4=\sum_{q=u,d,s,c,b}\bar{d}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)b, \tag{3}$$

$$O_5=\sum_{q=u,d,s,c,b}\bar{d}\gamma_\mu(1-\gamma_5)b\bar{q}\gamma^\mu(1+\gamma_5)q, \tag{4}$$

$$O_6=-2\sum_{q=u,d,s,c,b}\bar{d}(1-\gamma_5)q\bar{q}(1+\gamma_5)b. \tag{5}$$

The Wilson coefficients have the following values for $\mu=m_b$, with $m_b=5$ GeV in the leading-log approximation [9,11]:

$$c_1=1.11,\quad c_2=-0.26,$$
$$c_3=0.012,\quad c_4=-0.026, \tag{6}$$
$$c_5=0.008,\quad c_6=-0.033.$$

This is sufficient for our purpose, as our main purpose here is to show the importance of the O_4 and O_6 penguin operators relative to the O_1 and O_2 tree-level operators.

The factorization approximation is obtained by neglecting in the Lagrangian terms which are the product of two color-octet operators after Fierz reordering of the quark fields. The effective Lagrangian for non-leptonic decays are then given by Eq. (1) with c_i replaced by a_i. For $N_c=3$, we have

$$a_1=1.02,\quad a_2=0.07,$$
$$a_3=0.003,\quad a_4=-0.022, \tag{7}$$
$$a_5=-0.003,\quad a_6=-0.03.$$

Thus the effective coefficients a_3 and a_5 are one order of magnitude smaller than a_4 and a_6 and therefore we can safely neglect the contributions from O_3 and O_5 operators.

The quark currents required in the weak Hamiltonian (1) can be expressed in terms of the meson fields, as previously described explicitly in [4,16,22]. The operator O_6 can be rewritten as the product of the density operators. For the $\bar{d}(1-\gamma_5)q$ scalar and pseudoscalar quark density operator we use the CHPT result [24]. The explicit chiral symmetry breaking, to lowest order in the chiral expansion, is obtained by adding the quark mass term [24]

$$\mathcal{L}_s=\frac{f_\pi^2}{4}\{\text{tr}\,\mathcal{B}(MU^\dagger+UM^\dagger)\}, \tag{8}$$

where $M=\text{diag}(m_u,m_d,m_s)$ and $\mathcal{B}$ is a real constant that can be expressed in terms of quark and meson masses; e.g., to lowest order $m_{K^0}^2=\mathcal{B}(m_s+m_d)$ and $U=\exp(i2\Pi/f)$ where Π is a pseudoscalar meson matrix. Using Eq. (8) one can easily bosonize the density operators:

$$\bar{q}_i(1-\gamma_5)q_j=-\frac{f_\pi^2}{2}\mathcal{B}U_{ji}. \tag{9}$$

For the calculation of the density operator $\bar{q}(1+\gamma_5)b$ we use the relations [10]

$$\bar{q}\gamma_5 b=\frac{-i}{m_b}\partial_\alpha(\bar{q}\gamma^\alpha\gamma_5 b), \tag{10}$$

and

$$\bar{q}b=\frac{i}{m_b}\partial_\alpha(\bar{q}\gamma^\alpha b), \tag{11}$$

where m_q has been dropped since $m_q \ll m_b$.

The evaluation of the matrix elements $\langle M|\bar{q}(1+\gamma_5)b|B\rangle$ and $\langle MM|\bar{q}(1+\gamma_5)b|B\rangle$ can then be reduced to the evaluation of the matrix elements of the weak currents $\langle MM|\bar{q}\gamma_\mu\gamma_5 b|B\rangle$ and $\langle M|\bar{q}\gamma_\mu b|B\rangle$. Assuming factorization, we evaluate the matrix elements of the operator O_6:

$$\langle MMM|O_6|B\rangle=-2\sum_{u,d,s,c,b}\{\langle M|\bar{d}(1-\gamma_5)q|0\rangle$$
$$\times\langle MM|\bar{q}(1+\gamma_5)b|B\rangle$$
$$+\langle MM|\bar{d}(1-\gamma_5)q|0\rangle\langle M|\bar{q}(1+\gamma_5)b|B\rangle$$
$$+\langle MMM|\bar{d}(1-\gamma_5)q|0\rangle$$
$$\times\langle 0|\bar{q}(1+\gamma_5)b|B\rangle\}. \tag{12}$$

The matrix elements $\langle M|\bar{d}(1-\gamma_5)q|0\rangle$, $\langle MM|\bar{d}(1-\gamma_5)q|0\rangle$, and $\langle MMM|\bar{d}(1-\gamma_5)q|0\rangle$ are easily calculated using Eq. (9). For the calculation of the matrix elements $\langle M|\bar{q}\gamma_\mu b|B\rangle$ and $\langle MM|\bar{q}\gamma_\mu\gamma_5 b|B\rangle$ we generalize the results obtained in the analysis of D meson semileptonic decays described in detail in [16] and [22]. The matrix element $\langle M|\bar{q}\gamma_\mu(1-\gamma_5)b|B\rangle$ is given by [22,23]

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle = \left[(p_B+p')_\mu - \frac{m_H^2-m_M^2}{q^2}q_\mu\right]F_1(q^2) + \frac{m_H^2-m_M^2}{q^2}q_\mu F_0(q^2), \tag{13}$$

where $q=p_B-p'$ and $F_1(0)=F_0(0)$. The form factors are found to be [22,23]

$$F_1(q^2) = -\frac{f_B}{2} + g f_{B'*}\frac{m_{B'*}^{3/2}m_B^{1/2}}{q^2-m_{B'*}^2}, \tag{14}$$

and

$$F_0(q^2) = -\frac{f_B}{2} - g f_{B'*}\sqrt{\frac{m_B}{m_{B'*}}} + \frac{q^2}{m_B^2-m_M^2}\times\left[-\frac{f_B}{2} + g f_{B'*}\sqrt{\frac{m_{B'*}}{m_B}}\right], \tag{15}$$

where B'^* denotes the relevant vector meson pole and g is the B^*BM coupling constant.

To evaluate the matrix element $\langle M_1(p_1)M_2(p_2)|(\bar{q}_i b)_{V-A}|B^-(p_B)\rangle$ we will also use and generalize the results obtained previously in the analysis of the nonresonant $D^+\to\pi^+K^-l\nu_l$ decay width [16]. We write the matrix element $\langle M_1(p_1)M_2(p_2)|(\bar{u}b)_{V-A}|B^-(p_B)\rangle$ in the general form

$$\langle M_1(p_1)M_2(p_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|B^-(p_B)\rangle = ir(p_B-p_2-p_1)_\mu + iw_+(p_2+p_1)_\mu + iw_-(p_2-p_1)_\mu - 2h\epsilon_{\mu\alpha\beta\gamma}p_B^\alpha p_2^\beta p_1^\gamma. \tag{16}$$

The form factors $w_\pm^{nr}$ for the nonresonant decay are given in [4,16]:

$$w_+^{nr}(p_1,p_2) = -\frac{g}{f_1f_2}\frac{f_{B*}m_{B*}^{3/2}m_B^{1/2}}{(p_B-p_1)^2-m_{B*}^2}\left[1-\frac{p_1\cdot(p_B-p_1)}{2m_{B*}^2}\right] + \frac{f_B}{2f_1f_2} - \frac{\sqrt{m_B}\alpha_2}{2f_1f_2}\frac{1}{m_B^2}p_B\cdot(p_2-p_1), \tag{17}$$

$$w_-^{nr}(p_1,p_2) = \frac{g}{f_1f_2}\frac{f_{B*}m_{B*}^{3/2}m_B^{1/2}}{(p_B-p_1)^2-m_{B*}^2}\left[1+\frac{p_1\cdot(p_B-p_1)}{2m_{B*}^2}\right] + \frac{\sqrt{m_B}\alpha_1}{f_1f_2}. \tag{18}$$

The parameters $\alpha_{1,2}$ are defined in [22]. Note that both the α_1 and α_2 terms are important in Eqs. (17) and (18), which was previously overlooked [2]. Within this same framework [16,22] we evaluate r^{nr}:

$$r^{nr}(p_1,p_2) = -\frac{1+\tilde{\beta}}{f_1f_2}p_B\cdot(p_2-p_1)\sqrt{\frac{m_{B'}}{m_B}}\frac{f_{B'}}{(p_B-p_1-p_2)^2-m_{B'}^2} - \sqrt{\frac{m_B}{m_B}}\frac{4g^2f_{B''}m_{B'}^*m_{B'}}{f_1f_2}\frac{1}{(p_B-p_1-p_2)^2-m_{B''}^2}\frac{\left[p_1\cdot p_2 - \frac{1}{m_{B*'}^2}p_2\cdot(p_B-p_1)p_1\cdot(p_B-p_1)\right]}{(p_B-p_1)^2-m_{B'*}^2} + \frac{2g}{f_1f_2}\frac{f_{B'*}m_{B'*}}{(p_B-p_1)^2-m_{B'*}^2}\frac{p_1\cdot(p_B-p_1)}{m_{B*'}^2} + \frac{f_B}{2f_1f_2} + \frac{\alpha_2\sqrt{m_B}}{2m_B^2}p_B\cdot(p_2-p_1). \tag{19}$$

Here B', B'^*, B'' denote the relevant B meson poles, and $f_{1,2}$ denotes the pseudoscalar meson decay constants. The coupling $\tilde{\beta}$ has been analyzed in [23] and found to be close to zero and therefore will be neglected.

The matrix element of the operator O_4 can be evaluated straightforwardly using factorization:

$$\langle\pi^-\pi^+\pi^-|O_4|B^-\rangle_{nr} = \langle\pi^+\pi^-|\bar{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle_{nr}\langle\pi^-|\bar{d}\gamma^\mu(1-\gamma_5)u|0\rangle, \tag{20}$$

and the corresponding expression for the $B^-\to K^-K^+\pi^-$ matrix elements can simply be obtained by the replacement π^+ and π^- by K^+ and K^- respectively. Note that the matrix element $\langle MM|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|0\rangle$ is dominated by resonant contributions. Using the variables $s=(p_B-p_3)^2=(p_2+p_1)^2$, $t=(p_B-p_1)^2=(p_2+p_3)^2$ $u=(p_B-p_2)^2=(p_1+p_3)^2$ and the pseudoscalar meson decay constants f_i, we can then write the nonresonant decay matrix element of O_4 as

$$\langle M_1(p_1)M_2(p_2)\pi^-(p_3)|O_4|B^-(p_B)\rangle_{nr}=\left[f_\pi m_\pi^2 r^{nr}(s,t)+\frac{f_\pi}{2}(m_B^2-s-m_\pi^2)w_+^{nr}(s,t)\right.$$
$$\left.+\frac{f_\pi}{2}(2t+s-m_B^2-m_1^2-m_2^2-m_\pi^2)w_-^{nr}(t)\right] \tag{21}$$

where M_1, M_2 represent either π^+, π^- or K^+, K^-.

Using factorization the matrix elements of O_6 can be written as

$$\langle\pi^+(p_1)\pi^-(p_2)\pi^-(p_3)|O_6|B^-(p_B)\rangle_{nr}=-2\left\{\langle\pi^+(p_1)\pi^-(p_2)|\overline{u}\gamma_\mu b|B^-(p_B)\rangle\frac{p_3^\mu}{m_B}\langle\pi^-(p_3)|\overline{d}(1-\gamma_5)u|0\rangle\right.$$
$$+\langle\pi^+(p_1)\pi^-(p_2)\pi^-(p_3)|\overline{d}(1-\gamma_5)u|0\rangle\langle 0|\overline{u}\gamma_\mu b|B^-(p_B)\rangle\frac{p_B^\mu}{m_B}$$
$$\left.+\langle\pi^+(p_1)\pi^-(p_2)|\overline{u}(1-\gamma_5)u|0\rangle\langle\pi^-(p_3)|\overline{d}\gamma_\mu b|B^-(p_B)\rangle\frac{p_B^\mu-p_3^\mu}{m_B}+(p_1\leftrightarrow p_3)\right\}, \tag{22}$$

where we have assumed $m_b\simeq m_B$. The corresponding result for $B^-\to\pi^-K^+K^-$ can be straightforwardly obtained simply by replacing $\pi^+\pi^-$ by K^+K^-. Using the expressions for the matrix elements of the current and the density operators we find

$$\langle M_1(p_1)M_2(p_2)\pi^-(p_3)|O_6|B(p_B)\rangle_{nr}=-f_3\frac{\mathcal{B}}{m_B}\left\{\frac{1}{2}[m_3^2 r^{nr}+(m_B^2-m_3^2-s)w_+^{nr}+(2t+s-m_B^2-m_1^2-m_2^2-m_3^2)w_-^{nr}]\right.$$
$$\left.+[(m_B^2-m_3^2)F_0(s)]-\frac{4}{3}\frac{f_3 f_B}{f_1 f_2}m_B\right\}. \tag{23}$$

For the $B^-\to\pi^-\pi^+\pi^-$ decay there is an additional term with the replacement $s\leftrightarrow t$, since there are two identical pions in the final state in this case.

The nonresonant amplitudes for the $B^-\to M\overline{M}\pi^-$ $(M=\pi^-,K^-)$ decays can be written in terms of the following matrix elements:

$$\mathcal{M}_{nr}(B^-\to M\overline{M}\pi^-)=-\frac{G_F}{\sqrt{2}}\{V_{ub}V_{ud}^* a_1\langle M\overline{M}\pi^-|O_1|B^-\rangle-V_{tb}V_{td}^*(a_4\langle M\overline{M}\pi^-|O_4|B^-\rangle+a_6\langle M\overline{M}\pi^-|O_6|B^-\rangle)\}. \tag{24}$$

The matrix element $\langle M\overline{M}\pi^-|O_1|B^-\rangle$ $(M=\pi^-,K^-)$ was given in [4].

Contrary to the Cabibbo-Kobayashi-Maskawa- (CKM-)allowed cases in which the main contribution comes from the operator O_1, we notice that the CKM-forbidden decay $B^-\to K^0\overline{K}^0\pi^-$ occurs through the penguin operators O_4 and O_6. The nonresonant matrix elements are

$$\langle K^0(p_1)\pi^-(p_1)\overline{K}^0(p_2)\pi^-(p_3)|O_4|B^-(p_B)\rangle_{nr}=\frac{f_K}{2}[m_K^2 r^{nr}(s,t)+(m_B^2-t-m_K^2)w_+^{nr}(s,t)$$
$$+(2t+s-m_B^2-2m_K^2-m_\pi^2)w_-^{nr}(s,t)] \tag{25}$$

and

$$\langle K^0(p_1)\overline{K}^0(p_2)\pi^-(p_3)|O_6|B^-(p_B)\rangle_{nr}=-\mathcal{B}\frac{f_K}{m_B}\left\{r^{nr}(s,t)m_K^2+w_+^{nr}(s,t)(m_B^2-t-m_K^2)+w_-^{nr}(s,t)(2t+s-m_B^2-2m_K^2-m_\pi^2)\right.$$
$$\left.+[(m_B^2-m_\pi^2)F_0(s)]-\frac{4}{3}\frac{f_\pi f_B}{f_K^2}m_B\right\}. \tag{26}$$

TABLE I. The branching ratios for $B^- \to M\bar{M}\pi^-$, $(M=\pi^-,K^-)$ and BR$(B^- \to K^0\bar{K}^0\pi^-)$ for two cases of ρ and η parameters, taken from Ref. [13]. The second number in Table I, for $(M=\pi^-,K^-)$, gives the increase or decrease due to the penguin contributions.

ρ	η	g	BR$(\pi^-\pi^+\pi^-)$	BR$(K^+K^-\pi^-)$	BR$(K^0\bar{K}^0\pi^-)$
-0.15	0.23	0.2	$(1.96+0.13)\times 10^{-5}$	$(3.42+0.06)\times 10^{-5}$	2.57×10^{-6}
-0.15	0.23	0.23	$(2.23+0.12)\times 10^{-5}$	$(3.58+0.10)\times 10^{-5}$	3.06×10^{-6}
-0.15	0.33	0.2	$(3.41+0.17)\times 10^{-5}$	$(5.09+0.06)\times 10^{-5}$	2.68×10^{-6}
-0.15	0.33	0.23	$(3.88+0.20)\times 10^{-5}$	$(6.30+0.05)\times 10^{-5}$	3.18×10^{-6}
0.15	0.23	0.2	$(1.96+0.19)\times 10^{-5}$	$(3.42-0.04)\times 10^{-5}$	1.45×10^{-6}
0.15	0.23	0.23	$(2.23+0.21)\times 10^{-5}$	$(3.61-0.03)\times 10^{-5}$	1.55×10^{-6}
0.15	0.33	0.2	$(3.42+0.22)\times 10^{-5}$	$(5.95-0.04)\times 10^{-5}$	1.72×10^{-6}
0.15	0.23	0.23	$(3.88+0.25)\times 10^{-5}$	$(6.30-0.05)\times 10^{-5}$	1.85×10^{-6}

The nonresonant amplitude for the $B^-(p_B) \to K^0(p_1)\bar{K}^0(p_2)\pi^-$ decay can be written in terms of these matrix elements (25), (26):

$$\mathcal{M}_{nr}(B^- \to K^0\bar{K}^0\pi^-) = \frac{G_F}{\sqrt{2}} V_{tb}V_{td}^* \{ a_4 \langle K^0\bar{K}^0\pi^- | O_4 | B^- \rangle + a_6 \langle K^0\bar{K}^0\pi^- | O_6 | B^- \rangle \}. \tag{27}$$

The partial width for the nonresonant decay $B^- \to M\bar{M}\pi^-$ $(M=\pi^-,K^-,K^0)$ is given by

$$\Gamma_{nr}(B^- \to M\bar{M}\pi^-) = \frac{1}{(2\pi)^3} \frac{1}{32 m_B^3} \int |\mathcal{M}_{nr}|^2 \; ds \; dt. \tag{28}$$

In the numerical calculation of the branching ratios we follow the discussion of the input parameters given in [4]. From heavy quark symmetry we have used $f_B/f_D = \sqrt{m_D/m_B}$ with the reasonable choice $f_D \simeq 200$ MeV [23,25–27]. The B decay constant is then $f_B \simeq 128$ MeV. In [4] we found that the parameters $\alpha_1^{B\rho} = -0.13$ GeV$^{1/2}$ and $\alpha_2^{B\rho} = -0.36$ GeV$^{1/2}$ lead to the branching ratio BR$(B^- \to \pi^-\pi^+\pi^+)$ being smaller than the experimental upper limit [6] and we used this possibility. Here we also use the values of $\alpha_{1,2}$ as in [4]. And, as discussed in [4], here we also consider the range $0.2 \leq g \leq 0.23$.

For the CKM parameters, we shall use a typical values given by Browder *et al.* [13] in their analyses of *CP* asymmetry in quasi-inclusive charmless B decays. Thus we consider two cases: (I) $\rho=-0.15$, $\eta=0.23,0.33$ and (II) $\rho=0.15$, $\eta=0.23,0.33$, where ρ and η are the two parameters in the Wolfenstein parametrization [28] of the CKM matrix. We have $V_{ub}=A\lambda^3(\rho - i\eta)$, $V_{ud}=1-\lambda^2/2$, $V_{td}=A\lambda^3(1-\rho-i\eta)$, $V_{tb}=1$, with $A=0.81$ and $\lambda=0.22$.

The numerical value of $\mathcal{B}$ can be determined from $\mathcal{B}=(2m_K^2-m_\pi^2)/2m_s$. Taking $m_s(\mu=5 \text{ GeV})=150$ MeV, the same value used in [8] for the extraction of the effective Wilson coefficients, we find $\mathcal{B}=1.6$ GeV. Inspection of the contributions coming from the $O_{4,6}$ operators shows some cancellations occur among the combinations $a_4 O_4$ and $a_6 O_6$. One also can explicitly see the dependence on the Dalitz variables.

In Table I we present the penguin contributions of the operators $O_{4,6}$ to branching ratios for the $B^- \to \pi^-\pi^+\pi^-$ and $B^- \to K^-K^+\pi^-$ together with the dominant tree-level contribution of the operator O_1. Both numerical results for the Wolfenstein parameters [cases (I) and (II)], as well as the range of g, as discussed above, are presented. [We do not give the result for the Wolfenstein parameter $\eta=0.43$, used in [13] since it leads to the branching ratio for $B^- \to M\bar{M}\pi^-$, $(M=\pi^-,K^-)$ larger than experimental upper limits.] Since the penguin operators make only small contributions to the $B \to M\bar{M}\pi$ decays, a change in the CKM matrix element would not greatly affect the branching ratios. For example, as shown below, for the hypothetical case where all CKM matrix elements are taken to be real with the following values: $V_{ud}=0.975, V_{ub}=0.0033, V_{td}=0.007; V_{tb}=0.999$, we find

$$BR(B^- \to \pi^-\pi^-\pi^+) = (3.83+0.23)\times 10^{-5}, \quad g=0.2,$$

$$BR(B^- \to \pi^-\pi^-\pi^+) = (4.35+0.26)\times 10^{-5}, \quad g=0.23,$$

and for $B^- \to \pi^-K^+K^-$

$$BR(B^- \to \pi^-K^+K^-) = (6.67-0.52)\times 10^{-5}, \quad g=0.2,$$

$$BR(B^- \to \pi^-K^+K^-) = (7.05-0.61)\times 10^{-5}, \quad g=0.23.$$

The first number shows the leading contribution, while the second gives the increase or decrease due to the penguin contribution. With the same set of the CKM parameters, but with the opposite sign for the product $V_{td}V_{tb}=-0.007\times 0.999$ we find

$$BR(B^- \to \pi^-\pi^-\pi^+) = (3.83-0.06)\times 10^{-5}, \quad g=0.2,$$

$$BR(B^- \to \pi^-\pi^-\pi^+) = (4.35-0.06)\times 10^{-5}, \quad g=0.23$$

and for $B^- \to \pi^-K^+K^-$,

$$BR(B^- \to \pi^-K^+K^-) = (6.67+0.10)\times 10^{-5}, \quad g=0.2,$$

$$BR(B^- \to \pi^-K^+K^-) = (7.05+0.10)\times 10^{-5}, \quad g=0.23.$$

It is clear from these quantitative numerical results that the uncertainties coming from the input parameters give much larger uncertainties in the branching ratios than the contributions of the penguin operators. Interestingly the penguin contributions, while small, are less sensitive to the input parameters than the dominant tree-level contributions, which are quite sensitive to these input parameters. Since the amplitudes for the $B^-\to\pi^+\pi^-\pi^+$ and $B^-\to K^+K^-\pi^+$ decays receive rather small corrections from the penguin operators we do not expect significant changes in the *CP* violating asymmetry, which we have discussed in [4].

For the branching ratio for the CKM-suppressed nonresonant decay $B^-\to K^0\overline{K^0}\pi^-$ we find the range

$$1.45\times10^{-6}\leqslant \mathrm{BR}(B^-\to K^0\overline{K^0}\pi^-)\leqslant 3.18\times10^{-6}, \quad (29)$$

for $\rho,\ \eta$ as discussed above, and the range $0.2\leqslant g\leqslant 0.23$. In Table I we present this branching ratio for different combinations of parameters $\rho,\ \eta$ and g.

Measurement of this branching ratio is important as it provides information about the strength of the penguin interactions as given by the effective Wilson coefficients a_4 and a_6 in Eq. (7) .

It is interesting to note that in the factorization approximation, as mentioned earlier, this decay is entirely induced by the penguin interactions. Final state interactions (FSI) effects, could alter this, however we believe this is unlikely as data on color-suppressed B decays indicate that the branching ratio for B^0 decay into D^0 and a neutral light hadron is indeed suppressed. Thus we expect FSI would contribute at most a branching ratio for $K^0\overline{K}^0$ mode of the same order as the penguin terms. This could be checked in future measurements of this decay rate.

To summarize, we have quantitatively analyzed the penguin contributions to the nonresonant $B^-\to M\overline{M}\pi^-$ decay amplitudes ($M=\pi^-,K^-,K^0$), including the dependence on the Dalitz variables. We calculated the branching ratios for $B^-\to M\overline{M}\pi^-$ decays ($M=\pi^-,K^-$) including the penguin contributions and found that they only changed the branching ratios by less than 10%. However, while the penguin contributions are small and not very sensitive to the uncertainties in the input parameters, the corresponding uncertainties in the dominant tree-level contributions are considerably larger than the penguin contributions. We also found that the branching ratio for the CKM-suppressed nonresonant decay $B^-\to K^0\overline{K^0}\pi^-$ is entirely induced by penguin effects which produce a rather small branching ratio of the order 10^{-6}.

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