Covariant analysis of the light-front quark model

Wolfgang Jaus
Institut für Theoretische Physik der Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland

(Received 21 December 1998; published 9 August 1999)

A manifestly covariant formalism is used as a guide to construct a covariant extension of the light-front quark model. Our analysis demonstrates in detail that covariance necessarily requires the inclusion of zero-mode contributions. The main goal of this paper is to use this technique in order to extend the standard light-front formalism such that all form factors that are necessary to represent the Lorentz structure of a hadronic matrix element can be calculated on the same footing. The form factors that have been calculated in the standard approach are reproduced, except for those that describe transitions that involve vector mesons. The covariant approach permits also the calculation of the scalar form factor for transitions between pseudoscalar mesons, and the form factor $a_{-}(q^2)$ for transitions between pseudoscalar and vector mesons, which is not possible in the standard light-front formalism. The practical application of the covariant extension of the light-front quark model is successful only if the formulas for form factors are evaluated with standard light-front vertex functions. The latter violate the conditions for strict Lorentz covariance of the formalism. In order to explore the predictive power of this approach, we calculate various properties of pseudoscalar and vector mesons in the *u*-, *d*-, *s*-quark sector. We find good agreement with all available data for electroweak transitions. [S0556-2821(99)03215-4]

PACS number(s): 12.39.Ki, 13.20.−v, 13.25.−k, 13.40.−f

I. INTRODUCTION

The relativistic constituent quark model (RQM) is based on the light-front formalism [1] and provides a conceptually simple, phenomenological framework for the determination of hadronic form factors and coupling constants. It is an attractive feature of the light-front formalism that it permits a consistent relativistic treatment of a relativistic composite system. In the RQM hadrons are composed of valence quarks and the equation of motion of the bound $q\bar{q}$ meson and the bound qqq baryon in the light-front formalism is a relativistic Schrödinger equation with an effective confining potential. Instead of calculating these wave functions in terms of a phenomenological potential, very often one starts with a phenomenological wave function, which depends on a parameter $1/\beta$ which essentially determines the confinement scale, i.e., the ''size'' of the bound state. The only parameters of the model are the constituent masses of the quarks and the wave function parameter β, which can be fixed by a fit to the data. In this work we shall deal only with the quark model description of mesons.

In recent years the RQM has become a useful and popular tool to investigate various electroweak properties of light and heavy mesons [2–6], based upon relativistic approximation methods for the relevant matrix elements. In our past work we have explored the quality and power of the RQM in the *u*-, *d*-, *s*-quark sector [7] for which a large body of precise data exists. We found that the RQM permits a reliable prediction of data on the electroweak transitions of pseudoscalar and vector mesons (see also Ref. [6]).

However, only selected properties of $q\bar{q}$ mesons can be analyzed unambiguously in the RQM, for it is well known [8–10] that the light-front calculation of the matrix element of a one-body current generates a four-vector structure that is in general not covariant, since it contains a spurious dependence on the orientation of the light front. The light front is defined in terms of the lightlike four vector ω by the invariant equation $(\omega x)=0$. The special case $\omega=(1,0,0,-1)$ corresponds to the light front or null plane $(\omega x)=x^{+}=x^{0}+x^{3}=0$.

This problem is closely associated with the violation of rotational invariance in the computation of the matrix element of a one-body current [11,12]. Because of the fact that the angular momentum operator contains interactions, the current operator cannot satisfy the requirement of rotational invariance without containing interaction dependent parts, i.e., two-body currents. Moreover, this condition and current conservation impose essential dynamic consistency conditions on the representations of the current operator and the $q\bar{q}$ bound-state wave function. The matrix element of the full current, which is the sum of a one-body current plus two-body currents, is completely covariant. While such a precise treatment is beyond the limit of the phenomenology of the RQM using as input one-body currents, there is an exceptional case which will be treated in detail. The matrix element of a one-body current transforms covariantly only under kinematic Lorentz transformations, which keep the light-front invariant, but lacks complete Lorentz covariance. Consequently the matrix element acquires a spurious ω dependence. In practical applications of the light-front formalism it is usually assumed that this problem can be avoided by the rule, that hadronic form factors should be calculated only from the plus components of the matrix elements of the respective currents, which, as we shall show, are essentially free of spurious contributions for transitions involving pseudoscalar mesons, but this is no longer true for vector mesons.

In order to treat the complete Lorentz structure of a hadronic matrix element the authors of Ref. [8] have developed a method to identify and separate spurious contributions and to determine the physical, i.e., ω independent contributions to the hadronic form factors and coupling constants. In this work we shall use a manifestly covariant framework as a guide to develop a basically different technique to deal with this problem. A similar procedure has been used in Refs. [13] to investigate the relation between the standard covariant quantum field theory and light-front field theory. In our approach the structure of a $q\overline{q}$ meson bound state will be approximated by a covariant model vertex function, instead of a wave function, and a hadronic matrix element of a one-body current, that is represented by a one-loop diagram, can be computed in two different ways: In the conventional space-time formalism the matrix element is covariant and consists of a Feynman momentum loop integral, which can be evaluated by standard methods. The corresponding light-front matrix element can be obtained from the momentum loop integral by a light-front decomposition of the loop momentum, and carrying out the integration over the minus component ($p^-=p^0-p^3$) by means of contour methods. It is important to require that the contour can be chosen such that only the singularities associated with the quark propagators contribute, while the meson vertex function is free of singularities within the contour. There is indeed a class of covariant meson vertex functions that have this property; they are characterized by an asymmetry in the variables of the constituent $q\overline{q}$ pair. The integration over the minus component of the loop momentum defines the corresponding light-front vertex functions. If they are used to calculate hadronic matrix elements, both procedures must lead to the same form factors, consequently the ω dependence of the light-front approach and the mechanism that causes the disappearance of all ω dependent contributions can be studied in detail. In particular we shall show that covariance requires the inclusion of the effect of zero modes. Hadronic form factors and coupling constants can be calculated consistently in this covariant model in terms of asymmetric light-front vertex functions.

The main goal of the present work is to use this covariant technique in order to extend the conventional light-front formalism such that all form factors that are necessary to represent the Lorentz structure of a hadronic matrix element, can be calculated on the same footing, preserving the simplicity that distinguishes the light-front approach. It is important to note that the expressions for form factors that have been obtained in the RQM in the past [2–7] are reproduced in the covariant approach, except for those that describe transitions which involve mesons of spin 1, polarized in the longitudinal direction. The fact that transition form factors for particles of spin 1 or higher are sensitive to the violation of rotational invariance in the light-front formalism is well known, and has been discussed in the context of calculations of the electromagnetic form factors for the deuteron [12,14], and the ρ meson [15]. We shall investigate this problem for the first time for transitions between states of spin 1 and spin 0. In particular, we shall show that the conventional light-front formulas for the vector coupling constant f_V (see, e.g., Ref. [7]) and the axial-vector form factor $A_1(q^2)$ (see, e.g., Ref. [16]), which is one of the form factors for semileptonic transitions between pseudoscalar and vector mesons, contain unphysical admixtures. In this work formulas will be given that are free of spurious ω dependent contributions.

For practical applications it is of interest that, aside from a clean treatment of the subtleties in connection with particles of spin 1, the covariant approach permits also the calculation of the scalar form factor for transitions between pseudoscalar mesons, and the form factor $a_-(q^2)$ for transitions between pseudoscalar and vector mesons, which is not possible in the conventional light-front approach. These form factors are required, for instance, for the analysis of tauonic B decays.

In Sec. II we present a brief summary of the basic formalism and an outline of the covariant analysis. The latter is motivated by the fact that the Lorentz decomposition of a light-front amplitude, which represents a hadronic matrix element, necessarily requires for its construction the vector ω, i.e., it depends on the light front. In Sec. III we investigate a simple model to discover the mechanism that leads to the cancellation of all ω dependence of a light-front amplitude, whereby zero-mode contributions play a central role. A general method is developed that permits the calculation of the physical form factors. This program is performed in Sec. IV. In Sec. V we discuss the problem associated with the choice of the appropriate vertex function, and explore the predictive power of the covariant extension of the light-front quark model by calculating various properties of pseudoscalar and vector mesons in the u-, d-, s-quark sectors and comparing the predictions with the available data. Finally, we briefly discuss the heavy quark limit of our model. We conclude this work in Sec. VI with a summary of our analysis.

II. BASIC FORMALISM AND OUTLINE OF THE COVARIANT ANALYSIS

As an introduction of our covariant analysis we shall briefly compare the calculation of the pseudoscalar decay constant and of the electroweak form factors of pseudoscalar mesons in a manifestly covariant framework with the result of the corresponding calculation in the light-front formalism.

The composite meson state is represented by the $q\overline{q}$ bound-state vertex operator Γ, which in a covariant formalism is the solution of the Bethe-Salpeter equation

$$\Gamma = UG\Gamma, \tag{2.1}$$

where U is the irreducible $q\overline{q}$ kernel in a field-theoretic description of the $q\overline{q}$ interaction. The full statement of the symbolic operator product of Eq. (2.1) includes the integration over all four components of the relative momentum $p=p_1'-p_2$, where p_1',p_2 denote the four momenta of the constituent quarks with masses m_1' and m_2, respectively, and the total four momentum of the meson state is given by $p_1'+p_2=P'$, with $P'^2=M'^2$, where M' is the mass of the meson. The Green's function G describes the propagation of the two off-shell quarks. We assume that the solution of the Bethe-Salpeter equation has the form

$$\Gamma = \gamma_5 H_0(p_1'^2, p_2^2). \tag{2.2}$$

This simple ansatz is not essential for the covariant analysis, which could be based equally well on the most general vertex operator which has the matrix structure of γ_5 and

$\gamma_5(\gamma P')$. In writing down Eq. (2.2) we have omitted obvious color and flavor operators, and $H_0(p_1'^2,p_2^2)$ is the scalar vertex function.

The pseudoscalar decay constant f_P for a $q\overline{q}$ meson is given by the matrix element of the axial vector current, which we express in the general form

$$\langle 0|\overline{q}''\gamma_\mu q'|P'\rangle = iP'_\mu\sqrt{2}f_P. \tag{2.3}$$

The matrix element (2.3) is given in the one-loop approximation as a momentum integral

$$A_\mu=\frac{N_c}{(2\pi)^4}\int d^4p_1'\frac{H_0'}{N_1'N_2}\mathrm{tr}[\gamma_\mu\gamma_5(\not{p}_1'+m_1')\gamma_5(-\not{p}_2+m_2)]$$

$$=-\frac{N_c}{(2\pi)^4}\int d^4p_1'\frac{H_0'}{N_1'N_2}4[m_2P'_\mu+(m_1'-m_2)p'_{1\mu}], \tag{2.4}$$

where

$$N_1'=p_1'^2-m_1'^2+i\epsilon \quad \text{and} \quad N_2=p_2^2-m_2^2+i\epsilon,$$

and N_c denotes the number of colors. The pseudoscalar decay constant can be derived from Eq. (2.4) as

$$\sqrt{2}f_P=-i\frac{(P'A)}{P'^2}, \tag{2.5}$$

and we have used $(P'A)$ to denote the scalar product of the four vectors P'_μ and A_μ.

The most general form of the hadronic matrix element of the vector current must be expressed in terms of two form factors

$$\langle P''|\overline{q}''\gamma_\mu q'|P'\rangle = P_\mu F_1(q^2)+q_\mu F_2(q^2). \tag{2.6}$$

The pseudoscalar mesons have momenta P', P'', and $P=P'+P''$, $q=P'-P''$. The matrix element (2.6) is given, again in the one-loop approximation, as a momentum integral

$$B_\mu=i\frac{N_c}{(2\pi)^4}\int d^4p_1'\frac{H_0'H_0''}{N_1'N_1''N_2}S_\mu, \tag{2.7}$$

where

$$\begin{aligned}S_\mu&=\mathrm{tr}[\gamma_5(\not{p}_1''+m_1'')\gamma_\mu(\not{p}_1'+m_1')\gamma_5(-\not{p}_2+m_2)]\\&=2p'_{1\mu}[M'^2+M''^2-q^2-2N_2-(m_1'-m_2)^2\\&\quad-(m_1''-m_2)^2+(m_1'-m_1'')^2]\\&\quad+q_\mu[q^2-2M'^2+N_1'-N_1''+2N_2+2(m_1'-m_2)^2\\&\quad-(m_1'-m_1'')^2]\\&\quad+P_\mu[q^2-N_1'-N_1''-(m_1'-m_1'')^2]\end{aligned} \tag{2.8}$$

and

$$H_0'\equiv H_0'(p_1'^2,p_2^2) \quad \text{and} \quad H_0''\equiv H_0''(p_1''^2,p_2^2),$$

and it has been used that $p_1'+p_2=P'$, $p_1''+p_2=P''$, $p_1'-p_1''=q$, and $N_1''=p_1''^2-m_1''^2+i\epsilon$. The form factors in the one-loop approximation are given by

$$F_1(q^2)=\frac{q^2(PB)-(qP)(qB)}{q^2P^2-(qP)^2}$$

and

$$F_2(q^2)=\frac{P^2(qB)-(qP)(PB)}{q^2P^2-(qP)^2}. \tag{2.9}$$

In order to make a comparison with the corresponding one-loop expressions obtained in the light-front formalism it is necessary to introduce light-front momenta. The four-momentum of the meson of mass M' in terms of light-front components is $P'=(P'^-,P'^+,P'_\perp)$, where $P'^\pm=P'^0\pm P'^3$ and $P'^2=P'^+P'^--P_\perp'^2=M'^2$. The appropriate variables for the internal motion of the constituents, $(x,p'_\perp)$, are defined by

$$p_1'^+=xP'^+,\quad p_2^+=(1-x)P'^+$$

$$p'_{1\perp}=xP'_\perp+p'_{\perp-},\quad p_{2\perp}=(1-x)P'_\perp-p'_\perp,$$

and the kinematic invariant mass is

$$M_0'^2=\frac{p_\perp'^2+m_1'^2}{x}+\frac{p_\perp'^2+m_2^2}{1-x}. \tag{2.10}$$

The pseudoscalar decay constant f_P and the vector form factor $F_1(q^2)$ have been derived first for $q\overline{q}$ mesons, whose constituents have equal mass, by Terentev [1] using Hamiltonian light-front dynamics (see also Ref. [17]). We have extended the model of Ref. [1] and derived the formulas that are valid for any pseudoscalar meson in Refs. [7,16].

The pseudoscalar decay constant for the meson of mass M' is given by [7]

$$f_P=\sqrt{2}\frac{N_c}{8\pi^3}\int_0^1 dx\int d^2p'_\perp\frac{h_0'}{x(1-x)(M'^2-M_0'^2)}\times[(1-x)m_1'+xm_2]. \tag{2.11}$$

For the determination of form factors it is crucial to impose the condition $q^+=0$, which at this stage of the calculation means that form factors are known only for spacelike momentum transfer $q^2=-q_\perp^2\leqslant 0$. The vector form factor $F_1(q^2)$ for the transition between an initial pseudoscalar meson with internal variables and masses of its constituent quarks $(x,p'_\perp,m_1',m_2)$ and a final pseudoscalar meson with the corresponding quantities $(x,p''_\perp,m_1'',m_2)$ has been obtained in [16,17] as

$$F_1(q^2) = \frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp \frac{h'_0 h''_0}{(1-x)x^2(M'^2-M_0'^2)(M''^2-M_0''^2)} \times\{xM_0'^2+xM_0''^2+(1-x)q^2-(1-x)(m'_1-m''_1)^2 - x(m'_1-m_2)^2 - x(m''_1-m_2)^2\}, \tag{2.12}$$

where $M_0'^2$ is given by Eq. (2.10), $p''_\perp = p'_\perp-(1-x)q_\perp$ and

$$M_0''^2 = \frac{p_\perp''^2+(1-x)m_1''^2+xm_2^2}{x(1-x)}. \tag{2.13}$$

The light-front vertex functions h'_0 and h''_0 depend on the internal variables $(x,p'_\perp)$ and $(x,p''_\perp)$, respectively, and will be specified later.

In order to compare the results of the two methods we shall evaluate the four-momentum integrals (2.4) and (2.7) by use of the light-front decomposition of the four-momentum vector, where

$$d^4p'_1 = \frac{1}{2}P'^+dp_1'^-dxd^2p'_\perp. \tag{2.14}$$

In general, it is not possible that, given the vertex function $H_0(p_1'^2,p_2^2)$, a light-front vertex function $h_0(x,p'_\perp)$ can be found such that the one-loop formulas (2.5) and (2.9) agree with the corresponding light-front expressions (2.11) and (2.12). One expects that such an agreement can be achieved only if two-body currents are included such that the total current operator is compatible with the light-front vertex function. The interaction-dependent parts of the current are generated by the exchange of gluons between the constituent quarks, and the treatment of such a process goes beyond the valence quark picture, which is the basic assumption of the approach used in this paper. However, there is a well-known exception: If it is assumed that the vertex function H_0 has no poles in the upper complex $p_1'^-$ plane, then the covariant calculation of meson properties and the calculation in the framework of the light-front formalism give identical results already at the one-loop level. The four-momentum integrals (2.4) and (2.7), expressed in terms of light-front variables, are carried out by contour methods in the complex $p_1'^-$ plane. Closing the contour in the upper $p_1'^-$ plane [under the condition $q^+=0$ for the amplitude B_μ of Eq. (2.7)] ensures that the momentum integrals are given by the respective residues of the spectator quark pole, corresponding to putting quark 2 on the mass shell. We shall use $\hat p'_1$, $\hat p''_1$, and $\hat p_2$ to denote the restricted four vectors

$$\hat p_2 = \left(\frac{m_{2\perp}^2}{p_2^+}, p_2^+, p_{2\perp}\right), \quad \hat p'_1 = P'-\hat p_2, \quad \hat p''_1 = \hat p'_1 - q, \tag{2.15}$$

where $m_{2\perp}^2 = p_{2\perp}^2+m_2^2$. It follows from Eq. (2.15) that

$$\hat N_2 = \hat p_2^2 - m_2^2 = 0, \quad \hat N'_1 = \hat p_1'^2 - m_1'^2 = x(M'^2-M_0'^2), \quad \hat N''_1 = \hat p_1''^2 - m_1''^2 = x(M''^2-M_0''^2), \tag{2.16}$$

and the vertex operators at the spectator quark pole are given by

$$\hat\Gamma' = \gamma_5 H'_0(\hat p_1'^2,\hat p_2^2) \quad \text{and} \quad \hat\Gamma'' = \gamma_5 H''_0(\hat p_1''^2,\hat p_2^2). \tag{2.17}$$

Note that the contour integration of A_μ and B_μ is zero unless $0\leqslant x\leqslant 1$, and we shall denote the resulting expressions by $\hat A_\mu$ and $\hat B_\mu$. They are given by

$$\hat A_\mu = -i\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp \frac{h'_0}{(1-x)\hat N'_1} \times 4[m_2P'_\mu + (m'_1-m_2)\hat p'_{1\mu}], \tag{2.18}$$

$$\hat B_\mu = \frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp \frac{h'_0h''_0}{(1-x)\hat N'_1\hat N''_1}\hat S_\mu, \tag{2.19}$$

where

$$h'_0 \equiv h'_0(\hat N'_1) = H'_0(\hat p_1'^2,\hat p_2^2), \quad h''_0 \equiv h''_0(\hat N''_1) = H''_0(\hat p_1''^2,\hat p_2^2) \tag{2.20}$$

are the corresponding light-front vertex functions, and $\hat S_\mu$ is the trace S_μ, Eq. (2.8), expressed in terms of the restricted four vectors of Eq. (2.15).

A peculiar property of the integrals $\hat A_\mu$ and $\hat B_\mu$ is their dependence on the light front, defined by the lightlike four vector $\omega_\mu=(2,0,0_\perp)$, which is explicitly revealed by their decomposition into four vectors, as given by

$$\hat A_\mu = i\sqrt{2}(P'_\mu f_P + \omega_\mu g_P), \tag{2.21}$$

$$\hat B_\mu = P_\mu F_1(q^2) + q_\mu \hat F_2(q^2) + \omega_\mu F_3(q^2). \tag{2.22}$$

From the formulas for $\hat A_\mu$, Eq. (2.18), and $\hat B_\mu$, Eq. (2.19), it can be shown that $g_P\neq 0$ and $F_3(q^2)\neq 0$. Therefore, not only is $\hat A_\mu \neq A_\mu$ and $\hat B_\mu\neq B_\mu$, but since ω_μ is a fixed vector, the four-vector structure of $\hat A_\mu$ and $\hat B_\mu$ is obviously not covariant. In fact, in going from the manifestly covariant one-loop integrals A_μ, Eq. (2.4), and B_μ, Eq. (2.7), to the light-front integrals $\hat A_\mu$ and $\hat B_\mu$ by means of a $p_1'^-$ integration, the associated zero-mode problem has been ignored. We shall show in Sec. III that zero modes are required in order to eliminate the spurious ω dependence and to obtain a covariant result.

Nevertheless, one can already determine the pseudoscalar decay constant f_P and the vector form factor $F_1(q^2)$ from the plus components of $\hat A_\mu$ and $\hat B_\mu$, respectively,

$$\hat{A}^{+}=iP'^{+}\sqrt{2}f_P, \tag{2.23}$$

$$\hat{B}^{+}=P^{+}F_1(q^2), \tag{2.24}$$

and the result coincides with the standard expressions, Eqs. (2.11), (2.12), of the light-front formalism. They will be seen to be free of spurious contributions, which explains the special role of the plus component of the matrix element of a current in light-front calculations.

Hadronic matrix elements are often expressed in terms of $q\bar{q}$ bound-state wave functions, and for completeness sake we shall briefly describe their relationship to the respective vertex operators. The covariant Bethe-Salpeter wave function $\Phi(p_1',p_2)$ is related to the vertex operator defined in Eq. (2.2) by

$$\Phi(p_1',p_2)=S(p_1')\Gamma S(-p_2)=\frac{(\not{p}_1'+m_1')\gamma_5(-\not{p}_2+m_2)}{(p_1'^2-m_1'^2)(p_2^2-m_2^2)}H_0'(p_1'^2,p_2^2). \tag{2.25}$$

The light-front version of the covariant Bethe-Salpeter wave function is defined by

$$\Psi(x,p_\perp')=\frac{i}{2\pi}\int dp_1'^{-}\Phi(p_1',p_2) \tag{2.26}$$

with $p_1'+p_2=P'$ and $P_\perp'=0$. Under the condition that the vertex function H_0' has no poles in the upper complex $p_1'^{-}$ plane we find

$$\Psi(x,p_\perp')=\frac{(\hat{\not{p}}_1'+m_1')\gamma_5(-\hat{\not{p}}_2+m_2)}{(1-x)\hat{N}_1'P'^{+}}h_0'(\hat{N}_1'), \tag{2.27}$$

where the light-front vertex function h_0' is given by Eq. (2.20). This representation can be used, for example, to rederive the result found by Brodsky and Lepage [18] that the pseudoscalar decay constant f_P is determined by the $(\gamma P)\gamma_5$ part of the wave function (2.27):

$$\Psi(x,p_\perp')\rightarrow\frac{1}{\sqrt{2}}(\gamma P')\gamma_5\psi_{q\bar{q}}(x,p_\perp'), \tag{2.28}$$

where

$$\psi_{q\bar{q}}(x,p_\perp')=\frac{-1}{2\sqrt{2}}\mathrm{tr}\{\gamma^{+}\gamma_5\Psi(x,p_\perp')\}=\sqrt{2}\,\frac{(1-x)m_1'+xm_2}{(1-x)\hat{N}_1'}h_0'. \tag{2.29}$$

The pseudoscalar decay constant is obviously given by

$$f_P=\frac{N_c}{8\pi^3}\int_0^1 dx\int d^2p_\perp'\,\psi_{q\bar{q}}(x,p_\perp') \tag{2.30}$$

and this formula agrees with Eq. (2.11).

The form factor $\hat{F}_2(q^2)$ can be calculated from the transverse component of $\hat{B}_\mu$, but it contains ω dependent, spurious admixtures, and is different from $F_2(q^2)$. We shall work out a method which will permit us to cancel the spurious parts of the light-front amplitudes $\hat{A}_\mu$ and $\hat{B}_\mu$. The modified amplitudes are free of additional form factors like g_P and F_3 and the physical form factor $F_2(q^2)$ can be determined unambiguously in one-loop order.

III. A MANIFESTLY COVARIANT ANALYSIS

The formulas for f_P, Eq. (2.5), in terms of the amplitude A_μ, and for $F_1(q^2)$ and $F_2(q^2)$, Eq. (2.9), in terms of the amplitude B_μ have been derived from the respective one-loop Feynman diagrams, whereby a manifestly covariant framework has been prescribed. For the special class of vertex functions $H_0(p_1^2,p_2^2)$, which have no poles in the upper complex p_1^{-} plane, the associated light-front vertex functions can be defined uniquely, and the corresponding light-front formalism must be able to reproduce not only f_P and $F_1(q^2)$, as we have verified in Sec. II, but also the form factor $F_2(q^2)$. Moreover, it must be possible to prove exactly that all dependence on the light front, i.e., all ω dependence, disappears.

A. An explicit model calculation

We shall first develop a strategy to deal with the ω dependence of the light-front formalism in the context of a simple covariant model, which permits explicit analytic calculations at each step of the following analysis. The starting point is a multipole ansatz for the $q\bar{q}$ bound-state vertex function for a pseudoscalar meson

$$H_0(p_1^2,p_2^2)=\frac{g}{N_\Lambda^n}, \tag{3.1}$$

where $N_\Lambda=p_1^2-\Lambda^2+i\epsilon$, and Λ and g are constant parameters. The vertex function (3.1) is not symmetric in the four momenta of the constituent quarks, and can hardly be considered a realistic approximation of a $q\bar{q}$ bound state. We regard it only as a convenient cutoff prescription which makes the one-loop integrals finite. For our purpose it is sufficient to consider only the case of a monopole form of the vertex function, i.e., $n=1$, since our results do not depend on the value of n. Covariant models of this type have been used also in Refs. [15,19].

In principle the momentum integrals can be calculated in terms of the usual space-time components by the standard Feynman parameter method, and all our results can be checked in this manner. However, we are mainly interested here in the evaluation of the momentum integrals by use of light-front variables, and in the reason why they depend on ω. If, for example, the matrix element (2.6) is calculated in one-loop order with the model vertex function (3.1) one finds the amplitude

$$b_\mu = i\,\frac{N_c}{(2\pi)^4}\int d^4p_1' \frac{g'g''}{N_\Lambda' N_1' N_2 N_1'' N_\Lambda''} S_\mu, \tag{3.2}$$

where the trace S_μ is given by Eq. (2.8). If the $p_1'^-$ integration is carried out by contour methods as before, we arrive at the light-front amplitude $\hat{b}_\mu$, which is given by Eq. (2.19) with light-front vertex functions corresponding to the model (3.1)

$$h_0'(\hat{N}_1') = \frac{g'}{\hat{N}_\Lambda'} \quad \text{and} \quad h_0''(\hat{N}_1'') = \frac{g''}{\hat{N}_\Lambda''}, \tag{3.3}$$

where $\hat{N}_\Lambda' = \hat{N}_1' + m_1'^2 - \Lambda'^2$ and $\hat{N}_\Lambda'' = \hat{N}_1'' + m_1''^2 - \Lambda''^2$. The light-front amplitude $\hat{b}_\mu$ can be decomposed according to Eq. (2.22), i.e., it also depends on ω. We shall explain now why the momentum integrals b_μ and $\hat{b}_\mu$ are different.

Obviously there are terms of S_μ which depend linearly on N_2, such that the factor N_2 in the denominator of Eq. (3.2) is cancelled, and all poles are in the lower complex $p_1'^-$ plane at

$$p_1'^- = \frac{m_\perp^2 - i\epsilon}{p_1'^+}, \tag{3.4}$$

where $m_\perp^2 = m_1'^2 + p_{1\perp}'^2, \Lambda'^2 + p_{1\perp}'^2, m_1''^2 + p_{1\perp}''^2, \Lambda''^2 + p_{1\perp}''^2$. The expression for $\hat{b}_\mu$ is obtained by closing the contour in the upper complex $p_1'^-$ plane and contains no contribution from these poles. However, integrals of this type are associated with zero modes and have been discussed first by the authors of Ref. [20], who argued that these pole contributions are strictly zero for $p_1'^+ \neq 0$, but for $p_1'^+ = 0$ the pole (3.4) is at infinity, and cannot be avoided by closing the contour either from above or below. In this case the $p_1'^-$ integration must yield a result proportional to $\delta(p_1'^+)$. In Ref. [20] a method has been indicated by which a $p_1'^-$ integral can be represented as a sum of the contribution due to the standard contour method and a zero-mode contribution.

The following integral representation is used:

$$\frac{i}{p^2 - m^2 + i\epsilon} = \int_0^\infty d\alpha\, e^{i\alpha(p^2 - m^2 + i\epsilon)}. \tag{3.5}$$

The formal p^- integration of this expression results in δ functions:

$$\int dp^- \frac{i}{p^2 - m^2 + i\epsilon} = 2\pi \int_0^\infty d\alpha\, e^{-i\alpha m_\perp^2} \times \left\{ \frac{1}{p^+}\,\delta(\alpha) + \frac{1}{\alpha}\,\delta(p^+) \right\}, \tag{3.6}$$

$$\int dp^- \frac{p^-}{p^2 - m^2 + i\epsilon} = -2\pi \int_0^\infty d\alpha\, e^{-i\alpha m_\perp^2} \frac{1}{p^+} \times \left\{ \frac{1}{p^+}\,\delta'(\alpha) - \frac{1}{\alpha^2}\,\delta(p^+) \right\}. \tag{3.7}$$

For further applications it is useful to decompose the integrand of b_μ, Eq. (3.2), as follows:

$$\frac{1}{N_\Lambda' N_1' N_2 N_1'' N_\Lambda''} = \frac{1}{(\Lambda'^2 - m_1'^2)(\Lambda''^2 - m_1''^2)}\,\frac{1}{N_2}\left(\frac{1}{N_\Lambda'} - \frac{1}{N_1'}\right) \times \left(\frac{1}{N_\Lambda''} - \frac{1}{N_1''}\right). \tag{3.8}$$

Using Eqs. (3.6) and (3.7) and carrying out the α integrals gives

$$\frac{i}{2\pi}\int dp_1'^- \frac{1}{N_1' N_2 N_1''} = \frac{1}{p_2^+ \hat{N}_1' \hat{N}_1''}, \tag{3.9}$$

$$\frac{i}{2\pi}\int dp_1'^- \frac{p_1'^-}{N_1' N_2 N_1''} = \frac{\hat{p}_1'^-}{p_2^+ \hat{N}_1' \hat{N}_1''} - \frac{\delta(p_1'^+)}{p_2^+ (m_{1\perp}'^2 - m_{1\perp}''^2)}\, ln\,\frac{m_{1\perp}''^2}{m_{1\perp}'^2}, \tag{3.10}$$

i.e., one finds again the respective residues at the spectator quark pole in terms of the restricted quantities defined in Eqs. (2.15) and (2.16) and, as expected, a term proportional to $\delta(p_1'^+)$. Equation (3.10) shows that it is the dependence of the numerator on $p_1'^-$ which generates the zero-mode contribution proportional to $\delta(p_1'^+)$, which is absent, however, if $p_1'^-$ is combined with a factor proportional to $p_1'^+$. This can be seen more clearly, if the following integrals are considered:

$$\frac{i}{2\pi}\int dp_1'^- \frac{N_1'}{N_1' N_2 N_1''} = \frac{\hat{N}_1'}{p_2^+ \hat{N}_1' \hat{N}_1''}, \tag{3.11}$$

$$\frac{i}{2\pi}\int dp_1'^- \frac{N_2}{N_1' N_2 N_1''} = \frac{\hat{N}_2}{p_2^+ \hat{N}_1' \hat{N}_1''} + \frac{\delta(p_1'^+)}{m_{1\perp}'^2 - m_{1\perp}''^2}\, ln\,\frac{m_{1\perp}''^2}{m_{1\perp}'^2}. \tag{3.12}$$

Note that $\hat{N}_2 = 0$ and the integral (3.12) consists only of a zero-mode contribution. Combining all the terms of the decomposition (3.8) and using Eqs. (3.12) and (2.14) the complete momentum integral is given by

$$\frac{i}{(2\pi)^4}\int d^4p_1' \frac{N_2}{N_\Lambda' N_1' N_2 N_1'' N_\Lambda''} = \frac{1}{16\pi^2(\Lambda'^2 - m_1'^2)(\Lambda''^2 - m_1''^2)}\int_0^1 dy\, ln\,\frac{C_{11}^0 C_{\Lambda\Lambda}^0}{C_{1\Lambda}^0 C_{\Lambda 1}^0}, \tag{3.13}$$

where

$$C^0_{11}=C^0(m'_1,m''_1)=(1-y)m'^2_1+ym''^2_1-y(1-y)q^2,$$

$$C^0_{\Lambda\Lambda}=C^0(\Lambda',\Lambda''),C^0_{1\Lambda}=C^0(m'_1,\Lambda''),C^0_{\Lambda 1}=C^0(\Lambda',m''_1).$$

The same result is obtained if the four-momentum integral is carried out using standard Feynman parameters.

From the arguments given above it is plausible that the terms proportional to $p'_{1\mu}$, $q_\mu N_2$, and $p'_{1\mu}N_2$ in the integrands of the one-loop amplitudes A_μ, B_μ are either missed or not completely accounted for if the p'^-_1 integration is carried out by the contour method. The lost information is signaled by the appearance of a spurious ω dependence, which is expected to disappear if the respective zero-mode contributions are included correctly.

Note that the zero-mode contributions proportional to $\delta(p'^+_1)$ have been interpreted in [21] as residues of virtual pair creation processes in the limit $q^+\to 0$.

B. Effective inclusion of zero-mode contributions and necessary condition for covariance

We present in this subsection an alternative and more practical method to deal with the ω dependence of the light-front formalism, which is manifest in the decomposition of the amplitudes $\hat{A}_\mu$, Eq. (2.21), and $\hat{B}_\mu$, Eq. (2.22), account for the missing zero-mode contributions and thus restore the covariance of the amplitudes. For this purpose we shall investigate the various parts of the integrand of $\hat{B}_\mu$, which is essentially given by the trace $\hat{S}_\mu$. We start with the four vector $\hat{p}'_{1\mu}$, which is given in terms of the internal variables x and $p'_\perp$ by Eq. (2.15). However, in order to separate its ω dependent parts one needs its decomposition with regard to P, q, and ω, which is inferred from symmetry considerations to be

$$\int_0^1 dx\int d^2p'_\perp\frac{\Omega}{1-x}\hat{p}'_{1\mu}=\int_0^1 dx\int d^2p'_\perp\frac{\Omega}{1-x}\left(P_\mu A^{(1)}_1+q_\mu A^{(1)}_2+\frac{1}{(\omega P)}\omega_\mu C^{(1)}_1\right), \tag{3.14}$$

where

$$\Omega\equiv\Omega(\hat{N}'_1,\hat{N}''_1)=\frac{h'_0(\hat{N}'_1)h''_0(\hat{N}''_1)}{\hat{N}'_1\hat{N}''_1}. \tag{3.15}$$

An equation such as Eq. (3.14) will be written in the following as a relation between integrands as

$$\hat{p}'_{1\mu}\doteq P_\mu A^{(1)}_1+q_\mu A^{(1)}_2+\frac{1}{(\omega P)}\omega_\mu C^{(1)}_1. \tag{3.16}$$

The coefficients in Eqs. (3.14) and (3.16) are given by

$$A^{(1)}_1=\frac{(\omega\hat{p}'_1)}{(\omega P)}=\frac{x}{2},$$

$$A^{(1)}_2=\frac{1}{q^2}\left((\hat{p}'_1q)-(qP)\frac{(\omega\hat{p}'_1)}{(\omega P)}\right)=\frac{x}{2}-\frac{p'_\perp q_\perp}{q^2}, \tag{3.17}$$

$$C^{(1)}_1=(\hat{p}'_1P)-P^2A^{(1)}_1-(qP)A^{(1)}_2=-\hat{N}_2+Z_2,$$

where

$$Z_2=\hat{N}'_1+m'^2_1-m^2_2+(1-2x)M'^2+[q^2+(qP)]\frac{p'_\perp q_\perp}{q^2}. \tag{3.18}$$

Note that only the coefficient which is combined with ω_μ, namely $C^{(1)}_1$, depends on $\hat{p}'^-_1$. We have shown in Sec. III A that, while $\hat{N}'_1$ is given by Eq. (2.16), terms that contain $\hat{N}_2$ are associated with zero-mode contributions, which we can include exactly in this particular case by the replacement

$$\hat{N}_2\to Z_2. \tag{3.19}$$

This replacement is equivalent to setting $C^{(1)}_1=0$, and due to this prescription the decomposition (3.16) becomes covariant, i.e., free of any ω dependence. We shall prove in the Appendix that in the framework of the simple model (3.1) the prescription (3.19) accounts exactly for the relevant zero-mode contribution, given by Eq. (3.13), i.e., the integrated function Z_2, Eq. (A8), is equal to Eq. (3.13).

The amplitude $\hat{A}_\mu$ can be handled in the same way (except that there is no q dependence) and it follows that $g_P=0$.

We shall need also the tensor decomposition

$$\hat{p}'_{1\mu}\hat{p}'_{1\nu}\doteq g_{\mu\nu}A^{(2)}_1+P_\mu P_\nu A^{(2)}_2+(P_\mu q_\nu+q_\mu P_\nu)A^{(2)}_3+q_\mu q_\nu A^{(2)}_4+\frac{1}{(\omega P)}(P_\mu\omega_\nu+\omega_\mu P_\nu)B^{(2)}_1+\frac{1}{(\omega P)}(q_\mu\omega_\nu+\omega_\mu q_\nu)C^{(2)}_1+\frac{1}{(\omega P)^2}\omega_\mu\omega_\nu C^{(2)}_2, \tag{3.20}$$

where

$$A^{(2)}_1=-p'^2_\perp-\frac{(p'_\perp q_\perp)^2}{q^2},$$

$$A^{(2)}_2=(A^{(1)}_1)^2,$$

$$A^{(2)}_3=A^{(1)}_1A^{(1)}_2,$$

$$A^{(2)}_4=(A^{(1)}_2)^2-\frac{1}{q^2}A^{(2)}_1, \tag{3.21}$$

$$B^{(2)}_1=A^{(1)}_1C^{(1)}_1-A^{(2)}_1,$$

$$C^{(2)}_1=A^{(1)}_2C^{(1)}_1+\frac{(qP)}{q^2}A^{(2)}_1,$$

$$C^{(2)}_2=(C^{(1)}_1)^2+\left[P^2-\frac{(qP)^2}{q^2}\right]A^{(2)}_1.$$

The coefficient $B_1^{(2)}$ plays a special role since, on the one hand, it is combined with ω_μ, on the other hand, it contains the term $x\hat{N}_2$ which according to Eq. (3.12) takes the value

$$x\hat{N}_2=0, \tag{3.22}$$

since $x\,\delta(p_1'^{+})=0$, and which means that there are no zero-mode contributions associated with $B_1^{(2)}$, which is therefore given by its value at the spectator quark pole:

$$B_1^{(2)}=\frac{x}{2}Z_2-A_1^{(2)}, \tag{3.23}$$

i.e., the coefficient $B_1^{(2)}$ does not vanish. Note also that, if $\hat{p}_1'^{2}$ is calculated by contracting the tensor (3.20), Eq. (3.23) for $B_1^{(2)}$ must be used in order to obtain the result given by Eq. (2.16). The role of $B_1^{(2)}$ will be discussed below.

Again, the coefficient $C_1^{(2)}$ is cancelled if the corresponding zero-mode contribution is included, which can be achieved in this case by the replacement

$$A_2^{(1)}\hat{N}_2\to A_2^{(1)}Z_2+\frac{(qP)}{q^2}A_1^{(2)}, \tag{3.24}$$

which gives $C_1^{(2)}=0$. In this manner one finds how the term $\hat{p}'_{1\mu}\hat{N}_2$, which is part of the trace $\hat{S}_\mu$, is modified by the effective inclusion of the relevant zero-mode contribution:

$$\begin{aligned}\hat{p}'_{1\mu}\hat{N}_2&\doteq P_\mu A_1^{(1)}\hat{N}_2+q_\mu A_2^{(1)}\hat{N}_2+\omega_\mu C_1^{(1)}\hat{N}_2\\&\to q_\mu\left[A_2^{(1)}Z_2+\frac{(qP)}{q^2}A_1^{(2)}\right],\end{aligned} \tag{3.25}$$

where Eq. (3.22) and the prescription (3.24) have been used. The coefficient $C_2^{(2)}$ is of second order in $\hat{p}_1'^{-}$. It can be discussed in a similar way, but we do not explicitly consider here terms of higher order.

For the practical applications considered in this work we shall need also the decomposition of the following tensor product:

$$\begin{aligned}\hat{p}'_{1\mu}\hat{p}'_{1\nu}\hat{p}'_{1\alpha}\doteq{}&(g_{\mu\nu}P_\alpha+g_{\mu\alpha}P_\nu+g_{\nu\alpha}P_\mu)A_1^{(3)}+(g_{\mu\nu}q_\alpha+g_{\mu\alpha}q_\nu+g_{\nu\alpha}q_\mu)A_2^{(3)}+P_\mu P_\nu P_\alpha A_3^{(3)}\\&+(P_\mu P_\nu q_\alpha+P_\mu q_\nu P_\alpha+q_\mu P_\nu P_\alpha)A_4^{(3)}+(q_\mu q_\nu P_\alpha+q_\mu P_\nu q_\alpha+P_\mu q_\nu q_\alpha)A_5^{(3)}+q_\mu q_\nu q_\alpha A_6^{(3)}\\&+\frac{1}{(\omega P)}(P_\mu P_\nu\omega_\alpha+P_\mu\omega_\nu P_\alpha+\omega_\mu P_\nu P_\alpha)B_1^{(3)}\\&+\frac{1}{(\omega P)}[(P_\mu q_\nu+q_\mu P_\nu)\omega_\alpha+(P_\mu q_\alpha+q_\mu P_\alpha)\omega_\nu+(P_\nu q_\alpha+q_\nu P_\alpha)\omega_\mu]B_2^{(3)}\\&+\frac{1}{(\omega P)}(g_{\mu\nu}\omega_\alpha+g_{\mu\alpha}\omega_\nu+g_{\nu\alpha}\omega_\mu)C_1^{(3)}+\frac{1}{(\omega P)}(q_\mu q_\nu\omega_\alpha+q_\mu\omega_\nu q_\alpha+\omega_\mu q_\nu q_\alpha)C_2^{(3)}+O(\omega^2)+O(\omega^3),\end{aligned} \tag{3.26}$$

where we have omitted all terms that are of second and third order in ω_μ, and the coefficients are given by

$$\begin{aligned}&A_1^{(3)}=A_1^{(1)}A_1^{(2)},\quad A_2^{(3)}=A_2^{(1)}A_1^{(2)},\\&A_3^{(3)}=A_1^{(1)}A_2^{(2)},\quad A_4^{(3)}=A_2^{(1)}A_2^{(2)},\\&A_5^{(3)}=A_1^{(1)}A_4^{(2)},\quad A_6^{(3)}=A_2^{(1)}A_4^{(2)}-\frac{2}{q^2}A_2^{(1)}A_1^{(2)},\end{aligned} \tag{3.27}$$

$$\begin{aligned}&B_1^{(3)}=A_1^{(1)}B_1^{(2)}-A_1^{(1)}A_1^{(2)},\quad B_2^{(3)}=A_1^{(1)}C_1^{(2)}-A_2^{(1)}A_1^{(2)},\\&C_1^{(3)}=C_1^{(1)}A_1^{(2)},\quad C_2^{(3)}=C_1^{(1)}A_4^{(2)}+2\frac{(qP)}{q^2}A_2^{(1)}A_1^{(2)}.\end{aligned}$$

The ω dependent terms of Eq. (3.26) can be analyzed in analogy to the discussion of Eqs. (3.16) and (3.20). Again the coefficients $B_1^{(3)}$ and $B_2^{(3)}$ are not associated with zero-mode contributions and are given by their values at the spectator quark pole:

$$B_1^{(3)}=\frac{x}{2}(B_1^{(2)}-A_1^{(2)}), \tag{3.28}$$

$$B_2^{(3)}=\left(\frac{x}{2}-\frac{p'_\perp q_\perp}{q^2}\right)B_1^{(2)}+\frac{x}{2}\frac{(qP)}{q^2}A_1^{(2)}, \tag{3.29}$$

where $B_1^{(2)}$ is given by Eq. (3.23). The coefficients $C_1^{(3)}$ and $C_2^{(3)}$ are cancelled by their associated zero-mode contributions, which are accounted for by the replacements

$$\begin{aligned}&A_1^{(2)}\hat{N}_2\to A_1^{(2)}Z_2,\\&A_4^{(2)}\hat{N}_2\to A_4^{(2)}Z_2+2\frac{(qP)}{q^2}A_2^{(1)}A_1^{(2)}.\end{aligned} \tag{3.30}$$

The prescriptions (3.30) obviously give $C_1^{(3)}=C_2^{(3)}=0$. Consequently, the tensor $\hat{p}'_{1\mu}\hat{p}'_{1\nu}\hat{N}_2$ is modified in the following way:

$$\hat{p}'_{1\mu}\hat{p}'_{1\nu}\hat{N}_2 \doteq g_{\mu\nu}A_1^{(2)}\hat{N}_2 + P_\mu P_\nu A_2^{(2)}\hat{N}_2 + (P_\mu q_\nu + q_\mu P_\nu)A_3^{(2)}\hat{N}_2 + q_\mu q_\nu A_4^{(2)}\hat{N}_2 + \omega \text{ dependent terms} \rightarrow g_{\mu\nu}A_1^{(2)}Z_2 + q_\mu q_\nu\left\{A_4^{(2)}Z_2 + 2\frac{(qP)}{q^2}A_2^{(1)}A_1^{(2)}\right\}, \tag{3.31}$$

where Eq. (3.22) and the prescriptions (3.30) have been used.

We can now complete the expression of the light-front amplitude $\hat{B}_\mu$, as given by Eq. (2.19), by adding the effect of zero-mode contributions, which can be included by means of the replacements (3.19) and (3.25). However, the tensor decompositions that are used to construct the modified amplitude $\hat{B}_\mu$ still contain the B coefficients $B_1^{(2)}$, Eq. (3.23), $B_1^{(3)}$ and $B_2^{(3)}$, Eqs. (3.28), (3.29), and exact agreement with the covariant amplitude B_μ, Eq. (2.7), is possible only if the residual ω dependence, which is still manifest in terms of these B coefficients, can be proven to be spurious too. It was shown that zero modes do not affect the B coefficients, therefore a different mechanism must be operative to neutralize their effect. The covariant formalism solves this problem quite simply by means of the identities

$$\int_0^1 dx \int d^2p'_\perp \frac{\Omega}{1-x}B_2^{(2)} = \int_0^1 dx \int d^2p'_\perp \frac{\Omega}{1-x}B_1^{(3)} = \int_0^1 dx \int d^2p'_\perp \frac{\Omega}{1-x}B_2^{(3)} = 0, \tag{3.32}$$

where $\Omega \equiv \Omega(\hat{N}'_1, \hat{N}''_1)$ has been defined in Eq. (3.15). These identities can be proven for the model vertex function (3.3), as we show in the Appendix, but they are valid for general vertex functions, which must necessarily be functions of $\hat{N}'_1$ and $\hat{N}''_1$, as we have verified numerically.

Identities of the type given in Eq. (3.32) are, therefore, the necessary condition for the covariance of the light-front formalism. With this last step we have finally provided all the material required to derive unique and consistent light-front formulas for coupling constants and form factors that are independent of ω. We shall work out this program in the next section. In particular, we know now the conditions that restore the covariance of the light-front amplitude $\hat{B}_\mu$, Eq. (2.19), and can derive within the light-front formalism not only the form factor $F_1(q^2)$, but also $F_2(q^2)$, such that the original results, Eq. (2.9), are reproduced exactly.

IV. CALCULATION OF COUPLING CONSTANTS AND FORM FACTORS IN A MANIFESTLY COVARIANT FRAMEWORK

The technique, which was developed in the last section in a manifestly covariant framework, provides a general method for the analysis of the covariant structure of a one-loop amplitude in the light-front formulation. It permits us to identify and separate that part of the structure which depends on the light front, i.e., on the four vector ω. The spurious ω dependence can be eliminated exactly if the light-front vertex function obeys the identities given by Eq. (3.32). This method will be used to determine coupling constants and form factors for pseudoscalar and vector mesons. These calculations will be a straightforward application of the formulas and rules that have been derived in Sec. III.

A. The form factors for transitions between pseudoscalar mesons

The matrix element for transitions between pseudoscalar mesons, Eq. (2.6), must be expressed in terms of the form factors $F_1(q^2)$ and $F_2(q^2)$. The resulting one-loop amplitude in the light-front formalism, denoted as $\hat{B}_\mu$, is given by Eq. (2.19) and must be decomposed according to Eq. (2.22).

The vector form factor $F_1(q^2)$ can be calculated in the following way:

$$F_1(q^2) = \frac{(\omega\hat{B})}{(\omega P)}, \tag{4.1}$$

i.e., it is determined by $\hat{B}^+$. We have shown in Sec. III B that the vector form factor has no spurious ω dependence, it coincides with the result obtained in the standard light-front approach, Eq. (2.12).

The form factor $\hat{F}_2(q^2)$ can be calculated according to

$$\hat{F}_2(q^2) = \frac{1}{q^2}\left[(q\hat{B}) - (qP)\frac{(\omega\hat{B})}{(\omega P)}\right], \tag{4.2}$$

and depends on ω. The physical form factor $F_2(q^2)$, which is independent of ω, is obtained if the amplitude $\hat{B}_\mu$ in Eq. (4.2) is modified by including the effect of the relevant zero-mode contributions, which can be achieved by replacing all terms linear in $\hat{N}_2$ in the numerator of the integrand of $\hat{B}_\mu$ according to Eqs. (3.19) and (3.25). The result is

$$F_2(q^2)=\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp\frac{h'_0h''_0}{(1-x)\hat{N}'_1\hat{N}''_1}\Bigg\{-2x(1-x)M'^2-2p'^2_\perp-2m'_1m_2$$
$$+2(m''_1-m_2)[(1-x)m'_1+xm_2]+4\frac{(qP)}{q^2}\left(p'^2_\perp+2\frac{(p'_\perp q_\perp)^2}{q^2}\right)+4\frac{(p'_\perp q_\perp)^2}{q^2}$$
$$-2\frac{p'_\perp q_\perp}{q^2}[M''^2-(1-x)(q^2+(qP))-(1-2x)M'^2$$
$$+2xM'^2_0-2(m'_1-m_2)(m'_1+m''_1)]\Bigg\}. \tag{4.3}$$

The consistency of the expression for $F_2(q^2)$ can be checked easily, since for $M'=M''$ and $m'_1=m''_1$ and $\beta'=\beta''$ current conservation requires $F_2(q^2)=0$. This condition can be shown to be true, for all vertex functions that will be considered in Sec. V A by means of the substitution $p'_\perp=p_\perp+1/2(1-x)q_\perp$, which transforms the integrand of Eq. (4.3) into an odd function of $p_\perp$ and the momentum integral vanishes.

B. The form factors for transitions between pseudoscalar and vector mesons

The most general form of the matrix element for the transition between a pseudoscalar meson with four momentum P' and a meson with four momentum P'' and spin 1 is represented in terms of the appropriate form factors [16]:

$$\langle P'';1J_3|\bar{q}''\gamma_\mu(1-\gamma_5)q'|P'\rangle$$
$$=ig(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}P^\alpha q^\beta$$
$$-f(q^2)\varepsilon^*_\mu-a_+(q^2)(\varepsilon^*P)P_\mu-a_-(q^2)(\varepsilon^*P)q_\mu, \tag{4.4}$$

where $\varepsilon=\varepsilon(J_3)$ is the polarization vector of the vector meson with $(\varepsilon P'')=0$. The form factors defined in Eq. (4.4) are related to the convention used most frequently by

$$V(q^2)=-(M'+M'')g(q^2),$$
$$A_1(q^2)=-(M'+M'')^{-1}f(q^2),$$
$$A_2(q^2)=(M'+M'')a_+(q^2), \tag{4.5}$$

where M' and M'' are the masses of the initial and final mesons, respectively.

The matrix element (4.4) is given in one-loop approximation in analogy to Eq. (2.19) as a light-front integral

$$\hat{B}_{\mu\nu}\varepsilon^{*\nu}=\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp\frac{\{\hat{\Gamma}'(-\hat{p}\!\!/_2+m_2)\hat{\Gamma}''_\nu(\hat{p}\!\!/''_1+m''_1)\gamma_\mu(1-\gamma_5)(\hat{p}\!\!/'_1+m'_1)\}\hat{\varepsilon}^{*\nu}}{(1-x)\hat{N}'_1\hat{N}''_1}. \tag{4.6}$$

The pseudoscalar vertex operator $\hat{\Gamma}'=\hat{\Gamma}'(\hat{p}'_1,\hat{p}_2)$ has been defined in Eq. (2.17), and for the vector vertex operator $\hat{\Gamma}''_\nu=\hat{\Gamma}''_\nu(\hat{p}''_1,\hat{p}_2)$ for $^3S_1$-state mesons we use the ansatz

$$\hat{\Gamma}''_\nu\hat{\varepsilon}^{*\nu}=-h''_0\left\{\gamma_\nu-\frac{1}{D''}(\hat{p}''_1-\hat{p}_2)_\nu\right\}\hat{\varepsilon}^{*\nu}, \tag{4.7}$$

where the vertex function h''_0 is defined in analogy to Eq. (2.20). In a manifestly covariant formalism both h''_0 and D'' are necessarily functions of $\hat{N}''_1=x(M''-M''_0)$, and they will be specified later. We shall need only the transverse polarization vector

$$\hat{\varepsilon}(\pm)=\varepsilon(\pm)=\left(\frac{2}{P''^+}\varepsilon_\perp P''_\perp,0,\varepsilon_\perp\right),$$

$$\varepsilon_\perp(\pm)=\pm(1,\pm i)/\sqrt{2}. \tag{4.8}$$

The longitudinal polarization vectors $\varepsilon(0)$ and $\hat{\varepsilon}(0)$ have been given in Ref. [16], and we shall use only the property that they have a nonvanishing plus component.

With Eq. (4.7) the light-front integral (4.6) can be rewritten as

$$\hat{B}_{\mu\nu}\varepsilon^{*\nu}=-\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp\frac{h'_0h''_0}{(1-x)\hat{N}'_1\hat{N}''_1}\hat{S}_{\mu\nu}\hat{\varepsilon}^{*\nu}, \tag{4.9}$$

where

$$\hat{S}_{\mu\nu}=\mathrm{tr}\left\{\left(\gamma_\nu-\frac{1}{D''}(\hat{p}_1''-\hat{p}_2)_\nu\right)(\hat{\not p}_1''+m_1'')\gamma_\mu(1-\gamma_5)(\hat{\not p}_1'+m_1')\gamma_5(-\hat{\not p}_2+m_2)\right\}$$

$$=2i\epsilon_{\mu\nu\alpha\beta}\{\hat{p}_1'^{\alpha}P^{\beta}(m_1''-m_1')+\hat{p}_1'^{\alpha}q^{\beta}(m_1''+m_1'-2m_2)+q^{\alpha}P^{\beta}m_1'\}-\frac{1}{D''}(4\hat{p}_{1\nu}'-3q_\nu-P_\nu)i\epsilon_{\mu\alpha\beta\rho}\hat{p}_1'^{\alpha}q^{\beta}P^{\rho}$$

$$+2g_{\mu\nu}\{m_2(q^2-\hat{N}_1'-\hat{N}_1''-m_1'^2-m_1''^2)-m_1'(M''^2-\hat{N}_1''-\hat{N}_2-m_1''^2-m_2^2)$$

$$-m_1''(M'^2-\hat{N}_1'-\hat{N}_2-m_1'^2-m_2^2)-2m_1'm_1''m_2\}$$

$$+8\hat{p}_{1\mu}'\hat{p}_{1\nu}'(m_2-m_1')-2(P_\mu q_\nu+q_\mu P_\nu+2q_\mu q_\nu)m_1'+2\hat{p}_{1\mu}'P_\nu(m_1'-m_1'')$$

$$+2\hat{p}_{1\mu}'q_\nu(3m_1'-m_1''-2m_2)+2P_\mu\hat{p}_{1\nu}'(m_1'+m_1'')+2q_\mu\hat{p}_{1\nu}'(3m_1'+m_1''-2m_2)$$

$$+\frac{1}{2D''}(4\hat{p}_{1\nu}'-3q_\nu-P_\nu)\{2\hat{p}_{1\mu}'[M'^2+M''^2-q^2-2\hat{N}_2+2(m_1'-m_2)(m_1''+m_2)]$$

$$+q_\mu[q^2-2M'^2+\hat{N}_1'-\hat{N}_1''+2\hat{N}_2-(m_1'+m_1'')^2+2(m_1'-m_2)^2]+P_\mu[q^2-\hat{N}_1'-\hat{N}_1''-(m_1'+m_1'')^2]\}. \tag{4.10}$$

The decomposition of the amplitude (4.9) into four vectors is more complicated than the representation given in Eq. (4.4), since $\hat{B}_{\mu\nu}$ depends not only on the four vectors P and q, but also on ω:

$$\hat{B}_{\mu\nu}\varepsilon^{*\nu}=i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}P^{\alpha}q^{\beta}g(q^2)+i\epsilon_{\mu\rho\alpha\beta}\omega^{\rho}P^{\alpha}q^{\beta}((\varepsilon^*\omega)R_1(q^2)+(\varepsilon^*P)R_2(q^2))$$

$$-\varepsilon_\mu^*\hat{f}(q^2)-(\varepsilon^*P)P_\mu a_+(q^2)-(\varepsilon^*P)q_\mu\hat{a}_--(\varepsilon^*\omega)P_\mu R_3(q^2)$$

$$-(\varepsilon^*\omega)q_\mu R_4(q^2)-(\varepsilon^*P)\omega_\mu R_5(q^2)-(\varepsilon^*\omega)\omega_\mu R_6(q^2). \tag{4.11}$$

The form factors $R_i(q^2)$ are all spurious. In the standard light-front approach as, e.g., in Ref. [16], the form factors $g(q^2)$ and $a_+(q^2)$ are determined by the transverse decay mode according to the equation

$$\hat{B}_{\mu\nu}\omega^{\mu}\varepsilon^{*\nu}(\pm)=i\epsilon_{\mu\nu\alpha\beta}\omega^{\mu}\varepsilon^{*\nu}P^{\alpha}q^{\beta}g(q^2)-(\varepsilon^*P)(\omega P)a_+(q^2), \tag{4.12}$$

where we have used that $(\omega\varepsilon(\pm))=(\omega q)=\omega^2=0$. Equation (4.12) is free of spurious terms, and the expressions for $g(q^2)$ and $a_+(q^2)$ are identical with those given in Ref. [16], which we include here for the sake of completeness:

$$g(q^2)=-\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p_\perp'\frac{2h_0'h_0''}{(1-x)\hat{N}_1'\hat{N}_1''}\left\{(1-x)m_1'+xm_2+(m_1'-m_1'')\frac{p_\perp'q_\perp}{q^2}+\frac{2}{D''}\left[p_\perp'^2+\frac{(p_\perp'q_\perp)^2}{q^2}\right]\right\}, \tag{4.13}$$

$$a_+(q^2)=\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p_\perp'\frac{2h_0'h_0''}{(1-x)\hat{N}_1'\hat{N}_1''}\left\{(2x-1)[(1-x)m_1'+xm_2]-[2xm_2+m_1''+(1-2x)m_1']\frac{p_\perp'q_\perp}{q^2}\right.$$

$$\left.-2\frac{(1-x)q^2+p_\perp'q_\perp}{(1-x)q^2D''}(p_\perp'p_\perp''+[xm_2+(1-x)m_1'][xm_2-(1-x)m_1''])\right\}. \tag{4.14}$$

In our former work [16] we have attempted to derive the form factor $f(q^2)$ from the plus component of Eq. (4.11) for the longitudinal decay mode according to the equation

$$\hat{B}_{\mu\nu}\omega^{\mu}\varepsilon^{*\nu}(0)=i\epsilon_{\mu\nu\alpha\beta}\omega^{\mu}\varepsilon^{*\nu}P^{\alpha}q^{\beta}g(q^2)-(\varepsilon^*\omega)\hat{f}(q^2)-(\varepsilon^*P)(\omega P)a_+(q^2)-(\varepsilon^*\omega)(\omega P)R_3(q^2), \tag{4.15}$$

where we have used that $(\omega\varepsilon(0))=\varepsilon^+(0)\neq 0$. Since the form factors $g(q^2)$ and $a_+(q^2)$ are known, the form factor determined by means of Eq. (4.15) obviously is the combination $\hat{f}+(\omega P)R_3$ instead of f. Consequently, the formula for $A_1(q^2)$ used, e.g., in Refs. [2–6] contains spurious contributions. The physical form factor $f(q^2)$, which is independent of ω, is obtained if the light-front integral $\hat{B}_{\mu\nu}$ is modified by the inclusion of the effect of zero modes. This can be achieved if the trace (4.10) is rewritten by use of the tensor decompositions (3.16), (3.20), and (3.26), and by replacing all terms linear in $\hat{N}_2$ according to Eqs. (3.19), (3.25), and (3.31). Collecting all terms proportional to $g_{\mu\nu}$ gives the result

$$f(q^2)=\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp \frac{h'_0h''_0}{(1-x)\hat{N}'_1\hat{N}''_1}\Bigg\{2x(m_2-m'_1)(M'^2_0+M''^2_0)-4xm''_1M'^2_0$$
$$+2(1-x)m'_1(qP)+2m_2q^2-2xm_2(M'^2+M''^2)+2(m'_1-m_2)(m'_1+m''_1)^2$$
$$+8(m'_1-m_2)\left[p'^2_\perp+\frac{(p'_\perp q_\perp)^2}{q^2}\right]+2(m'_1+m''_1)(q^2+(qP))\frac{p'_\perp q_\perp}{q^2}$$
$$-4\frac{q^2p'^2_\perp+(p'_\perp q_\perp)^2}{q^2D''}\left[2xM'^2+2xM'^2_0-q^2-(qP)-2(q^2+(qP))\frac{p'_\perp q_\perp}{q^2}-2(m'_1-m''_1)(m'_1-m_2)\right]\Bigg\}. \quad (4.16)$$

The physical form factor $a_-(q^2)$ is obtained in the same manner. Collecting all terms proportional to $q_\mu P_\nu$ and $q_\mu q_\nu$ and combining them in the appropriate way gives the form factor $a_-(q^2)$ as

$$a_-(q^2)=\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp \frac{h'_0h''_0}{(1-x)\hat{N}'_1\hat{N}''_1}\Bigg\{2(2x-3)[(1-x)m'_1+xm_2]$$
$$-8(m'_1-m_2)\left[\frac{p'^2_\perp}{q^2}+2\frac{(p'_\perp q_\perp)^2}{q^4}\right]-[(14-12x)m'_1-2m''_1-(8-12x)m_2]\frac{p'_\perp q_\perp}{q^2}$$
$$+\frac{4}{D''}\Bigg([M'^2+M''^2-q^2+2(m'_1-m_2)(m''_1+m_2)](A_3^{(2)}+A_4^{(2)}-A_2^{(1)})+Z_2(3A_2^{(1)}-2A_4^{(2)}-1)$$
$$+\frac{1}{2}[x(q^2+(qP))-2M'^2-2p'_\perp q_\perp-2m'_1(m''_1+m_2)-2m_2(m'_1-m_2)](A_1^{(1)}+A_2^{(1)}-1)$$
$$+(qP)\left[\frac{p'^2_\perp}{q^2}+\frac{(p'_\perp q_\perp)^2}{q^4}\right](4A_2^{(1)}-3)\Bigg)\Bigg\}. \quad (4.17)$$

C. The vector decay constant

The vector decay constant f_V is defined by the matrix element of the vector current

$$\langle 0|\overline{q}''\gamma_\mu q'|P;1J_3\rangle=\varepsilon_\mu(J_3)\sqrt{2}f_V. \quad (4.18)$$

The matrix element can be represented in one-loop order by a light-front momentum integral, which we shall denote as $\hat{A}_{\mu\nu}\varepsilon^\nu$:

$$\hat{A}_{\mu\nu}\varepsilon^\nu=\frac{N_c}{16\pi^3}\int_0^1 dx\int d^2p'_\perp \frac{h'_0}{(1-x)\hat{N}'_1}\hat{s}_{\mu\nu}\hat{\varepsilon}^\nu, \quad (4.19)$$

where

$$\hat{s}_{\mu\nu}=tr\left\{\gamma_\mu(\hat{\not{p}}'_1+m'_1)\left[\gamma_\nu-\frac{1}{D'}(\hat{p}'_1-\hat{p}_2)_\nu\right](-\hat{\not{p}}_2+m_2)\right\}, \quad (4.20)$$

and we have used the vertex operator for ${}^3S_1$-state mesons, Eq. (4.7). The decomposition of the integral $\hat{A}_{\mu\nu}\varepsilon^\nu$, Eq. (4.18) into four vectors depends on the light front in the following way:

$$\hat{A}_{\mu\nu}\varepsilon^\nu=\sqrt{2}\{\varepsilon_\mu\hat{f}_V+(\varepsilon\omega)P'_\mu r_1+(\varepsilon\omega)\omega_\mu r_2\}, \quad (4.21)$$

where the terms proportional to r_1 and r_2 are spurious. The standard procedure (e.g., in Ref. [16]) uses the plus component of Eq. (4.21) for the longitutinal decay mode to evaluate the vector decay constant, and picks up the combination $\hat{f}_V+P'^+r_1$, which obviously contains spurious contributions, instead of f_V. The physical coupling constant f_V is obtained, if the ω dependence of the light-front integral $\hat{A}_{\mu\nu}\varepsilon^\nu$ is removed by including the appropriate zero-mode contributions. The necessary prescriptions are analogous to those derived in Sec. III B, except that there is no q dependence. In the present case one requires only the replacement

$$\hat{N}_2\rightarrow\hat{N}'_1+m'^2_1-m_2^2+(1-2x)M'^2.$$

The result for f_V is

$$f_V=\frac{N_c}{8\pi^3}\int_0^1 dx\int d^2p'_\perp \frac{\sqrt{2}h'_0}{(1-x)\hat{N}'_1}\Bigg\{xM'^2_0-m'_1(m'_1-m_2)$$
$$-p'^2_\perp+\frac{m'_1+m_2}{D'}p'^2_\perp\Bigg\}. \quad (4.22)$$

D. The q^2 dependence of the hadronic form factors

Form factors have been calculated in this section in the light-front approach under the condition $q^+=0$, i.e., for spacelike momentum transfer $q^2=-q_\perp^2\leqslant 0$, whereas physical decays occur in the timelike region $q^2\geqslant 0$. However, we have proposed in Ref. [2] to rewrite the form factor as an explicit function of q^2 and analytically continue from timelike to spacelike momentum transfer. For the multipole form (3.1) of the vertex function this method is seen to reproduce exactly the form factor that can be derived, in this case, also by the standard space-time methods for all values q^2. We shall use this procedure in practical applications in order to determine hadronic form factors for physical values of q^2.

V. APPLICATIONS

A. Choice of light-front vertex function

It has been shown in Sec. III B that if the light-front vertex functions h'_0 and h''_0 are functions of $\hat{N}'_1$ and $\hat{N}''_1$, respectively, the identities (3.32) guarantee that the formulas for coupling constants and form factors given in Sec. IV are entirely free of spurious ω contributions. As an ansatz for the vertex function one could choose, for instance, a multipole form [as in Eq. (3.1), but with N_Λ replaced by $\hat{N}_\Lambda$] or an exponential form

$$h'_0=N'\exp\left(\frac{\hat{N}'_1}{\alpha'}\right), \tag{5.1}$$

where α' and N' are appropriate parameters.

Using the formulas of Sec. IV we have tentatively calculated various properties of light mesons on the basis of the exponential form (5.1) and setting $D''=M_V+m'_1+m_2$ in Eq. (4.7) for the vertex operator of a vector meson of mass M_V. The results are rather unsatisfactory and cannot compete with those given, e.g., in Refs. [6,7]. For instance, it is not possible to obtain an acceptable approximation for the electromagnetic form factor of the pion for low values of q^2. Even if a more general pseudoscalar vertex operator than the one given in Eq. (2.17) is used, the results are not essentially improved.

It seems that the main problem in the manifestly covariant calculation of meson properties is the inevitable occurrence of light-front vertex functions, like h'_0 of Eq. (5.1), which are functions of $\hat{N}'_1=x(M'^2-M_0'^2)$, and that are not symmetric in the variables of the two quarks. This property is a consequence of the asymmetric treatment of the constituent quarks of the $q\bar{q}$ bound state by means of a vertex function like Eq. (3.1).

The picture becomes quite different and surprisingly accurate if the standard light-front vertex functions, which are symmetric in the variables of the constituent $q\bar{q}$ pair, are used instead. Different choices are possible [6], but for the calculation reported in Sec. V B we have preferred to use the vertex operators which we have derived for ${}^1S_0$- and ${}^3S_1$-state mesons in Ref. [7], and which are given by Eqs. (2.17) and (4.7) with

$$h'_0=\left[\frac{M_0'^4-(m_1'^2-m_2^2)^2}{4M_0'^3}\right]^{1/2}\frac{M'^2-M_0'^2}{[M_0'^2-(m'_1-m_2)^2]^{1/2}}\,\phi(M_0'^2), \tag{5.2}$$

$$D''=M''_0+m''_1+m_2, \tag{5.3}$$

and a similar equation for h''_0. The orbital wave function is assumed to be a simple function of the kinematic invariant mass as

$$\phi(M_0'^2)=N'\exp(-M_0'^2/8\beta'^2), \tag{5.4}$$

where N' is the normalization constant and the parameter $1/\beta'$ determines the confinement scale.

The formulas for coupling constants and form factors have been derived in a manifestly covariant framework. However, if these formulas are evaluated with the symmetric light-front vertex function (5.2), the covariance conditions (3.32) are violated, i.e., the integrals of Eq. (3.32) are nonzero. Consequently, some residual ω dependence is introduced into these expressions if Eqs. (5.2) and (5.3) are used for the vertex function. This remaining ω dependence is minimal in the sense that only the B coefficients $B_n^{(m)}$ in the tensor decompositions (3.20) and (3.26) are included in the form factors. The practical application of this approach will be discussed in Sec. V B.

For the sake of completeness, we mention that we have tried also to eliminate all ω dependence by simply omitting all B coefficients that are implicitly contained in the light-front formulas for coupling constants and form factors. Due to the identities (3.32) the formulas without B coefficients are exactly equivalent to the original formulas, given in Sec. IV, in a manifestly covariant framework, but give different results if the standard light-front vertex function, Eqs. (5.2) and (5.3), is used. With this approach the data can be fitted only partially, in particular we found that the values for the coupling constants $g_{\rho\pi\pi}$ and $g_{K^*K\pi}$ are about 20% lower than the experimental data.

B. Pionic and kaonic processes

In Refs. [6,7] the standard light-front quark model has been investigated and the predicted electroweak properties of pseudoscalar and vector mesons in the u-, d-, s-quark sector were found to be in good agreement with the experimental data. The covariant analysis of the light-front formalism presented in this work extends the standard approach, and permits the calculation of all form factors, that represent the Lorentz structure of a hadronic matrix element, on the same footing. We shall update some of the results of our past work [7], however, we are mainly interested in the quality of the additional predictions that are possible on the basis of the formulas, collected in Sec. IV, and as emphasized in Sec. V A, we shall use for these calculations the standard light-front vertex function, given by Eqs. (5.2) and (5.3).

The values of the free parameters of the light-front quark model, that have been chosen in Ref. [7], have to be modified, since the covariant approach leads to a different formula for the vector decay constant f_V, Eq. (4.22). Therefore, we fix again the parameters $m=m_u=m_d$, β_π and β_ρ by fitting the pion decay constant $f_\pi=92.4\pm 0.2$ MeV [22], the ρ decay constant $f_\rho/M_\rho=150$ MeV [23], and the charge radius of the pion [24], which will be given below.

Similarly, the parameters m_s, β_K, and β_{K*} are fixed by fitting the decay constant $f_K=113.4\pm 1.1$ MeV [22], the K^* decay constant $f_{K*}/M_{K*}=152$ MeV [25], and the $K\pi$-transition radius [26]. We shall use the values for masses and wave function parameters listed in Table I.

A comparison of the values for quark masses with the parametrization of Ref. [7] shows that only the value for $m=m_u=m_d$ is slightly changed, and it might be interesting to note that the corresponding modification of the electromagnetic form factor of the pion $F_\pi(q^2)$ leads to an almost perfect agreement between the prediction and the data for small values of q^2. The experimental data have been reanalyzed in Ref. [24], based on the parametrization

$$F_\pi(q^2)=\frac{1}{1-q^2/\Lambda_1^2+sq^4/\Lambda_2^4}. \tag{5.5}$$

In the following we compare the results of the fit to the data with the values for the parameters Λ_1 and Λ_2, which we have derived from $F_1(q^2)$, Eq. (2.12), by taking the first and second derivative at zero-momentum transfer:

$$\Lambda_1=\begin{cases}719.3 \ \text{MeV} & (\text{this work}),\\ 720\pm 4 \ \text{MeV} & (\text{Ref. [24]}),\end{cases}$$

$$s\Lambda_2=\begin{cases}-1473 \ \text{MeV} & (\text{this work}),\\ -(1420^{+690}_{-190}) \ \text{MeV} & (\text{Ref. [24]}).\end{cases}$$

The rates for the radiative transitions $V\to P\gamma$, which we have calculated in Ref. [7], are modified also due to the changed values for the vector meson parameters β_V. The rate is given by

$$\Gamma=\frac{1}{3}\alpha g_{VP\gamma}^2\left[\frac{M_V^2-M_P^2}{2M_V}\right]^3, \tag{5.6}$$

where the coupling constant $g_{VP\gamma}=g(0)$ can be calculated with Eq. (4.13). We have summarized the results for a selected set of radiative transitions in Table II. We note, in particular, that the predicted rate for the transition $\rho^+\to\pi^+\gamma$ is in agreement with the experimental result of Ref. [27], but disagrees with the average value of the Particle Data Group [22].

In Ref. [2] we have investigated also the pionic decays $\rho^+\to\pi^+\pi^0$ and $K^{*+}\to(K\pi)^+$, and have calculated the coupling constant $g_{VP\pi^0}$ by means of a soft pion theorem due to Das, Mathur, and Okubo [28]

$$4|g_{VP\pi^0}|=|f(0)-(M_V^2-M_P^2)a_+(0)|, \tag{5.7}$$

where $a_+(0)$ and $f(0)$ are given by Eqs. (4.14) and (4.16). [Note, that if this relation refers to $g_{\rho^+\pi^+\pi^0}$ the factor 4 in Eq. (5.5) must be replaced by the factor 2.] The comparison of the predicted coupling constants with the experimental data gives an impression of the quality of the new formula for $f(q^2)$. The numerical results are

$$g_{\rho^+\pi^+\pi^0}=6.02 \quad (6.06\pm 0.01),$$

$$g_{K^*K\pi}\equiv\sqrt{3}g_{K^{*+}K^+\pi^0}=5.54 \quad (5.57\pm 0.03).$$

The experimental coupling constants are given in parentheses. The values for coupling constants, which we have calculated in Ref. [2] in the standard light-front approach are about 10% smaller, and the differences are even larger if the alternative methods, mentioned in Sec. V A, are used.

Next, we shall consider the case of K_{l3} decays, $K^+\to\pi^0 l^+\nu$ and $K^0\to\pi^+l^-\nu$, whose hadronic structure is, according to Eq. (2.6), described by the form factors $F_1(q^2)$ and $F_2(q^2)$, which are given in the light-front quark model by Eqs. (2.12) and (4.3). Very often K_{l3} decays are analyzed in terms of the form factors $F_1(q^2)$ and $F_0(q^2)$. The scalar form factor $F_0(q^2)$ is defined by

$$F_0(q^2)=F_1(q^2)+\frac{q^2}{M_{K^0}^2-M_{\pi^+}^2}F_2(q^2). \tag{5.8}$$

The first derivatives of $F_1(q^2)$ and $F_0(q^2)$ determine the vector radius $r_{1,K\pi}$ and the scalar radius $r_{0,K\pi}$, respectively, which are usually accounted for by the slope parameters

$$\lambda_+=\frac{1}{6}r_{1,K\pi}^2M_{\pi^+}^2 \quad \text{and} \quad \lambda_0=\frac{1}{6}r_{0,K\pi}^2M_{\pi^+}^2. \tag{5.9}$$

The ratio of the form factors at $q^2=0$ is denoted by ξ

$$\xi=F_2(0)/F_1(0), \tag{5.10}$$

and the quantities λ_0, λ_+, and ξ are, according to Eqs. (5.8) and (5.10), related by

$$\lambda_0=\lambda_++\frac{M_{\pi^+}^2}{M_{K^0}^2-M_{\pi^+}^2}\xi. \tag{5.11}$$

TABLE I. Quark masses m_Q and wave function parameters β_P and β_V for $(q,\overline{Q})$ pseudoscalar and vector mesons. The light quark mass is $m_q=m_{u,d}=0.26$ GeV.

$(q\overline{Q})$ meson	m_Q (GeV)	β_P (GeV)	β_V (GeV)
π,ρ	0.26	0.3088	0.2600
K,K^*	0.37	0.3884	0.2770

TABLE II. Rates and decay constants for the radiative decays $V\to P\gamma$.

$V\to P\gamma$	$g_{VP\gamma}$ (GeV^{-1})	Γ_{theor} (keV)	Γ_{expt} (keV)
$\rho^+\to\pi^+\gamma$	0.810	81.3	68±7 [22]
			81±4±4 [27]
$K^{*+}\to K^+\gamma$	0.867	54.4	50±5 [22]
$K^{*0}\to K^0\gamma$	−1.314	124.3	117±10 [22]

TABLE III. The parameters for K_{l3} decays.

	This work	ChPT [29]	Experiment
$F_1(0)$	0.9620	0.961 ± 0.008 [31]	
λ_+	0.0269	0.031 ± 0.003	$0.0279\pm0.0027(K^0_{e3})$ [26]
			$0.0286\pm0.0022(K^+_{e3})$ [22]
λ_0	0.0161	0.017 ± 0.004	
ξ	-0.125	-0.16 ± 0.06	$-0.11\pm0.09(K^0_{\mu3})$ [22]
			$-0.33\pm0.14(K^+_{\mu3})$ [22]

In Table III we compare the results of our calculation with the data, wherever possible, and with the corresponding results obtained by the use of chiral perturbation theory (ChPT), where we rely on the detailed discussions in Ref. [29]. We note in particular, that the recent calculation of higher-order corrections [30] indicates an even larger range of uncertainty of the ChPT predictions for λ_+ and λ_0, than the one quoted in Table III.

Finally, we consider the Callan-Treiman relation [32], which is also a soft-pion result, that relates the form factors of K_{l3} and K_{l2} decays, namely

$$[F_1(M_K^2)+F_2(M_K^2)]_{M_\pi^2=0}=\frac{f_K}{f_\pi}=1.227\pm0.012. \tag{5.12}$$

The form factors $F_1(q^2)$ and $F_2(q^2)$ are given by Eqs. (2.12) and (4.3), and can be analytically continued from timelike to spacelike momentum transfer using the method proposed in Ref. [2]. The relation (5.12) involves the form factors continued to zero pion mass. The form factor $F_1(q^2)$ does not explicitly depend on the pion mass, and we find $F_1(M_K^2)=1.4963$. The form factor $F_2(q^2)$ has an explicit dependence on M_π, and we find

$$F_2(M_K^2)=-0.2441 \xrightarrow{M_\pi^2=0} -0.2753.$$

While the dependence of the constituent mass m and the parameter β_π on M_π is unknown, it seems reasonable to assume that these quantities are gently varying functions of M_π^2. Consequently, the light-front quark model predicts

$$[F_1(M_K^2)+F_2(M_K^2)]_{M_\pi^2=0}=1.221$$

in agreement with Eq. (5.12).

C. Covariance in the heavy quark limit

In Ref. [33] a covariant light-front model for heavy mesons was constructed within the framework of the heavy quark effective theory. An important feature of this covariant model is the requirement, that the vertex function for a heavy meson must be a function of (vp_q), where v_μ is the four velocity of the heavy meson with mass M_H, i.e., the four momentum of the meson is $P'=M_Hv$, and p_q is the four momentum of the light quark with mass m_q, which is on its mass-shell, i.e., $p_q^2=m_q^2$. The mass of the heavy quark is m_Q.

It is easy to prove that this condition for covariance, found in [33], is equivalent to the requirement, which we established in Sec. III B that the light-front vertex function of a manifestly covariant formalism must necessarily be a function of $\hat{N}_1'$. Since we can rewrite $\hat{N}_1'$ in the following way:

$$\hat{N}_1'=x(M_H^2-M_0^2)=xM_H^2-\frac{p_\perp'^2+m_q^2}{1-x}-m_Q^2+m_q^2$$
$$=-2M_H(vp_q)+M_H^2-m_Q^2+m_q^2, \tag{5.13}$$

where we have used the relation

$$(vp_q)=\frac{p_\perp'^2+m_q^2+(1-x)^2M_H^2}{2(1-x)M_H},$$

$\hat{N}_1'$ is indeed a linear function of (vp_q).

If heavy meson properties and transitions between heavy mesons are calculated in the framework of the covariant extension of the light-front formalism in combination with the standard light-front vertex function, Eqs. (5.2) and (5.3), the exponential form (5.4) of the vertex function guarantees that in the limit where the heavy quark mass goes to infinity, the light-front integrals receive contributions only for values of $1-x$ in the neighborhood of $(1-x)\simeq m_q/m_Q$, where it is justified to set $M_H^2-M_0'^2=\hat{N}_1'/x$ equal to $\hat{N}_1'$. Then, the reasoning that led to Eq. (5.2) can be used also to prove that the light-front vertex function h_0', Eq. (5.2), becomes a function of (vp_q). We can at once conclude that, while the extended light-front quark model lacks manifest covariance if used to describe light mesons as we did in Sec. V B, manifest covariance is recovered in the heavy quark limit.

Therefore, the covariant analysis presented in this work provides also an ideal framework to derive unique expressions for form factors in the heavy quark limit.

VI. CONCLUDING REMARKS

The RQM, based originally on the light-front formalism, has been extended in Ref. [2] to the treatment of decay processes with a timelike momentum transfer. In the present work we have further extended the range of applicability of the RQM and used a manifestly covariant formalism as a guide to derive formulas for all form factors that are required to represent the Lorentz decomposition of the hadronic matrix elements of the electroweak current in one-loop order. However, the practical application of this covariant extension of the light-front quark model is successful only if the formulas for form factors are evaluated with standard light-front vertex functions, which are symmetric in the variables of the constituent $q\bar{q}$ pair. The latter violate the conditions for the strict Lorentz covariance of the formalism. We have indicated that manifest covariance is recovered in the heavy quark limit, since the vertex functions become asymmetric in the variables of the heavy-light $q\bar{q}$ pair in accordance with the conditions for covariance.

In order to explore the predictive power of this approach, we have calculated various properties of pseudoscalar and vector mesons in the u-, d-, s-quark sector. The good agreement with the data for electroweak transitions, which has been found on the basis of the standard light-front methods in Ref. [7], is not only confirmed, but even improved, since some form factors for transitions which involve vector mesons are modified in the covariant analysis.

We have derived also those form factors which cannot be evaluated in the standard approach. We have, for example, calculated the scalar form factor for K_{l3} decays and found that the prediction is consistent with the available experimental data. Likewise, the form factor $a_-(q^2)$, which appears in the Lorentz decomposition of the matrix element for transitions between pseudoscalar and vector mesons, has been calculated. Both form factors are relevant, e.g., for the analysis of the properties of semileptonic decays of B mesons in the channel $B\to M\tau\nu_\tau$, where $M=\pi,\rho,D,D^*$, which will be measured at the B factories which are currently under construction. The investigation of this decay channel has been started in Ref. [34], where the covariant extension of the light-front quark model has been applied to the decay $B\to\pi\tau\nu_\tau$, and excellent agreement with the results of lattice calculations and light-cone sum rules has been found. We plan to investigate further hadronic properties in this manner.

Note added. After the completion of this work I became aware of several publications that are related to my paper. A simple connection between the covariant Feynman formalism and time-ordered perturbation theory in the infinite momentum frame has been investigated by Schmidt [35]. A discussion of the problems with vertex functions to be used in a covariant analysis is given too. Brodsky and Hwang [36] find that zero-mode contributions are not only crucial to obtain the correct results for electroweak form factors, but to provide a new perspective on the physics of semileptonic decays. Their result for the charge form factor of a neutral composite system composed of two charged scalars as derived from the minus component of the one-loop amplitude is equivalent to Eq. (3.10). The zero-mode contribution appears in [36] as the contribution of the annihilation of the $q\bar{q}$ pair of the initial Fock state to the electromagnetic current in the limit $q^+\to 0$; in this work this contribution is effectively accounted for by means of the prescription (3.19). The charge form factor can be calculated also from the plus component of the amplitude, and the prescription (3.19) guarantees that the two determinations of the charge form factor are equal. Zero modes are discussed also by Choi and Ji [37]. In Ref. [38] the form factors for K_{l3} decays are investigated. The method used to calculate $F_1(q^2)$ is the same as in my work, but the approach proposed to determine the second form factor gives $\hat{F}_2(q^2)$, as defined in Eq. (2.22) of the present paper, and not the physical form factor $F_2(q^2)$. A related light-cone formalism for the calculation of spin-1 form factors has been given by Brodsky and Hiller [39].

ACKNOWLEDGMENT

I would like to thank D. Wyler for helpful discussions.

APPENDIX: THE PRESCRIPTION (3.19) AND THE IDENTITIES (3.32) IN THE MODEL OF SEC. III A

In the framework of the model vertex function of Eq. (3.1) the prescription (3.19) states that the integrated function Z_2, denoted by $I[Z_2]$, is equal to the zero-mode contribution, given by Eq. (3.13). The integral $I[Z_2]$ is defined by

$$I[Z_2]=\frac{1}{16\pi^3}\int_0^1 dx\int d^2p'_\perp\frac{Z_2}{(1-x)\hat{N}'_1\hat{N}''_1\hat{N}'_\Lambda\hat{N}''_\Lambda},\quad \text{(A1)}$$

where Z_2 is defined by Eq. (3.18), $\hat{N}'_\Lambda=\hat{N}'_1+m_1'^2-\Lambda'^2$ and $\hat{N}''_\Lambda=\hat{N}''_1+m_1''^2-\Lambda''^2$.

The following identity is useful:

$$\frac{1}{\hat{N}'_1\hat{N}''_1\hat{N}'_\Lambda\hat{N}''_\Lambda}=\frac{1}{(\Lambda'^2-m_1'^2)(\Lambda''^2-m_1''^2)}\times\left(\frac{1}{\hat{N}'_1}-\frac{1}{\hat{N}'_\Lambda}\right)\left(\frac{1}{\hat{N}''_1}-\frac{1}{\hat{N}''_\Lambda}\right).\quad \text{(A2)}$$

We evaluate the $p'_\perp$ integral by the standard Feynman parameter method. Combining denominators, changing the momentum variable to $p_\perp=p'_\perp-y(1-x)q_\perp$, and eliminating terms linear in $p_\perp$ by symmetric integration gives

$$\begin{aligned}I[Z_2]=&\frac{1}{16\pi^2(\Lambda'^2-m_1'^2)(\Lambda''^2-m_1''^2)}\int_0^1 dx\int_0^1 dy(1-x)\\&\times\Bigg\{(\Lambda'^2-m_1'^2)\left(\frac{1}{C_{\Lambda\Lambda}}-\frac{1}{C_{\Lambda 1}}\right)\\&+[m_1'^2-m_2^2+(1-2x)M'^2-y(1-x)(q^2+(qP))]\\&\times\left(\frac{1}{C_{11}}+\frac{1}{C_{\Lambda\Lambda}}-\frac{1}{C_{1\Lambda}}-\frac{1}{C_{\Lambda 1}}\right)\Bigg\},\end{aligned}\quad \text{(A3)}$$

where

$$\begin{aligned}C_{11}&=C(m_1',m_1'')\\&=(1-x)(1-y)m_1'^2+(1-x)ym_1''^2+xm_2^2\\&\quad -x(1-x)[(1-y)M'^2+yM''^2]-(1-x)^2y(1-y)q^2,\end{aligned}$$

$$C_{\Lambda\Lambda}=C(\Lambda',\Lambda''),\quad C_{1\Lambda}=C(m_1',\Lambda''),\quad C_{\Lambda 1}=C(\Lambda',m_1'').\quad \text{(A4)}$$

For the final step it is helpful to change variables from *x,y* to u,v, where the two sets are related by

$$x=uv,\quad y=\frac{1-u}{1-uv},\quad (1-x)dxdy=ududv.\quad \text{(A5)}$$

The function C_{11}, expressed in the new variables, is

$$\begin{aligned}C_{11}=&u(1-v)m_1'^2+(1-u)m_1''^2+uvm_2^2-uv[u(1-v)M'^2\\&+(1-u)M''^2]-u(1-u)(1-v)q^2,\end{aligned}\quad \text{(A6)}$$

and the integrand of Eq. (A3) can be cast into a simple form, if it is used that

$$\begin{aligned}
u[m_1'^2-m_2^2+(1-2x)M'^2-y(1-x)(q^2+(qP))] \\
=u[m_1'^2-m_2^2+(1-2uv)M'^2-(1-u)(q^2+(qP))] \\
=-\frac{dC_{11}}{dv}\equiv-C_{11}'=-C_{1\Lambda}' \\
=-C_{\Lambda\Lambda}'-u(\Lambda'^2-m_1'^2) \\
=-C_{\Lambda 1}'-u(\Lambda'^2-m_1'^2).
\end{aligned} \tag{A7}$$

The integration of Eq. (A3) with respect to v can now be performed with the result

$$I[Z_2]=\frac{1}{16\pi^2(\Lambda'^2-m_1'^2)(\Lambda''^2-m_1''^2)}\int_0^1 du\ \ln\frac{C_{11}^0 C_{\Lambda\Lambda}^0}{C_{1\Lambda}^0 C_{\Lambda 1}^0}, \tag{A8}$$

where $C_{11}^0=C_{11|v=0}$, etc. This expression for $I[Z_2]$ is the same integral as given by Eq. (3.13). The prescriptions (3.25) and (3.31) can be proven in the same manner.

This procedure can be used also to prove the identities of Eq. (3.32). It will be sufficient to consider the first identity, which means that the integrated function $B_1^{(2)}$ [$B_1^{(2)}$ is defined by Eq. (3.23)], which we shall denote in analogy to Eq. (A1) by $I[B_1^{(2)}]$, must be equal to zero. We split the integral as follows:

$$I[B_1^{(2)}]=\frac{1}{2}I[xZ_2]-I[A_1^{(2)}], \tag{A9}$$

where $A_1^{(2)}$ has been defined in Eq. (3.21). The integral $I[A_1^{(2)}]$ can be calculated by the Feynman parameter method, as indicated above, with the result

$$I[A_1^{(2)}]=\frac{1}{32\pi^2(\Lambda'^2-m_1'^2)(\Lambda''^2-m_1''^2)} \times\int_0^1 dx\int_0^1 dy(1-x)\ln\frac{C_{11}C_{\Lambda\Lambda}}{C_{1\Lambda}C_{\Lambda 1}}. \tag{A10}$$

The integral $I[xZ_2]$ is given by Eq. (A3) with an additional factor of x in the integrand. The substitution (A5) and the modification of the integrand according to Eq. (A7) permits an easy partial integration in the variable v with the result

$$I[Z_2]=\frac{1}{16\pi^2(\Lambda'^2-m_1'^2)(\Lambda''^2-m_1''^2)} \times\int_0^1 du\int_0^1 dvu\ \ln\frac{C_{11}C_{\Lambda\Lambda}}{C_{1\Lambda}C_{\Lambda 1}}, \tag{A11}$$

and if Eq. (A10) for $I[A_1^{(2)}]$ is changed according to Eq. (A5) we find the result

$$I[B_1^{(2)}]=0. \tag{A12}$$

The remaining identities of Eq. (3.32) can be proven in the same manner.

[1] M. V. Terent'ev, Yad. Fiz. **24**, 207 (1976) [Sov. J. Nucl. Phys. **24**, 106 (1976)]; V. J. Berestetsky and M. V. Terent'ev, *ibid.* **24**, 1044 (1976) [**24**, 547 (1976)]; **25**, 653 (1977) [**25**, 347 (1977)].

[2] W. Jaus, Phys. Rev. D **53**, 1349 (1996); **54**, 5904(E) (1996).

[3] P. J. O'Donnell and G. Turan, Phys. Rev. D **56**, 295 (1997).

[4] D. Melikhov, Phys. Rev. D **53**, 2460 (1996); Phys. Lett. B **380**, 363 (1996).

[5] H. Y. Cheng, C. Y. Cheung, and C. W. Hwang, Phys. Rev. D **55**, 1559 (1997).

[6] H. M. Choi and C. R. Ji, Phys. Rev. D **56**, 6010 (1997); Nucl. Phys. **A618**, 291 (1997).

[7] W. Jaus, Phys. Rev. D **44**, 2851 (1991).

[8] V. A. Karmanov and A. V. Smirnov, Nucl. Phys. **A546**, 691 (1992); J. Carbonell, B. Desplanques, V. A. Karmanov, and J. F. Mathiot, Phys. Rep. **300**, 215 (1998).

[9] M. G. Fuda, Ann. Phys. (N.Y.) **197**, 265 (1990).

[10] A. Szczepaniak, C. R. Ji, and S. R. Cotanch, Phys. Rev. D **52**, 5284 (1995).

[11] H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) **112**, 94 (1978).

[12] L. L. Frankfurt and M. I. Strikman, Nucl. Phys. **B148**, 107 (1979); Phys. Rep. **76**, 215 (1981); L. L. Frankfurt, I. L. Grach, L. A. Kondratyuk, and M. I. Strikman, Phys. Rev. Lett. **62**, 387 (1989); L. L. Frankfurt, T. Frederico, and M. Strikman, Phys. Rev. C **48**, 2182 (1993); B. D. Keister, Phys. Rev. D **49**, 1500 (1994).

[13] S. J. Chang and S. K. Ma, Phys. Rev. **180**, 1506 (1969); J. B. Kogut and D. E. Soper, Phys. Rev. D **1**, 2901 (1970); N. E. Ligterink and B. L. G. Bakker, *ibid.* **52**, 5954 (1995); N. C. J. Schoonderwoerd, B. L. G. Bakker, and V. A. Karmanov, Phys. Rev. C **58**, 3093 (1998).

[14] V. A. Karmanov, Nucl. Phys. **A608**, 316 (1996); V. A. Karmanov and A. V. Smirnov, *ibid.* **A575**, 520 (1994).

[15] J. P. B. C. de Melo and T. Frederico, Phys. Rev. C **55**, 2043 (1997).

[16] W. Jaus, Phys. Rev. D **41**, 3394 (1990).

[17] P. L. Chung, F. Coester, B. D. Keister, and W. N. Polyzou, Phys. Rev. C **37**, 2000 (1988).

[18] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[19] C. M. Shakin and Wei-Dong Sun, Phys. Rev. C **51**, 2171 (1995).

[20] S. J. Chang, R. G. Root, and T. M. Yan, Phys. Rev. D **7**, 1133 (1973); T. M. Yan, *ibid.* **7**, 1780 (1973).

[21] J. P. B. C. de Melo, J. H. O. Sales, T. Frederico, and P. U. Sauer, Nucl. Phys. **A631**, 574c (1998).

[22] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[23] The branching ratio $\mathrm{Br}(\tau\to\rho\nu_\tau)=(25.02\pm0.16)\%$ [22] leads to the value $f_\rho/M_\rho=147.3\pm0.7$ MeV for the vector decay constant for the charged rho meson, which is smaller than the value for the neutral rho meson $f_\rho/M_\rho=152.9\pm3.6$ MeV. In

our analysis we shall use $f_\rho / M_\rho = 150$ MeV.

[24] C. Erkal and M. G. Olsson, J. Phys. G **13**, 1355 (1987).

[25] The branching ratio $\mathrm{Br}(\tau \rightarrow K^* \nu_\tau) = (1.28 \pm 0.08)\%$ [22] leads to the value $f_{K*}/M_{K*} = 152 \pm 5$ MeV. In our analysis we shall use the central value $f_{K*}/M_{K*} = 152$ MeV.

[26] V. K. Birulev *et al.*, Nucl. Phys. **B182**, 1 (1981).

[27] L. Capraro *et al.*, Nucl. Phys. **B288**, 659 (1987).

[28] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. **19**, 1067 (1967).

[29] J. Bijnens, G. Colangelo, G. Ecker, and J. Gasser, in *The Second DAΦNE Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (INFN, Rome, Italy, 1995), p. 315; A. Pich, Rep. Prog. Phys. **58**, 563 (1995).

[30] J. Bijnens, G. Colangelo, and G. Ecker, Phys. Lett. B **441**, 437 (1998).

[31] H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).

[32] C. Callan and S. Treiman, Phys. Rev. Lett. **16**, 153 (1966).

[33] H. Y. Cheng, C. Y. Cheung, C. W. Hwang, and W. M. Zhang, Phys. Rev. D **57**, 5598 (1998).

[34] U. Haberthuer, Diplomarbeit ETHZ, Zurich 1998.

[35] M. G. Schmidt, Phys. Rev. D **9**, 408 (1974).

[36] S. J. Brodsky and D. S. Hwang, Nucl. Phys. **B543**, 239 (1999).

[37] H. M. Choi and C. R. Ji, Phys. Rev. D **58**, 071901 (1998).

[38] H. M. Choi and C. R. Ji, Phys. Rev. D **59**, 034001 (1999).

[39] S. J. Brodsky and J. R. Hiller, Phys. Rev. D **46**, 2141 (1992).