Nonleptonic charmless 2-body B decays in the perturbative QCD approach

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With the generalized factorization approximation, we calculate the branching ratios and CP asymmetries in B meson decays into two charmless pseudoscalar mesons. We give a new estimation of the matrix elements of $(S+P)(S-P)$ current products with the perturbative QCD method instead of the equation of motion. We find that our results are comparatively smaller than those in the literature. [S0556-2821(99)03515-8]

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I. INTRODUCTION

Penguin diagrams play an important role in charmless B decays and direct CP violation. They can provide not only the necessary different loop effects of internal u and c quarks [1], but also dominate the branching ratios of many modes of charmless B decays, such as $B\to\pi K$, $B\to KK$, and $B\to K\eta'$.

As we know, the standard theoretical framework of studying nonleptonic B decays is based on the effective Hamilton approach and the factorization approximation. The effective Hamiltonian is expressed by a sum of the products of a series of Wilson coefficients and four-quark operators. Unfortunately, we have many difficulties in calculating matrix elements of the four-quark operators directly in exclusive nonleptonic B decays, such as B to two pseudoscalar meson processes. So we have to use the factorization assumption, usually the Bauer-Stech-Wirbel (BSW) model [2]. Then the mesonic matrix elements are factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. However in the BSW model, the factorization involves the contributions of Fierz transformations of the four-quark operators. Using the Fierz rearrangement, one can find that the current-current product $(S+P)(S-P)$ matrix elements should be taken into account. The general method to deal with $(S+P)(S-P)$ matrix elements is to transform them into $(V-A)(V-A)$ matrix elements by using equation of motion. Then one can find that $(S+P)(S-P)$ matrix elements are very sensitive to the masses of light quarks. But the current masses of light quarks are not determined precisely. Obviously, it brings large uncertainty for estimating the $(S+P)(S-P)$ matrix elements and the branching ratios of charmless B decays. As pointed out by Ali, Kramer, and Lü [3], varying the light quarks masses by $\pm 20\%$ yields variation of up to $\pm 25\%$ in some selected decay modes (such as $B^{\pm}\to K^{\pm}\eta'$ and $\overline{B}^0\to\overline{K}^0\eta'$). In this work, instead of using equation of motion we will try to apply the perturbative QCD (PQCD) method to recalculate the ratio of the $(S+P)(S-P)$ to $(V-A)(V-A)$ matrix elements at leading twist approximation, which is not sensitive to the masses of light quarks. We think that it might have less uncertainties than the results obtained by using the quark equation of motion. So, it is necessary to recalculate the branching ratios for B meson charmless decays by using PQCD method and compare with those by using equation of motion. On the other hand, nonfactorizable effects in charmless B decays cannot be neglected. To compensate it for this, the general approach is to replace the number of colors N_c by a phenomenological color parameter $N_c^{\rm eff}$. We will discuss the differences while $N_c^{\rm eff}$ equal 2, 3, ∞, respectively.

This work is organized as follows. Section II gives the framework of calculation including the effective Hamiltonian and fatorization approximation. Section III is devoted to the PQCD method to estimate the ratio of the $(S+P)(S-P)$ to $(V-A)(V-A)$ matrix elements. In Sec. IV, we calculate the branching ratios of charmless B decays into two pseudoscalar mesons and their CP asymmetries using the method mentioned above. We also give some discussions of the numerical results. Section V is for the concluding remarks.

II. CALCULATIONAL FRAMEWORK

The $|\Delta B|=1$ effective Hamiltonian is

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}\left[\sum_{q=u,c} v_q\left(C_1(\mu)Q_1^q(\mu)+C_2(\mu)Q_2^q(\mu)+\sum_{k=3}^{10} C_k(\mu)Q_k(\mu)\right)\right]+\text{H.c.}, \tag{1}$$

where $v_q=V_{qb}V_{qd}^*$ (for $b\to d$ transition) or $v_q=V_{qb}V_{qs}^*$ (for $b\to s$ transition) and $C_i(\mu)$ are Wilson coefficients which have been evaluated to next-to-leading order approximation. In Eq. (1), the four-quark operators Q_i are given by

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$$Q_1^u=(\overline{u}_\alpha b_\beta)_{V-A}(\overline{q}_\beta u_\alpha)_{V-A}, \quad Q_1^c=(\overline{c}_\alpha b_\beta)_{V-A}(\overline{q}_\beta c_\alpha)_{V-A},$$

$$Q_2^u=(\overline{u}_\alpha b_\alpha)_{V-A}(\overline{q}_\beta u_\beta)_{V-A}, \quad Q_2^c=(\overline{c}_\alpha b_\alpha)_{V-A}(\overline{q}_\beta c_\beta)_{V-A},$$

$$Q_3=(\overline{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\overline{q}'_\beta q'_\beta)_{V-A},$$

$$Q_4=(\overline{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\overline{q}'_\alpha q'_\beta)_{V-A},$$

$$Q_5=(\overline{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\overline{q}'_\beta q'_\beta)_{V+A},$$

$$Q_6=(\overline{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\overline{q}'_\alpha q'_\beta)_{V+A},$$

$$Q_7=\frac{3}{2}(\overline{q}_\alpha b_\alpha)_{V-A}\sum_{q'}e_{q'}(\overline{q}'_\beta q'_\beta)_{V+A},$$

$$Q_8=\frac{3}{2}(\overline{q}_\beta b_\alpha)_{V-A}\sum_{q'}e_{q'}(\overline{q}'_\alpha q'_\beta)_{V+A},$$

$$Q_9=\frac{3}{2}(\overline{q}_\alpha b_\alpha)_{V-A}\sum_{q'}e_{q'}(\overline{q}'_\beta q'_\beta)_{V-A},$$

$$Q_{10}=\frac{3}{2}(\overline{q}_\beta b_\alpha)_{V-A}\sum_{q'}e_{q'}(\overline{q}'_\alpha q'_\beta)_{V-A}, \tag{2}$$

with Q_1^q and Q_2^q being the tree operators, Q_3-Q_6 the QCD penguin operators, and Q_7-Q_{10} the electroweak penguin operators. With the renormalization group method, we can evolove the renormalization scheme independent Wilson coefficients $\overline{C}_i(\mu)$ from the scale $\mu=m_W$ to $\mu=5.0$ GeV $\approx m_B$, which are [5]

$$\overline{C}_1=-0.313, \quad \overline{C}_2=1.150, \quad \overline{C}_3=0.017, \quad \overline{C}_4=-0.037,$$

$$\overline{C}_5=0.010, \quad \overline{C}_6=-0.046, \quad \overline{C}_7=-0.001\alpha_{em},$$

$$\overline{C}_8=0.049\alpha_{em}, \quad \overline{C}_9=-1.321\alpha_{em}, \quad \overline{C}_{10}=0.267\alpha_{em}. \tag{3}$$

So we can express the physical amplitude as follows:

$$\langle \mathbf{Q}^T(\mu)\cdot\mathbf{C}(\mu)\rangle \equiv \langle \mathbf{Q}^T\rangle_0\cdot\mathbf{C}'(\mu), \tag{4}$$

where $\langle \mathbf{Q}^T\rangle_0$ denote the tree level matrix elements and

$$C_1'=\overline{C}_1, \quad C_2'=\overline{C}_2, \quad C_3'=\overline{C}_3-\frac{P_s}{3}, \quad C_4'=\overline{C}_4+P_s,$$

$$C_5'=\overline{C}_5-\frac{P_s}{3}, \quad C_6'=\overline{C}_6+P_s, \quad C_7'=\overline{C}_7+P_e,$$

$$C_8'=\overline{C}_8, \quad C_9'=\overline{C}_9+P_e, \quad C_{10}'=\overline{C}_{10}, \tag{5}$$

$$P_s=\frac{\alpha_s}{8\pi}\overline{C}_2(\mu)\left(\frac{10}{9}-G(m_q,q^2,\mu)\right),$$

$$P_e=\frac{\alpha_{em}}{9\pi}[3\overline{C}_1(\mu)+\overline{C}_2(\mu)] \times\left(\frac{10}{9}-G(m_q,q^2,\mu)\right),$$

$$G(m_q,q^2,\mu)=-4\int_0^1 dx\,x(1-x)\ln\left(\frac{m_q^2-x(1-x)q^2}{\mu^2}\right). \tag{6}$$

As noted in the Introduction, we have to calculate the matrix elements of the four-quark operators by using the factorization assumption. Here we apply the BSW model [2]. However, nonfactorizable effects are not negligible in the process of B to two light mesons. We will use the simpliest approach to compensate it by using only one color parameter $N_c^{\rm eff}$, even if there is no reason why using only one single parameter $N_c^{\rm eff}$ to explain the branching ratios of all kind of different modes.

For illustration, we give the amplitude of $B_u^-\to K^-\eta'$ as an example:

$$\begin{aligned}\langle K^-\eta'|\mathcal{H}_{\rm eff}|B_u^-\rangle &= \frac{G_F}{\sqrt{2}}\sum_{q=u,c}v_q\Bigg\{(a_1\delta_{qu}+a_3+a_9)M_{suu}^{K^-\eta'}\\ &+\left(a_2\delta_{qu}+2a_4-2a_6-\frac{a_8}{2}+\frac{a_{10}}{2}\right)M_{uus}^{\eta'K^-}\\ &+\left(a_4-a_6+a_3+\frac{a_8}{2}+\frac{a_9}{2}-\frac{a_{10}}{2}\right)M_{sss}^{\eta'K^-}\\ &+(a_2\delta_{qc}-a_8+a_{10})M_{ccs}^{\eta'K^-}+(-2a_5-2a_7)X_{suu}^{K^-\eta'}\\ &+(-2a_5+a_7)X_{sss}^{\eta'K^-}\Bigg\},\end{aligned} \tag{7}$$

where

$$a_{2i-1}=C_{2i}'+\frac{C_{2i-1}'}{N_c^{\rm eff}}, \quad a_{2i}=C_{2i-1}'+\frac{C_{2i}'}{N_c^{\rm eff}}, \tag{8}$$

and

$$M_{q_1q_2q_3}^{PP'}=\langle P|(\overline{q}_1q_2)_{V-A}|0\rangle\langle P'|(\overline{q}_3b)_{V-A}|B\rangle,$$

$$X_{q_1q_2q_3}^{PP'}=\langle P|(\overline{q}_1q_2)_{S+P}|0\rangle\langle P'|(\overline{q}_3b)_{S-P}|B\rangle. \tag{9}$$

We will use the following parametrization for decay constants and form factors:

$$\langle 0|V_\mu - A_\mu|P(q)\rangle = i f_P q_\mu ,$$

$$\langle P_2(q_2)|V_\mu - A_\mu|P_1(q_1)\rangle = F_+^{P_1\to P_2}(q_-^2) q_{+\mu} + F_-^{P_1\to P_2}(q_-^2) q_{-\mu}, \quad (10)$$

where $q_\pm = q_1 \pm q_2$, and we use the monopole dominance assumption for the q_-^2 dependence of the form factors,

$$F_+^{P_1\to P_2}(q_-^2) \approx \frac{F_+^{P_1\to P_2}(0)}{1-q_-^2/m_{\rm pol}^2},$$

$$F_-^{P_1\to P_2}(q_-^2) \approx -\frac{m_1-m_2}{m_1+m_2} F_+^{P_1\to P_2}(q_-^2). \quad (11)$$

Then we can obtain

$$M_{q_1q_2q_3}^{PP'} = -i f_P F_+^{B\to P'}(m_P^2) \frac{m_B - m_{P'}}{m_B + m_{P'}} [(m_B + m_{P'})^2 - m_P^2], \quad (12)$$

$$X_{q_1q_2q_3}^{PP'} = \frac{m_P^2}{[(m_1+m_2)(m_3-m_b)]} M_{q_1q_2q_3}^{PP'}, \quad (13)$$

where Eq. (13) is derived from the equation of motion and m_i presents the mass of the light quark q_i, respectively ($i = 1,2,3$).

Calculations in this framework have been discussed in detail in some papers [3,4] involving the branching ratios and CP asymmetries in nonleptonic charmless two-body B decays. In these papers the uncertainties resulting from the renormalization scale dependence, nonfactorizable contributions and the input parameters (α_s, quark masses and form factors) have been worked out. Further penguin effects and the strong sensitivity of the CP asymmetries to the Cabibbo-Kobayashi-Maskawa (CKM) parameters (ρ, η) have been discussed there. In these uncertainties, the uncertainty of light quark masses is mainly showed in the part of $(S+P)(S-P)$ matrix elements. Sometimes they are dominant terms in some modes of charmless B decays, such as $B_u^- \to K^-\eta'$, in which the term $X_{ssu}^{\eta' K^-}$ is enhanced by the factor $m_{\eta'}/m_s$. Then it motivates us to give a new estimation of the matrix elements $X_{q_1q_2q_3}^{PP'}$ to cancel the uncertainty of the light quark masses.

III. PQCD METHOD

Brodsky *et al.* [6] has pointed out that the factorization formula of PQCD can be applied to the exclusive B decays into light mesons for the large momentum transfers. One can write the amplitude as a convolution of a hard-scattering quark-gluon amplitudes $\phi(x,Q^2)$ which describe the fractional longitudinal momentum distribution of the quark and antiquark in each meson. An important feature of this formalism is that, at high momentum transfer, long-range final state interactions between the outgoing hardrons can be neglected. In the case of nonleptonic weak decays the mass squared of the heavy meson m_H^2 establishes the relevant momentum scale $Q^2 \sim m_H^2$, so that for a sufficiently massive initial state the decay amplitude is of the order of $\alpha_s(Q^2)$, even without including loop corrections to the weak Hamiltonian. The dominant contribution is controlled by single gluon exchange.

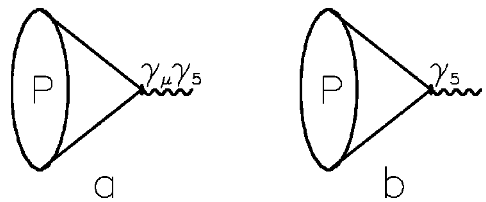

FIG. 1. Diagrams for the matrix elements $\langle P|\overline{q}_1\gamma_\mu\gamma_5 q_2|0\rangle$ (a) and $\langle P|\overline{q}_1\gamma_5 q_2|0\rangle$ (b).

We intend to apply the PQCD method to estimate those hadronic matrix elements such as $(V-A)(V-A)$ and $(S+P)(S-P)$ at the leading twist approximation. The wave function of B meson and flavor $SU(3)$ singlet or octet pseudoscalar mesons are taken as

$$\Psi_B(x) = \frac{1}{\sqrt{2}}\frac{I_C}{\sqrt{3}}\phi_B(x)(\not p + m_B)\gamma_5,$$

$$\Psi_P(y) = \frac{1}{\sqrt{2}}\frac{I_C}{\sqrt{3}}\phi_P(y)(\not q + m_P)\gamma_5, \quad (14)$$

where I_C is an identity in color space. In QCD, the integration of the distribution amplitude is related to the meson decay constant

$$\int \phi_P(y)dy = \frac{1}{2\sqrt{6}} f_P, \quad \int \phi_B(x)dx = \frac{1}{2\sqrt{6}} f_B. \quad (15)$$

Then we can write down the amplitude of Fig. 1 as

$$\langle P|\overline{q}_1\gamma_\mu\gamma_5 q_2|0\rangle_{\rm PQCD} = 3\times\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\int dy\ \phi_P(y) \times {\rm Tr}[\gamma_5\ (\not q + m_P)\gamma_\mu\gamma_5] = f_P q_\mu,$$

$$\langle P|\overline{q}_1\gamma_5 q_2|0\rangle_{\rm PQCD} = 3\times\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\int dy\ \phi_P(y) \times {\rm Tr}[\gamma_5\ (\not q + m_P)\gamma_5] = f_P m_P. \quad (16)$$

In a consistent way, we can use perturbative QCD to estimate the matrix elements such as $\langle P|\overline{q}_l\gamma_\mu b|B\rangle$ and $\langle P|\overline{q}_l b|B\rangle$ (Figs. 2,3), where q_l denotes light quark field operator and we have neglected the fermi motion of quarks, while the gluons in Figs. 2,3 are hard because

$$k^2 = [xp - (1-y)q]^2 \simeq -x(1-y)m_B^2 \sim 1\ \ {\rm GeV}^2 \quad (17)$$

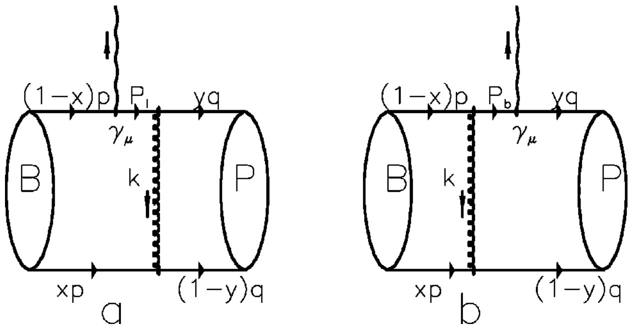

FIG. 2. Leading twist diagrams in QCD for the matrix elements $\langle P|\overline{q_l}\gamma_\mu b|B\rangle$.

(here we using mean values $\langle y\rangle\sim\frac{1}{2},\langle x\rangle\sim\epsilon_B$, with $\epsilon_B\sim0.05-0.1$ and $x^2\ll1$) so we can neglect the $\mathcal{O}(x^2m_B^2)$ term and use perturbative QCD method to calculate the amplitude. It turns out to be

$$\langle P|\overline{q_l}\gamma_\mu b|B\rangle_{\rm PQCD} = -\frac{2}{3}g^2\int dxdy\,\phi_B(x)\phi_P(y) \times\left\{\frac{Tr[\gamma_5(\not q+m_P)\gamma^\nu \not P_l\gamma_\mu(\not p+m_B)\gamma_5\gamma_\nu]}{k^2P_l^2} + \frac{\mathrm{Tr}[\gamma_5(\not q+m_P)\gamma_\mu(\not P_b+m_b)\gamma^\nu(\not p+m_B)\gamma_5\gamma_\nu]}{k^2(P_b^2-m_b^2)}\right\}. \quad (18)$$

In order to get quantitative estimation, we take the wave functions as [6,7]

$$\phi_B(x)=\frac{f_B}{2\sqrt{6}}\delta(x-\epsilon_B),\quad \phi_P(y)=\sqrt{\frac{3}{2}}f_Py(1-y). \quad (19)$$

(here ϵ_B is the peaking position of the B-meson wave function, typically $\langle\epsilon_B\rangle\sim m_B-m_b/m_B$). We get

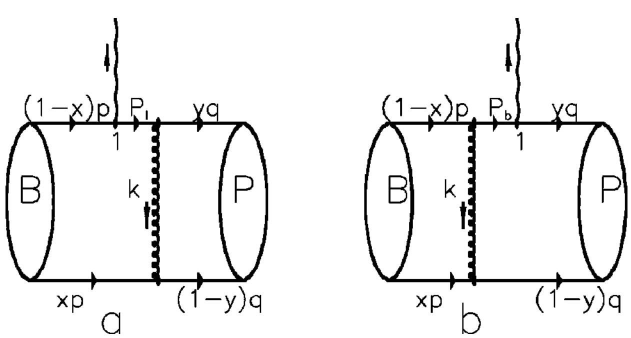

FIG. 3. Leading twist diagrams in QCD for the matrix elements $\langle P|\overline{q_l}b|B\rangle$.

TABLE I. The PQCD estimations about the element $\langle\pi^-|\overline{s}\gamma_\mu b|B^-\rangle$.

$F_+^{B_u^-\to\pi^-}$	$\epsilon_B=0.05$	$\epsilon_B=0.06$	$\epsilon_B=0.07$	$\epsilon_B=0.08$
$mb=5.0$ GeV	0.21	0.17	0.14	0.12
$mb=4.9$ GeV	0.26	0.21	0.18	0.15
$mb=4.8$ GeV	0.19	0.15	0.13	0.11

$$\langle P|\overline{q_l}\gamma_\mu b|B\rangle_{\rm PQCD}=F_+^{B\to P}(Q^2)(p+q)_\mu + F_-^{B\to P}(Q^2)(p-q)_\mu, \quad (20)$$

where

$$F_+^{B\to P}(Q^2)=-\frac{8\pi\alpha_s}{3}f_Pf_B\left\{-\frac{m_Pm_B}{\epsilon_B^2m_B^4} - \int dy\, y\frac{m_b(m_P-2m_B)+y(m_B^2-2m_Pm_B)}{\epsilon_Bm_B^2(ym_B^2-m_b^2)}\right\}, \quad (21)$$

$$F_-^{B\to P}(Q^2)=-\frac{8\pi\alpha_s}{3}f_Pf_B\left\{-\frac{m_B(\epsilon_B-m_P)}{\epsilon_B^2m_B^4} - \int dy\, y\frac{2m_b-4m_P-y(m_B-2m_P)}{\epsilon_Bm_B(ym_B^2-m_b^2)}\right\}. \quad (22)$$

Here, $Q^2=(p-q)^2$. So we can obtain the matrix element $M_{q_1q_2q_3}^{P'P}$. We can also get the matrix element $\langle P|\overline{q_l}b|B\rangle$ as

$$\langle P|\overline{s}b|B\rangle_{\rm PQCD} = -\frac{2}{3}g^2\int dxdy\,\phi_B(x)\phi_P(y) \times\left\{\frac{\mathrm{Tr}[\gamma_5(\not q+m_P)\gamma^\nu\not P_l(\not p+m_B)\gamma_5\gamma_\nu]}{k^2P_l^2} + \frac{\mathrm{Tr}[\gamma_5(\not q+m_P)(\not P_b+m_b)\gamma^\nu(\not P+m_B)\gamma_5\gamma_\nu]}{k^2(P_b^2-m_b^2)}\right\}$$

TABLE II. The PQCD estimations about the ratio of the matrix element $X_{sdd\ \rm PQCD}^{\overline{K}^0\pi^-}$ to $M_{sdd\ \rm PQCD}^{\overline{K}^0\pi^-}$.

$\mathcal{R}=\frac{X_{sdd\ \rm PQCD}^{\overline{K}^0\pi^-}}{M_{sdd\ \rm PQCD}^{\overline{K}^0\pi^-}}$	$\epsilon_B=0.05$	$\epsilon_B=0.06$	$\epsilon_B=0.07$	$\epsilon_B=0.08$
$mb=5.0$ GeV	−0.025	−0.025	−0.026	−0.026
$mb=4.9$ GeV	−0.025	−0.026	−0.026	−0.026
$mb=4.8$ GeV	−0.026	−0.026	−0.027	−0.027

TABLE III. Branching ratio in 10^{-5} and CP asymmetries in %. ''QCD'' and ''EW'' present the QCD penguin and EW penguin effects, respectively, and ''Dirac'' presents the results with the equation of motion.

$N_c^{\rm eff}=2$	Branching ratio				CP Asymmetry		
Decay mode	TR	QCD	EW	Dirac	QCD	EW	Dirac
$B_u^-\to\pi^0\pi^-$	0.53	0.53	0.53	0.54	0.03	1.9	3.5
$B_u^-\to\pi^-\eta'$	0.14	0.19	0.19	1.54	16.4	0.05	0.2
$B_u^-\to\pi^-\eta$	0.42	0.57	0.57	1.48	16.0	0.1	0.6
$B_u^-\to K^0K^-$		0.032	0.032	0.065	12.9	13.3	12.3
$B_u^-\to\pi^0K^-$	0.038	0.090	0.19	0.405	−39.0	−23.3	−14.8
$B_u^-\to K^-\eta'$	0.17	0.42	0.38	1.38	−11.5	−16.7	−7.8
$B_u^-\to K^-\eta$	0.024	0.063	0.038	0.025	17.8	−7.4	−10.4
$B_u^-\to\overline{K^0}\pi^-$		0.35	0.33	0.72	−0.3	−0.3	−0.3
$\overline{B_d^0}\to\pi^+\pi^-$	0.64	0.76	0.83	0.77	13.2	13.2	18.1
$\overline{B_d^0}\to\pi^0\pi^0$	0.012	7.9×10^{-3}	6.8×10^{-3}	6.94×10^{-3}	−42.7	−46.8	−51.5
$\overline{B_d^0}\to\pi^0\eta'$	1.6×10^{-3}	7.0×10^{-3}	5.0×10^{-3}	5.0×10^{-3}	−27.8	−36.0	−36.0
$\overline{B_d^0}\to\pi^0\eta$	3.1×10^{-4}	0.019	0.019	0.34	0.8	0.8	8.6
$\overline{B_d^0}\to\eta'\eta'$	1.1×10^{-3}	1.2×10^{-3}	1.1×10^{-3}	8.63×10^{-3}	7.5	5.6	12.2
$\overline{B_d^0}\to\eta\eta$	9.4×10^{-3}	7.6×10^{-3}	8.1×10^{-3}	8.47×10^{-3}	2.2	4.8	35.5
$\overline{B_d^0}\to\eta\eta'$	0.056	0.058	0.056	0.094	9.3	9.3	95.7
$\overline{B_d^0}\to K^0\overline{K^0}$		0.042	0.041	0.065	12.8	12.6	12.3
$\overline{B_d^0}\to\pi^+K^-$	0.048	0.16	0.23	0.46	−31.7	−28.5	−18.2
$\overline{B_d^0}\to\pi^0\overline{K^0}$	1.3×10^{-3}	0.22	0.14	0.38	5.3	8.7	3.9
$\overline{B_d^0}\to\overline{K^0}\eta'$	0.017	0.42	0.36	1.47	−2.9	−10.8	−5.4
$\overline{B_d^0}\to\overline{K^0}\eta$	2.4×10^{-3}	0.013	3.6×10^{-3}	2.8×10^{-3}	13.7	−45.2	2.3
$\overline{B_s^0}\to\pi^-K^+$	0.68	0.81	0.81	0.89	13.1	13.1	18.1
$\overline{B_s^0}\to\pi^0K^0$	0.020	0.012	0.012	0.012	−40.9	−44.7	−45.8
$\overline{B_s^0}\to K^0\eta'$	8.2×10^{-3}	0.063	0.060	1.16	62.2	60.6	19.5
$\overline{B_s^0}\to K^0\eta$	0.022	0.019	0.019	0.34	−8.7	−4.9	34.9
$\overline{B_s^0}\to K^0\overline{K^0}$		0.35	0.33	0.75	−0.3	−0.3	−0.3
$\overline{B_s^0}\to K^-K^+$		0.31	0.35	0.75	−0.3	−0.3	−0.3
$\overline{B_s^0}\to\pi^0\eta'$	6.1×10^{-4}	6.1×10^{-4}	4.3×10^{-3}	4.3×10^{-3}	0	0	0
$\overline{B_s^0}\to\pi^0\eta$	4.6×10^{-4}	4.6×10^{-4}	3.3×10^{-3}	3.3×10^{-3}	0	0	0
$\overline{B_s^0}\to\eta'\eta'$	7.4×10^{-3}	0.11	0.11	0.41	−3.7	−3.8	−1.7
$\overline{B_s^0}\to\eta\eta$	8.2×10^{-4}	0.078	0.078	0.11	4.8	4.6	3.2
$\overline{B_s^0}\to\eta\eta'$	3.1×10^{-4}	0.33	0.33	0.82	1.4	1.2	0.4

$$=-\frac{8\pi}{3}\alpha_s f_B f_P \times\left\{\frac{(-2m_P(1-2\epsilon_B)+\epsilon_B m_B)m_B+m_P^2}{\epsilon_B^2 m_B^3}-\int dyy \times\frac{m_B(m_P+m_b)-2m_B^2-4m_b m_P+y m_P m_B}{\epsilon_B m_B(y m_B^2-m_b^2)}\right\}. \tag{23}$$

In Ref. [8], as an example, the authors calculated the numerical results of the matrix element $\langle K^-|\overline{s}\gamma_\mu b|B^-\rangle$ using the above framework, where they applied $\alpha_s\simeq0.38$, $f_B=200$ MeV, and $f_K=160$ MeV. One finds that their results are sensitive to the values of parameters ϵ_B and m_b, and seem small compared with the BSW result. We also compute the matrix element $\langle\pi^-|\overline{s}\gamma_\mu b|B^-\rangle$ and list the numerical results in Table I, where we take $\alpha_s=0.38$, $f_B=0.2$ GeV, and $f_\pi=0.13$ GeV.

We can see that the results are very sensitive to the values of parameter ϵ_B and m_b, and smaller than the BSW result which is about 0.29 [14]. As mentioned in Ref. [8], the PQCD results are comparatively small in many cases.

However, the ratio

$$\mathcal{R}=\frac{X^{PP'}_{q_1q_2q_3\ \rm PQCD}}{M^{PP'}_{q_1q_2q_3\ \rm PQCD}} \tag{24}$$

is insensitive to the parameters ϵ_B and m_b. So it is more

TABLE IV. Branching ratio in 10^{-5} and CP asymmetries in %. ''QCD'' and ''EW'' present the QCD penguin and EW penguin effects, respectively, and ''Dirac'' presents the results with the equation of motion.

$N_c^{\rm eff}=3$	Branching ratio				CP Asymmetry		
Decay mode	TR	QCD	EW	Dirac	QCD	EW	Dirac
$B_u^-\to\pi^0\pi^-$	0.42	0.42	0.42	0.43	0.04	1.9	3.8
$B_u^-\to\pi^-\eta'$	0.10	0.15	0.15	1.56	18.9	0.04	0.2
$B_u^-\to\pi^-\eta$	0.31	0.46	0.46	1.39	19.2	0.2	0.7
$B_u^-\to K^0K^-$		0.037	0.036	0.075	12.8	12.9	12.1
$B_u^-\to\pi^0K^-$	0.031	0.12	0.22	0.46	-32.3	-20.4	-12.9
$B_u^-\to K^-\eta'$	5.9×10^{-3}	0.42	0.38	1.44	-11.0	-15.9	-7.2
$B_u^-\to K^-\eta$	0.021	0.067	0.039	0.021	15.4	-11.1	19.8
$B_u^-\to\overline{K^0}\pi^-$		0.41	0.40	0.84	-0.3	-0.3	-0.3
$\overline{B_d^0}\to\pi^+\pi^-$	0.71	0.85	0.85	0.92	13.3	13.3	18.3
$\overline{B_d^0}\to\pi^0\pi^0$	8.8×10^{-4}	2.8×10^{-3}	1.4×10^{-3}	3.6×10^{-3}	-31.8	-50.3	-30.3
$\overline{B_d^0}\to\pi^0\eta'$	1.2×10^{-4}	9.3×10^{-3}	6.9×10^{-3}	6.9×10^{-3}	3.2	2.3	2.3
$\overline{B_d^0}\to\pi^0\eta$	2.2×10^{-5}	0.023	0.023	0.38	10.4	10.8	10.8
$\overline{B_d^0}\to\eta'\eta'$	8.1×10^{-5}	1.1×10^{-4}	7.8×10^{-5}	0.013	28.3	0.3	1.1
$\overline{B_d^0}\to\eta\eta$	6.8×10^{-4}	4.1×10^{-4}	7.9×10^{-4}	0.017	13.2	61.7	12.1
$\overline{B_d^0}\to\eta\eta'$	4.0×10^{-3}	4.7×10^{-3}	4.2×10^{-3}	0.073	42.2	42.8	21.0
$\overline{B_d^0}\to K^0\overline{K^0}$		0.049	0.048	0.076	12.4	12.5	12.1
$\overline{B_d^0}\to\pi^+K^-$	0.053	0.23	0.25	0.50	-30.4	-29.1	-18.7
$\overline{B_d^0}\to\pi^0\overline{K^0}$	9.7×10^{-5}	0.24	0.15	0.42	1.1	1.8	0.7
$\overline{B_d^0}\to\overline{K^0}\eta'$	1.2×10^{-3}	0.43	0.38	1.56	-1.1	-7.2	-3.7
$\overline{B_d^0}\to\overline{K^0}\eta$	1.7×10^{-4}	0.015	2.8×10^{-3}	4.9×10^{-3}	2.8	-67.9	-78.5
$\overline{B_s^0}\to\pi^-K^+$	0.75	0.90	0.90	0.98	13.3	13.3	18.3
$\overline{B_s^0}\to\pi^0K^0$	1.4×10^{-3}	4.3×10^{-3}	2.0×10^{-3}	2.2×10^{-3}	-32.6	-52.3	-49.6
$\overline{B_s^0}\to K^0\eta'$	5.9×10^{-4}	0.043	0.041	1.2	30.9	29.0	13.3
$\overline{B_s^0}\to K^0\eta$	1.6×10^{-3}	1.3×10^{-3}	9.3×10^{-4}	0.29	-32.6	-38.1	19.8
$\overline{B_s^0}\to K^0\overline{K^0}$		0.41	0.40	0.88	-0.3	-0.3	-0.3
$\overline{B_s^0}\to K^-K^+$		0.37	0.39	0.82	-0.3	-0.3	-0.3
$\overline{B_s^0}\to\pi^0\eta'$	4.4×10^{-5}	4.4×10^{-5}	5.3×10^{-3}	5.3×10^{-3}	0	0	0
$\overline{B_s^0}\to\pi^0\eta$	3.3×10^{-5}	3.3×10^{-5}	4.0×10^{-3}	4.0×10^{-3}	0	0	0
$\overline{B_s^0}\to\eta'\eta'$	5.3×10^{-4}	0.11	0.11	0.42	-1.4	-1.5	-0.8
$\overline{B_s^0}\to\eta\eta$	5.9×10^{-5}	0.092	0.092	0.13	0.9	0.7	0.5
$\overline{B_s^0}\to\eta\eta'$	2.2×10^{-5}	0.39	0.39	0.93	0.06	-0.1	-0.2

reliable because of cancelation of the main uncertainties. We list our computation in Table II.

The ratio by using equation of motion is

$$\mathcal{R}=\frac{X_{sdd}^{\overline{K^0}\pi^-}}{M_{q_1q_2q_3}^{K^0\pi^-}}=\frac{m_{K^0}^2}{(m_s+m_d)(m_d-m_b)}\simeq-0.30 \tag{25}$$

and is about one order of magnitude larger than the PQCD estimation. As mentioned in our Introduction, the matrix elements of $(S+P)(S-P)$ four quarks operator are very important in some decay modes of B mesons, such as B to η' and other mesons. So it is necessary to recalculate the branching ratios and CP asymmetries for two-body charmless B decays by using the PQCD method instead of the equation of motion.

IV. BRANCHING RATIOS AND CP ASYMMETRIES

In the B rest frame, the two body decay width is

$$\Gamma(B\to PP')=\frac{1}{8\pi}|\langle PP'|H_{\rm eff}|B\rangle|^2\frac{|p|}{m_B^2}, \tag{26}$$

where

$$|p|=\frac{\{[m_B^2-(m_P+m_{P'})^2][m_B^2-(m_P-m_{P'})^2]\}^{1/2}}{2m_B} \tag{27}$$

is the magnitude of the momentum of the particle P or P'. The corresponding branching ratio is given by

TABLE V. Branching ratios in unit of 10^{-5} and CP asymmetry in unit of %. ''QCD'' and ''EW'' present the QCD penguin and EW penguin effects, respectively, and ''Dirac'' presents the results with the equation of motion.

$N_c^{\rm eff}=\infty$	Branching ratio				CP Asymmetry		
Decay mode	TR	QCD	EW	Dirac	QCD	EW	Dirac
$B_u^-\to\pi^0\pi^-$	0.23	0.23	0.24	0.25	0.05	1.9	4.7
$B_u^-\to\pi^-\eta'$	0.045	0.082	0.082	1.60	25.2	0.2	0.2
$B_u^-\to\pi^-\eta$	0.14	0.27	0.27	1.25	29.5	0.5	0.9
$B_u^-\to K^0K^-$		0.048	0.049	0.10	12.5	12.3	11.9
$B_u^-\to\pi^0K^-$	0.019	0.18	0.28	0.58	-21.1	-14.9	-9.5
$B_u^-\to K^-\eta'$	0.040	0.42	0.38	1.56	-9.0	-13.6	-6.0
$B_u^-\to K^-\eta$	0.022	0.079	0.054	0.024	10.0	-8.8	20.9
$B_u^-\to\overline{K^0}\pi^-$		0.54	0.56	1.12	-0.3	-0.3	-0.3
$\overline{B_d^0}\to\pi^+\pi^-$	0.86	1.00	1.00	1.12	13.6	13.6	18.7
$\overline{B_d^0}\to\pi^0\pi^0$	0.017	0.034	0.032	0.039	39.1	42.0	46.4
$\overline{B_d^0}\to\pi^0\eta'$	2.3×10^{-3}	0.020	0.016	0.017	61.3	78.0	78.0
$\overline{B_d^0}\to\pi^0\eta$	4.5×10^{-4}	0.034	0.031	0.47	26.1	29.2	14.7
$\overline{B_d^0}\to\eta'\eta'$	1.6×10^{-3}	1.8×10^{-3}	1.8×10^{-3}	0.026	-6.3	-7.3	-6.5
$\overline{B_d^0}\to\eta\eta$	0.013	0.017	0.018	0.070	-1.3	-0.6	1.1
$\overline{B_d^0}\to\eta\eta'$	0.079	0.080	0.080	0.21	-8.1	-8.1	-24.1
$\overline{B_d^0}\to K^0\overline{K^0}$		0.063	0.064	0.099	12.3	12.1	11.9
$\overline{B_d^0}\to\pi^+K^-$	0.065	0.31	0.29	0.58	-28.4	-30.4	-19.5
$\overline{B_d^0}\to\pi^0\overline{K^0}$	1.9×10^{-3}	0.27	0.18	0.52	-5.9	-8.5	-4.4
$\overline{B_d^0}\to\overline{K^0}\eta'$	0.024	0.44	0.42	1.77	3.1	0.5	-0.3
$\overline{B_d^0}\to\overline{K^0}\eta$	3.4×10^{-3}	0.027	6.8×10^{-3}	1.5×10^{-3}	-7.8	-63.9	-79.6
$\overline{B_s^0}\to\pi^-K^+$	0.91	1.10	1.10	1.19	13.5	13.6	18.7
$\overline{B_s^0}\to\pi^0K^0$	0.029	0.055	0.052	0.049	36.6	39.2	36.4
$\overline{B_s^0}\to K^0\eta'$	0.012	0.021	0.020	1.27	-48.6	-50.1	1.7
$\overline{B_s^0}\to K^0\eta$	0.031	0.037	0.035	0.24	14.2	9.2	-16.4
$\overline{B_s^0}\to K^0\overline{K^0}$		0.54	0.56	1.17	-0.3	-0.3	-0.3
$\overline{B_s^0}\to K^-K^+$		0.48	0.45	0.96	-0.3	-0.3	-0.3
$\overline{B_s^0}\to\pi^0\eta'$	8.7×10^{-4}	8.7×10^{-4}	9.3×10^{-4}	9.3×10^{-4}	0	0	0
$\overline{B_s^0}\to\pi^0\eta$	6.5×10^{-4}	6.5×10^{-4}	7.0×10^{-3}	7.0×10^{-3}	0	0	0
$\overline{B_s^0}\to\eta'\eta'$	0.011	0.10	0.10	0.43	5.1	5.0	1.4
$\overline{B_s^0}\to\eta\eta$	1.2×10^{-3}	0.12	0.12	0.17	-4.4	-4.5	-3.5
$\overline{B_s^0}\to\eta\eta'$	4.4×10^{-4}	0.50	0.50	1.2	-1.9	-2.0	-1.2

$$\mathcal{B}_{\rm BR}(B\to PP')=\frac{\Gamma(B\to PP')}{\Gamma_{\rm tot}}. \tag{28}$$

The direct CP asymmetry $\mathcal{A}_{CP}$ for B meson decays into PP' is defined as

$$\mathcal{A}_{CP}=\frac{\Gamma(B\to PP')-\Gamma(\overline{B}\to\overline{PP'})}{\Gamma(B\to PP')+\Gamma(\overline{B}\to\overline{PP'})}. \tag{29}$$

In our numerical calculation, we use the Wolfstein parametrization for the CKM matrix

$$V_{\rm CKM}=\begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3A(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & \lambda^2A \\ \lambda^3A(1-\rho-i\eta) & -\lambda^2A & 1 \end{bmatrix} +\mathcal{O}(\lambda^4) \tag{30}$$

and we take [9]

$$A=0.823\pm0.033,\quad \lambda=0.220,\quad \rho=0.160,\quad \eta=0.336. \tag{31}$$

Otherwise we take all parameters such as meson decay constants and form factors needed in our calculation as follows [5,13]: $f_\pi = 0.13$ GeV, $f_K = 0.160$ GeV, $f^u_{\eta'} = f^d_{\eta'} = 0.049$ GeV, $f^u_\eta = f^d_\eta = 0.092$ GeV, $f^s_{\eta'} = 0.12$ GeV, $f^s_\eta = -0.105$ GeV, $f^c_{\eta'} = -0.0063$ GeV, $f^c_\eta = -0.0024$ GeV, and $F_+^{B_u^- \to \pi^-}(0) = 0.29$, $F_+^{B_u^- \to K^-}(0) = 0.32$, $F_+^{B_u^- \to \eta'}(0) = 0.254/\sqrt{6}$, $F_+^{\bar{B}_s^- \to \eta'}(0) = 2 \times 0.282/\sqrt{6}$, $F_+^{B_u^- \to \eta}(0) = 0.307/\sqrt{3}$, $F_+^{\bar{B}_s^- \to \eta}(0) = -0.335/\sqrt{3}$. Here we apply the flavor wave functions of η' and η as [13]

$$|\eta'\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle + 2|s\bar{s}\rangle}{\sqrt{6}},$$

$$|\eta\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle - |s\bar{s}\rangle}{\sqrt{3}}. \tag{32}$$

We give the numerical results of the branching ratios and CP asymmetries for B charmless decays in Tables III–V. As a comparison, the results by using the equation of motion are also listed in the tables where we take $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 150$ MeV, and $m_b = 5.0$ GeV. In the calculation, we have neglected the contributions of W annihilation, W exchange, and spacelike penguin diagrams.

From the tables, we can see the following features.

(i) For most of charmless B decays, the contributions of penguin diagrams are important.

(ii) Comparing the results of the PQCD method with those by using equation of motion, one can find a large difference between them. In the modes of $B \to \pi K$ and $B \to KK$, the branching ratios predicted by using equation of motion is larger than those of the PQCD method by about a factor 2. While final state involving η', the factor would be more large. Obviously, CP asymmetries are also affected by these differences. In our computation, we find that the ratio of $X^{PP'}_{q_1 q_2 q_3}$ to $M^{PP'}_{q_1 q_2 q_3}$ predicted by the PQCD method is not of m_P^2 dependence such as the estimation by use of equation of motion. So while m_P is large, the distinctions between two methods are more obvious.

(iii) In many decay modes, the branching ratios are sensitive to the color parameter $N_c^{\rm eff}$, such as $\bar{B}^0_d \to \pi^0 \pi^0$, $\eta\eta(\eta'\eta')$, and $\eta\eta'$. Otherwise, the value of $N_c^{\rm eff}$ affects CP asymmetries more largely than branching ratios in some modes, for example, $\bar{B}^0_s \to K^0 \eta'$, in which the CP asymmetry ranges from 60.6 to -50.1% for $N_c^{\rm eff}$ ranging from 2 to ∞. This is because a_i are sensitive to $N_c^{\rm eff}$ which gives the different strong phases.

(iv) Our results are smaller than those in Refs. [5,13]. In some decay modes such as $B \to K\eta'$, our results are one order of magnitude smaller than the results of the experiments [10]. Because we did not consider the contributions of other mechanisms, such as $b \to sg^* \to s\eta'$ via QCD abnormaly [12], $b \to sgg \to s\eta'$ [11], etc. In Refs. [8,15], the authors gave the numerical results involved the contributions of the new mechanisms, which fit the experiments very well.

V. CONCLUDING REMARKS

In this paper, we recalculate the decays of B to two charmless pseudoscalar mesons with conventional method (the standard effective weak Hamiltonian and the BSW model). Instead of using equations of motion, we use an alternative method to estimate the hadronic matrix elements $(S+P)(S-P)$ and obtain comparatively smaller results. In some modes, which are penguin dominant, such as $B \to \pi K$, the branching ratios that we predicted seem to be a little bit smaller than the lower limits of the experiments of CLEO [10]. But they are derived in the factorization approach, many mechanisms are not considered in this work such as final state interactions. Especially in the modes of $B \to \pi K$ or KK, FSI could yield dominant contribution to the decay width [3]. So we need to study more uncertainties in nonleptonic charmless B decays in the future.

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