Beyond the adiabatic approximation: The impact of thresholds on the hadronic spectrum

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In the adiabatic approximation, most of the effects of $q\bar{q}$ loops on spectroscopy can be absorbed into a static interquark potential. I develop a formalism which can be used to treat the residual nonadiabatic effects associated with the presence of nearby hadronic thresholds for heavy quarks. I then define a potential which includes additional high energy corrections to the adiabatic limit which would be present for finite quark masses. This ''improved'' potential allows a systematic low energy expansion of the impact of thresholds on hadronic spectra. [S0556-2821(99)06315-8]

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I. INTRODUCTION

The valence quark model is surprisingly successful in describing mesons and baryons as $q\bar{q}$ and qqq systems moving in effective potentials. The surprise comes in part because hadrons are so strongly coupled to their (real and virtual) decay channels that each nearby channel ought to shift a hadron's mass by $\Delta m \sim \Gamma_{typical}$, thereby totally disrupting the valence quark model's spectroscopy.

A simple resolution of this conundrum has been proposed in a series of papers [1,2] examining the effects of ''unquenching the quark model,'' i.e., allowing extra $q\bar{q}$ pairs to bubble up in valence quark states. This bubbling dresses the valence hadrons with a certain class of meson loop diagrams [3]. [These papers also address how the Okubo-Zweig-Iizuka (OZI) rule [4] survives unquenching; in this paper I will exclusively consider flavor nonsinglet states for which such OZI-violation is not an issue.] The proposed resolution is an extension of the idea [5] that in the absence of light quarks the heavy quarkonium potential $V_0^{adiabatic}(r) \sim b_0 r$ is the adiabatically evolving ground state energy $E_0(r)$ of the purely gluonic QCD Hamiltonian in the presence of a static color triplet source Q and color anti-triplet sink $\bar{Q}$ separated by a distance r. Once n_f light quarks are introduced into this Hamiltonian, two major changes occur:

(1) $E_0(r)$ will be shifted to $E_{n_f}(r)$ by ordinary second order perturbation theory, and

(2) $E_{n_f}(r)$ will no longer be isolated from all other adiabatic surfaces: once pair creation can occur, the $Q\bar{Q}$ flux tube can break to create states $(Q\bar{q})_\alpha(q\bar{Q})_\beta$ with adiabatic energy surfaces that are constant in r at the values $\epsilon_\alpha + \epsilon_\beta$ (ϵ_i is the i^{th} eigenvalue of the $Q\bar{q}$ system, with the heavy quark mass m_Q subtracted).

Despite the latter complication, in the weak pair creation limit the flux-tube-like adiabatic surface $E_{n_f}(r)$ can be tracked through the level crossings that occur when $E_{n_f}(r) = \epsilon_\alpha + \epsilon_\beta$ and *identified as the renormalized $Q\bar{Q}$ adiabatic potential* $V_{n_f}^{adiabatic}(r) = E_{n_f}(r)$. In Ref. [1] it is shown that for large r, $V_{n_f}^{adiabatic}(r)$ remains linear, so that the net effect of the pairs is simply to renormalize the string tension. Since quark modelers determined their string tension from experiment, *the quark model heavy quarkonium potential already included the effect of meson loops to leading order in the adiabatic approximation, i.e., $b_{n_f} = b$, the physical string tension.*

Note that a similar renormalization occurs at short distances: in lowest order $\alpha_s^{(0)} \to \alpha_s^{(n_f)} = 12\pi/[(33 - 2n_f)\ln(Q^2/\Lambda^2_{QCD})]$. The renormalization of the string tension by $q\bar{q}$ loops is quite similar, though complicated by the existence of the open channels corresponding to adiabatic level crossings. It should also be stressed that the possibility of subsuming $q\bar{q}$ loops into b_{n_f} only occurs if one sums over a huge set of hadronic loop diagrams (real and virtual) [1]. No simple truncation of the sum over loops, as is often attempted in hadronic effective theories, is generally possible. Consider, for example, the simplest orbital splitting $a_2(1320) - \rho(770)$. Summing the mass shifts associated with the known decay modes of these states would significantly change their absolute masses and violently alter their splitting. Preserving them requires a large renormalization of the string tension and summing over loop graphs involving many high mass (i.e., virtual) channels, since $q\bar{q}$ creation inside the original $Q\bar{Q}$ state is dual to a very large tower of $(Q\bar{q})_\alpha(q\bar{Q})_\beta$ intermediate states.

Although the renormalization $V_0^{adiabatic} \to V_{n_f}^{adiabatic}$ will capture the bulk of the effect of ''unquenching'' in heavy quarkonia, $E_{n_f}(r)$ deviates quite substantially from linearity near level crossings [1] . Both this fact and explicit modelling suggest that for phenomenologically relevant quark masses substantial nonadiabatic effects will remain after renormalization, and in particular that states near thresholds to which they are strongly coupled should be expected to deviate from their potential model positions. This paper is devoted to developing a method for addressing these residual effects. This is straightforward as $m_Q \to \infty$, but I will show that for finite m_Q it is essential to go beyond the naive adiabatic approximation to define an ''improved'' interquark potential which includes the high energy part of the corrections to the adiabatic limit.

II. THE FORMALISM IN THE ADIABATIC LIMIT

To deal with violations of the adiabatic approximation, we can closely imitate the normal methods of mass renormalization. For very massive quarks Q and $\overline{Q}$, the effects of all hadronic loop graphs can be subsumed into

$$V_{n_f}^{adiabatic}(r)=V_0^{adiabatic}(r)+\sum_{\alpha\beta}\Delta V_{\alpha\beta}^{adiabatic}(r) \tag{1}$$

where $V_0^{adiabatic}(r)$ is the ''purely gluonic'' static $Q\overline{Q}$ potential, and $\Delta V_{\alpha\beta}^{adiabatic}(r)$ is the shift in this static potential generated by the channel $\alpha\beta$. Here the subscript on $V_0^{adiabatic}$ is used to denote that it *is* purely gluonic; we have suppressed additional labels to identify which gluonic adiabatic surface $V_0^{adiabatic}$ represents (the normal meson surface, the first $\Lambda=\pm 1$ hybrid surface, etc.) since our discussion applies identically to them all. For the low-lying thresholds of interest to us here, $\Delta V_{\alpha\beta}^{adiabatic}(r)$ will typically have a strength of order Λ_{QCD} and a range of order Λ_{QCD}^{-1}. This range arises because $\psi_\alpha(\vec{r}_{q\overline{Q}})$ and $\psi_\beta(\vec{r}_{qQ})$ are localized at relatively small $|\vec{r}_{\overline{q}Q}|$ and $|\vec{r}_{q\overline{Q}}|$ for low-lying states so that for large r the production of such states by the point-like creation of a $q\overline{q}$ pair is strongly damped by the rapidly falling tails of their confined wave functions; conversely, for small r the created q and $\overline{q}$ are easily accommodated into the ''heart'' of their respective wave functions.

Let us now compare the adiabatic Hamiltonian for the $Q\overline{Q}$ system [6]

$$H_{adiabatic}=\frac{p^2}{2\mu_{Q\overline{Q}}}+V_{n_f}^{adiabatic} \tag{2}$$

(with μ_{ij} the reduced mass of m_i and m_j) with the two channel Hamiltonian $H^{(\alpha\beta)}$ that is the penultimate step in generating $H_{adiabatic}$ in the sense that all channels *except* $\alpha\beta$ have been integrated out:

$$H^{(\alpha\beta)}=\begin{bmatrix} \frac{p^2}{2\mu_{Q\overline{Q}}}+V_{n_f}^{adiabatic}-\Delta V_{\alpha\beta}^{adiabatic} & H_{(\alpha\beta)}^{q\overline{q}} \\ H_{(\alpha\beta)}^{q\overline{q}} & \frac{p_\rho^2}{2\mu_{\alpha\beta}}+\epsilon_\alpha+\epsilon_\beta \end{bmatrix} \tag{3}$$

where $H_{(\alpha\beta)}^{q\overline{q}}$ is an interaction which couples the $Q\overline{Q}$ system to *the single channel* $(Q\overline{q})_\alpha(q\overline{Q})_\beta$ with the matrix elements dictated by the underlying pair creation Hamiltonian $H_{pc}^{q\overline{q}}$. In the adiabatic limit we must recover $H_{adiabatic}$ from $H^{(\alpha\beta)}$, but $H^{(\alpha\beta)}$ contains the full dynamics of the coupling of the $Q\overline{Q}$ system to the channel $\alpha\beta$. With the superscript on $H^{(\alpha\beta)}$ we are making explicit that $H^{(\alpha\beta)}$ has the channel $\alpha\beta$ removed from the $Q\overline{Q}$ adiabatic potential and added back in full via $H_{(\alpha\beta)}^{q\overline{q}}$. We could in general remove any subset of n channels from $V_{n_f}^{adiabatic}(r)$ and add them back in dynamically as part of an $(n+1)$-channel problem. In the limit of taking all channels we would recover the original full unquenched Hamiltonian. However, since our treatment is in lowest-order perturbation theory, the effects of the individual channels are additive, and Eq. (3) with just an individual channel $(\alpha\beta)$ selected for study is sufficient for our purposes.

Note that the hadronic multichannel version of our unquenched Hamiltonian is an appropriate representation of $q\overline{q}$ pair creation in a confined system. When the pair is created, the $(Q\overline{q}q\overline{Q})$ system has three relative coordinates which we may take to be $\vec{\rho}$, the separation between the center of mass of meson β and that of meson α, and the two intrameson coordinates $\vec{r}_{\overline{q}Q}\equiv\vec{r}_{\overline{q}}-\vec{r}_Q$ and $\vec{r}_{q\overline{Q}}\equiv\vec{r}_q-\vec{r}_{\overline{Q}}$. Since we ignore the residual final state interaction between the color singlets $(Q\overline{q})_\alpha$ and $(q\overline{Q})_\beta$, the eigenstates of this sector are mesons in relative plane waves, corresponding to the entry $H_{22}^{(\alpha\beta)}$ in Eq. (3). Thus, with $\vec{p}_\rho$ canonically conjugate to $\vec{\rho}$, the three quantum labels $(\vec{p}_\rho,\alpha,\beta)$ replace the three labels $(\vec{\rho},\vec{r}_{\overline{q}Q},\vec{r}_{q\overline{Q}})$.

The main goal of this paper is to describe the relation between the eigenvalues of the adiabatic Hamiltonian (2) and the dynamic Hamiltonian (3). If we define

$$H_0=\begin{bmatrix} \frac{p^2}{2\mu_{Q\overline{Q}}}+V_{n_f}^{adiabatic} & 0 \\ 0 & \frac{p_\rho^2}{2\mu_{\alpha\beta}}+\epsilon_\alpha+\epsilon_\beta \end{bmatrix} \tag{4}$$

and

$$H_{pert}=\begin{bmatrix} -\Delta V_{\alpha\beta}^{adiabatic} & H_{(\alpha\beta)}^{q\overline{q}} \\ H_{(\alpha\beta)}^{q\overline{q}} & 0 \end{bmatrix} \tag{5}$$

and denote the $Q\overline{Q}$ eigenvalues of H_0 and $H^{(\alpha\beta)}$ by E_i^0 and $E_i^{(\alpha\beta)}$, respectively, then since $H^{(\alpha\beta)}=H_0+H_{pert}$, by second order perturbation theory $\Delta E_i^{(\alpha\beta)}=E_i^{(\alpha\beta)}-E_i^0$ is given by

$$\Delta E_i^{(\alpha\beta)} = -\langle \psi_0^i | \Delta V_{\alpha\beta}^{adiabatic} | \psi_0^i \rangle + \int d^3q \frac{|\langle \alpha\beta(\vec{q}) | H_{(\alpha\beta)}^{q\bar{q}} | \psi_0^i \rangle|^2}{E_i^0 - \left(\epsilon_\alpha + \epsilon_\beta + \frac{q^2}{2\mu_{\alpha\beta}} \right)} \quad (6)$$

$$\equiv -\Delta E_i^{adiabatic(\alpha\beta)} + \Delta E_i^{dynamic(\alpha\beta)}, \quad (7)$$

where $|\psi_0^i\rangle$ is the i^{th} eigenstate of H_0. *This simple equation is the main focus of this paper.* It represents the correction to the adiabatic approximation for the $Q\bar{Q}$ energy eigenvalues from a full dynamical versus an adiabatic treatment of the channel $(\alpha\beta)$. In what follows I will first show explicitly that $\Delta E_i^{(\alpha\beta)} \rightarrow 0$ as expected in the limit $m_Q \rightarrow \infty$. I will then define an improved effective potential $V_{n_f}^{improved}$ which incorporates ''trivial'' high energy corrections to the adiabatic approximation, but which is essential for incorporating threshold effects in a systematic low energy expansion for finite m_Q.

I begin by defining precisely $\Delta V_{\alpha\beta}^{adiabatic}$ in Eq. (1). If $|\alpha\beta(\vec{\rho})\rangle$ denotes an $\alpha\beta$ state with relative coordinate $\vec{\rho}$, then as $m_Q \rightarrow \infty$

$$\langle \alpha\beta(\vec{\rho}) | H_{pc}^{q\bar{q}} | Q\bar{Q}(\vec{r}) \rangle \equiv \langle \alpha\beta(\vec{\rho}) | H_{(\alpha\beta)}^{q\bar{q}} | Q\bar{Q}(\vec{r}) \rangle = c_{\alpha\beta}(\vec{r})\, \delta^3(\vec{\rho} - \vec{r}) \quad (8)$$

since the $Q\bar{Q}$ relative coordinate is frozen in the adiabatic approximation by definition and since $\vec{r}_{\bar{Q}} - \vec{r}_Q \rightarrow \vec{\rho}$ as $m_Q \rightarrow \infty$. Thus

$$\langle Q\bar{Q}(\vec{r}\,') | \Delta V_{\alpha\beta}^{adiabatic} | Q\bar{Q}(\vec{r}) \rangle \equiv \int d^3\rho \frac{\langle Q\bar{Q}(\vec{r}\,') | H_{(\alpha\beta)}^{q\bar{q}} | \alpha\beta(\vec{\rho}) \rangle \langle \alpha\beta(\vec{\rho}) | H_{(\alpha\beta)}^{q\bar{q}} | Q\bar{Q}(\vec{r}) \rangle}{br - (\epsilon_\alpha + \epsilon_\beta)} \quad (9)$$

$$= \delta^3(\vec{r}\,' - \vec{r}) \frac{|c_{\alpha\beta}(\vec{r})|^2}{br - (\epsilon_\alpha + \epsilon_\beta)} \quad (10)$$

$$\equiv \delta^3(\vec{r}\,' - \vec{r})\, \Delta V_{\alpha\beta}^{adiabatic}(\vec{r}) \quad (11)$$

for br far from $\epsilon_\alpha + \epsilon_\beta$; see Ref. [1] for a discussion of how the poles in Eq. (10) are to be handled. Given this $\Delta V_{\alpha\beta}^{adiabatic}(\vec{r})$, by definition

$$\Delta E_i^{adiabatic(\alpha\beta)} = \langle \psi_0^i | \Delta V_{\alpha\beta}^{adiabatic} | \psi_0^i \rangle \quad (12)$$

$$= \int d^3r \frac{|\psi_0^i(\vec{r})|^2 |c_{\alpha\beta}(\vec{r})|^2}{br - (\epsilon_\alpha + \epsilon_\beta)}. \quad (13)$$

I now show how $\Delta E_i^{adiabatic(\alpha\beta)}$ approximates the true shift

$$\Delta E_i^{dynamic(\alpha\beta)} \equiv \int d^3q \frac{|\langle \alpha\beta(\vec{q}) | H_{(\alpha\beta)}^{q\bar{q}} | \psi_0^i \rangle|^2}{E_i^0 - \left(\epsilon_\alpha + \epsilon_\beta + \frac{q^2}{2\mu_{\alpha\beta}} \right)} \quad (14)$$

even for ''nearby'' thresholds as $m_Q \rightarrow \infty$. Denote by $\langle v \rangle_i$ the expectation value of the variable v in the state $|\psi_0^i\rangle$. In the limit $m_Q \rightarrow \infty$, each of $\langle p^2 \rangle_i / 2\mu_{Q\bar{Q}}$, $b\langle r \rangle_i$, and $q^2/2\mu_{\alpha\beta}$ vanishes like $(\Lambda_{QCD}/m_Q)^{1/3}\Lambda_{QCD}$ and so is small compared to $\epsilon_\alpha + \epsilon_\beta$, which is of order Λ_{QCD}, but large compared to the corrections to $\epsilon_\alpha + \epsilon_\beta$, which are of order Λ_{QCD}/m_Q. [In the general power law potential $c_n r^n$, they each behave like $(\Lambda_{QCD}/m_Q)^{n/n+2}\Lambda_{QCD}$, i.e., they vanish for any confining $(n>0)$ potential.] For $q^2/2\mu_{\alpha\beta}$, this statement is nontrivial: it relies on the behavior of the numerator of Eq. (14). Using Eq. (8),

$$\langle \alpha\beta(\vec{q}) | H_{(\alpha\beta)}^{q\bar{q}} | Q\bar{Q}(\vec{p}) \rangle = \frac{1}{(2\pi)^3} \int d^3r\, e^{i(\vec{p} - \vec{q}) \cdot \vec{r}} c_{\alpha\beta}(\vec{r}) \quad (15)$$

$$\equiv \tilde{c}_{\alpha\beta}(\vec{p} - \vec{q}), \quad (16)$$

so, even though $|\vec{p}| \sim (\Lambda_{QCD}^2 m_Q)^{1/3} \rightarrow \infty$, $|\vec{p} - \vec{q}|$ must be of order Λ_{QCD} since $\tilde{c}_{\alpha\beta}$ is a light quark object. After writing $E_i^0 = \langle p^2 \rangle_i / 2\mu_{Q\bar{Q}} + b\langle r \rangle_i$, we can therefore Taylor series expand:

$$\Delta E_i^{dynamic(\alpha\beta)} \simeq -\left(\frac{1}{\epsilon_\alpha + \epsilon_\beta} \right) \int d^3q |\langle \alpha\beta(\vec{q}) | H_{(\alpha\beta)}^{q\bar{q}} | \psi_0^i \rangle|^2 \left(1 + \left(\frac{1}{\epsilon_\alpha + \epsilon_\beta} \right) \left[\frac{\langle p^2 \rangle_i - q^2}{2\mu_{Q\bar{Q}}} + b\langle r \rangle_i \right] + \cdots \right) \quad (17)$$

$$\simeq -\left(\frac{1}{\epsilon_\alpha + \epsilon_\beta} \right) \int d^3p' \int d^3q \int d^3p\, \phi_0^{i*}(\vec{p}\,') \tilde{c}_{\alpha\beta}^*(\vec{p}\,' - \vec{q}) \tilde{c}_{\alpha\beta}(\vec{p} - \vec{q}) \phi_0^i(\vec{p}) \times \left(1 + \left(\frac{1}{\epsilon_\alpha + \epsilon_\beta} \right) \left[\frac{\langle p^2 \rangle_i - q^2}{2\mu_{Q\bar{Q}}} + b\langle r \rangle_i \right] + \cdots \right), \quad (18)$$

where $\phi_0^i(\vec{p}) \equiv 1/(2\pi)^{3/2} \int d^3r\, e^{-i\vec{p} \cdot \vec{r}} \psi_0^i(\vec{r})$. Noting that

$$\int d^3s\tilde{c}_{\alpha\beta}(\vec{s})=c_{\alpha\beta}(\vec{0}) \tag{19}$$

and that except for the $\tilde{c}$ the factors of the integrand are slowly varying functions, we can approximate

$$\tilde{c}_{\alpha\beta}(\vec{s})\simeq c_{\alpha\beta}(\vec{0})\,\delta^3(\vec{s})+\cdots \tag{20}$$

to obtain

$$\Delta E_i^{dynamic(\alpha\beta)}\simeq-\left(\frac{|c_{\alpha\beta}(\vec{0})|^2}{\epsilon_\alpha+\epsilon_\beta}\right)\int d^3p|\phi_0^i(\vec{p})|^2\left(1+\left(\frac{1}{\epsilon_\alpha+\epsilon_\beta}\right)\left[\frac{\langle p^2\rangle_i-p^2}{m_Q}+b\langle r\rangle_i\right]+\cdots\right) \tag{21}$$

$$\simeq-\left(\frac{|c_{\alpha\beta}(\vec{0})|^2}{\epsilon_\alpha+\epsilon_\beta}\right)\int d^3p|\phi_0^i(\vec{p})|^2\left[1+\left(\frac{br}{\epsilon_\alpha+\epsilon_\beta}\right)\right] \tag{22}$$

$$\simeq\int d^3p\frac{|\phi_0^i(\vec{p})|^2|c_{\alpha\beta}(\vec{0})|^2}{br-(\epsilon_\alpha+\epsilon_\beta)} \tag{23}$$

$$\simeq\int d^3r\frac{|\psi_0^i(\vec{r})|^2|c_{\alpha\beta}(\vec{0})|^2}{br-(\epsilon_\alpha+\epsilon_\beta)}. \tag{24}$$

This expression differs slightly from $\Delta E_i^{adiabatic(\alpha\beta)}$ in Eq. (13): it has $|c_{\alpha\beta}(\vec{r})|^2\rightarrow|c_{\alpha\beta}(\vec{0})|^2$. However, for low-lying states, $\langle|c_{\alpha\beta}(\vec{r})|^2/|c_{\alpha\beta}(\vec{0})|^2\rangle_i-1\sim(\Lambda_{QCD}/m_Q)^{2/3}$ which is negligible as $m_Q\rightarrow\infty$ compared to $\langle br/(\epsilon_\alpha+\epsilon_\beta)\rangle_i\sim(\Lambda_{QCD}/m_Q)^{1/3}$ which we retained. [The physics behind this approximation is simply that $|c_{\alpha\beta}(\vec{r})|^2$ reflects light quark scales while $|\psi_0^i(\vec{r})|^2$ reflects short distance scales as $m_Q\rightarrow\infty$.] Thus to leading order as $m_Q\rightarrow\infty$, $\Delta E_i^{adiabatic(\alpha\beta)}=\Delta E_i^{dynamic(\alpha\beta)}$ for low-lying states, as we set out to prove.

The deviation $\Delta E_i^{(\alpha\beta)}$ of the energy of the state i from its value in the adiabatic potential $V_{n_f}^{adiabatic}$ due to the residual dynamical effects of the channel $(Q\bar{q})_\alpha(q\bar{Q})_\beta$ thus has the property that $\Delta E_i^{(\alpha\beta)}\rightarrow 0$ channel by channel as $m_Q\rightarrow\infty$. The utility of this formalism for heavy quarkonium arises from not only this property, but also that it allows one to consistently focus on low-lying thresholds. The latter feature is based on the fact that the full shift $\Delta E_i\equiv\Sigma_{\alpha\beta}\Delta E_i^{(\alpha\beta)}$ may receive significant ''random'' contributions from strategically placed low mass channels, but for fixed large m_Q the $\Delta E_i^{(\alpha\beta)}\rightarrow 0$ rapidly as $\epsilon_\alpha+\epsilon_\beta$ gets large. This rapid convergence occurs because, first, as $\epsilon_\alpha+\epsilon_\beta\rightarrow\infty$, E_i^0 and $q^2/2\mu_{\alpha\beta}$ in Eq. (14) and br in Eq. (13), which were already small with respect to $\epsilon_\alpha+\epsilon_\beta$ even for low mass channels, become negligible. In this limit $\Delta E_i^{(\alpha\beta)}$ is therefore trivially zero independent of the accuracy of the approximations inherent in Eqs. (17)–(24). Moreover, the factor $|c_{\alpha\beta}(\vec{r})|^2$ in the numerator of each of $\Delta E_i^{adiabatic(\alpha\beta)}$ and $\Delta E_i^{dynamic(\alpha\beta)}$ rapidly approaches zero as $\epsilon_\alpha+\epsilon_\beta\rightarrow\infty$ since for low-lying states $|\psi_i^0\rangle$ there is little kinetic energy in the initial wave function and the pair creation process can only create momenta of order Λ_{QCD}. We can therefore expect this formalism to provide a rapidly converging low-energy expansion of the effects of pair creation on heavy quarkonia.

III. AN IMPROVED QUARKONIUM POTENTIAL

While suitable for heavy quarkonia, the framework of Sec. II has serious shortcomings for light quark spectroscopy. Because the eigenvalues $\epsilon_{\underline{\alpha}}^{(m_Q)}$ and $\epsilon_\beta^{(m_Q)}$ and the matrix elements $\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle$ for finite m_Q are only qualitatively related to their $m_Q\rightarrow\infty$ counterparts ϵ_α, ϵ_β, and $\langle\alpha\beta(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle$, $\Delta V_{\alpha\beta}^{adiabatic}(r)$ is not in this case an accurate approximation to the effects of the channel $(Q\bar{q})_\alpha(q\bar{Q})_\beta$. As a result, the $\Delta E_i^{(\alpha\beta)}$ will not be small, i.e., the critical separation of the effects of the channel $\alpha\beta$ into large adiabatic and small residual dynamical effects will fail. If it were only for a few nearby channels, this failure would not be so serious, but while E_i^0 and $q^2/2\mu_{\alpha\beta}$ in the finite m_Q analogue of Eq. (14) and br in Eq. (13) can still be neglected as $\epsilon_\alpha+\epsilon_\beta\rightarrow\infty$, since $\epsilon_{\underline{\alpha}}^{(m_Q)}+\epsilon_{\underline{\beta}}^{(m_Q)}\neq\epsilon_\alpha+\epsilon_\beta$ and $\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle\neq\langle\alpha\beta(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle$, the finite m_Q analogue of $\Delta E_i^{dynamical(\alpha\beta)}$ will not trivially approach $\Delta E_i^{adiabatic(\alpha\beta)}$ in this high energy limit. Moreover, while the matrix elements $\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle$ may still be expected to cut off high mass channels, since the momenta in low mass states and in the pair creation process are comparable, these channels will be cut off more slowly in the finite m_Q analogue of $\Delta E_i^{dynamical(\alpha\beta)}$ than in $\Delta E_i^{adiabatic(\alpha\beta)}$. These shortcomings make the $m_Q\rightarrow\infty$ framework far less useful in light quark systems since $\Delta E_i\equiv\Sigma_{\alpha\beta}\Delta E_i^{(\alpha\beta)}$ will converge only marginally faster than the ''brute force'' sum $\Sigma_{\alpha\beta}\Delta E_i^{dynamic(\alpha\beta)}$.

I will now show that it is possible to define an improved effective quarkonium potential $V_{n_f}^{improved}$ which leads to energy shifts $\delta E_i^{(\alpha\beta)}$ which vanish as $\epsilon_\alpha+\epsilon_\beta\rightarrow\infty$ *for any* m_Q

and so give smaller and more rapidly converging corrections to the quark model spectroscopy built on $V_{n_f}^{improved}$ than the $\Delta E_i^{(\alpha\beta)}$. The price to be paid for this important feature is that the universal (flavor-independent) adiabatic quarkonium potential $V_{n_f}^{adiabatic}$ must be replaced by a flavor-dependent effective potential $V_{n_f}^{improved}$ built out of $V_0^{adiabatic}$ plus flavor-dependent contributions $\Delta V_{\alpha\beta}^{improved}$.

The basic idea is very simple. For any m_Q [6], the shift in the energy of the state $|\psi_0^{i(m_Q)}\rangle$ due to channel $\alpha\beta$ is given by the generalization of Eq. (14), namely

$$\Delta E_i^{dynamic(\alpha\beta)(m_Q)} \equiv \int d^3q \frac{|\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})|H_{(\alpha\beta)}^{q\bar{q}}|\psi_0^{i(m_Q)}\rangle|^2}{E_i^{0(m_Q)} - \left(\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)} + \dfrac{q^2}{2\mu_{\alpha\beta}}\right)} \tag{25}$$

where the superscripts (m_Q) denote quantities at finite m_Q in contrast to those previously defined for $m_Q \to \infty$.

I begin by examining the limit [9] $\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)} \gg E_i^{0(m_Q)}$ and $q^2/2\mu_{\alpha\beta}$, each of which are for m_Q comparable to Λ_{QCD} themselves of order Λ_{QCD}. In this limit we have

$$\Delta E_i^{dynamic(\alpha\beta)(m_Q)} \xrightarrow{\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)} \to \infty} \langle \psi_0^{i(m_Q)}|\Delta \tilde{V}_{\alpha\beta}^{(m_Q)}|\psi_0^{i(m_Q)}\rangle \tag{26}$$

where

$$\Delta \tilde{V}_{\alpha\beta}^{(m_Q)} = \frac{-1}{\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)}} \int d^3q H_{(\alpha\beta)}^{q\bar{q}} |\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})\rangle\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})| H_{(\alpha\beta)}^{q\bar{q}} \tag{27}$$

is an m_Q-dependent but $|\psi_0^{i(m_Q)}\rangle$-independent effective potential operator. Thus

$$\langle Q\bar{Q}(\vec{r}\,')|\Delta \tilde{V}_{\alpha\beta}^{(m_Q)}|Q\bar{Q}(\vec{r})\rangle = \frac{-1}{\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)}} \int d^3q \langle Q\bar{Q}(\vec{r}\,')|H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})\rangle\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})|H_{(\alpha\beta)}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle \tag{28}$$

$$= \frac{-1}{\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)}} \int d^3q d^3\rho' d^3\rho \langle Q\bar{Q}(\vec{r}\,')|H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho}\,')\rangle \times \frac{e^{i\vec{q}\cdot(\vec{\rho}\,'-\vec{\rho})}}{(2\pi)^3}\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{(\alpha\beta)}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle \tag{29}$$

$$= \frac{-1}{\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)}} \int d^3\rho \langle Q\bar{Q}(\vec{r}\,')|H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})\rangle\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{(\alpha\beta)}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle \tag{30}$$

which can be compared to Eq. (9). I next introduce the finite m_Q analogue of Eq. (8). As $m_Q \to \infty$, the form of Eq. (8) is model independent, with dynamical effects residing in $c_{\alpha\beta}(\vec{r})$. However, for finite m_Q even the form of the analogue to Eq. (8) becomes model-dependent. The key to extending the utility of the heavy quarkonium framework down to light quark masses is to make a "local approximation"

$$\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle \simeq c_{\alpha\beta}^{(m_Q)}(\vec{r})\,\delta^3(\vec{\rho} - \eta\vec{r}) \tag{31}$$

where as $m_Q \to \infty$, $\eta \to 1$ and

$$c_{\alpha\beta}^{(m_Q)}(\vec{r}) \to c_{\alpha\beta}(\vec{r}), \tag{32}$$

the right hand side being the function defined in the adiabatic limit by Eq. (8). Note that $c_{\alpha\beta}^{(m_Q)}(\vec{r})$ involves at the microscopic level overlap integrals between $|Q\bar{Q}(\vec{r})\rangle$ and $|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})\rangle$ with wave functions $\psi_\alpha^{(m_Q)}(\vec{r}_{q\bar{Q}})$ and $\psi_\beta^{(m_Q)}(\vec{r}_{\bar{q}Q})$ for finite m_Q, while $c_{\alpha\beta}(\vec{r})$ involves the heavy quark limits $\psi_\alpha(\vec{r}_{q\bar{Q}})$ and $\psi_\beta(\vec{r}_{\bar{q}Q})$ of these wave functions. In the simplest and most common models [1,7,8], the "local approximation" is exact and automatic with $\eta = m_Q/(m_Q + m_q)$, corresponding to $q\bar{q}$ pair creation that is pointlike and instantaneous. There are, of course, other possibilities, both local and nonlocal [10]; the latter need to be ''localized'' *via* an approximation of the form of Eq. (31) in order that one may define their improved effective potentials.

With Eq. (31) used in Eq. (30), we have

$$\langle Q\bar{Q}(\vec{r}\,')|\Delta\tilde{V}^{(m_Q)}_{\alpha\beta}|Q\bar{Q}(\vec{r})\rangle=\delta^3(\vec{r}\,'-\vec{r})\left[\frac{-\eta^{-3}|c^{(m_Q)}_{\alpha\beta}(\eta\vec{r})|^2}{\epsilon^{(m_Q)}_{\alpha}+\epsilon^{(m_Q)}_{\beta}}\right] \tag{33}$$

$$\equiv\delta^3(\vec{r}\,'-\vec{r})\Delta\tilde{V}^{improved}_{\alpha\beta}(\vec{r}). \tag{34}$$

By construction, $\Delta\tilde{V}^{improved}_{\alpha\beta}$ gives a $\Delta E_i^{improved(\alpha\beta)}$ which converges to $\Delta E_i^{dynamic(\alpha\beta)(m_Q)}$ for models in which the local approximation (31) is exact. In all cases [10], once the local approximation (31) is made, $\Delta\tilde{V}^{improved}_{\alpha\beta}$ will give a more accurate approximation to the high energy effects of $H^{q\bar{q}}_{pc}$ than $\Delta V^{adiabatic}_{\alpha\beta}(\vec{r})$, so its use leads to a more rapidly converging approximation to the effects of thresholds on the hadronic system. We can actually improve matters even further if we incorporate the additional convergence available even for low mass thresholds as $m_Q\to\infty$ by defining [9]

$$\Delta V^{improved}_{\alpha\beta}(\vec{r})\equiv\left[\frac{\eta^{-3}|c^{(m_Q)}_{\alpha\beta}(\eta\vec{r})|^2}{br-(\epsilon^{(m_Q)}_{\alpha}+\epsilon^{(m_Q)}_{\beta})}\right]. \tag{35}$$

For $\epsilon^{(m_Q)}_{\alpha}+\epsilon^{(m_Q)}_{\beta}$ large, br is negligible so this approximation is no worse than that leading to $\Delta\tilde{V}^{improved}_{\alpha\beta}(\vec{r})$, but $\Delta V^{improved}_{\alpha\beta}(\vec{r})$ also approaches our old $\Delta V^{adiabatic}_{\alpha\beta}(\vec{r})$ as $m_Q\to\infty$ and so gives a good approximation to $\Delta E_i^{dynamic(\alpha\beta)(m_Q)}$ for all $\alpha\beta$ in this limit.

Given these features, we can improve upon Eqs. (2) and (3) by defining

$$H_{improved}=\frac{p^2}{2\mu_{Q\bar{Q}}}+V^{improved}_{n_f} \tag{36}$$

and

$$H^{(\alpha\beta)}_{improved}=\begin{bmatrix}\frac{p^2}{2\mu_{Q\bar{Q}}}+V^{improved}_{n_f}-\Delta V^{improved}_{\alpha\beta} & H^{q\bar{q}}_{(\alpha\beta)}\\ H^{q\bar{q}}_{(\alpha\beta)} & \frac{p_\rho^2}{2\mu_{\alpha\beta}}+\epsilon^{(m_Q)}_{\alpha}+\epsilon^{(m_Q)}_{\beta}\end{bmatrix} \tag{37}$$

along with the analogue of Eq. (7)

$$\delta E_i^{(\alpha\beta)}\equiv-\langle\psi_0^{i(m_Q)}|\Delta V^{improved}_{\alpha\beta}|\psi_0^{i(m_Q)}\rangle+\Delta E_i^{dynamic(\alpha\beta)(m_Q)}. \tag{38}$$

The $\delta E_i^{(\alpha\beta)}$ now approach zero both in the strict adiabatic limit $m_Q\to\infty$ for all $\alpha\beta$ and also in the limit $\epsilon^{(m_Q)}_{\alpha}+\epsilon^{(m_Q)}_{\beta}\gg\Lambda_{QCD}$ for all m_Q. They therefore allow a systematic *low energy* expansion of the impact of thresholds on the spectra of *all* quarkonia.

IV. CONCLUSIONS

I have presented here a formalism for calculating the nonadiabatic component $\Delta E_i^{\alpha\beta}$ of the mass shift of a valence heavy $Q\bar{Q}$ state i from the hadronic loop process $i\to\alpha\beta\to i$, i.e., the component of this process that cannot be absorbed into the renormalized heavy quarkonium potential. The resulting formula was shown to have the expected property that $\Delta E_i^{(\alpha\beta)}\to 0$ as $m_Q\to\infty$. The formula is also very simple and, when combined with a pair creation model like the flux-tube-breaking model [7] or the $^3P_0$ model [8], should provide a quick method of estimating the influence of nearby thresholds on the spectra of heavy quarkonia.

I have also shown how to define an ''improved'' quarkonium potential which incorporates nonadiabatic effects associated with high mass thresholds for any m_Q. When this potential is identified with the empirical quark model potential, the deviations $\delta E_i^{(\alpha\beta)}$ of the spectrum from the potential model predictions due to thresholds have the property that they vanish both as $m_Q\to\infty$ for all $\alpha\beta$ and also as the mass $\epsilon^{(m_Q)}_{\alpha}+\epsilon^{(m_Q)}_{\beta}$ of the threshold $\alpha\beta$ gets large for any m_Q. This improved potential therefore allows a systematic low energy expansion of the impact of thresholds on hadronic spectra.

This advantage has a price: the ''improved'' potential has the characteristic that it violates the rule of flavor independence. While this rule is valid in the heavy quark limit and to leading order in perturbative QCD for light quarks, violations are to be expected. Indeed, though obscured by possible relativistic corrections, there are indications from quark models that the best empirical potentials are system-dependent [11].

An important step not taken here is to calculate the $\Delta E_i^{(\alpha\beta)}$ and $\delta E_i^{(\alpha\beta)}$ for selected channels to assess numerically how rapidly each converges as $\epsilon^{(m_Q)}_{\alpha}+\epsilon^{(m_Q)}_{\beta}\to\infty$ [9], and to quantify the m_Q-dependence of $V^{improved}_{n_f}$. Quark models seem to constrain this mass dependence to be surprisingly weak [12]. Assuming that the approach defined here passes quantitative tests such as these, it will then be interesting to apply it to a number of outstanding phenomenological issues. Among these are the threshold shifts in the $c\bar{c}$ and $b\bar{b}$ systems and the $\Lambda(1520)-\Lambda(1405)$ problem. It will also be amusing to study heavy-light systems to see explicitly how groups of states conspire to maintain the spectroscopic relations required by heavy quark symmetry [13] as $m_Q \to\infty$, and to quantify the importance of heavy-quark-symmetry-breaking pair creation effects residing in the $\delta E_i^{(\alpha\beta)}$ compared to their valence potential model counterparts [14].

Finally, I note that while this paper is couched in the language of the nonrelativistic quark model, there is nothing in the proposed general framework that would prevent its being transferred to either a relativistic quark model or to field theory.

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[1] P. Geiger and N. Isgur, Phys. Rev. D **41**, 1595 (1990).

[2] P. Geiger and N. Isgur, Phys. Rev. D **44**, 799 (1991); Phys. Rev. Lett. **67**, 1066 (1991); Phys. Rev. D **47**, 5050 (1993); P. Geiger, *ibid.* **49**, 6003 (1993); P. Geiger and N. Isgur, *ibid.* **55**, 299 (1997).

[3] Lowest order meson loop Feynman graphs can have two distinct topologies at the quark level: those with a closed $q\bar{q}$ loop and those where only the (relativistically) propagating valence lines appear. Both types of graphs, when analyzed in a hadronic basis, have mesons ''in the air.'' In mesons, the latter class of graphs either correspond to η' meson loops (and so are dual to Z graphs) or to direct ''s-channel'' $\omega-\phi$-mixing-type OZI violations [2]. In baryons, such graphs correspond to a variety of meson loops, but all are dual to Z graphs. Given these dualities, one can imagine that non-closed-$q\bar{q}$-loop diagrams are subsumed into the valence quark model. We focus here on the pure $q\bar{q}$ loop processes which are not otherwise taken into account in the quark model.

[4] S. Okubo, Phys. Lett. **5**, 165 (1963); Phys. Rev. D **16**, 2336 (1977); G. Zweig, CERN Report No. 8419 TH 412, 1964; reprinted in *Developments in the Quark Theory of Hadrons*, edited by D. B. Lichtenberg and S. P. Rosen (Hadronic, Nonantum, Massachusetts, 1980); J. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. **35**, 1061 (1966); J. Iizuka, Prog. Theor. Phys. Suppl. **37-38**, 21 (1966).

[5] For a review of the adiabatic approximation in the context of the flux tube model [based on N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985) and Refs. [1,2]], see N. Isgur, in *Proceedings of Few Body Problems in Physics*, Williamsburg, VA, 1994, AIP Conf. Proc. No. 334 edited by F. Gross, (AIP, New York, 1995), p. 3. For some early discussions of incorporating the nonperturbative effects of $q\bar{q}$ loops (which were all in the context of $c\bar{c}$ spectroscopy and decays), see J. Kogut and L. Susskind, Phys. Rev. Lett. **34**, 767 (1975); E. Eichten *et al.*, *ibid.* **36**, 500 (1976); and E. C. Poggio and H. J. Schnitzer, *ibid.* **41**, 1344 (1978). These discussions are all different, and each introduces essential elements of the adiabatic approximation adopted here.

[6] The considerations of this paper are trivially extended to mesons $Q_1\bar{Q}_2$ with arbitrary masses m_1 and m_2.

[7] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).

[8] L. Micu, Nucl. Phys. **B10**, 521 (1969); Jonathan L. Rosner, Phys. Rev. Lett. **13**, 689 (1969); E. W. Colglazier and J. L. Rosner, Nucl. Phys. **B27**, 349 (1971); W. P. Peterson and J. L. Rosner, Phys. Rev. D **6**, 820 (1972); **7**, 747 (1973); A. Le Yaouanc, L. Oliver, O. Pene, and J.-C. Raynal, *ibid.* **8**, 2233 (1973); Phys. Lett. **71B**, 397 (1977); **72B**, 57 (1977); W. Roberts and B. Silvestre-Brac, Few-Body Syst. **11**, 171 (1992); P. Geiger and E. S. Swanson, Phys. Rev. D **50**, 6855 (1994); Fl. Stancu and P. Stassart, *ibid.* **38**, 233 (1988); **39**, 343 (1989); **41**, 916 (1990); **42**, 1521 (1990); S. Capstick and W. Roberts, *ibid.* **47**, 1994 (1993); **49**, 4570 (1994).

[9] One could instead expand Eq. (14) in $\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+q^2/2\mu_{\alpha\beta}$. This would lead to a Yukawa-like nonlocality in $|\vec{\rho}\,'-\vec{\rho}|$ which, using a local approximation to this nonlocality and an analogue to Eq. (31), would give an alternative definition of $\Delta V_{\alpha\beta}^{improved}(\vec{r})$. This version of $\Delta V_{\alpha\beta}^{improved}(\vec{r})$ would not contain the adiabatic limit as $m_Q\to\infty$, and so probably converges more slowly than Eq. (35).

[10] If the pair creation occurs by creating a gap of classical length $2m_d/b$ in the flux tube, then $\eta=m_Q/(m_Q+m_q)+2m_q^2/(m_Q+m_q)br$. In other models the pair creation might be intrinsically nonlocal (for an example, see Ref. [1]); in such cases the definition of $\Delta V_{\alpha\beta}^{improved}$ requires that one make a local approximation of the form of Eq. (31) to the model's actual matrix element $\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle$.

[11] The explicit considerations given here for mesons are trivially extended to baryons and to both meson and baryon hybrids. There is no reason to expect exact equality of the ''improved'' string tensions in these diverse systems. Indeed, S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986) note that the baryon string tension seems to differ by about 20% from the meson string tension.

[12] See, for example, the unified description of mesons in S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[13] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991).

[14] For an initial study of the relative importance of threshold and valence effects on spectroscopy, see N. Isgur, Phys. Rev. D **57**, 4041 (1998) where the influence of low-lying S-wave thresholds on the $s_l^{\pi_l}=\frac{1}{2}^+$ and $\frac{3}{2}^+$ heavy quark mesons is considered.