PHYSICAL REVIEW D, VOLUME 60, 053010

Radiative neutrino decay in media

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(Received 25 March 1999; published 10 August 1999)

In this paper we determine the decay rate of a heavy neutrino into a lighter one by the emission, or the absorption, of a photon in the presence of an ultrarelativistic heat bath. Our method is based on the generalization of the optical theorem at finite temperature and density. Differently from previous works on this subject, our approach allows one to account for dispersive and dissipative electromagnetic properties of the medium. Some inconsistencies that are present in the literature on the subject are pointed out and corrected here. We shortly discuss the relevance of our results for neutrino evolution in the early universe. [S0556-2821(99)00617-7]

PACS number(s): 13.35.Hb

I. INTRODUCTION

Most neutrinos in the universe are expected to be produced in the presence of environments characterized by very high temperatures and densities. In such media neutrinos behave quite differently from the way they do in vacuum [1]. The coherent interaction of neutrinos with the particles belonging to the medium can amplify or induce effects that are otherwise very feeble or absent in the vacuum. This can be crucial in order to disentangle possible anomalous properties of the neutrinos that may testify for new physics present beyond the standard model. Mikheyev-Smirnov-Wolfenstein (MSW) [2] neutrino resonant oscillations give a well-known example of such a kind of effect.

Neutrino electromagnetic properties are also expected to be significantly affected by the presence of media. In fact, because of their coherent interaction with charged leptons and nucleons in the medium, neutrinos acquire an effective coupling to the electromagnetic field. This effect was investigated by several authors who showed how even massless neutrinos passing through the matter acquire an effective charge [3] and, in a charge asymmetric medium, also an effective magnetic dipole moment [4]. Several intriguing consequences of medium induced neutrino electromagnetic couplings have been studied in the literature, e.g., plasmon decay in $\nu\bar{\nu}$ pairs [5] and Cherenkov emission of neutrinos [6].

In this paper we focus on the radiative decay of a heavy neutrino into a lighter one. Our underling particle physics model consists of a minimal extension of the standard model which allows for nonvanishing neutrino masses and mixing. In the ambit of this model, neutrino radiative decay is extremely suppressed in vacuum because of the Glashow-Iliopoulos-Maiani (GIM) cancellation [7]. However, if $T < m_\tau$, this is not the case in a heat bath due to the different thermal populations of the three electrically charged leptons. As a consequence, the radiative neutrino decay rate in the presence of hot media exceeds the decay rate in vacuum by many order of magnitudes. This was first showed by D'Olivo, Nieves, and Pal (DNP) [8] (see also Ref. [9]), who computed the decay rate both in the case of a nonrelativistic plasma (NR) ($T \ll m_e$) and of an ultrarelativistic plasma (UR). Recently, DNP's work has been extended by Nieves and Pal [10] (NP) who showed that the decay rate may be further enhanced due to the Bose-Einstein stimulation in the production of low-energy photons ($\omega \ll T$).

In this paper we reconsider critically the work reported in Refs. [8,10]. Although we agree with DNP and NP concerning the huge neutrino decay rate amplification occurring in a medium we do not, however, agree with the method and part of the results presented in their papers. Our main disagreement concerns the properties of the photon produced by the neutrino decay that was assumed to be on the light-cone ($K^2=0$) by DNP and NP. Such an assumption is equivalent to disregard any dispersive property of photons in the thermal bath. It is well known, however, that in the presence of a plasma photons behaves more similar to collective excitations (usually dubbed *plasmons*) than free particles. While neglecting collective properties of photons may be a reasonable approximation in the high frequency regime this is, however, inconsistent when photons are low energy or, so-called, soft. Below we will point out some unphysical results which come out by neglecting plasma effects in the soft region and show how such a problem is solved by accounting for the correct photon dispersion relation.

Photon dissipative properties need also to be taken into account. In a heat bath low-energy photons are continuously absorbed and re-emitted, mainly by Landau damping, on the heat-bath free charges. As a consequence, in such environment the photon asymptotic state is a ill-defined quantity. To avoid such a problem we recourse here to a treatment that is based on finite temperature field theory and on the generali-

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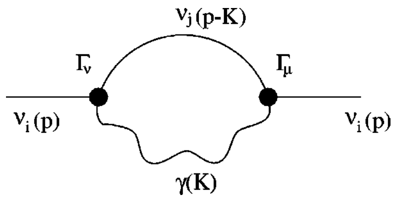

FIG. 1. Feynman diagram contributing to the heavy neutrino self-energy Σ.

zation of the optical theorem to such a framework [11]. The basic quantities in this approach are the Green functions which have the advantage to be well defined quantities even at finite temperature and density [12]. We will show that following this method the effect of the nontrivial plasmon dispersion and the role of the Landau damping spring out naturally from our equations. A similar approach has been previously applied by other authors to determine the chirality-flip rate of Dirac neutrinos in a degenerate plasma [13,14] and in the early universe [15].

In this paper we only consider the case of an UR nondegenerate electron-positron plasma which may be relevant for the study of neutrino evolution in the early universe. Although we find an additional contribution to heavy neutrino decay given by the presence of electromagnetic thermal fluctuations in the plasma, we show that conventional weak processes are generally dominant. This situation may, however, change if one consider larger extensions of the standard model which allow, for example, for the presence of large transition magnetic moment of the neutrinos. Quite independently from these considerations, we hope that our results and the methods used in our work will help to avoid further mistakes in the subject and be useful to study similar processes taking place in the presence of high density plasma.

The paper is organized as follows. In Sec. II we present our general method. Section III contains the computation of the decay rate in the case in which the plasmon is on shell. The generalization of this result to the case with the off-shell plasmon is discussed in Sec. IV. Finally in Sec. V we briefly discuss the relevance of our result for neutrino evolution in the early universe and summarize our conclusions.

II. THE GENERAL METHOD

It was first shown by Weldon [11] that in a heat-bath the decay rate Γ_d of a fermionic particle species (having the energy E), and the rate of the inverse process Γ_i, are related to the imaginary part of the fermion self-energy Σ by the following relation:

$$\mathrm{Im}\{\overline{u}(p)\Sigma(p)u(p)\} = -E(\Gamma_i+\Gamma_d). \tag{1}$$

This result is the generalization of the optical theorem for finite temperature and density. It is understood that Σ includes corrections due to the effects of the medium on the field propagators.

The contribution to Σ that is relevant for the decay $\nu_i \to \nu_j \gamma$ comes from the Feynman diagram represented in Fig. 1. This is evident by cutting that diagram along its vertical symmetry axis. It is understood that other loop diagrams provide the dominant contribution to Σ and to its imaginary part. In other words, as we already mentioned in the Introduction, $\nu_i \to \nu_j \gamma$ is not the main ν_i decay channel. We are, however, able to disentangle the contribution of $\nu_i \to \nu_j \gamma$ to the total decay rate by isolating the contribution to the self-energy (Σ_{rad}) of the diagram represented in Fig. 1.

To determine $\mathrm{Im}\,\Sigma_{\mathrm{rad}}$ we need some finite-temperature cutting rules. These rules have been already determined by Weldon [11], who used an imaginary-time formalism, and Kobes and Semenoff [16], who worked in a real-time formalism (RTF). We adopt here the latter approach. Following Ref. [16] we get

$$\mathrm{Im}\,\Sigma_{\mathrm{rad}}(p) = -\frac{\epsilon(p_0)}{2\sin 2\phi_p}\int \frac{d^4K}{(2\pi)^4}\Gamma^{\nu *}(K)[S^-(p-K) \times D^+_{\mu\nu}(K) + S^+(p-K)D^-_{\mu\nu}(K)]\Gamma_\mu(K), \tag{2}$$

where

$$\frac{1}{2}\sin 2\phi_p = \frac{e^{\beta|p_0|/2}}{e^{\beta|p_0|}+1} \tag{3}$$

and the expression for the effective vertex $\Gamma_\mu(K)$ will be given below. Here $K=(k_0,\vec{k})$ is the photon four-momentum in the medium rest frame and $\epsilon(p_0)$ is the sign function. We observe that a positive k_0 accounts for the process with emission of a photon (energy given to the medium) whereas a negative k_0 takes care of the inverse process (energy lost by the medium). This gives us a criterion to disentangle the different contributions of Eq. (1) to Γ_d and Γ_i by properly choosing the k_0 integration interval.

For the sake of simplicity we assume the light neutrino to be massless. In the RTF the off-diagonals component of its propagator are

$$S^{\pm}(p-K) = -2\pi i(\hat{p}-\hat{K})\{\theta[\pm(p_0-k_0) - n_F(|p_0-k_0|)]\}\delta[(p-K)^2] \tag{4}$$

where $n_{F(B)}$ is the Fermi-Dirac (Bose-Einstein) distribution function. In the rest frame of the medium (and in the Landau gauge) the photon propagator can be decomposed into

$$D^{\pm}_{\mu\nu}(K) = D^{\pm}_T P_{\mu\nu}(K) + D^{\pm}_L Q_{\mu\nu}(K). \tag{5}$$

The transverse and longitudinal projector operators are defined by

$$P_{\mu\nu}(K) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\delta_{\mu i}\delta_{\nu j}, \quad Q_{\mu\nu}(K) = e_\mu(K)e_\nu(K), \tag{6}$$

where $e_\mu(K)=(k,k_0\vec{k}/k)/\sqrt{K^2}$ is the polarization versor. The coefficients $D^{\pm}_{T,L}$ are given by [12,17]

$$D^{\pm}_{T,L}=2[\theta(\pm k_0-n_B(|k_0|)]\bigg(\pi\delta[K^2-\mathrm{Re}\,\Pi_{T,L}(K)]\delta_{Kr}[\mathrm{Im}\,\Pi_{T,L}(K)]-\frac{\mathrm{Im}\,\Pi_{T,L}(K)}{[K^2-\mathrm{Re}\,\Pi_{T,L}(K)]^2+[\mathrm{Im}\,\Pi_{T,L}(K)]^2}\bigg), \qquad (7)$$

where δ_{Kr} is a Kronecker function. These expressions include the one-loop thermal corrections to the photon propagator. It is known [18] that for soft photons ($k_0,k\lesssim eT$) these corrections cannot be neglected and that a meaningful perturbative expansion can be obtained performing a proper resummation of the hard-thermal loops contributing to the photon polarization tensor.

Equation (7) has been written in such a way in order to distinguish an *on-shell* part, corresponding to the propagation of the transversal and longitudinal modes of a plasmon, and an *off-shell* part associated to Landau damping. The fluctuation-dissipation theorem shed light on the physical nature of off-shell plasmons: they are thermal fluctuations of the electromagnetic field [17]. As suggested by the Breit-Wigner form of the off-shell part of $D^{\pm}_{T,L}$, we can think of such a kind of electromagnetic thermal fluctuations as resonances which are continuously produced from the free charges in the plasma by Cherenkov emission, and absorbed by the inverse process.

The general expression of the polarization tensor in a isotropic plasma is

$$\Pi_{\mu\nu}(K)=\Pi_T(K)P_{\mu\nu}(K)+\Pi_L(K)Q_{\mu\nu}(K), \qquad (8)$$

where, for an UR plasma, we have [19]

$$\mathrm{Re}\,\Pi_T(K)=\frac{3\omega_P^2}{2}\left[\frac{k_0^2}{k^2}+\left(1-\frac{k_0^2}{k^2}\right)\frac{k_0}{2k}\ln\left|\frac{k_0+k}{k_0-k}\right|\right],$$
$$\mathrm{Re}\,\Pi_L(K)=3\omega_P^2\left(1-\frac{k_0^2}{k^2}\right)\left[1-\frac{k_0}{2k}\ln\left|\frac{k_0+k}{k_0-k}\right|\right] \qquad (9)$$

and

$$\mathrm{Im}\,\Pi_T(K)=-\frac{3\omega_P^2}{2}\pi\left(1-\frac{k_0^2}{k^2}\right)\frac{k_0}{2k}\theta(k^2-k_0^2),$$
$$\mathrm{Im}\,\Pi_L(K)=3\omega_P^2\pi\left(1-\frac{k_0^2}{k^2}\right)\frac{k_0}{2k}\theta(k^2-k_0^2). \qquad (10)$$

In the UR limit, the plasma frequency is given by $\omega_P=eT/3$.

Plasmon dispersion relations are determined by solving $K^2=\mathrm{Re}\,\Pi_{T,L}(K)$. Analytic forms of the solutions can be obtained in some suitable limits [12]. In the $k\to 0$ limit transversal and longitudinal dispersion relations are, respectively,

$$k_0^2=\omega_P^2+\frac{6}{5}k^2 \quad \text{transversal},$$
$$k_0^2=\omega_P^2+\frac{3}{5}k^2 \quad \text{longitudinal}, \qquad (11)$$

whereas in the hard limit ($k_0,k\gg\omega_P$) one gets

$$k_0^2=\frac{3}{2}\omega_P^2+k^2 \quad \text{transversal},$$
$$k_0^2=k^2+4k^2e^{-2k^2/3\omega_P} \quad \text{longitudinal}. \qquad (12)$$

III. THE ON-SHELL PLASMON CASE

We now give some more details of our computation. As we discussed in the Introduction, in a medium the neutrino-photon coupling is mediated by the weak interaction of the neutrino with the free charges in the plasma. For definiteness we only consider here a nondegenerate electron-positron plasma at temperatures below the muon mass. From Refs. [3,20] we know that the effective medium induced neutrino electromagnetic vertex is given by

$$\Gamma_\mu(K)=U_{ie}\frac{G_F}{e}\Pi_{\mu\nu}(K)\gamma^\nu L, \qquad (13)$$

where U is the lepton mixing matrix and $L\equiv\frac{1}{2}(1-\gamma_5)$.

By substituting Eqs. (13), (4), and the *on-shell* part of Eq. (7) in Eq. (2), we find that the decay rate of the heavy neutrino into the lighter one and a transversal plasmon is

$$\Gamma_T^{\mathrm{on}}=\frac{1}{2\sin 2\phi_p}\frac{G_F^2}{4Ee^2}|U_{ie}^*U_{je}|^2\int\frac{d^4K}{(2\pi)^4}[\mathrm{Re}\,\Pi_T(K)]^2\times\delta[K^2-\mathrm{Re}\,\Pi_T(K)]\delta[(p-K)^2]\{\epsilon(p_0-k_0)\times[\theta(-k_0)+n_F(|p_0-k_0|)]+\epsilon(p_0-k_0)[\theta(k_0)+n_B(|k_0|)]\}P_{\alpha\beta}\,\mathrm{Tr}\{(\hat{p}-m)\gamma^\beta(\hat{p}-\hat{K})\gamma^\alpha L\}, \qquad (14)$$

where m is the heavy neutrino mass. The decay rate of the heavy neutrino into a longitudinal plasmon is readily obtained by replacing the subscript T with L and the projector $P_{\alpha\beta}$ with $Q_{\alpha\beta}$ in Eq. (14). In the square brackets in the right side of Eq. (14) we can distinguish a term accounting for the Bose-Einstein stimulated photon emission and a Pauli-blocking term for the production of the light neutrino. Since generally the latter is much smaller than the former we disregard the Pauli-blocking term in the following. We also approximate the factor $1/\sin 2\phi_p$ with the unity.

Using $\delta((p-K)^2)$ and $\delta(K^2-\mathrm{Re}\,\Pi_T(K))$ to suppress respectively the angular and the temporal parts of the $d^4K/(2\pi)^4$ integration we get

$$\Gamma_T^{\mathrm{on}}=\frac{9G_F^2}{32\pi e^2}|U_{ie}^*U_{je}|^2\omega_P^5 f_T(v). \tag{15}$$

Here v is the velocity of the heavy neutrino with respect to the medium rest frame. Since it is generally impossible to find an analytical expression for the plasmon dispersion relation to be used in Eq. (14) it is crucial to separate the kinematical function $f(v)$ in a soft and in a hard components. In the case the plasmon is transversal we have

$$f_T(v)=f_T^{\mathrm{hard}}(v)+f_T^{\mathrm{soft}}(v) \tag{16}$$

$$f_T^{\mathrm{soft}}(v)\simeq\frac{8}{9}\,\frac{1-v^2}{\tilde{m}^2 v}\int_{\tilde{k}_{\min}^{\mathrm{soft}}}^{\tilde{k}_{\max}^{\mathrm{soft}}}d\tilde{k}[1+n_B(1)]\left\{\tilde{E}^2-\frac{1}{2}(1+\tilde{m}^2)-\frac{1}{\tilde{k}^2}\left[\tilde{E}-\frac{1}{2}(1+\tilde{m}^2)\right]\right\}\theta(1-\tilde{k}) \tag{17}$$

and

$$f_T^{\mathrm{hard}}(v)\simeq\frac{1-v^2}{\tilde{m}^2 v}\int_{\tilde{k}_{\min}^{\mathrm{hard}}}^{\tilde{k}_{\max}^{\mathrm{hard}}}\frac{d\tilde{k}}{\tilde{k}}[1+n_B(\tilde{k})]\times\left\{\tilde{E}-\frac{\tilde{k}^2}{2}-\frac{\tilde{m}^2}{4\tilde{k}}\right\}\theta(\tilde{k}-1), \tag{18}$$

where E is the energy of the heavy neutrino and tilted quantities have been normalized to ω_P so that $f_T(v)$ is a dimensionless function. The integration limits are

$$\tilde{k}_{\max}^{\mathrm{hard}}=\frac{\tilde{m}}{2}\sqrt{\frac{1+v^2}{1-v^2}},\quad \tilde{k}_{\min}^{\mathrm{hard}}=\frac{\tilde{m}}{2}\sqrt{\frac{1-v^2}{1+v^2}}, \tag{19}$$

in the hard limit, and

$$\tilde{k}_{\max}^{\mathrm{soft}}=\tilde{m}\sqrt{\frac{1+v^2}{1-v^2}}-1,\quad \tilde{k}_{\min}^{\mathrm{soft}}=\left|\tilde{m}\sqrt{\frac{1-v^2}{1+v^2}}-1\right|, \tag{20}$$

in the soft one. For the sake of clarity, only a zero order expansion of $\mathrm{Re}\,\Pi_T(K)$ in $\tilde{k}$ has been reported in the expressions (18) and (17). However, a more complete second order expansion has been used in the computations that we performed to get Fig. 2. We also have to mention that in our numerical integrations some more suitable exponential cutoff functions have been used instead of the step functions appearing in the integrals in Eqs. (18) and (17). It is worthwhile to observe that Eq. (18) coincides with the result found by NP but the infrared cutoff function present in our expression. Such a difference, however, is not a minor point. In fact, the introduction of the IR cutoff is crucial in order to avoid the incorrect use of the hard plasmon dispersion relation ($K^2\simeq 0$) when $k\to 0$. This kinematical region is certainly reached when the decaying neutrino is ultrarelativistic as it is evident from Eq. (19) that $\tilde{k}_{\min}^{\mathrm{hard}}\to 0$ when $v\to 1$. Indeed, by using the dispersion relation $K^2\simeq 0$, an unphysical result was found in Ref. [10] consisting in a nonvanishing neutrino decay rate in the limit $v\to 1$. As we discussed, our approach solves this problem by properly separating the hard and the soft contributions to $f(v)$. From Fig. 2 the reader can see as, in our case, the decay rate drops to zero when $v\to 1$. This result is physically convincing since no rest frame is available for the neutrino to decay in when it travels at the speed of light.

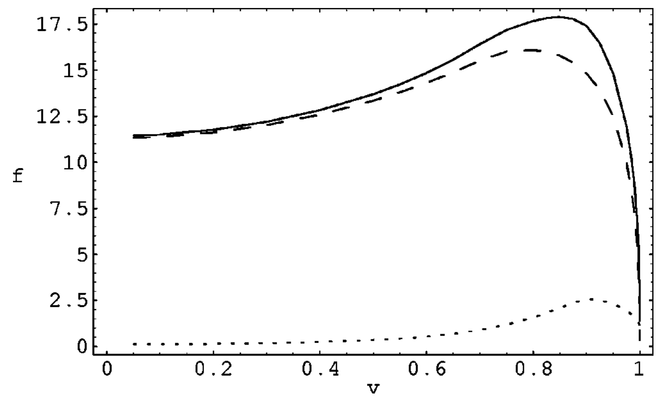

FIG. 2. In this figure the dashed, dotted and continuous lines represents, respectively, the functions $f_T^{\mathrm{hard}}(v)$, $f_T^{\mathrm{soft}}(v)$, and the sum of the two. Here we fixed $\tilde{m}=5$.

From Fig. 2 we see also that in spite of the large enhancement of the Bose-Einstein factor $n_B(|k_0|)=(e^{\beta|k_0|}-1)^{-1}\simeq T/k_0\gg 1$ that one gains in the soft limit, the soft contribution to the total decay rate is subdominant with respect to the hard one. This is due to the smaller integration interval available in Eq. (17) with respect to that we have in Eq. (18). Not surprisingly, such a situation becomes even more pronounced for higher values of the decaying neutrino mass. The maximum value of f_T^{soft} is reached at the velocity $v_*\equiv(m^2-\omega_P^2)/(m^2+\omega_P^2)$, where $\tilde{k}_{\min}^{\mathrm{soft}}=0$.

In the case the plasmon is longitudinal only the soft term contributes to the total decay rate. In fact, as we can see from the second of the equations (12) K^2, hence also $\mathrm{Re}\,\Pi_L(K)$, vanishes in the hard limit. Due to angular momentum conservation, $\Gamma_L(K)$ is suppressed by a factor $(\omega_P/m)^2$ with respect to the soft contribution to $\Gamma_T(K)$. For this reason the contribution of longitudinal plasmons to the total decay rate is subdominant and we do not report the expression of $\Gamma_L(K)$ here.

IV. THE OFF-SHELL PLASMON CASE

In this section we discuss the contribution of thermal fluctuations of the electromagnetic field to the neutrino decay.

As we discussed in the Sec. II the effect of these fluctuations is accounted by the second term on the right side of Eq. (7). Differently from the case considered in the previous section, plasmons associated to thermal fluctuations are not *on shell* hence they do not obey to any dispersion relation. As a consequence, in this case we have to deal with a double integration both over k and k_0. Indeed, the expressions for the decay rate look in this case

$$\Gamma^{\mathrm{off}}_{T,L}=\frac{9G_F^2}{32\pi e^2}|U^*_{ie}U_{je}|^2\omega_P^5 g_{T,L}(v). \tag{21}$$

The expressions of the kinematical functions $g_{T,L}(v)$ are

$$g_T(v)\simeq 2\tilde{T}\,\frac{1-v^2}{\tilde{m}^2 v}\int_0^\infty d\tilde{k}_0\int_{\tilde{k}^{\mathrm{soft}}_{\mathrm{min}}}^{\tilde{k}^{\mathrm{soft}}_{\mathrm{max}}}\frac{d\tilde{k}}{\tilde{k}^2}\Bigg[\tilde{E}^2-\frac{(\tilde{K}^2+\tilde{m}^2)}{2} - \frac{[\tilde{E}\tilde{k}_0-(1/2)(\tilde{K}^2+\tilde{m}^2)]^2}{\tilde{k}^2}\Bigg] \times\tilde{K}^2 A_T(\tilde{k}_0,\tilde{k})\,\theta(\tilde{k}^2-\tilde{k}_0^2) \tag{22}$$

and

$$g_L(v)\simeq\frac{\tilde{T}}{2}\,\frac{1-v^2}{\tilde{m}^2 v}\int_0^\infty d\tilde{k}_0\int_{\tilde{k}^{\mathrm{soft}}_{\mathrm{min}}}^{\tilde{k}^{\mathrm{soft}}_{\mathrm{max}}}\frac{d\tilde{k}}{\tilde{k}^4}\left\{\left[2\tilde{E}-\tilde{k}_0\left(1+\frac{\tilde{m}^2}{\tilde{K}^2}\right)\right]^2 -(\tilde{K}^2-\tilde{m}^2)\tilde{k}^2\right\}\tilde{K}^4 A_L(\tilde{k}_0,\tilde{k})\,\theta(\tilde{k}^2-\tilde{k}_0^2), \tag{23}$$

where

$$A_{T,L}(\tilde{k}_0,\tilde{k})\equiv\frac{[\mathrm{Re}\,\Pi_{T,L}(K)]^2+[\mathrm{Im}\,\Pi_{T,L}(K)]^2}{[K^2-\mathrm{Re}\,\Pi_{T,L}(K)]^2+[\mathrm{Im}\,\Pi_{T,L}(K)]^2}. \tag{24}$$

Since Landau damping is effective only for soft photons we can neglect here any contribution coming from the hard part of the photon spectrum. This allowed us to perform the substitution $[1+n_B(k_0)]\to T/k_0$ in Eqs. (22),(23). The functions $g_{T,L}(v)$ have been computed numerically and their plots are reported in Fig. 3. From this figure we see that as with the soft contribution to $\Gamma^{\mathrm{on}}_{T,L}$, $\Gamma^{\mathrm{off}}_{T,L}$ reach as its maximum value when $v=v_*$. This result is not unexpected since the functions $A_{T,L}(K)$ take their maximum values when K^2 approaches $\mathrm{Re}\,\Pi_{T,L}(K)$.

We have, however, a crucial difference between the off-shell and the on-shell contributions to the total decay rate. Whereas radiative neutrino decay is kinematically forbidden if $K^2>m^2$ this is clearly not the case whenever the photon is off shell. Note that even in the hard limit, where $K^2\simeq\frac{3}{2}\omega_P^2$ [see Eq. (12)], the *on-shell* decay cannot take place unless $m^2>\frac{3}{2}\omega_P^2$. This was not noted in Refs. [8–10]. We have to mention, however, that although Γ^{off} remains different from zero it is typically very small when $m\ll\omega_P$.

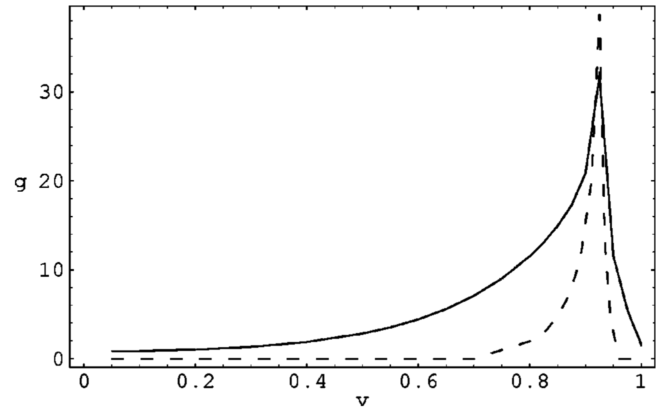

FIG. 3. In this figure the dashed and the continuous lines represent, respectively, the functions $g_T(v)$, $g_L(v)$. Again, we fixed $\tilde{m}=5$.

V. DISCUSSION

Among other possible applications, the results presented in the previous sections are of obvious interest for the study of neutrino evolution in the early universe. For example, the reader may wonder if medium induced radiative decay can give rise to a significative depletion of a heavy neutrino species and if such an effect can have any relevant consequence for the big bang nucleosynthesis (BBN). In order to answer this question, we need to compare the results of our previous sections with the rate of the weak processes which keep neutrinos in thermodynamical equilibrium with the heat bath. This rate is typically $\Gamma_{\mathrm{weak}}\simeq G_F^2T^5$. For definitess we consider the decay of tau neutrinos, having a mass in the MeV range, into massless electronic neutrinos. Looking at Eqs. (15) and (21) it is clear at glance that medium induced neutrino radiative decay cannot compete with standard weak processes. This result is not too unexpected since the neutrino-photon coupling is in our case mediated by plasma effects which gives rise to a $(\omega_P/T)^4\simeq 10^{-4}$ suppression factor in the radiative decay rate with respect to Γ_{weak}. This suppression is only partially compensated by the Bose-Einstein statistical enhancement responsible for a T/ω_P amplification factor. It should be noted that this situation is even more pronounced when the temperature drops and the electron-positron plasma becomes NR. In fact, in this limit $\omega_P\approx 4\pi\alpha n_e/m_e$ which is exponentially suppressed as T is decreased.

A different scenario may be realized in the case one considers a larger extension of the standard model which allows for the presence of a large neutrino transition magnetic moment. In this case, in fact, the dominant neutrino-photon coupling is independent on ω_P. It was shown in Ref. [15] that if a neutrino species has a Dirac neutrinos magnetic moment of the order of $10^{-11}\ \mu_B$, its interaction with electromagnetic plasma fluctuations should have relevant consequences on BBN. If the neutrino is of Majorana type and has a transition magnetic moment of the same order of magnitude, we expect that a similar process should give rise to rapid spin-flavor precession during the same epoch. Such a process can be interpreted as a neutrino radiative decay with emission, or absorption, of an off-shell plasmon and be treated by using the same method that we presented in Sec. IV. In our opinion, further investigations should be addressed to determine the relevance of this effect for the BBN and the supernova physics. It is noticeable that a Majorana magnetic moment of the order of $10^{-11}\ \mu_B$ provides a viable solution to the solar neutrino problem [21].

VI. CONCLUSIONS

In this paper we reconsidered the issue of the decay of a massive neutrino into a lighter one by the emission (or the absorption) of a photon in the presence of a ultrarelativistic electron-positron plasma. Differently from previous works on the subject we accounted here for the dispersion relation of the photon in the medium and considered the contribution to the decay of off-shell photons related to to thermal fluctuations (Landau damping). In both cases we got not negligible corrections. Although we considered here only a minimal extension of the standard model, which allows neutrino masses and mixing, we expect that more interesting applications of our results should be found considering larger extensions.

ACKNOWLEDGMENTS

The work of D.G. have been supported by the TMR network Grant No. ERBFMRXCT960090 of the European Union, and that of V.S. by INTAS Grant Nos. 96-0659 and RFFR 97-02-16501.

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