

# Implication of the running mass of the $\rho^0$ meson for the dilepton mass spectrum and the $\mu^+\mu^-/e^+e^-$ ratio in $K^+\rightarrow\pi^+l^+l^-$ decays

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We make an attempt to resolve the discrepancy of the observed  $e^+e^-$  mass spectrum in the  $K^+\rightarrow\pi^+e^+e^-$  decay with that predicted by meson dominance. To this end we investigate the properties of the  $\rho^0$  propagator. We use dispersion relations to evaluate the running mass  $m_\rho^2(t)$  of the  $\rho^0$  resonance without adjustable parameters. To improve the convergence of the dispersion integral, the momentum dependence of strong vertices is taken from the flux-tube-breaking model of Kokoski and Isgur. The obtained behavior of  $m_\rho^2(t)$  at small momentum squared  $t$  makes the  $K^+\rightarrow\pi^+e^+e^-$  form factor rise faster with increasing  $t$  than in the original meson dominance calculation and more in agreement with the published data. As a consequence, the meson dominance prediction of the  $\mu^+\mu^-/e^+e^-$  ratio changes slightly, from 0.224 to 0.236. We do not see any possibility to accommodate into the meson dominance approach an even steeper  $e^+e^-$  spectrum, indicated by the preliminary data of the E865 Collaboration at BNL AGS. [S0556-2821(99)02215-8]

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The decays  $K^+\rightarrow\pi^+l^+l^-$  ( $l=e,\mu$ ) have been the subject of intensive theoretical studies since the late 1950s (see [1–3] and references therein). A picture of the later theoretical developments can be gained by inspecting [4–7] and papers cited there. The decay  $K^+\rightarrow\pi^+l^+l^-$  was experimentally observed in 1975 in its  $e^+e^-$  mode [8] and in 1997 in the  $\mu^+\mu^-$  mode [9]. Other experiments include a more precise measurement of the  $e^+e^-$  mode by the BNL-E777 Collaboration [10], unpublished  $e^+e^-$  data of the BNL-E851 Collaboration [11], and the current BNL-E865 experiment, capable of measuring both modes [12] with high precision and statistics.

Today, it is customary to interpret experimental results in the framework of chiral perturbation theory [5,13,7]. Unfortunately, this theoretical framework contains free parameters: one in the  $p^4$  order [5], two in the  $p^6$  order [7]. This, on the one hand, diminishes the predictive power of the theory but, on the other hand, gives more room to experimentalists when trying to fit theoretical formulas.

On the contrary, as we have shown recently [14], meson dominance offers a parameter-free description of the  $K^+\rightarrow\pi^+l^+l^-$  decays. The relevant Feynman diagram is shown in Fig. 1. The corresponding formula for the differential decay rate in dilepton mass  $M$  has the form generally expected for the one-photon approximation, namely,

$$\frac{d\Gamma(K^+\rightarrow\pi^+l^+l^-)}{dM} = CM\lambda^{3/2}(m_{K^+}^2, m_{\pi^+}^2, M^2) \times \sqrt{1 - \frac{4m_l^2}{M^2}} \left(1 + \frac{2m_l^2}{M^2}\right) |F(M^2)|^2, \quad (1)$$

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with  $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$  and the form factor given by

$$F(t) = \frac{m_\rho^2}{m_\rho^2 - t}. \quad (2)$$

The normalization constant  $C$  is not given by first principles, but can be determined using data other than those on the  $K^+\rightarrow\pi^+l^+l^-$  decays themselves, concretely, from the experimental information about the  $\tau^-\rightarrow\pi^-\pi^-\pi^+\nu_\tau$  and  $K^+\rightarrow\mu^+\nu_\mu$  decay. In Ref. [14], we used the decay rate of the  $\tau^-\rightarrow a_1^-\nu_\tau$  decay, the  $a_1(1270)$  decay width, and the  $K^+\rightarrow\mu^+\nu_\mu$  branching fraction.<sup>1</sup> In this way we obtained  $B(K^+\rightarrow\pi^+e^+e^-) \approx 3.1 \times 10^{-7}$ , not in contradiction with experiment  $(2.74 \pm 0.23) \times 10^{-7}$  [15]. The approximative character of our result was caused by the badly known  $a_1$  decay width.

Formula (1) makes a definite prediction for the  $\mu^+\mu^-/e^+e^-$  branching ratio even if  $C$  is badly known. The number is 0.224 with an error which is negligible under the circumstances because the ratio is a function of the masses of participating particles only. We use it and the experimental  $e^+e^-$  branching fraction to predict

$$B(K^+\rightarrow\pi^+\mu^+\mu^-) = (6.2 \pm 0.5) \times 10^{-8}, \quad (3)$$

in agreement with the later measurement [9] of  $(5.0 \pm 1.0) \times 10^{-8}$ .<sup>2</sup>

<sup>1</sup>We use the terminology of [15]. The branching fraction  $B$  is the ratio of a partial decay rate to the total decay rate; the branching ratio is the ratio of two partial decay rates or, equivalently, of two branching fractions.

<sup>2</sup>We follow the convention of [15], where the statistical, systematic, and theoretical errors given in [9] are summed quadratically.

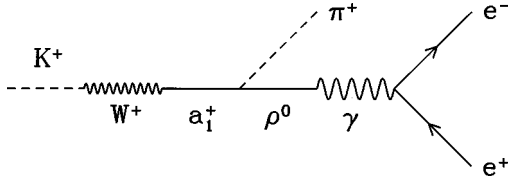


FIG. 1. Matrix element of the decay  $K^+ \rightarrow \pi^+ e^+ e^-$  in the meson dominance approach.

The  $t$  dependence of the form factor (2) can be, for the purpose of comparing with data, characterized by the slope variable

$$\lambda(t) = m_{\pi^+}^2 \frac{dF(t)}{dt}, \quad (4)$$

which is equal to 0.033 at  $t=0$  and reaches 0.053 at the upper kinematic boundary  $t \approx 0.125 \text{ GeV}^2$ . In Ref. [10] the data were fit by a linear approximation to the form factor [ $\lambda \equiv \lambda(0)$ ]

$$F(t) = 1 + \lambda \frac{t}{m_{\pi^+}^2}, \quad (5)$$

with  $\lambda = 0.105 \pm 0.035$  (stat)  $\pm 0.015$  (syst).

This result became a little surprising after the experimental value of  $B(K^+ \rightarrow \pi^+ \mu^+ \mu^-)$  was published [9]. In fact, if one assumes the  $\mu/e$  universality and validity of Eq. (5), the values of  $B(K^+ \rightarrow \pi^+ e^+ e^-)$ ,  $B(K^+ \rightarrow \pi^+ \mu^+ \mu^-)$ , and  $\lambda$  must match together. And they do not match very well. Even for  $\lambda = 0.055$  (mean value minus both errors) the ‘‘predicted’’ value of the  $\mu^+ \mu^-/e^+ e^-$  branching ratio is equal to 0.235, which should be compared to the experimental  $0.18 \pm 0.04$  (using [15], errors summed quadratically). The disagreement rises with  $\lambda$ . The preliminary data of the E865 experiment [12] indicate that the fault is not on the side of  $\lambda$ .

We therefore take for granted that the experimental value of  $\lambda$  indicates, despite its large errors, that the meson dominance form factor (2) is too flat. In the following, we will try to find the possible origin of this discrepancy and a way to improve the situation without introducing unnatural assumptions and free parameters.

When writing Eq. (1) with form factor given by Eq. (2), the essential assumption was that the  $\rho^0$  propagator in Fig. 1 can be written in a free-vector-particle form

$$-iG_0^{\mu\nu}(q) = \frac{-g^{\mu\nu} + q^\mu q^\nu / m_\rho^2}{t - m_\rho^2 + i\epsilon}, \quad (6)$$

where  $m_\rho$  is the mass of the  $\rho^0$  resonance, as is seen in the hadronic production experiments. The general expression for the interacting-vector-resonance propagator is a little more complicated. It reads (see, e.g., [16])

$$-iG^{\mu\nu}(q) = \frac{-g^{\mu\nu} + \omega(t)q^\mu q^\nu / t}{t - m_\rho^2(t) + im_\rho \Gamma(t)}, \quad (7)$$

where  $\Gamma(t)$  is the total width of the  $\rho$  resonance with off-shell mass  $\sqrt{t}$ , normalized at  $t = m_\rho^2$  to the nominal width  $\Gamma_\rho$ . Furthermore,  $m_\rho^2(t)$  is the running mass squared and  $\omega(s)$  is a complex function which reflects the properties of the one-particle-irreducible bubble.

The propagator (6), which is usually used in meson dominance calculations, differs from Eq. (7) in three respects.

(1) A simplified structure of the  $q^\mu q^\nu$  term. This is not important, because this term does not contribute anyhow due to the transverse  $\rho\pi\pi$  vertex.

(2) The absence of a finite imaginary part, which is justified, since most of our  $t$  region lies below the  $\pi\pi$  threshold. In a small window between the latter and the end of the  $t$  interval it is negligible. Nevertheless, we will include it in what follows.

(3) The only real difference is in replacing the running mass  $m_\rho(t)$  with the nominal mass  $m_\rho$ , which is generally allowed only in a close vicinity of the resonance point.

We will concentrate our effort on the last issue and study the consequences of replacing the nominal mass of the  $\rho^0$  resonance by its running mass in the denominator in Eq. (2). To be more concrete, we will write the modified form factor in the form

$$F(t) = \frac{m_\rho^2(0)}{m_\rho^2(t) - t - im_\rho \Gamma(t)}. \quad (8)$$

It follows from the causality of the propagator (7) that above the  $\pi\pi$  threshold  $t_0$ ,  $m_\rho^2(t)$  and  $m_\rho \Gamma(t)$  are boundary values of the real and imaginary parts, respectively, of a function analytic in the cut  $t$  plane. We can therefore write a once-subtracted dispersion relation [16]

$$m_\rho^2(t) = m_\rho^2 - \frac{t - m_\rho^2}{\pi} \mathcal{P} \int_{t_0}^{\infty} \frac{m_\rho \Gamma(t')}{(t' - t)(t' - m_\rho^2)} dt', \quad (9)$$

where the symbol  $\mathcal{P}$  denotes the principal value. To proceed further, we must find all important contributions to the variable width  $\Gamma(t)$ . Without any doubt, we start with  $\rho^0 \rightarrow \pi^+ \pi^-$ . Other candidates are, ordered according to rising thresholds,  $\rho^0 \rightarrow \eta \pi^+ \pi^-$ ,  $\rho^0 \rightarrow \omega \pi^0$ , and  $\rho^0 \rightarrow K^+ K^-$  and  $K^0 \bar{K}^0$ . The relative importance of those channels can further be assessed by comparing the abundance of their isotopic companions in the  $\tau^-$  decays. This suggests that of those three, the  $\omega \pi^0$  final state will be the most important, while the  $\eta \pi^+ \pi^-$  one the least important. The results of actual calculations confirm this estimate. Furthermore, inspection of the  $\tau$ -decay fractions shows that there is no other important hadronic channel with quantum numbers of the  $\rho^0$  meson. In addition, we assume that possible channels with thresholds above the  $\tau^-$  mass may be neglected. The results obtained below seem to validate this assumption.

Now we are going to describe our calculation in more detail. Let us start with the most important contribution to  $\Gamma(t)$ , which is the  $\rho^0 \rightarrow \pi^+ \pi^-$  decay. We write the  $\rho\pi\pi$  vertex in the form

$$V^\mu = f_{\rho\pi\pi} (p^{*\mu}) (p_{\pi^+}^\mu - p_{\pi^-}^\mu), \quad (10)$$

where  $p^*$  is the pion momentum in the  $\rho$  rest frame. Instead of the usual coupling constant we have introduced the strong form factor. Its momentum dependence was taken from the flux-tube-breaking model of Kokoski and Isgur [17]. We thus write

$$f_{\rho\pi\pi}(p^{*2}) = g_{\rho\pi\pi} \exp\left\{-\frac{p^{*2}}{12\beta^2}\right\}, \quad (11)$$

with  $\beta=0.4$ . We must confess that our original motivation for borrowing Eq. (11) from [17] was technical: we just wanted to ensure good convergence of the dispersion integral. But it appeared later that a very reasonable result for  $m_\rho^2(t)$ , which we will present below, could not be achieved without assuming Eq. (11) or with a very different value of the parameter  $\beta$ . Our opinion is now that the flux-tube-breaking model ansatz (11) reflects correctly the real dynamics of the  $\rho\pi\pi$  vertex. We will use the same parametrization for all strong form factors.

Using Eqs. (10) and (11) we easily arrive at the formula

$$\Gamma_{\rho^0 \rightarrow \pi^+ \pi^-}(t) = \frac{g_{\rho\pi\pi}^2 p^{*3}}{6\pi} \frac{1}{t} \exp\left\{-\frac{p^{*2}}{6\beta^2}\right\}, \quad (12)$$

where  $p^* = \sqrt{t/4 - m_\pi^2}$ . The coupling constant was determined from the condition

$$\Gamma_{\rho^0 \rightarrow \pi^+ \pi^-}(m_\rho^2) = \Gamma_\rho = (150.7 \pm 1.1) \text{ MeV}, \quad (13)$$

with the result  $g_{\rho\pi\pi}^2 = 41.7 \pm 0.3$ . Formula (12) can be used, with obvious modifications also for  $\rho^0 \rightarrow K\bar{K}$ . Here, the coupling constant can be determined from the  $\tau^- \rightarrow K^- K^0 \nu_\tau$  branching fraction. We refer the reader to Ref. [14]. Taking into account the modifications connected with present usage of the momentum-dependent strong form factors and assuming that  $\rho^-$  and  $\rho^0$  decay to their corresponding  $K\bar{K}$  systems with the same rate, we get  $g_{\rho^0 K^+ K^-}^2 + g_{\rho^0 K^0 \bar{K}^0}^2 = 28.2 \pm 5.1$ .

The  $\rho\omega\pi$  vertex is taken in the form<sup>3</sup>

$$V^{\mu\nu} = \frac{f_{\rho\omega\pi}(p^{*2})}{m_\rho} \epsilon^{\mu\alpha\nu\beta} p_{\rho,\alpha} p_{\omega,\beta}, \quad (14)$$

with the same momentum dependence of the strong form factor as in Eq. (11). The coupling constant can be determined from the decay rate  $\Gamma(\omega \rightarrow \pi^0 \gamma) = (7.2 \pm 0.4) \times 10^{-4}$  GeV assuming the usual vector-meson-dominance form of the coupling between  $\rho^0$  and  $\gamma$ . The result is  $g_{\rho\omega\pi}^2 = 155 \pm 8$ . The contribution to  $\Gamma(t)$  is given by the formula

$$\Gamma_{\rho^0 \rightarrow \omega \pi^0}(t) = \frac{g_{\rho\omega\pi}^2 p^{*3}}{12\pi} \frac{1}{t} \exp\left\{-\frac{p^{*2}}{6\beta^2}\right\}, \quad (15)$$

with  $p^{*2} = \lambda(t, m_\omega^2, m_{\pi^0}^2)/(4t)$ .

The last contribution to  $\Gamma(t)$  we consider is the decay  $\rho^0 \rightarrow \eta \pi^+ \pi^-$ . We will consider this as a two-step process:  $\rho^0 \rightarrow \eta \rho^0$  followed by the decay  $\rho^0 \rightarrow \pi^+ \pi^-$ . The mass squared of the parent  $\rho^0$  is  $t$ ; that of the daughter  $\rho^0$  is  $s < t$ . Thanks to the daughter decay matrix element being transverse, the decay rate of the whole process factorizes into two parts [18]. The first of them is given by formula (15) with obvious modifications; the second one contains a Breit-Wigner term with a decay rate of  $\rho^0 \rightarrow \pi^+ \pi^-$ . The only new element is the  $\rho\eta\rho$  coupling constant, which is determined from  $\Gamma(\rho^0 \rightarrow \eta\gamma) = (3.6 \pm 1.3) \times 10^{-5}$  GeV as  $g_{\rho\eta\rho}^2 = 55 \pm 21$ . The contribution to  $\Gamma(t)$  is given by

$$\begin{aligned} \Gamma_{\rho^0 \rightarrow \eta \pi^+ \pi^-}(t) &= \frac{g_{\rho\eta\rho}^2 g_{\rho\pi\pi}^2}{36\pi^3 t} \\ &\times \int_{2m_{\pi^+}}^{\sqrt{t}-m_\eta} \frac{(p_\pi^* p_\eta^*)^3}{(s-m_\rho^2)^2 + m_\rho^2 \Gamma^2(s)} \\ &\times \exp\left\{-\frac{p_\pi^{*2} + p_\eta^{*2}}{6\beta^2}\right\} d\sqrt{s}, \quad (16) \end{aligned}$$

where

$$p_\eta^* = \frac{\lambda^{1/2}(t, s, m_\eta^2)}{2\sqrt{t}}, \quad (17)$$

$$p_\pi^* = \sqrt{\frac{s}{4} - m_{\pi^+}^2}. \quad (18)$$

It seems to be a sort of conundrum that the right hand side contains the same quantity, the contribution to which we aim to determine, namely,  $\Gamma(s)$ . Under different circumstances we would be forced to repeat the whole procedure several times in search of a self-consistent solution. Fortunately, here it shows that the result depends only little on the form of  $\Gamma(s)$ . We compared the case of fixed width  $\Gamma_\rho$  with the case of  $\Gamma_{\rho\pi\pi}(s)$  and found only tiny differences. We picked the result of the latter choice.

Now we have collected all pieces and can add them to form the total  $\Gamma(t)$  and evaluate the dispersion integral. To be sure that we have things under control, we proceeded in a less straightforward way. We first took the basic ( $\rho^0 \pi^+ \pi^-$ ) contribution alone and determined  $m_\rho^2(t)$ . Then we did the same thing for the basic contribution combined with three other contributions taken individually and compared the changes against the basic contribution alone. In this way we determined the following sequence of contributions (most important first):  $\omega \pi^0$ ,  $K\bar{K}$ ,  $\eta \pi^+ \pi^-$ . Then we started again and added the contributions cumulatively, in the order just shown. The resulting  $m_\rho^2(t)$ 's are depicted in Fig. 2. We can see that the procedure of adding contributions converges to a very reasonable result: a wide plateau, the derivative at  $t = m_\rho^2$  almost vanishing. Our input parameters, coupling constants, have relatively large errors. So it would be possible to vary them within limits in an effort to find an even better solution (characterized by the vanishing derivative at  $t$

<sup>3</sup>The coupling constants here were made dimensionless, contrary to [14], by introducing the  $\rho$  mass in the denominator.

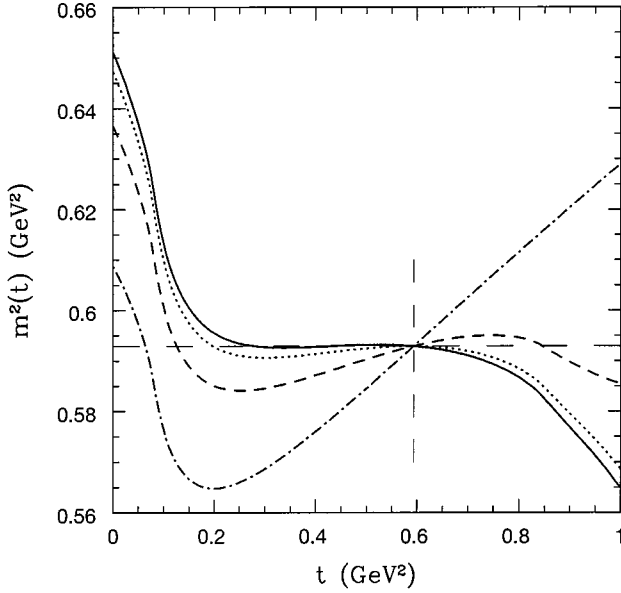


FIG. 2. Running mass squared of the  $\rho^0$  meson for different inputs to the dispersion relation: (i)  $\rho^0 \rightarrow \pi^+ \pi^-$  only (dash-dotted curve), (ii)  $\rho^0 \rightarrow \omega \pi^0$  added (dashed curve); (iii) also  $\rho^0 \rightarrow K \bar{K}$  added (dotted curve), (iv) the final curve (solid curve) after the  $\rho^0 \rightarrow \eta \pi^+ \pi^-$  contribution has been added.

$= m_\rho^2$ ). Another possibility would be to vary  $\omega$  a little around the breaking-flux-tube model [17] preferred solution  $\beta=0.4$ . We made only one try in that direction. We found that the derivative vanished if the coupling constants squared of all three additional contributions were diminished by 8%. But the behavior of the running mass squared in the region which interests us most [ $0 < t < (m_{K^+} - m_{\pi^+})^2$ ] did not change by that move at all. We therefore believe that our determination of  $m_\rho^2(t)$  at low  $t$  is stable and trustable.

Before we draw conclusions about the  $K^+ \rightarrow \pi^+ l^+ l^-$  form factor, we must mention one correction we should make in order to be consistent with the formalism we used in our dispersion relation evaluation of the  $\rho^0$  running mass. We should include the same momentum dependence of strong form factors also into our basic diagram, Fig. 1. Here, it applies to the  $a_1 \rho \pi$  vertex and leads to the following modification of the form factor (8):

$$F(t) = \frac{m_\rho^2(0)}{m_\rho^2(t) - t - i m_\rho \Gamma(t)} \exp \left\{ \frac{t(2m_{K^+}^2 + 2m_{\pi^+}^2 - t)}{48m_{K^+}^2 + \omega^2} \right\}. \quad (19)$$

Anyhow, to see the effect of the running mass alone, in Fig. 3 we present three curves: the old meson dominance form factor calculated from Eq. (2) (dashed curve); the form factor coming from the running mass with the vertex correction ignored, Eq. (8) (dash-dotted curve); and the form factor reflecting both effects, Eq. (19) (solid curve). The latter is what we consider the final product of our study.

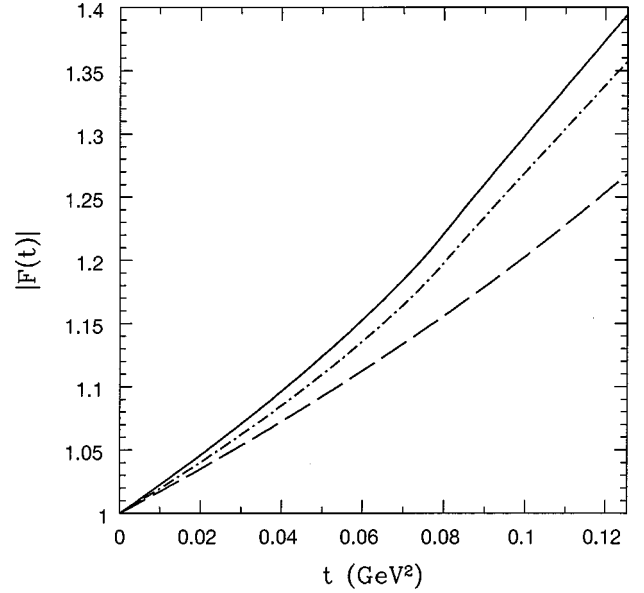


FIG. 3.  $K^+ \rightarrow \pi^+ e^+ e^-$  form factor as a function of  $t$ : (i) constant  $\rho^0$  mass, Eq. (2) (dashed curve); (ii) running  $\rho^0$  mass, Eq. (8) (dash-dotted curve); (iii) running  $\rho^0$  mass and the  $a_1 \rho \pi$  vertex correction, Eq. (19) (solid curve).

Figure 4 brings the same information but in a form which is better suited for comparison with the experimental mass spectra. It shows the dependence of the form factor squared on the dilepton mass.

The form factor calculated from Eq.(19) has a much steeper  $t$  dependence than the original form factor (2). It is characterized by  $\lambda=0.043$  at  $t=0$  and  $\lambda=0.073$  at the largest  $t$ . The  $e^+ e^- / \mu^+ \mu^-$  branching ratio calculated using Eq. (1) with Eq. (19) is 0.236. Using the experimental branching fraction of the  $e^+ e^-$  mode [15] we get a new prediction for the  $\mu^+ \mu^-$  mode,

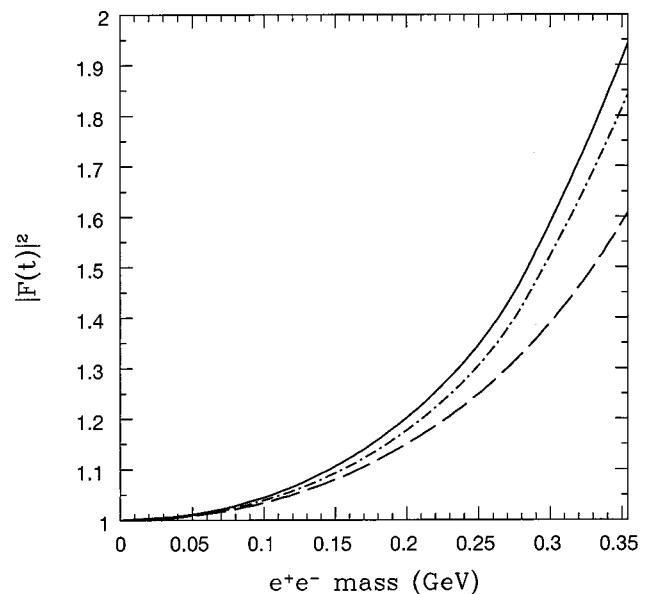


FIG. 4. Same as Fig. 3, but with the  $K^+ \rightarrow \pi^+ e^+ e^-$  form factor squared as a function of the dielectron mass.

$$B(K^+ \rightarrow \pi^+ \mu^+ \mu^-) = (6.5 \pm 0.6) \times 10^{-8}, \quad (20)$$

which differs only little from the original one (3). The ‘‘effective  $\lambda$ ’’ of our form factor, defined as the value of  $\lambda$  in linear parametrization (5) that leads to the same  $\mu^+ \mu^- / e^+ e^-$  branching ratio, is 0.057 (in the original version of the meson dominance calculation [14] it was 0.039). From the above we conclude that the meson dominance model with our new form factor is consistent with the shape of the  $e^+ e^-$  mass distribution as measured in experiment [10].

A much different story is the comparison with the preliminary data [12] of the E865 experiment at the Brookhaven National Laboratory Alternating Gradient Synchrotron. Their 10 000  $K^+ \rightarrow \pi^+ e^+ e^-$  events yielded a preliminary result of the form factor parameter of  $\lambda = 0.20 \pm 0.02$ . If this value is confirmed, the meson dominance model of the  $K^+ \rightarrow \pi^+ l^+ l^-$  decays will be ruled out, despite its success with a parameter-free calculation of the branching fractions.

Let us conclude with a general comment. It is a little unfortunate that the role of the  $K^+ \rightarrow \pi^+ l^+ l^-$  decays is sometimes shrunk to a testing ground of chiral perturbation theory and other clues are not followed. Here I mean mainly the importance of these decays in studying the behavior of the electromagnetic form factor induced by the  $\rho^0$  resonance at small  $t$ . In my opinion, there are only two kinds of pro-

cesses that are able to perform this task. Besides  $K^+ \rightarrow \pi^+ l^+ l^-$  decays these are  $\omega \rightarrow \pi^0 l^+ l^-$  Dalitz decays.<sup>4</sup> Concerning the latter, the only  $e^+ e^-$  experiment performed [19] had low statistics and was unable to provide the mass spectrum. The  $\mu^+ \mu^-$  experiment [20] showed that the dimuon mass spectrum disagreed with the vector meson dominance hypothesis. The parallel with  $K^+ \rightarrow \pi^+ l^+ l^-$  is interesting. But the kaon decays we consider here are unique in populating mainly the region below the  $\pi\pi$  threshold, whereas the dilepton mass spectrum of the  $\omega$  Dalitz decays spans to much higher values. The  $K^+ \rightarrow \pi^+ l^+ l^-$  decays can serve as a unique magnifying glass for studying the behavior of the  $\rho$ -induced electromagnetic form factor at small  $t$ .

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<sup>4</sup>Dalitz decays of  $\phi$ , which can, in principle, serve for the same purposes, have not been observed yet.

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