Vector-meson magnetic dipole moment effects in radiative τ decays

G. López Castro and G. Toledo Sánchez

Departamento de Física, Centro de Investigación y de Estudios, Avanzados del IPN, Apartado, Postal 14-740,
07000 México, Distrito Federal, México

We study the possibility that the magnetic dipole moment of light charged vector mesons could be measured from their effects in $\tau^- \rightarrow V^- \nu_\tau \gamma$ decays. We conclude that the energy spectrum and angular distribution of photons emitted at small angles with respect to vector mesons is sensitive to the effects of the magnetic dipole moment. Model-dependent contributions and photon radiation off other electromagnetic multipoles are small in this region. We also compute the effects of the magnetic dipole moment on the integrated rates and photon energy spectrum of these τ lepton decays. [S0556-2821(99)04315-5]

I. INTRODUCTION

The magnetic dipole moment (μ) and electric quadrupole moment ($\mathcal{Q}$) of vector mesons ($J^P = 1^-$), and more generally of spin-one elementary and hadronic particles, have not been measured yet. In the case of vector mesons, even upper bounds have not been reported by experiments up to now. The very short lifetime of vector mesons do not allow one to use vector-meson-electron scattering (as in the case of the deuteron) or the spin-precession technique [1] (as is useful for hyperons) to measure their electromagnetic static properties. Instead, only photoproduction experiments or radiative decays may eventually be able to measure these multipoles [2]. In this paper we study the sensitivity of $\tau^- \rightarrow V^- \nu_\tau \gamma$ decays to the magnetic dipole moment of the vector meson V^- and identify an observable associated with this decay that could provide a measurement of this important property.

Measurements of the magnetic dipole and electric quadrupole moments of the W gauge bosons would provide a significant test for the gauge structure of the standard model of particle interactions, while the corresponding multipoles of vector mesons contain information on the structure of these hadrons and, ultimately, about the dynamics of strong interactions. At present, more restrictive bounds on the magnetic dipole and electric quadrupole moments of the $W^\pm$ gauge bosons are being provided from experiments at the CERN $e^+ e^-$ collider LEPII [3] and Fermilab Tevatron colliders [4]. These bounds are consistent with the predictions based on the standard model, while the corresponding experimental information for vector mesons is absent. The simplest of the spin-one systems, whose multipoles have been measured with good precision, is the deuteron ($J^P = 1^+$): $\mu_D = 0.85741 \mu_N$, $\mathcal{Q}_D = 0.2859$ fm^2 [5]. This information confirms the picture that the deuteron can be viewed as a weakly bound state of a neutron and a proton whose electromagnetic properties can be understood, for instance, in terms of a model for low-energy interactions of baryons and mesons [6].

In a previous paper we have considered the possibility of measuring the magnetic dipole moment of light vector mesons $\rho^\pm$ and $K^{*\pm}$ by looking at the energy and angular distribution of photons emitted in their two-body radiative decays ($V^\pm \rightarrow P^\pm P^0 \gamma$, where P denotes a pseudoscalar meson)

[7]. We have found that this observable is more sensitive to the effects of the magnetic dipole moment than their corresponding integrated rates [8]. Indeed, measurements of the photon spectrum in the kinematical region where this observable is dominated by the emission off the magnetic dipole moment, would provide a determination of this property within $\Delta \mu = \pm 0.5$ (in units of $e/2m_V$), if the photon spectrum is measured with a precision of around 25% [7].

In the present paper we are concerned with the possibility of measuring the magnetic dipole moment of the vector mesons $\rho^\pm$, $K^{*\pm}$ in the radiative decays of the τ lepton, namely $\tau^- \rightarrow V^- \nu_\tau \gamma$ (henceforth our discussion will be focused on the ρ^- vector meson, but everything applies to the K^{*-} meson with the appropriate changes in flavor indices). Among the motivations for this study we find the following: (a) charged vector mesons are produced in a clean way through the decay $\tau^- \rightarrow V^- \nu_\tau$ and the ρ^- accounts for almost 25% of all τ decays [9], therefore the corresponding decay with an accompanying photon can be expected at a fraction of a percent level and, (b) there are several recent studies of the electromagnetic vertex of light vector mesons that provide a computation of their electromagnetic multipoles [10,11].

An implicit shortcoming of dealing with vector mesons (and unstable particles in general) is that the separation of the production and decay processes of a resonance are in general model dependent. Actually, only the full S-matrix amplitude that involves the production and decay of an intermediate resonance is physically meaningful. Therefore, the consideration of truncated processes such as the radiative decays $V^+ \rightarrow P^+ P^0 \gamma$ [7,8] or the radiative production of a resonance as in the present study necessarily involves a factorization approximation. The effects of the vector meson magnetic dipole moment in the full production and decay processes (for example, $\tau^- \rightarrow \rho^- \nu_\tau \gamma \rightarrow \pi^- \pi^0 \nu_\tau \gamma$ in the present case) and the difficulties associated with a gauge-invariant formulation of the electromagnetic vertex of an unstable vector particle [12] goes beyond the purpose of the present paper [13].

This paper is organized as follows. In Sec. II we analyze the effects of the magnetic dipole moment of the vector mesons ρ^- or K^{*-} in the energy and angular spectrum of photons in $\tau^- \rightarrow (\rho^-, K^{*-}) \nu_\tau \gamma$ decays. These effects in the cor-

responding integrated rates and photon energy spectra are given in Sec. III. Conclusions are summarized in Sec. IV and the explicit formulas for the differential decay rates of τ lepton radiative decays are provided in the Appendix.

II. EFFECTS IN THE ENERGY AND ANGULAR DISTRIBUTION OF PHOTONS

Let us start by setting our notations. The relevant electromagnetic vertex of the charged vector meson V is defined as follows [the flow of momenta and Lorentz indices are chosen according to $V_\alpha^+(q_1) \rightarrow V_\beta^+(q_2)\gamma_\mu(k)$]:

$$\Gamma^{\alpha\beta\mu} = g^{\alpha\beta}(q_1+q_2)^\mu \alpha(k^2) + (k^\alpha g^{\mu\beta} - k^\beta g^{\mu\alpha})\beta(k^2) + (q_1+q_2)^\mu k^\beta k^\alpha \gamma(k^2). \quad (1)$$

The form factors $\alpha(k^2)$, $\beta(k^2)$, and $\gamma(k^2)$ at $k^2=0$ are related (see, for example, [14]) to the electric charge q (in units of e), the magnetic dipole moment μ (in units of $e/2m_V$), and electric quadrupole moment $\mathcal{Q}$ (in units of e/m_V^2) through the relations $\alpha(0)=q=1$, $\beta(0)=\mu$, and $\gamma(0)=(1-\mu-\mathcal{Q})/2m_V^2$.

The value $\beta(0)=2$ for the giromagnetic ratio can be considered as the canonical value [14] and it appears in a natural way in a model where the isomultiplets of vector mesons couple through non-Abelian Yang-Mills terms, in an analogous way to the $W^\pm$ gauge boson couplings in the standard model. Also, according to several authors the canonical value $\beta(0)=2$ can be considered as the gauge condition for spin-one particles [15]. However, the Yang-Mills nature of the vector mesons is far from being proved and, therefore, substantial deviations from the canonical value may be expected. For instance, the estimates of Refs. [10,11], indicate that the magnetic moments of the light vector mesons are in the range $\beta(0)=2.2-3.0(2.37)$ for the $\rho^-(K^{*-})$ mesons.

Following the discussion in our previous paper [7], we do not consider the effect of the form factor $\gamma(k^2)$ because it

enters the amplitude at the same order in the photon energy as does the model-dependent contributions [16] (for example, $\tau^- \rightarrow \pi^- \nu_\tau \rightarrow \rho^- \gamma \nu_\tau$ in the present case), and we expect them to be suppressed for the special kinematical configuration to be discussed below (see, also, Ref. [7]).

The four-momenta and the polarization four vectors (ϵ_μ^* and η_ν^*) of the vector particles are chosen as $\tau^-(p) \rightarrow V^-(q,\eta)\nu_\tau(p')\gamma(k,\epsilon^*)$. Up to terms of order k^0, the gauge-invariant amplitude for the process $\tau^- \rightarrow \rho^- \nu_\tau \gamma$ is given by

$$\mathcal{M} = \frac{eg_\rho G_F V_{ud}}{\sqrt{2}} \left\{ \bar{u}(p')O^\alpha u(p)\left(\frac{q\cdot\epsilon^*}{q\cdot k} - \frac{p\cdot\epsilon^*}{p\cdot k}\right)\eta_\alpha^* \right.$$
$$+ \bar{u}(p')O^\alpha \frac{\not{k}\not{\epsilon}^*}{2p\cdot k}u(p)\eta_\alpha^* + \frac{\beta(0)}{2q\cdot k}\bar{u}(p')O^\alpha u(p)$$
$$\times (k_\alpha \epsilon_\lambda^* - k_\lambda \epsilon_\alpha^*)\eta_\lambda^* + \frac{m_\tau}{m^2}\left(1 - \frac{\beta(0)}{2}\right)$$
$$\left. \times \bar{u}(p')(1+\gamma_5)u(p)\left(\epsilon_\lambda^* - \frac{q\cdot\epsilon^*}{q\cdot k}k_\lambda\right)\eta_\lambda^* \right\}, \quad (2)$$

where m_τ and m denote the masses of the τ lepton and the vector meson, respectively. G_F and V_{ud} denote the Fermi constant and the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element, respectively, while $O^\alpha \equiv \gamma^\alpha(1-\gamma_5)$. The constant g_ρ denotes the $W-\rho^-$ strength coupling. As stated by Low's theorem [16], this amplitude depends only on the static electromagnetic properties of the particles, the parameters of the nonradiative process, and it is model independent.

It is interesting to look at the structure of the squared amplitude in terms of the independent Lorentz scalars of the process. If we sum over polarizations of fermions and the vector meson we obtain [an overall factor $(eg_\rho G_F |V_{ud}|/\sqrt{2})^2$ is omitted]:

$$\sum_{\text{pols}} |\mathcal{M}|^2 = \frac{4m_\tau^2}{r}\left|\frac{p\cdot\epsilon}{p\cdot k} - \frac{q\cdot\epsilon}{q\cdot k}\right|^2 \left\{(1-r)(1+2r) + 4\frac{(p\cdot k)^2}{m_\tau^4}\left(1 - \frac{\beta(0)}{2}\right)^2\right\}$$

$$+ 8\epsilon\cdot\epsilon^* \frac{q\cdot k}{rm^2}\left(1 - \frac{\beta(0)}{2}\right)^2 (1+r) + 4\epsilon\cdot\epsilon^*\left\{\left(-1 + \frac{q\cdot k}{p\cdot k}\right)\left[1 - \beta(0)\frac{p\cdot k}{q\cdot k} - \frac{1}{r}\left(1 - \frac{\beta(0)}{2}\right)\right]^2\right.$$

$$\left. + \left(1 - \frac{1}{r}\frac{q\cdot k}{p\cdot k}\right)\left[\frac{1}{r}\left(1 - \frac{\beta(0)}{2}\right)^2 - \beta(0)\right] + \left(1 - \frac{1}{r}\right)\left[\left(\frac{\beta(0)}{2}\right)^2 + \frac{q\cdot k}{p\cdot k}\right]\right\}, \quad (3)$$

where we have introduced the dimensionless quantity $r \equiv (m/m_\tau)^2$. This squared amplitude satisfies the Burnett and Kroll theorem [17], i.e., does not contain terms of $O(k^{-1})$. Equation (3) takes a very simple form when $\beta(0)=2$.

When we sum over photon polarizations, the first term in Eq. (3) becomes proportional to the infrared factor (we will

place in the rest frame of the τ lepton):

$$\sum_{\text{pols }\epsilon} \left|\frac{p\cdot\epsilon}{p\cdot k} - \frac{q\cdot\epsilon}{q\cdot k}\right|^2 = \frac{|\vec{q}|^2\sin^2\theta}{\omega^2(E-|\vec{q}|\cos\theta)^2}, \quad (4)$$

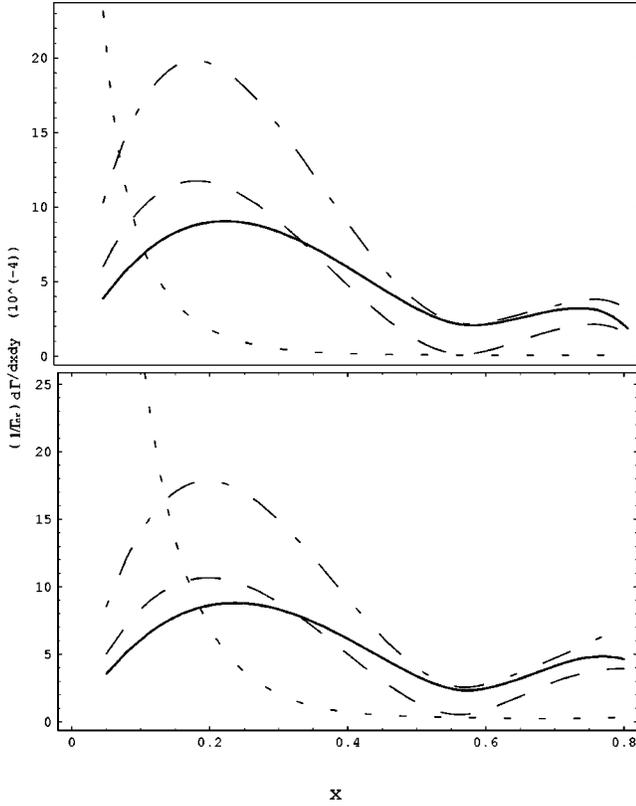where θ is the angle between the photon and the vector me-

FIG. 1. Differential photon spectrum for $\tau \to \rho \nu \gamma$ decay. See description after Eq. (5).

FIG. 2. Same as in Fig. 1 for the $\tau \to K^* \nu \gamma$ decay.

son three momenta, $E (\omega)$ is the energy of the vector meson (the photon), and $E = \sqrt{\vec{q}^2 + m^2}$ in this frame. This term is suppressed for small values of the angle θ, therefore, the photon radiation off the electric charges of the τ lepton and the vector meson can be suppressed for this angular configuration. In the same way, it can be shown that this small angle configuration allows to suppress the interference terms of $O(k^0)$ that comes from the interference between the quadrupole (or model-dependent) terms with the emission off the charges of τ and V.

Therefore, it becomes convenient to split the unpolarized differential decay rate into two terms. The first one, which is associated to the first term in Eq. (3), vanishes when $\theta = 0, \pi$ (collinear photons). This term reduces to the radiation emitted from the electric charges of the τ^- and V^- when $\beta(0) = 2$. The second term [remaining terms in Eq. (3)], does not vanish for collinear photons and contains a sensitive dependence on the magnetic dipole moment of the vector meson. If we denote these two terms with subscripts 1 and 2, respectively, we can write the normalized [to the nonradiative decay rate $\Gamma_{nr} = \Gamma(\tau^- \to V^- \nu_\tau)$] double differential decay rate as follows:

$$\frac{d\Gamma}{\Gamma_{nr} dx dy} = \frac{1}{\Gamma_{nr}} \sum_{\pm} \left(\frac{d\Gamma_1^{\pm}}{dx dy} + \frac{d\Gamma_2^{\pm}}{dx dy} \right), \qquad (5)$$

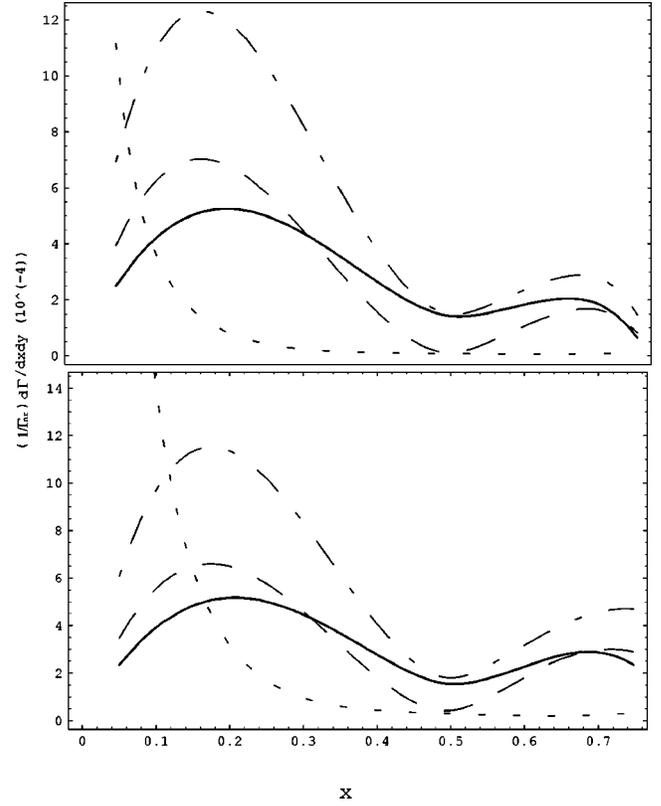where we have introduced the dimensionless variables x

$= 2\omega/m_\tau$ and $y = \cos \theta$. The explicit expressions for the two contributions to the differential decay rate are given in the Appendix.

The normalized rates given above do not depend on the g_ρ coupling and the CKM matrix element. Their only dependence is on the magnetic dipole moment $\beta(0)$. Following the previous discussion, in Fig. 1 (ρ^- meson) and Fig. 2 (K^{*-}) we plot the differential decay rates of Eq. (5) as a function of the photon energy (x) for fixed values of the angle θ [$\theta = 10^0 (20^0)$ corresponds to the upper (lower) half of the plots] and three different values of $\beta(0)$. The short-dashed line in the different plots denotes the first term of Eq. (5), while the other lines refer to the second term, for the different values of $\beta(0)$: $\beta(0) = 1$ (solid line), $\beta(0) = 2$ (long-dashed line), and $\beta(0) = 3$ (long–short-dashed line).

As is evident from these plots, the model-independent terms of order k^0 dominate over the radiation due to the electric charges [terms of $O(k^{-2})$ in the squared amplitude] in the region of photon energies $0.2 < x < 0.5$ for the $\tau^- \to \rho^- \nu_\tau \gamma$ decay and $0.2 < x < 0.4$ for the $\tau^- \to K^{*-} \nu_\tau \gamma$ case. Therefore, it is precisely in this kinematical region where the measurement of the photon spectrum may provide a determination of the vector meson magnetic moment with a reasonable accuracy.

The curves also exhibit a dip at $x = 0.5667$ for the ρ^- case and $x = 0.4991$ for the case of K^{*-} production. The position of this dip is independent of y and the value of $\beta(0)$. Furthermore, the dip is more pronounced when $\beta(0) = 2$. At first sight this dip could be associated to a *null* radiation amplitude [18], however, it can be checked that it corre-
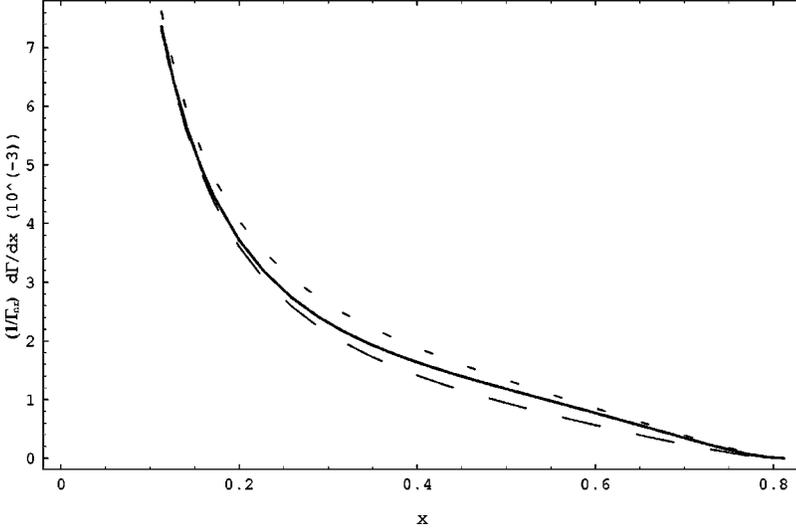
FIG. 3. Normalized photon energy spectrum of the decay $\tau \to \rho \nu \gamma$ for $\beta(0) = 1$ (solid line), 2 (long-dashed), and 3 (short-dashed).

sponds to the special kinematical configuration where the vector meson remains at rest. Indeed, the dominant contribution to the differential decay rate is the one evaluated at the E^- solution (see the Appendix) and this rate vanishes when $v^- = 0$ or, equivalently, $x = 1 - \sqrt{r}$ [i.e., the position of the dip in the photon energy spectrum is independent of y and $\beta(0)$].

III. EFFECTS IN THE INTEGRATED RATE AND SINGLE PHOTON SPECTRUM

For completeness, we also provide the dependence of the decay rate and the photon energy spectrum for the process $\tau^- \to (\rho^-, K^{*-}) \nu_\tau \gamma$ upon the magnetic dipole moment of the vector meson. The general expression for the branching ratio is

$$
\begin{aligned}
B(&\tau \to V^- \nu_\tau \gamma; \omega > \omega_{\min}) \\
&= \frac{\alpha}{2\pi} \frac{r B(\tau^- \to V^- \nu_\tau)}{m_\tau^2 (1-r)^2 (1+2r)} \\
&\quad \times \int_{\omega_{\min}}^{(m_\tau^2 - m^2)/2m_\tau} d\omega \int_{E^{\min}}^{E^{\max}} dE F(E, \omega),
\end{aligned}
$$
(6)

where

TABLE I. Branching ratios (in %) for the $\tau^- \to (\rho^-, K^{*-}) \nu_\tau \gamma$ decays as a function of $\beta(0)$ and the lower cutoff photon energy $\omega_{\min}$ (in MeV).

Channel	$\beta(0)$	$\omega_{\min} = 15$	30	60	90
ρ^-	1	0.07618	0.05900	0.04268	0.03374
	2	0.07332	0.05614	0.03984	0.03094
	3	0.07981	0.06260	0.04616	0.03705
K^{*-}	1	0.00246	0.00187	0.00131	0.00101
	2	0.00243	0.00184	0.00128	0.00098
	3	0.00267	0.00207	0.00151	0.00119

$$
\begin{aligned}
F(E, \omega) &= \frac{8\omega^2 (E^2 - m^2) - 2(A + 2E\omega)^2}{\omega^2 A^2 r} C(\omega) \\
&\quad + 8\left(1 + \frac{A}{2m_\tau \omega}\right) \left\{1 - \frac{1}{r}\left(1 - \frac{\beta(0)}{2}\right)\right. \\
&\quad \left. + \frac{2\beta(0) m_\tau \omega}{A}\right\}^2 - 8\left(1 + \frac{A}{2rm_\tau \omega}\right) \\
&\quad \times \left[\frac{1}{r}\left(1 - \frac{\beta(0)}{2}\right)^2 - \beta(0)\right] + 8\left(\frac{1}{r} - 1\right)\left[\left(\frac{\beta(0)}{2}\right)^2\right. \\
&\quad \left. - \frac{A}{2m_\tau \omega}\right] + \frac{8A}{m^2}\left(1 + \frac{1}{r}\right)\left(1 - \frac{\beta(0)}{2}\right)^2,
\end{aligned}
$$
(7)

and

$$
A = m_\tau^2 + m^2 - 2m_\tau \omega - 2m_\tau E,
$$

$$
C(\omega) = m_\tau^2 (1-r)(1+2r) + 4\omega^2 \left(1 - \frac{\beta(0)}{2}\right)^2,
$$

$$
\begin{aligned}
E^{\max, \min} &= \frac{(m_\tau - \omega)(m_\tau^2 - 2m_\tau \omega + m^2) \pm \omega(m_\tau^2 - 2m_\tau \omega - m^2)}{2(m_\tau^2 - 2m_\tau \omega)},
\end{aligned}
$$
(8)

and $\alpha \equiv e^2/4\pi \approx 1/137$ denotes the fine-structure constant. Thus, the branching ratio for the radiative decays depends only upon the minimum photon cutoff energy $\omega_{\min}$ the experiment is able to discriminate and also depends on the magnetic dipole moment of vector meson $\beta(0)$.

In Table I we display the branching ratios for $\tau^- \to (\rho^-, K^{*-}) \nu_\tau \gamma$ decays for three different values of $\beta(0)$ and four realistic values of the photon cutoff energy $\omega_{\min}$. The branching ratios of the nonradiative processes $\tau^- \to V^- \nu_\tau$ were taken from Ref. [9]. As expected, the branching ratios in Table I are of order 10^{-2} with respect to their nonradiative
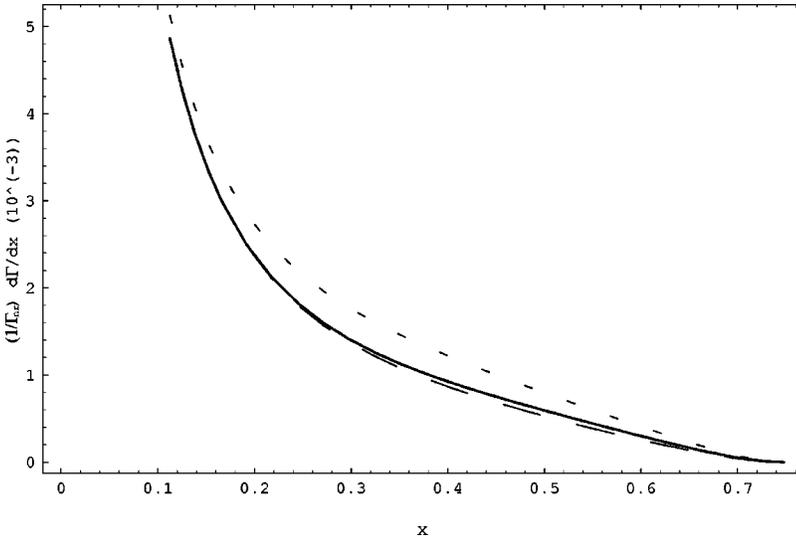
FIG. 4. Same description as in Fig. 3 for the $\tau \to K^* \nu \gamma$ decay.

counterparts. We observe that these branching ratios approach their minimum values when $\beta(0)$ is close to its canonical value $\beta(0) = 2$. We observe also that these ratios become more sensitive to the effect of $\beta(0)$ when $\omega_{\min}$ increases. This happens because the radiation due to the electric charges is more important at the lower end of the photon energy spectrum, while higher electromagnetic multipoles are more important for higher energies of the radiated photon.

Finally, we can also compute the effects of $\beta(0)$ in the photon energy spectrum. The single photon energy spectrum is obtained from Eq. (6) after performing the integration only over E. The plots for the normalized photon energy spectrum $(1/\Gamma_{\mathrm{nr}}) d\Gamma/dx$, where $x \equiv 2\omega/m_\tau$, for values of $\omega \geq 100$ MeV are shown in Fig. 3 (for $\tau \to \rho \nu_\tau \gamma$) and Fig. 4 (for $\tau \to K^* \nu_\tau \gamma$). The solid, long-dashed, and short-dashed lines correspond, respectively, to $\beta(0) = 1$, 2, and 3. These single photon spectra are more sensitive in the region of intermediate photon energies. However, as in the case of the integrated rates, this observable is not as sensitive to the effects of the magnetic dipole moment as the double differential photon spectrum discussed in Sec. II.

$\to (\rho^-, K^{*-}) \nu_\tau \gamma$ decays. Our main interest is on the energy and angular distribution of the emitted photon. We observe that this differential decay rate is sensitive to the effects of the magnetic dipole moment when the photon is emitted at small angles with respect to the vector meson. This effect is more important for intermediate photon energies where radiation off the magnetic dipole moment dominates over the radiation due to the electric charges and the electric quadrupole moment of vector mesons and the other model-dependent contributions. For completeness, we have computed also the effects of the magnetic dipole moment on the corresponding branching ratios and photon energy spectrum of these decays and found that they are less sensitive to these effects.

Since vector mesons are detected through their decays into two pseudoscalar mesons, the present study would require to reconstruct the invariant masses of the two pseudoscalars in a region close to the vector meson mass. On the other hand, it requires one to be able to detect photons emitted at small angles with respect to the vector meson. Those experimental capabilities would allow a measurement of the magnetic dipole moments from the most favored region in the differential decay rate with a reasonable accuracy.

IV. SOME CONCLUSIONS

In this paper we have explored the effects of the magnetic dipole moments of vector mesons on the τ^-

ACKNOWLEDGMENTS

APPENDIX

The expressions for the differential decay rates appearing in Eq. (5) are

$$\frac{d\Gamma_1^{\pm}}{\Gamma_{\mathrm{nr}} dx\, dy} = \frac{2\alpha x(1-y^2)}{\pi |(2-x)v^{\pm} + xy|(1-r)} \frac{(E^{\pm})^3 (v^{\pm})^4}{(T^{\pm})^2} \tag{A1}$$

and

$$\frac{d\Gamma_2^{\pm}}{\Gamma_{nr}dxdy} = \frac{2x\alpha E^{\pm}(v^{\pm})^2}{\pi|(2-x)v^{\pm}+xy|(1-r)^2(1+2r)}\left\{\left((1-y^2)\left(\frac{xE^{\pm}v^{\pm}}{T^{\pm}}\right)^2 + \frac{1}{r}T^{\pm}(1+r)\right)\left(1-\frac{\beta(0)}{2}\right)^2\right.$$

$$\left.+\frac{r}{x}(T^{\pm}+x)\left[1+\frac{\beta(0)x}{T^{\pm}}-\frac{1}{r}\left(1-\frac{\beta(0)}{2}\right)\right]^2 + \frac{1}{x}(T^{\pm}+xr)\left[\beta(0)-\frac{1}{r}\left(1-\frac{\beta(0)}{2}\right)^2\right] + \frac{(1-r)}{x}\left(x\left(\frac{\beta(0)}{2}\right)^2 - T^{\pm}\right)\right\}.$$

$$(A2)$$

In the above expressions, $\alpha \equiv e^2/4\pi$ is the fine-structure constant, $v^{\pm} = \sqrt{1-r/(E^{\pm})^2}$ and

$$T^{\pm} = 1+r-x-2E^{\pm},$$

$$E^{\pm} = \frac{1}{(x-2)^2-x^2y^2}\{(2-x)(1+r-x)\pm xy\sqrt{(1-r-x)^2-x^2r(1-y^2)}\}. \qquad (A3)$$

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