Gravitationally violated U(1) symmetry and neutrino anomalies

Anjan S. Joshipura
Theoretical Physics Group, Physical Research Laboratory, Navarangpura, Ahmedabad, 380 009, India
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The current searches for neutrino oscillations seem to suggest an approximate $L_e - L_\mu - L_\tau$ flavor symmetry. This symmetry implies a pair of degenerate neutrinos with a mass m_0 and large leptonic mixing. We explore the possibility that gravitational interactions break this global symmetry. The Planck scale suppressed breaking of the $L_e - L_\mu - L_\tau$ symmetry is shown to lead to the right amount of splitting between the degenerate neutrinos needed in order to solve the solar neutrino problem. The common mass m_0 of the pair can be identified with the atmospheric neutrino scale. A concrete model is proposed in which the smallness of m_0 and the hierarchy in the solar and atmospheric neutrino scales get linked to hierarchies in the weak, grand unification, and Planck scales. [S0556-2821(99)04011-4]

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The pattern of neutrino masses and mixing as suggested by the present experimental evidence and hints seems quite different from the one in the quark sector. The oscillations of ν_μ of the atmospheric origin require a large $\nu_\mu - \nu_\tau$ mixing and a very small difference $\Delta_A \sim 10^{-3}$ eV^2 in their squared masses [1]. The solar neutrino anomalies require [2] a much smaller mass scale $\Delta_S \sim 10^{-6}$ eV^2 [Mikheyev-Smirnov-Wolfenstein (MSW) [3] conversion] or $\Delta_S \sim 10^{-10}$ eV^2 (vacuum oscillations [4]). The latter alternative can reconcile the solar anomaly only if mixing involving ν_e is large.

The conventional seesaw models based on grand unified theories link the masses and mixing of leptons to that in the quark sector [5]. This link does not seem to be fully supported by the experiments and one must either admit a variety of textures [6,7] in right-handed neutrino masses or look for some alternative [8] schemes.

The presently available information on the solar neutrinos do not seem to choose unambiguously [2] between the MSW or the vacuum oscillation solutions, although the MSW conversion with a large angle seems to be disfavored [2] by the recent [9] day night asymmetry measurements at SuperKamiokande. Thus the MSW mechanism most likely requires small mixing of ν_e. It is then intriguing why large mixing is preferred in the $\nu_\mu - \nu_\tau$ system with less hierarchical Δ_A. The vacuum solution seems more natural from the point of view of this theoretical prejudice, but in this case one has a problem of accounting for a very large hierarchy, $\Delta_S/\Delta_A \sim 10^{-7}$. This note is devoted to discussion of these issues.

Let us suppose that both the solar and the atmospheric neutrino oscillations are described by maximal ($\sim \pi/4$) mixing among relevant states. This hypothesis is shown [10] to lead through unitarity to a unique structure for the mixing matrix U given by

$$U = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ c/\sqrt{2} & c/\sqrt{2} & -s \\ s/\sqrt{2} & s/\sqrt{2} & c \end{pmatrix}, \tag{1}$$

where, $c = \cos\theta$, $s = \sin\theta \sim 1/\sqrt{2}$. This structure can describe the solar and atmospheric neutrino observations successfully if $\Delta_{23} \equiv m_{\nu_3}^2 - m_{\nu_2}^2 \sim 10^{-3}$ eV^2 and $\Delta_{12} \equiv m_{\nu_2}^2 - m_{\nu_1}^2 \sim 10^{-10}$ eV^2. It also implies that ν_e does not oscillate at the atmospheric scale in accordance with the findings at SuperKamiokande. This U together with neutrino masses can be used to determine the structure of the light neutrino mass matrix in basis with diagonal charged lepton masses. This was done [10] in case of the hierarchical masses. Since large mixing may be intimately related to pseudo-Dirac structure, let us suppose that a pair of neutrinos are (almost) degenerate with masses m_0 and $-m_0$. This common mass may be identified with the atmospheric neutrino scale. For a fixed U as given in Eq. (1), one has three physically distinct possibilities corresponding to m_i $(i=1,2,3)$ values

$$(a)\ (m_0, -m_0, 0) \quad (b)\ (m_0, 0, -m_0) \quad (c)\ (0, m_0, -m_0).$$

This implies the following neutrino mass matrices M_ν for the three light states:

$$(a)\ m_0 \begin{pmatrix} 0 & c & s \\ c & 0 & 0 \\ s & 0 & 0 \end{pmatrix} \quad (b)\ \frac{m_0}{2} \begin{pmatrix} 1 & c & s \\ c & 1-3s^2 & 3cs \\ s & 3cs & 1-3c^2 \end{pmatrix}, \tag{2}$$

$$(c)\ \frac{m_0}{2} \begin{pmatrix} 1 & -c & -s \\ -c & 1-3s^2 & 3cs \\ -s & 3cs & 1-3c^2 \end{pmatrix}. \tag{3}$$

Of these, the texture in (a) seems more interesting as it does not presuppose any relations among matrix elements of M_ν. Moreover, this texture follows from a simple $L_e - L_\mu - L_\tau$ symmetry. Conversely, bimaximal mixing may be regarded [6] as a consequence of the $L_e - L_\mu - L_\tau$ symmetry imposed in the leptonic sector [11]. One still needs to understand the origin of m_0 and of much smaller splitting Δ_S between (almost) degenerate pair. The splitting may arise due to small breaking of the $L_e - L_\mu - L_\tau$ symmetry. This can be parametrized [6] in terms of a small parameter ϵ leading to

$$M_\nu = m_0 \begin{pmatrix} \epsilon & c & s \\ c & \epsilon & \epsilon \\ s & \epsilon & \epsilon \end{pmatrix}, \tag{4}$$

where different entries are meant to denote the order of magnitudes of the breaking term. This leads to

$$\Delta_S \sim 4 m_0 \epsilon. \tag{5}$$

When m_0 is identified with the atmospheric scale (~ 0.03 eV), the above equation implies

$$\begin{aligned} &\epsilon \sim (10^{-4} - 10^{-5}) \\ &\text{for} \quad \Delta_S \sim (10^{-5} - 10^{-6})\ \text{eV}^2 \quad (\text{MSW}), \\ &\epsilon \sim (10^{-9} - 10^{-10}) \\ &\text{for} \quad \Delta_S \sim (10^{-10} - 10^{-11})\ \text{eV}^2 \quad (\text{vacuum}). \end{aligned} \tag{6}$$

What could be the origin of such small values for m_0 and ϵ? It is indeed possible to link these scales to the hierarchies among the known scales namely M_{weak}, $M_H \sim M_{GUT}$, and M_{Planck}.

Let us consider the standard $SU(2)\times U(1)$ model without addition of any right-handed neutrinos. Neutrino masses are generated through the following Yukawa couplings when an $SU(2)$-triplet Higgs field is introduced:

$$-\mathcal{L}_\nu = \tfrac{1}{2} f_{ij} L_i^T \Delta L_j + \text{c.c.} \tag{7}$$

Here Δ refers to the 2×2 matrix for the triplet Higgs field. We have suppressed the Lorentz indices in the above equation. i,j refer to the generation indices. We also impose the $L_e - L_\mu - L_\tau$ symmetry. Nonzero vacuum expectation value (vev) for the neutral component of Δ then leads to structure as in Eq. (2a) with

$$m_0 = (f_{12}^2 + f_{13}^2)^{1/2} \langle \Delta^0 \rangle; \quad \tan\theta = \frac{f_{13}}{f_{12}}.$$

θ could be naturally large for $f_{12} \sim f_{13}$. The smallness of m_0 may appear unnatural in $SU(2)\times U(1)$ theory. But small triplet vev and, hence, m_0 may result if theory contains heavy scales such as M_{GUT}. This follows from the induced vev mechanism which implies [5,12]:

$$m_0 \sim \langle \Delta^0 \rangle \sim \frac{M_W^2}{M_H}.$$

$M_H \sim 10^{15}$ GeV then leads to the required value $m_0 \sim 10^{-2}$ eV.

The symmetry $L_e - L_\mu - L_\tau$ must be regarded as a global symmetry in the present context, since it is not possible to gauge this symmetry in standard model (SM) without introducing right-handed neutrinos. Such global symmetries are known to be unstable against gravitational effects [13,14] and would be broken. We assume that this breaking is manifested in the low-energy theory through higher-dimensional operators suppressed by the Planck mass M_P. One could write the following symmetry breaking nonrenormalizable terms:

$$\begin{aligned} O_1 &= \frac{1}{2} \beta_{1ij} \nu_i^T \nu_j \left(\frac{\phi^{0*2}}{M_P} \right), \\ O_2 &= \frac{1}{2} \beta_{2ij} \nu_i^T \nu_j \Delta^0 \left(\frac{\eta}{M_P} \right), \\ O_3 &= \frac{1}{2} \beta_{3ij} \nu_i^T \nu_j \Delta^0 \left(\frac{\eta^\dagger \eta}{M_P^2} \right). \end{aligned} \tag{8}$$

Here we have introduced $SU(2)\times U(1)$ singlet field η which is assumed to obtain large $\sim M_H$ vev. ϕ^0 (Δ^0) corresponds to the neutral component of the $SU(2)\times U(1)$ doublet (triplet) Higgs field. The dimensionless couplings β_{mij} ($m=1,2,3$) break the $L_e - L_\mu - L_\tau$ symmetry.

The operator O_1 is the familiar one introduced, for example, in [15]. The above operators lead to the ϵ parameters in Eq. (4) and, hence, to the splitting among the degenerate pair. One, respectively, gets, for the operators $O_1 - O_3$,

$$\begin{aligned} (\Delta m^2)_1 &\sim 4\beta_1 m_0 \frac{M_{weak}^2}{M_P} \sim \beta_1 (10^{-7}\ \text{eV}^2) \left(\frac{m_0}{10^{-2}\ \text{eV}} \right), \\ (\Delta m^2)_2 &\sim 4\beta_2 m_0^2 \left(\frac{\langle \eta \rangle}{M_P} \right) \\ &\sim \beta_2 (4.0\times 10^{-6}\ \text{eV}^2) \left(\frac{m_0^2}{10^{-3}\ \text{eV}^2} \right) \left(\frac{\langle \eta \rangle}{10^{16}\ \text{GeV}} \right), \\ (\Delta m^2)_3 &\sim 4\beta_3 m_0^2 \left(\frac{\langle \eta \rangle}{M_P} \right)^2 \\ &\sim \beta_3 (4.0\times 10^{-9}\ \text{eV}^2) \left(\frac{m_0^2}{10^{-3}\ \text{eV}^2} \right) \left(\frac{\langle \eta \rangle}{10^{16}\ \text{GeV}} \right)^2, \end{aligned} \tag{9}$$

where $(\Delta m^2)_i$ denotes splitting of the degenerate states induced by O_i. Here we have assumed that parameters f in Eq. (7) are of O(1) and have identified $f\langle \Delta^0 \rangle \sim \langle \Delta^0 \rangle = m_0$.

The operator O_1 gives a splitting which is somewhat larger (smaller) than the scale needed for the vacuum (MSW) solution to the solar neutrino problem. The second and the third operators can generate scales relevant for the MSW and the vacuum solutions, respectively, if the vev for η is at or near the grand unification scale. While MSW is a natural and appealing solution to the solar neutrino problem, it cannot be implemented in the present context for two reasons. Firstly, the large angle solution obtained here from the $L_e - L_\mu - L_\tau$ symmetry seems to be disfavored experimentally as already mentioned. More importantly, the said symmetry implies a mixing angle of $\pi/4$ degree for which the MSW effect does not occur. The corrections to this mixing angle induced due to ϵ are too small to change it appreciably [6]. Thus, in spite of the possibility of generating the MSW scale naturally, vacuum solution is to be preferred in the present context. This solution can be realized easily in a simple model to which we now turn.

We extend the SM by adding two additional Higgs fields namely, an $SU(2)$-triplet Δ and a singlet η. In addition, we impose $L_e-L_\mu-L_\tau$ and a Z_3 symmetry with the charge assignment $(1,-1,1,1)$ for the fields (u^c,d^c,ϕ,η). The rest of the fields are assumed to carry zero charge under Z_3. All the scalar fields are also assumed to be neutral under $L_e-L_\mu-L_\tau$. The Yukawa couplings in Eq. (7) generate the required $L_e-L_\mu-L_\tau$ symmetric mass matrix. The smallness of Δ_0 arises as follows. Consider the following scalar potential containing a heavy $\sim M_{GUT}$ and the electroweak scale:

$$V=\mu^2\phi^\dagger\phi+M_\Delta^2\ \mathrm{Tr}\Delta^\dagger\Delta+M_\eta^2\eta^\dagger\eta+\lambda(\phi^\dagger\phi)^2 +\lambda_\Delta\ \mathrm{Tr}(\Delta^\dagger\Delta)^2+\lambda_\eta(\eta^\dagger\eta)^2+\cdots -[\beta\phi^T\Delta\phi\eta+\gamma\eta^3+\mathrm{c.c.}]. \tag{10}$$

The terms not explicitly written in the above equations correspond to some of the quartic terms involving Δ and crossed quartic terms [16] for the doublet field. We assume that all the mass scales except the one (namely μ^2) associated with the $SU(2)$ doublet field in the above equations are large, i.e., $\sim M_{GUT}$. For M_η^2 negative, the vev for η is driven to a large scale, while Δ^0 vev can be small for $M_{\Delta^2}>0$. Minimization of Eq. (10) gives,

$$u\sim-\frac{M_\eta^2}{\lambda_\eta},$$

$$\omega\sim\frac{\beta v^2u}{2M_\Delta^2}\sim\frac{\beta v^2}{2M_H},$$

$$v^2\sim-\frac{\mu^2}{\lambda+(\beta^2/2\lambda_\eta)(M_\eta^2/M_\Delta^2)}, \tag{11}$$

where, $\langle\phi^0\rangle\equiv v/\sqrt{2}$, $\langle\Delta^0\rangle\equiv\omega/\sqrt{2}$, $\langle\eta^0\rangle\equiv u/\sqrt{2}$. The choice $M_H\sim M_\Delta\sim10^{15}$ GeV leads to $\langle\Delta^0\rangle\sim10^{-2}$ eV very close to the atmospheric mass scale $m_0\sim0.03$ eV.

Note that in the absence of the cubic term in η, the above scalar potential has a global symmetry under which Δ and η carry opposite charges. The cubic term allowed here by the Z_3 symmetry makes the would be Goldstone boson, the Majoron, massive with mass at the grand unification scale. The Majoron would also obtain mass through higher-dimensional terms [17] like η^6/M_p^2. This mass would be of the order $10^{-3}M_{GUT}$ for the natural value of the parameters. Such heavy scalar in the presence of large symmetry breaking scale $\langle\eta\rangle$ may appear to cause cosmological problem [17]. In the present case, the coupling of η to the heavy triplet field Δ provides an effective decay channel for the Majoron provided M_{Δ_I} (I referring to the *CP*-odd component) is sufficiently lighter than the Majoron. The quartic coupling β in Eq. (10) is seen to lead to

$$\Gamma(\eta_I\rightarrow\phi\Delta_I)\sim\frac{\beta^2M_W^2}{M_{\eta_I}8\pi}.$$

The above provides a fast decay channel for η_I with lifetime of order $\sim10^{-13}$ sec provided its mass is around 10^{14} GeV. This could sufficiently dilute the relic density of the massive Majoron avoiding problem with the cosmology.

The structure of the $L_e-L_\mu-L_\tau$ breaking higher-dimensional operator induced by gravitational effect is governed by the gauge symmetries of the model. It was realized that this is true, even if the gauge symmetry of the low-energy world is a discrete [18] one. The gauged discrete symmetries may arise in the low-energy theory as a remnant of some continuous gauge symmetries, if the Higgs fields responsible for its breaking are invariant under a discrete subgroup. Such discrete symmetries are then required to satisfy the discrete anomaly constraints [19]. These constraints derived in [19] are given for Z_N group as

$$SU(M)^2XZ_N:\ \ \Sigma T_iq_i=\tfrac{1}{2}p_MN,$$

$$Z_N^3:\quad \Sigma q_i^3=mN+\delta nN^3/8, \tag{12}$$

where, $\delta=0\,(1)$ for N odd (even). The corresponding anomalies involving $U(1)$ factors do not impose any significant restrictions on the low-energy theory [19]. It is easily verified that the discrete Z_3 imposed here indeed satisfies these constraints with $p_3=p_2=0, m=-1$ in case of the three fermionic generations. This symmetry may then be imposed as an additional constraint in deciding the structure of the allowed higher-dimensional terms. One sees that of the three operators in Eq. (8), only dimension six operator is invariant under the Z_3. As mentioned in Eq. (9), this operator can lead to the right splitting between the degenerate pairs to account for the solar neutrino deficit through vacuum oscillations. It is indeed remarkable that one could relate both the solar and the atmospheric scales to the other known scales this way.

The imposition of a discrete symmetry above is somewhat *ad hoc* and may be dispensed with, if the coefficient $\beta_{1,2}$ associated with dimension five terms are small instead of being O(1). Specifically, one requires $\beta_1\sim10^{-3},\beta_2\sim10^{-4}$ in Eq. (9) in order to account for the vacuum value for Δ_S. This suppression need not be as unnatural as it may look. A familiar example of such suppression [14,20] is provided in case of the breaking of the Peccei-Quinn (PQ) symmetry [21] induced by the wormhole effects [13]. It is found that if the global symmetry in question is spontaneously broken at a scale f, then coefficients characterizing its gravitational breaking are suppressed by the wormhole action. Such suppression is typically expected [20] to be f/M_P. Thus in our case, spontaneous breaking of the $L_e-L_\mu-L_\tau$ symmetry around the grand unified theory (GUT) scale may account for the required suppression in $\beta_{1,2}$.

We have restricted ourselves so far to the SM. Many of the present considerations can be generalized to the $SU(5)$ model with some modifications. The triplet Δ may be part of a 15 dimensional representation (denoted by the same symbol) of $SU(5)$ and the role of the singlet field may be played by the adjoint (A) representation used for breaking the $SU(5)$ symmetry. A straightforward generalization of the $L_e - L_\mu - L_\tau$ symmetry would be to assume a family dependent $U(1)$ symmetry and assign charges $(1,-1,-1)$, respectively, to three generations of the $\overline{5}$-plet of fermions leaving rest of the fields neutral under it. The couplings $\overline{5}_i\overline{5}_j\Delta$ then lead to the neutrino masses as in Eq. (7) if the triplet component of the 15-plet has a small vev. Such vev could follow [5] from a term in the scalar potential coupling, the 5-dimensional Higgs field $\overline{H}$ to Δ:

$$\beta \overline{H}_a \overline{H}_b \Delta^{ac} A_c^b , \tag{13}$$

where, a,b refer to the $SU(5)$ indices. This term is analogous to the last term in Eq. (10). As in that case, the vevs for the doublet component of $\overline{H}$ and the adjoint field induce a vev for the triplet in 15.

The splitting among neutrinos is accounted for by the following dim 5 operators:

$$\frac{\beta_{1ij}}{2M_P}\overline{5}_{ia}\overline{5}_{jb}H^a H^b \quad \frac{\beta_{2ij}}{2M_P}\overline{5}_{ia}\overline{5}_{jb}\Delta^{ac}A_c^b . \tag{14}$$

These are analogous to operators $O_{1,2}$ in Eq. (8) and can account for the vacuum oscillation scale provided the coefficients $\beta_{1,2}$ are suppressed.

In the exact $U(1)$ symmetric limit, the down quarks remain massless, while the mass matrix for the up-quark is not restricted by the imposed $U(1)$ symmetry. The former can obtain masses from the $U(1)$ breaking terms. These are characterized by the following dimension five operators

$$\frac{\Gamma_{ij}^d}{M_P}\overline{5}_{ai}10_j^{ab}H_c A_b^c . \tag{15}$$

The adjoint field will acquire a vev at the grand unification scale

$$\langle A_a^a \rangle \sim \frac{M_X}{g_{GUT}},$$

where M_X is mass of the $SU(5)$ gauge Boson. For $M_X \sim 10^{16}$ the above operator leads to a contribution of $\leqslant$O(0.1 GeV) which is right for the description of the strange quark mass, but falls short of the value of the b quark mass.

Let us consider an alternative possibility in which one assigns nontrivial $U(1)$-charges also to Higgs fields and the 10-plets of fermions. Take as an example, the $U(1)$ assignment $(0,-1/3,-1/3)$ for three 10-plets. The $\overline{5}$ of Higgs field $\overline{H}$ and the adjoint are assumed, respectively, to carry the charges $-2/3$ and 4/3. The $\overline{H}$ charge is specifically chosen to obtain the right structure for the quark masses. The charge for A is fixed by requiring that the term in Eq. (13) be allowed by the $U(1)$. One now obtains the following mass matrices for the up and down quarks in the absence of the gravitational breaking of the symmetry:

$$M_d = \begin{pmatrix} 0 & m_1' & m_2' \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad M_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_1 & m_2 \\ 0 & m_2 & m_3 \end{pmatrix}, \tag{16}$$

where $m_{1,2}'$ ($m_{1,2,3}$) are parameters determining down (up-quark) masses. It is seen that the b, c, and t quarks acquire masses at this stage. The higher-dimensional terms displayed in Eq. (15) can now account for the strange and down quark masses. Similarly, one could write dimension 5 operator analogous to Eq. (15) giving mass to the up-quark. Thus a large part of quark masses and mixings may actually be due to the gravitational breaking of the $U(1)$ symmetry. This symmetry is, however, not strong enough to make definitive predictions on theses masses and mixings.

The symmetry $U(1)$ does not remain exact in the last example, but is spontaneously broken around the GUT scale. This may be a welcome feature as such breaking can possibly account for suppression [20] in the magnitudes of the coefficient $\beta_{1,2}$ of the higher-dimensional term.

In summary, we have underlined the role that the $L_e - L_\mu - L_\tau$ symmetry can play in generating leptonic mixing structure desired on experimental grounds. The presence of a heavy scale M_H in theory then accounts for the atmospheric mass scale. Planck scale suppressed breaking of the symmetry seems to be in the correct range to provide a solution to the solar anomaly as well. The role such breaking can play in generation of neutrino masses has been emphasized previously [15]. Here we have shown that the Planck scale along with M_{weak} and a $M_H \sim M_{GUT}$ can account for all the observed features of the solar and atmospheric anomalies provided neutrino mass structure is approximately $L_e - L_\mu - L_\tau$ symmetric.

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