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# Intrinsic charm contribution to the proton spin

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The charm quark contribution to the first moment of $g_1(x,Q^2)$ is calculated using a heavy mass expansion of the divergence of the singlet axial vector current. It is shown to be small. [S0556-2821(99)50515-8]

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The size of a possible intrinsic charm contribution in the proton has been the topic of intensive discussions [1–4] for many years. It is therefore a natural question to investigate the *polarized* intrinsic charm distribution in the nucleon [5]. Recently one of us and collaborators argued [6] that earlier treatments of the polarized charm contribution to the $\eta$ and $\eta'$ [7,8] were incorrect. In this contribution we extend and adopt that analysis to the nucleon. More precisely we shall focus on the intrinsic charm contribution to the first moment of the spin structure function $g_1(x,Q^2)$. This is known to be intimately related to the gluonic axial anomaly [9–12]. It may be expressed as the forward limit of $G_A^{(0)}(t)$, the form factor in the proton matrix element of the singlet axial vector current:

$$\langle N(p_2,\lambda_2)|j_{5\mu}^{(0)}(0)|N(p_1,\lambda_1)\rangle = \overline{u}_N^{(\lambda_2)}(p_2)(G_A^{(0)}(t)\gamma_\mu\gamma_5 - G_P^{(0)}(t)q_\mu\gamma_5)u_N^{(\lambda_1)}(p_1), \tag{1}$$

where $q=p_2-p_1$ and $t=q^2$. The singlet pseudoscalar form factor does not acquire a Goldstone pole at $t=0$, even in the chiral limit, contrary to the matrix elements of the octet currents. In this limit, there exist eight massless pseudoscalar mesons serving as Goldstone bosons. However, the ninth pseudoscalar, the $\eta'$ meson, remains massive, due to the mixing with the QCD ghost pole.

This fact allows us to relate the forward matrix element of the axial vector current to the (slightly) off-forward one of its divergence:

$$\lim_{t\to 0}\langle N(p_2,\lambda_2)|\partial^\mu j_{5\mu}^{(0)}(0)|N(p_1,\lambda_1)\rangle = 2m_N G_A(0)\overline{u}_N^{(\lambda_2)}(p_2)\gamma_5 u_N^{(\lambda_1)}(p_1), \tag{2}$$

$m_N$ being the proton mass. The divergence of the singlet axial vector current in turn contains a normal and an anomalous piece,

$$\partial^\mu j_{5\mu}^{(0)} = 2i\sum_q m_q\overline{q}\gamma_5 q - \left(\frac{N_f\alpha_s}{4\pi}\right)G_{\mu\nu}^a\widetilde{G}^{\mu\nu,a}, \tag{3}$$

where $N_f$ is the number of flavors. The two terms at the right-hand side (RHS) of the last equation are known to cancel in the limit of infinite quark mass [11–13]. This is the so-called cancellation of physical and regulator fermions, related to the fact that the anomaly may be regarded as a usual mass term in the infinite mass limit, up to a sign, resulting from the subtraction in the definition of the regularized operators.

Consequently, one should expect that the contribution of infinitely heavy quarks to the first moment of $g_1$ is zero. This is exactly what happens in a perturbative calculation of the triangle anomaly graph [12]. One may wonder, what is the size of this correction for large, but finite masses and how does it compare with the purely perturbative result. To answer this question, one should calculate the RHS of Eq. (3) for heavy fermions. The leading coefficient is of the order $m^{-2}$, and its calculation was addressed recently by two groups [7] and [8] who came up with results differing by a factor of 6. However, the operator $f_{abc}G_{\mu\nu}^a\widetilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c$ appearing in both treatments, does not satisfy some basic properties, such that both calculations seem to be flawed. (i) It is not a divergence of a local operator, therefore it is not clear that its forward matrix element (2) will vanish. (ii) It makes no contact with the calculation of the triangle diagram in momentum space [14,12] being essentially non-Abelian.

The recent contribution [6] corrected this result and arrived at the expression

$$\partial^\mu j_{5\mu}^c = \frac{\alpha_s}{48\pi m_c^2}\partial^\mu R_\mu, \tag{4}$$

where

$$R_\mu = \partial_\mu(G_{\rho\nu}^a\widetilde{G}^{\rho\nu,a}) - 4(D_\alpha G^{\nu\alpha})^a\widetilde{G}_{\mu\nu}^a. \tag{5}$$

(Here we use the conventions $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\varepsilon_{0123}=1$.) This result is an explicit 4-divergence and has the Abelian limit. The crucial observation made in Ref. [6] was that the heavy quark mass expansion of the divergency of the axial vector current [see Eq. (4)] does not contain truly non-Abelian operators of the type $f_{abc}G_{\mu\nu}^a\widetilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c$. This observation implies that the result (4) can also be obtained by a $1/m$ expansion of the triangle diagram contribution. Actually the result of [6] demonstrates that in order $m_c^{-2}$ the entire result (5) can be restored from the venerable triangle diagram. The diagrams with the larger number of ''legs'' give only contributions to the non-Abelian part of the result (5). Indeed, computing the forward matrix element of operator (5) between two virtual gluon states we get the following expressions:

$$\langle p|\frac{\alpha_s}{48\pi m_c^2}R_\mu|p\rangle=-i\frac{\alpha_s}{12\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_c^2}. \qquad (6)$$

On the other hand the result of a calculation of the triangle diagram with massive fermions (see, e.g., [11]) has the form

$$\langle p|\bar{c}\gamma_\mu\gamma_5 c|p\rangle=i\frac{\alpha_s}{2\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda \times\left\{1-\int_0^1 dx\frac{2m_c^2(1-x)}{m_c^2-p^2x(1-x)}\right\} =-i\frac{\alpha_s}{12\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_c^2}+O\left(\frac{1}{m_c^4}\right). \qquad (7)$$

This expression coincides exactly with the result (6). In order to complete the proof it is enough to consider the off-forward matrix element of the operator (4) between two gluons at zero virtuality and compare the result with the expression for the triangle diagram for $\langle p'|\partial^\mu\bar{c}\gamma_\mu\gamma_5 c|p\rangle$. It is easy to check that again the results coincide. Let us stress once more that the very possibility to extract the form of *nonperturbative* gluonic operator from the $1/m$ expansion of the triangle diagram is only due to the *proof* (see Ref. [6]) of absence of truly non-Abelian operators in the heavy quark mass expansion of divergency of the axial vector current to the order $1/m^2$.

Up to this moment, we derived the perturbative coefficient (which, as usual, may get the corrections of the order $\alpha_s/\pi$). We are now ready to go beyond perturbation theory by considering the matrix elements of the derived operator relation between hadronic states.

The proton matrix element of $R_\mu$ takes a form analogous to that of (1):

$$\langle N(p_2,\lambda_2)|R_\mu(0)|N(p_1,\lambda_1)\rangle =\overline{u}_N^{(\lambda_2)}(p_2)(G_A^R(t)\gamma_\mu\gamma_5-G_P^R(t)q_\mu\gamma_5)u_N^{(\lambda_1)}(p_1). \qquad (8)$$

It is crucial, that because of the explicit gauge invariance of $R_\mu$ the zero mass ghost pole does not contribute. This makes an apparent difference with respect to the massless case, when the divergence of the gauge-dependent topological current $K_\mu$ appears and the ghost pole contribution does not allow us to deduce the relation between the matrix elements of the currents starting from the relation for their divergencies [13]. In the case under investigation only the contribution of the massive $\eta'$ meson may appear so that

$$\lim_{t\to 0}\langle N(p_2,\lambda_2)|\partial_\mu R_\mu(0)|N(p_1,\lambda_1)\rangle =2m_N G_A^R(0)\overline{u}_N^{(\lambda_2)}(p_2)\gamma_5 u_N^{(\lambda_1)}(p_1). \qquad (9)$$

The contribution of the charm to the forward matrix element can be obtained by substituting (1, 8) into the proton matrix elements of Eq. (4), giving in the forward limit,

$$\langle N(p,\lambda)|j_{5\mu}^{(c)}(0)|N(p,\lambda)\rangle =\frac{\alpha_s}{48\pi m_c^2}\langle N(p,\lambda)|R_\mu(0)|N(p,\lambda)\rangle. \qquad (10)$$

In deriving this expression we used (2, 9). Note that the first term in $R_\mu$ does not contribute to the forward matrix element because of its gradient form, while the contribution of the second one is rewritten, by making use of the equation of motion, as matrix element of the operator

$$\langle N(p,\lambda)|j_{5\mu}^{(c)}(0)|N(p,\lambda)\rangle =\frac{\alpha_s}{12\pi m_c^2}\langle N(p,\lambda)|g\sum_{\rm f=u,d,s}\bar{\psi}_f\gamma_\nu\tilde{G}_\mu{}^\nu\psi_f|N(p,\lambda)\rangle \equiv\frac{\alpha_s}{12\pi m_c^2}2m_N^3 s_\mu f_S^{(2)}. \qquad (11)$$

The parameter $f_S^{(2)}$ was determined before in calculations of the power corrections to the first moment of the singlet part of the $g_1$ part of which is given by exactly the quark-gluon-quark matrix element we got. Note that within our $1/m_c$ approximation the $c$ contribution to the flavor sum can be neglected. QCD-sum rule calculations gave $f_S^{(2)}=\frac{9}{5}(f^{(2)}(\text{proton})+f^{(2)}(\text{neutron}))=0.09$[1] [15], estimates using the renormalon approach led to $f_S^{(2)}=\pm 0.02$ [16] and calculations in the instanton model of the QCD vacuum [17] give a result very close to that of QCD sum rule [15].

Inserting these numbers we get finally for the charm axial vector constant the estimate

$$\bar{G}_A^c(0)=-\frac{\alpha_s}{12\pi}f_S^{(2)}\left(\frac{m_N}{m_c}\right)^2\approx -5\times 10^{-4}, \qquad (12)$$

with probably a 100% uncertainty (see, e.g., [18]). As the mass term in the triangle diagram is coming from the region of transverse momenta of the order $m_c$, this should be the correct scale of both $\alpha_s$ and $f_S^{(2)}$. Because this scale is not far from the typical hadronic scale at which $f_S^{(2)}$ was estimated we can neglect evolution effects. Note that this contribution is of nonperturbative origin (therefore we call it intrinsic), so that it is sensitive to large distances, as soon as the factorization scale is larger than $m_c$. If the scale is also larger than $m_b$, one can immediately conclude that the nonperturbative bottom contribution is further suppressed by the factor

[1] Note that here we use a convention for the $\varepsilon$ tensor which differs by sign from that of [15].

$(m_c/m_b)^2 \sim 0.1$. This may be compared with the perturbative contribution of heavy quarks [19], which is zero at the low normalization scale and appearing as a consequence of evolution. Our result is just the $1/m^2$ correction, mentioned and neglected in that reference. The perturbative result is of the same order $10^{-3}$ for charmed and bottom quarks, so that our correction is important in the former, and negligible in the latter case.

Let us note, that the naive application of our approach to the case of strange quarks gives for their contribution to the first moment of $g_1$ roughly $-5\times10^{-2}$, which is compatible with the experimental data. The possible applicability of a heavy quark expansion for strange quarks in a similar problem was discussed earlier [20] in the case of the vacuum condensates of heavy quarks. That analysis was also related to the anomaly equation for heavy quarks, however, for the trace anomaly, rather than the axial one.

Let us summarize. We have related the nonperturbative contribution of charm quarks to the nucleon spin (at scale $m_c$) to the singlet twist-4 coefficent appearing, e.g., in the Ellis-Jaffe sum rule. Numerically it is found to be very small, contrary to the suggestion of [5,8]. We would like to note that in a recent paper [21] it was shown that the perturbative $\Delta c$ contribution is also very small. We see this as further support for our result.

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