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Unique mass texture for quarks and leptons

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Texture specific quark mass matrices which are Hermitian and hierarchical are examined in detail. In the case of texture 6 zero matrices, out of sixteen possibilities examined by us, none is able to fit the low energy data (LED), for example, $|V_{us}|=0.2196\pm0.0023$, $|V_{cb}|=0.0395\pm0.0017$, $|V_{ub}/V_{cb}|=0.08\pm0.02$, and $|V_{td}|$ lies in the range 0.004–0.014 (PDG). Similarly none of the 32 texture 5 zero mass matrices considered is able to reproduce the LED. In particular, the latest data from CERN LEP regarding $|V_{ub}/V_{cb}|=(0.093\pm0.016)$ rules out all of them. In the texture 4 zero case, we find that there is a unique texture structure for U and D mass matrices which is able to fit the data. [S0556-2821(99)50415-3]

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The raison d'être for the existence of three well separated families of charged fermions remains incomprehensible in the context of present day high energy physics. The mystery regarding the fermion masses has further deepened with the observation of ''neutrino oscillations'' by the SuperKamiokande Collaboration [1] implying nonzero masses for the neutrinos and, thus, giving for the first time a strong signal for physics beyond the standard model. In the absence of any deeper understanding of fermion masses, attempts have been made on the one hand to understand arbitrary standard model Yukawa couplings of the fermions from more fundamental theories such as grand unified theories (GUTs) [2], composite models [3], left right symmetric models [4], etc. On the other hand, attempts have been made to discover phenomenological quark mass matrices, which are in tune with the low energy data (LED). In this regard, specific *Ansätze* for the quark mass matrices have been tried, with a fair degree of success [5–13], to explain quark mixing matrix. Similarly phenomenological neutrino mass matrices have been considered [13,14] which attempt to accommodate simultaneously the solar neutrino problem (SNP), atmospheric neutrino problem (ANP), and neutrino oscillations observed by the Liquid Scintillation Neutrino Detector (LSND).

The purpose of the present Rapid Communication is to find possible textures [15] for mass matrices which are able to accommodate LED regarding quark mixtures as well as neutrino oscillations required to explain SNP, ANP, and LSND oscillations. To this end, we carry out a detailed and exhaustive analysis of a large number of texture specific quark mass matrices (almost 52) and try to find out a set of quark mass matrices which can accommodate LED. After having restricted the number of quark mass matrices, we assume a similar texture structure for neutrino mass matrices and study its implications for three neutrino anomalies.

Before one can take an extensive analysis of mass matrices, it should be borne in mind that the number of free parameters available with the general mass matrices is larger than the physical observables. For example, if no restrictions are applied, there are 36 free parameters to describe 10 physical observables, i.e., 6 quark masses, 3 mixing angles, and one CP violating phase. Therefore to develop viable phenomenological quark mass matrices one has to limit the number of free parameters in the mass matrices. It is, therefore, desirable to invoke certain broad guiding principles based on general considerations [9,10] borne out of experimental data [16] and insight gained from the past such analyses [7–12]. These guidelines constrain the present analysis to a manageable number of mass matrices by restricting the number of free parameters of general quark mass matrices. In fact, it is also the purpose of present Rapid Communication to consolidate and reiterate these guiding principles.

In this context, we first make use of the polar decomposition theorem of matrix algebra, by which one can always express a general mass matrix as a product of Hermitian and a unitary matrix. Therefore, without loss of generality, we can consider quark mass matrices to be Hermitian as the unitary matrix can be absorbed in the right handed quark fields. This immediately brings down the number of free parameters from 36 to 18.

The hierarchical pattern of quark masses as well as those of mixing angles immediately suggests that one should start with mass matrices whose elements follow hierarchy. This is borne out of several past and present analyses [10–17]. One can think of mass matrices whose elements do not exhibit hierarchy, but nevertheless are still able to reproduce the quark masses; however, such mass matrices do not satisfy the criterion of ''naturalness'' proposed recently by Peccei and Wang [9]. Therefore, we assume that the elements of Hermitian quark mass matrices follow the pattern:

$$M=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{1}$$

$$a_{11}<|a_{12}|\sim|a_{13}|\ll a_{22}\sim|a_{23}|\ll a_{33}.$$

The famous Fritzsch *Ansatz* and subsequent generalizations [7,8,11,12] have all considered hierarchical matrices.

By using polar decomposition theorem, though the number of parameters is brought down to 18, it is still larger than the number of observables; therefore it needs to be brought down further. In this context, following Weinberg [18] and Fritzsch [5], the strategy has been to assume that mass ma-

TABLE I. Expressions for CKM matrix elements V_{us}, V_{cb}, and V_{ub} corresponding to M_i $(i=u,d)$ listed in column I. The symbols used in the table are $a=\sqrt{m_u/m_c}$, $b=\sqrt{m_c/m_t}$, $c=\sqrt{m_d/m_s}$, $d=\sqrt{m_s/m_b}$, $A_i=|A_i|e^{i\alpha_i}$, $\phi=\alpha_u-\alpha_d$.

	M_i	V_{us}	V_{cb}	V_{ub}
I	$\begin{pmatrix} 0 & A_i & 0 \\ A_i^* & D_i & 0 \\ 0 & 0 & C_i \end{pmatrix}$	$c-ae^{i\phi}$	0	0
II	$\begin{pmatrix} 0 & 0 & A_i \\ 0 & D_i & 0 \\ A_i^* & 0 & C_i \end{pmatrix}$	0	0	$-cde^{i\phi}+ab$
III	$\begin{pmatrix} 0 & A_i & \|B_i\| \\ A_i^* & 0 & 0 \\ \|B_i\| & 0 & C_i \end{pmatrix}$	$a-ce^{i\phi}$ $+abd$	$-d+acd^3e^{i\phi}$ $+b$	$-ad-cd^3e^{i\phi}$ $+ab$
IV	$\begin{pmatrix} 0 & A_i & 0 \\ A_i^* & 0 & \|B_i\| \\ 0 & \|B_i\| & C_i \end{pmatrix}$	$-ce^{i\phi}+a$ $+abd$	$-acd^3e^{i\phi}+d$ $-b$	$-cd^3e^{i\phi}+ad$ $-ab$

trices for fermions have certain ''textures'' imposed on them by some underlying symmetries or these could be purely phenomenological *Ansätze*. These textures allow one to derive some interesting ''predictions'' which can then be compared with the experiments. Therefore, in order to keep free parameters under control, one therefore starts with texture specific *Ansätze*.

Before proceeding further, it is perhaps natural to ask whether the assumptions of Hermiticity, hierarchy, and textures are preserved when one scales down from GUT scale to low energy as the mass matrices are usually derived at the GUT scale. This question has been examined in detail [8,9,10] and it has been shown that the hierarchical structure of mass matrices is not affected, whereas the texture structure and Hermiticity is broken to a minor extent leaving phenomenological consequences unaltered. The texture structure, however, is maintained by the renormalization group (RG) equations if it is ensured by additional symmetry. We, therefore, consider at low energy phenomenological texture specific mass matrices which are Hermitian and hierarchical.

A well determined quark mixing matrix [16], in particular the knowledge of elements $|V_{us}|$, $|V_{cb}|$, and $|V_{ub}/V_{cb}|$, leads to the vital clues for the possible structure of mass matrices. In this context, a survey of some of the past analyses [5–11] as well as our own investigations [12] suggest that $|V_{us}|$ is given by $\sqrt{m_d/m_s}$ and a very small correction term $\sqrt{m_u/m_c}$ while $|V_{cb}|$ is given by m_s/m_b and a correction term m_c/m_t. Thus $|V_{us}|$ is controlled largely by D type of quark mass matrices, whereas in the case of $|V_{cb}|$ both U and D sectors contribute significantly. These facts can give us vital clues about the positioning of the texture zeros in the phenomenological mass matrices. For the sake of simplicity, it is desirable that the U and D type of the mass matrices be taken to have identical texture structures [7,8].

Keeping in mind the above broad guidelines, one could start with the specific texture structures in the U type and D type of mass matrices [10]. Before going into the detailed structures of 3×3 hierarchical mass matrices, one would like to note that in a phenomenological *Ansatz*, the (1,1) element can always be taken to be equal to zero, because a nonzero (1,1) element leads only to the rescaling of the lightest quark masses in both U and D type of mass matrices [7]. It can be very easily seen that the maximum number of texture zeros which can be considered for U or D type of mass matrices have to be three. More than three would lead to at least one of the quark masses to be zero. Therefore we start with texture 3 zero type of matrices. In order to have nontrivial mixings of three generations, as well as keeping in mind the guidelines mentioned above, only the following nontrivial texture 3 zero structures are possible:

$$\begin{aligned} &M_3(\text{I}): && \text{with texture zeros at (1,1), (1,3), and (2,3),} \\ &M_3(\text{II}): && \text{with texture zeros at (1,1), (1,2), and (2,3),} \\ &M_3(\text{III}): && \text{with texture zeros at (1,1), (2,2), and (1,3),} \\ &M_3(\text{IV}): && \text{with texture zeros at (1,1), (2,2), and (2,3),} \end{aligned} \tag{2}$$

where (1,1), etc., correspond to zero at the position of first row and first column of mass matrix and so on. In general M_u and M_d could be any of the four matrices mentioned above, resulting in 16 combinations. However, if M_u and M_d are taken to have parallel texture structure, we are left with only four possibilities. All these matrices have been diagonalized exactly. The corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrices can be found easily [12] and checked against the experimental values of CKM matrix elements, for example, $|V_{us}|=0.2196\pm0.0023$,$|V_{cb}|=0.0395\pm0.0017$,$|V_{ub}/V_{cb}|=0.08\pm0.02$, $|V_{td}|$ lies in the range 0.004–0.013 [16]. In Table I, we have summarized the expressions for the CKM matrices corresponding to the four possibilities mentioned above. Without going into the details

TABLE II. Expressions for CKM matrix elements V_{us}, V_{cb}, and V_{ub} corresponding to M_u and M_d listed in column I and II, respectively. The symbols used are as defined for Table I and $B_i=|B_i|e^{i\alpha_i}$, $R_t=D_u/m_t$, $R_t'=1-R_t$, $R_t''=b^2+R_t$, $R_b=D_d/m_b$, $R_b'=1-R_b$, $R_b''=d^2+R_b$.

	M_u	M_d	V_{us}	V_{cb}	V_{ub}
I	$\begin{pmatrix} 0 & A_u & 0 \\ A_u^* & D_u & \lvert B_u\rvert \\ 0 & \lvert B_u\rvert & C_u \end{pmatrix}$	$\begin{pmatrix} 0 & A_d & 0 \\ A_d^* & 0 & \lvert B_d\rvert \\ 0 & \lvert B_d\rvert & C_d \end{pmatrix}$	$-ce^{i\phi}+aR_t'^{1/2}+adR_t''^{1/2}$	$-acd^3e^{i\phi}+dR_t'^{1/2}-R_t''^{1/2}$	$cd^3e^{i\phi}+adR_t'^{1/2}-aR_t''^{1/2}$
II	$\begin{pmatrix} 0 & \lvert A_u\rvert & B_u \\ \lvert A_u\rvert & 0 & 0 \\ B_u^* & 0 & C_u \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert A_d\rvert & 0 \\ \lvert A_d\rvert & D_d & B_d \\ 0 & B_d^* & C_d \end{pmatrix}$	$ca-R_b'^{1/2}-abR_b''^{1/2}e^{i\phi}$	$-cd^2\left(\frac{R_b''}{R_b'}\right)^{1/2}+aR_b''^{1/2}+b(R_b')^{1/2}e^{i\phi}$	$-acd^2\left(\frac{R_b''}{R_b'}\right)^{1/2}-R_b''^{1/2}+abR_b'^{1/2}e^{i\phi}$
III	$\begin{pmatrix} 0 & 0 & A_u \\ 0 & D_u & \lvert B_u\rvert \\ A_u^* & \lvert B_u\rvert & C_u \end{pmatrix}$	$\begin{pmatrix} 0 & A_d & 0 \\ A_d^* & 0 & \lvert B_d\rvert \\ 0 & \lvert B_d\rvert & C_d \end{pmatrix}$	$c\left(1-\frac{(ab)^2}{R_t}\right)^{1/2}e^{i\phi}+a\left(\frac{R_t''R_t'}{R_t}\right)^{1/2}+ad[R_t-(ab)^2]^{1/2}$	$acd^3\left(\frac{R_t''}{R_t}\right)^{1/2}e^{i\phi}+d\left(\frac{R_t'[R_t-(ab)^2]}{R_t}\right)^{1/2}-R_t''^{1/2}$	$-cd^3\left(1-\frac{(ab)^2}{R_t}\right)^{1/2}e^{i\phi}+ad\left(\frac{R_t''R_t'}{R_t}\right)^{1/2}-a[R_t-(ab)^2]^{1/2}$
IV	$\begin{pmatrix} 0 & A_u & 0 \\ A_u^* & D_u & 0 \\ 0 & 0 & C_u \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & A_d \\ 0 & D_d & \lvert B_d\rvert \\ A_d^* & \lvert B_d\rvert & C_d \end{pmatrix}$	$c\left(\frac{R_b''}{R_b}\right)^{1/2}e^{i\phi}+a\left(\frac{R_b'[R_b-(cd)^2]}{R_b}\right)^{1/2}$	$acd^2\left(\frac{R_b'}{R_b}\right)^{1/2}e^{i\phi}-\left(\frac{R_b''[R_b-(cd)^2]}{R_b}\right)^{1/2}$	$cd^2\left(\frac{R_b'}{R_b}\right)^{1/2}e^{i\phi}+a\left(\frac{R_b''[R_b-(cd)^2]}{R_b}\right)^{1/2}$
V	$\begin{pmatrix} 0 & 0 & \lvert A_u\rvert \\ 0 & D_u & B_u \\ \lvert A_u\rvert & B_u^* & C_u \end{pmatrix}$	$\begin{pmatrix} 0 & A_d & \lvert B_d\rvert \\ A_d^* & 0 & 0 \\ \lvert B_d\rvert & 0 & C_d \end{pmatrix}$	$\left(1-\frac{(ab)^2}{R_t}\right)^{1/2}+ac\left(\frac{R_t''R_t'}{R_t}\right)^{1/2}e^{i\phi}-ad[R_t-(ab)^2]^{1/2}$	$ad\left(\frac{R_t''}{R_t}\right)^{1/2}+cd^3\left(\frac{R_t'[R_t-(ab)^2]}{R_t}\right)^{1/2}e^{i\phi}-R_t''$	$-d\left(\frac{R_t-(ab)^2}{R_t}\right)^{1/2}+acd^3\left(\frac{R_t''R_t'}{R_t}\right)^{1/2}e^{i\phi}-a[R_t-(ab)^2]^{1/2}$

of the methodology for analyzing such CKM matrices, we refer the reader to our earlier work [12]. However, we would like to emphasize that the quark masses taken for the CKM matrix analysis correspond to masses at 1 GeV [19], for example, $m_u=0.0051\pm0.0015$ GeV, $m_d=0.0089\pm0.0026$ GeV, $m_s=0.175\pm0.055$ GeV, $m_c=1.35\pm0.05$ GeV, $m_b=5.3\pm0.1$ GeV, and $m_t=300\pm50$ GeV.

From Table I, it is clear that possibilities I and II are completely ruled out. In the case of III possibility $|V_{cb}|$ cannot be fitted even after full variation of input parameters, as can be checked from the expression given in the table. The possibility IV is the famous Fritzsch *Ansatz*, which again gives $|V_{cb}|$ much above the present experimental value, as has also been pointed out in a large number of earlier analyses. For the sake of completeness, we have also carried out investigations of the possibilities where M_u and M_d do not have parallel structures. Twelve such possibilities are there, which are not listed here. We find that none of the structures is viable. It may be noted that in texture 6 zero structures, the Fritzsch *Ansatz* is perhaps the best and that sets the tone for future modifications.

A strict adherence to parallel texture structure for M_u and M_d type of mass matrices rules out texture 5 zero mass matrices; however, such matrices have been discussed in the literature. Therefore, we have also included here a discussion of such matrices. Texture 5 zero mass matrices would have either of the two, M_u or M_d, being of texture 2 zero type. As texture 3 zero matrices have already been listed, therefore we list all the possible texture 2 zero mass matrices compatible with the guidelines enunciated above:

$$\begin{aligned} M_2(\text{I}):&\quad \text{with zeros at (1,1) and (1,2)},\\ M_2(\text{II}):&\quad \text{with zeros at (1,1) and (1,3)},\\ M_2(\text{III}):&\quad \text{with zeros at (1,1) and (2,3)},\\ M_2(\text{IV}):&\quad \text{with zeros at (1,1) and (2,2)}. \end{aligned} \tag{3}$$

All these matrices are exactly diagonalizable, except for $M_2(\text{IV})$ where diagonalization can be achieved perturbatively [17]. To obtain texture 5 zero mass matrices, one has to combine any of the matrices given in Eq. (2) with any of the matrices in Eq. (3). This leads to 32 possibilities for the texture 5 zero mass matrices. These possibilities include the five such examples discussed by Ramond, Roberts, and Ross (RRR) [10]. The examples considered by RRR are ruled out by the present data [16]; particularly, the recent measure-

TABLE III. Expressions for CKM matrix elements V_{us}, V_{cb}, and V_{ub} corresponding to M_i ($i=u,d$) listed in column I. The symbols used are as defined for Tables I and II and $\Delta_b=1-2R_b$, $\Delta_t=1-2R_t$.

	M_i	V_{us}	V_{cb}	V_{ub}
I	$\begin{pmatrix} 0 & 0 & A_i \\ 0 & D_i & \lvert B_i\rvert \\ A_i^* & \lvert B_i\rvert & C_i \end{pmatrix}$	$-c\left[\left(1-\frac{(ab)^2}{R_t}\right)\frac{R_b''}{R_b}\right]^{1/2} e^{i\phi}$ $+\left[\frac{R_t''R_t'R_b'}{R_t}\left(1-\frac{(cd)^2}{R_b}\right)\right]^{1/2}$ $+a\{R_b''[R_t-(ab)^2]\}^{1/2}$	$acd^2\left[\frac{R_t''R_b'}{R_tR_b}\right]^{1/2} e^{i\phi}$ $+\left(\frac{[R_t-(ab)^2]R_t'[R_b-(cd)^2]R_b''}{R_bR_t}\right)^{1/2}$ $-(R_b'R_t'')^{1/2}$	$-cd^2\left(\frac{[R_t-(ab)^2]R_b'}{R_tR_b}\right)^{1/2} e^{i\phi}$ $+\left(\frac{R_t''R_t'[R_b-(cd)^2]R_b''}{R_bR_t}\right)^{1/2}$ $-a\{R_b'[R_t-(ab)^2]\}^{1/2}$
II	$\begin{pmatrix} 0 & A_i & \lvert B_i\rvert \\ A_i^* & D_i & 0 \\ \lvert B_i\rvert & 0 & C_i \end{pmatrix}$	$\frac{1}{bd}\{[(ab)^2-R_t]R_t'R_b'R_b''\}^{1/2}$ $-\frac{R_t'R_b'}{bd}\left(\frac{R_t''[R_b-(cd)^2]}{\Delta_t\Delta_b}\right)^{1/2} e^{i\phi}$ $+\frac{R_b''}{bd}\left(\frac{R_t'R_b'R_t''[R_t-(ab)^2]}{\Delta_t\Delta_b}\right)^{1/2}$	$-\frac{1}{b}(R_t''R_t'R_b'R_b'')^{1/2}$ $+\frac{R_t'R_b''}{b}\left(\frac{[R_t-(ab)^2][R_b-(cd)^2]}{\Delta_t\Delta_b}\right)^{1/2} e^{i\phi}$ $+\frac{R_t''R_b'}{b}\left(\frac{R_t'R_b'}{\Delta_t\Delta_b}\right)^{1/2}$	$-\frac{1}{b}\{[(ab)^2-R_t]R_t'R_b'R_b''\}^{1/2}$ $-\frac{R_t'R_b''}{b}\left(\frac{R_t''[(cd)^2-R_b]}{\Delta_t\Delta_b}\right)^{1/2} e^{i\phi}$ $+\frac{R_b'}{b}\left(\frac{R_t'R_b'R_t''[(ab)^2-R_t]}{\Delta_t\Delta_b}\right)^{1/2}$
III	$\begin{pmatrix} 0 & A_i & B_i \\ A_i^* & 0 & D_i \\ B_i^* & D_i & C_i \end{pmatrix}$	$ce^{i\phi}-a$ $+abd$	$2acde^{i\phi}+d$ $-b$	$2cde^{i\phi}-ad$ $-ab$
IV	$\begin{pmatrix} 0 & A_i & 0 \\ A_i^* & D_i & \lvert B_i\rvert \\ 0 & \lvert B_i\rvert & C_i \end{pmatrix}$	$-ce^{i\phi}+a(R_t'R_b')^{1/2}$ $+a(R_b''R_t'')^{1/2}$	$-acd^2\left[\frac{R_b''}{R_b'}\right]^{1/2} e^{i\phi}$ $+(R_t'R_b'')^{1/2}$ $-(R_t''R_b')^{1/2}$	$cd^2\left(\frac{R_b''}{R_b'}\right)^{1/2} e^{i\phi}$ $+a(R_t'R_b'')^{1/2}$ $-a(R_b'R_t'')^{1/2}$

ments of $\lvert V_{ub}/V_{cb}\rvert$ at the CERN e^+e^- collider LEP [20] rules these out unambiguously. In Table II, we present some of the examples of M_u and M_d constituting texture 5 zero matrices and not considered by RRR.

Proceeding in the same manner as that of texture 6 zero matrices, we find that all the cases considered in Table II are ruled out. In particular the possibility V is ruled out as $\lvert V_{us}\rvert$ cannot be reproduced. In the case I, III, and IV, $\lvert V_{us}\rvert$ can be reproduced but $\lvert V_{cb}\rvert$ cannot be fitted. In the case of possibility II, $\lvert V_{us}\rvert$ can be reproduced for large values of R_b; however, that makes $\lvert V_{cb}\rvert$ too large to fit the data. Similarly, we have exactly derived the CKM matrix for the rest of the texture 5 zero possibilities and found that none of these is able to reproduce the full CKM matrix, even after full variation of all the parameters.

After having ruled out the texture 5 zero mass matrices, it is natural to consider texture 4 zero matrices. In Table III, we have listed texture 4 zero mass matrices, where M_u and M_d are respectively 2 zero type and have parallel structures.

Following the procedure outlined earlier, one can easily see that possibilities I, II, and III are not able to reproduce $\lvert V_{us}\rvert$, $\lvert V_{cb}\rvert$, and $\lvert V_{ub}/V_{cb}\rvert$simultaneously; therefore we are left with only the possibility IV. In Table IV, we have presented the numerical values of $\lvert V_{us}\rvert$, $\lvert V_{cb}\rvert$, $\lvert V_{ub}/V_{cb}\rvert$, and $\lvert V_{td}\rvert$as a function of independent parameters R_t and R_b. To limit the number of possibilities we have considered only those cases where $R_b=R_t$. A quick look at Table IV shows that the calculated values of $\lvert V_{us}\rvert$, $\lvert V_{cb}\rvert$, and $\lvert V_{td}\rvert$ are well within the experimental bounds [16]. The details of the CKM phenomenology with CKM matrix derived from texture 4 zero mass matrices will be discussed elsewhere.

After having found a viable texture structure for quark mass matrices, it is natural to ask whether or not a similar texture could be used for neutrino mass matrices. In particular, one would like to examine whether such a texture can generate the required appearance and disappearance probabilities for explaining the three neutrino anomalies. In this context, recently an analysis has been carried out by Barenboim and Scheck [21]. In particular, they find a mixing matrix, which is able to provide a simultaneous fit to the SNP, ANP, and LSND oscillations, for example,

TABLE IV. Calculated values of $\lvert V_{us}\rvert$, $\lvert V_{cb}\rvert$, $\lvert V_{ub}/V_{cb}\rvert$, and $\lvert V_{td}\rvert$ corresponding to M_u and M_d of IV row in Table III.

R_t	R_b	$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}/V_{cb}\rvert$	$\lvert V_{td}\rvert$
0.075	0.075	0.22	0.044	0.0829	0.009
0.080	0.080	0.22	0.043	0.085	0.009
0.085	0.085	0.22	0.042	0.087	0.009
0.090	0.090	0.22	0.041	0.089	0.009
0.950	0.950	0.22	0.040	0.091	0.008
0.100	0.100	0.22	0.039	0.094	0.008
0.105	0.105	0.22	0.038	0.096	0.008
0.110	0.110	0.22	0.037	0.099	0.008

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$$\begin{bmatrix} 0.793 & 0.566 & 0.226 \\ -0.601 & 0.662 & 0.447 \\ 0.103 & -0.490 & 0.865 \end{bmatrix}.$$

In our analysis based on texture 4 zero mass matrices, the above matrix can be reproduced by considering the following eigenvalues of neutrino masses: $m_1 = 0.53\times 10^{-3}$ eV, $m_2 = 0.1\times 10^{-1}$ eV, and $m_3 = 0.5$ eV. Without going into the details (to be discussed elsewhere), we would like to emphasize that hierarchical neutrino masses can generate the required mixing matrix within the texture four zero scenario.

In conclusion, we would like to mention that an extensive analysis of a large number of texture specific quark mass matrices, which are Hermitian and hierarchical, has been carried out. Interestingly, there are no quark mass matrices with texture 6 zero structure and texture 5 zero structure which can fit LED. In the case of texture 4 zero matrices, there is a unique texture with parallel texture structure for M_u and M_d which fits the data. When a similar texture structure is assumed for neutrinos, we are able to reproduce a mixing matrix which can accommodate the solar neutrino problem, the atmospheric neutrino problem, and the oscillations observed at LSND.

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[1] SuperKamiokande Collaboration, Y. Fukuda *et al.*, Phys. Lett. B **433**, 9 (1988); **436**, 33 (1998); T. Kajita, talk presented at Neutrino-98, Takayama, Japan, 1998.

[2] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979); J. Harvey, P. Ramond, and D. Reiss, *ibid.* **92B**, 309 (1980); S. Dimopoulos, L. J. Hall, and S. Raby, Phys. Rev. Lett. **68**, 1984 (1992); Phys. Rev. D **45**, 4192 (1992); P. Ramond, in *On Klauder's Path: A Field Trip*, edited by G. Emch, G. Hegerfeldt, and L. Streit (World Scientific, Singapore, 1992); H. Arason, D. J. Castano, P. Ramond, and E. J. Piard, Phys. Rev. D **47**, 232 (1993); G. F. Giudice, Mod. Phys. Lett. A **7**, 2429 (1992).

[3] For a review, see E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981), and references therein; J. C. Pati, Phys. Lett. B **228**, 228 (1989); J. C. Pati, M. Cvetic, and H. Sharatchandra, Phys. Rev. Lett. **58**, 851 (1987); K. S. Babu, J. C. Pati, and H. Stremnitzer, Phys. Lett. B **256**, 206 (1991); Phys. Rev. Lett. **67**, 1688 (1991).

[4] J. Pati and A. Salam, Phys. Rev. Lett. **31**, 661 (1973); R. Mohapatra and J. Pati, Phys. Rev. D **11**, 566 (1975); G. Senjanovic and R. Mohapatra, *ibid.* **12**, 1502 (1975); H. Fritzsch and P. Minkowski, Nucl. Phys. **B103**, 61 (1976); M. Beg, R. Budny, R. Mohapatra, and A. Sirlin, Phys. Rev. Lett. **38**, 1252 (1977).

[5] H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979); Phys. Lett. **166B**, 423 (1986).

[6] B. Stech, Phys. Lett. **130B**, 189 (1983); M. Shin, *ibid.* **145B**, 285 (1984); Phys. Lett. B **191**, 464 (1987); M. Gronau, R. Johnson, and J. Schechter, Phys. Rev. Lett. **54**, 2176 (1985); L. Lavoura, Phys. Lett. B **228**, 245 (1989); T. Ito, N. Okamura, and M. Tanimoto, Phys. Rev. D **58**, 077301 (1998); M. Baillargeon, F. Boudjema, C. Hamzaoui, and J. Lindig, hep-ph/9809207.

[7] C. H. Albright and M. Lindner, Z. Phys. C **44**, 673 (1989); C. H. Albright, Phys. Lett. B **246**, 451 (1990); **246**, 451 (1990).

[8] H. Fritzsch and Z. Z. Xing, Phys. Lett. B **353**, 114 (1995); Z. Z. Xing, J. Phys. G **23**, 1563 (1997).

[9] R. Peccei and K. Wang, Phys. Rev. D **53**, 2712 (1996).

[10] P. Ramond, R. G. Roberts, and G. G. Ross, Nucl. Phys. **B406**, 19 (1993); hereafter referred to as RRR.

[11] Suraj N. Gupta and J. M. Johnson, Phys. Rev. D **44**, 2110 (1991); S. Rajpoot, Mod. Phys. Lett. A **7**, 309 (1992).

[12] P. S. Gill and M. Gupta, Mod. Phys. Lett. A **9**, 2461 (1994); J. Phys. G **21**, 1 (1995); Pramana, J. Phys. **45**, 333 (1995); Int. J. Mod. Phys. A **11**, 333 (1996); J. Phys. G **23**, 1 (1997); Phys. Rev. D **56**, 3143 (1997); Mod. Phys. Lett. A **13**, 2445 (1998).

[13] Kyungsik Kang and Sin Kyu Kang, Phys. Rev. D **56**, 1511 (1997); K. Kang, S. K. Kang, C. S. Kim, and S. M. Kim, hep-ph/9808419, and references therein.

[14] H. Fritzsch and Z. Z. Xing, Phys. Lett. B **372**, 265 (1996); P. S. Gill and M. Gupta, Phys. Rev. D **57**, 3971 (1998).

[15] The term texture zeros has been used in the sense of P. Ramond *et al.* in Ref. [10]. For example, texture four zeros corresponds to the number of diagonal zeros plus half of the symmetrically placed off diagonal zeros in *U* and *D* type mass matrices considered in Eq. (3).

[16] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[17] Andrija Rasin, Phys. Rev. D **58**, 096012 (1998).

[18] S. Weinberg, Trans. NY Acad. Sci. **38**, 185 (1977).

[19] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982); S. Narison, Phys. Lett. B **197**, 405 (1987).

[20] CLEO Collaboration, A. Bean *et al.*, Phys. Rev. Lett. **70**, 2681 (1993); CLEO Collaboration, J. P. Alexander *et al.*, *ibid.* **77**, 5000 (1996).

[21] G Barenboim and F. Scheck, Phys. Lett. B **440**, 332 (1998).