Verification of Einstein's principle of equivalence using a laser gyroscope in terrestrial conditions

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It is shown that the use of a laser gyroscope permits us to verify Einstein's equivalence principle for photons with an accuracy up to 10^{-16}. It is proposed to carry out an experiment with the laser gyroscope mounted on the mechanical gyroscopic platform. Since the mechanical gyroscopic platform, on the whole, consists of nucleons, then the laser gyroscope can be regarded as mounted in a nonrotational frame of reference for the nucleons with a high level of accuracy. If the equivalence principle with respect to rotation is violated on a certain level of accuracy, then this frame of reference will be a rotational one for photons moving inside the laser gyroscope and the laser gyroscope will register the presence of this relative rotation.
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Einstein's principle of equivalence is considered to be one of the pillars on which stands the modern theory of gravitation. In accordance with this principle, the gravitational interaction in Einstein's theory is defined by the metric tensor of the pseudo-Riemannian space-time $g_{\mu\nu}$ and this tensor is unique for all forms of matter.

Like all other fundamental laws of physics, the principle of equivalence has been subjected to numerous attempts of experimental verification and in each successive attempt, its validity has been established with an increasing accuracy. In particular, the latest measurements of the ratio of inertial and passive gravitational masses in a laboratory scale have established [1] the validity of this principle with a relative accuracy of 10^{-12}. However, this seemingly high accuracy may not be sufficient to establish the universal validity of the principle of equivalence.

Therefore, a number of experiments are being planned to verify the validity of this principle with accuracies higher than before. One such experiment could be carried out within the framework of the Stanford-NASA Gravity Probe-B Relativity Gyroscope Program which is being planned in the USA. This program [2] basically aims at measuring the geodesic and frame dragging precessions of the mechanical superconducting gyroscope located in the polar circular orbit. It is suggested [3] that experimental data obtained from this program can also be used for verifying the principle of equivalence with an enhanced accuracy of $10^{-13}-10^{-14}$. However, the design and construction of such a superconducting gyroscope to be mounted on a polar satellite represents a task of immense technical complexity which in the opinion of the authors and co-ordinators of the project Gravity Probe-B may not be accomplished at least until the beginning of the next century.

In this regard, we would like to propose an experiment much more simple from the technical point of view, which aims at determining the validity of the principle of equivalence using a laser gyroscope. As will be evident from the following argument, the principle of equivalence can be verified with an enhanced accuracy up to 10^{-16} without any major upgrading of the existing laser gyroscope.

In order to prove this statement we shall calculate the accuracy of measuring the parameters which could violate the weak principle of equivalence in an analogous manner to the calculations carried out [3] in project Gravity Probe-B.

In accordance to this work, one of the most general phenomenological models that violates the Einstein's equivalence principle is the Ni's $\chi-g$ framework for electromagnetically coupled particles [4,5].

In a world containing only particles coupled to an electromagnetic field, the $\chi-g$ Lagrangian density is

$$L=-\frac{\sqrt{-g}}{16\pi c}\chi^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}-\frac{\sqrt{-g}}{c^2}A_\mu J^\mu - \sum_A m_A c\sqrt{1-\frac{v_A^2}{c^2}}\delta^3(\vec{r}_A-\vec{r}),$$

where

$$\chi^{\alpha\beta\mu\nu}=-\chi^{\beta\alpha\mu\nu}=-\chi^{\alpha\beta\nu\mu}=\chi^{\mu\nu\alpha\beta}. \tag{1}$$

The system of Maxwell's tensor equations in this case takes the form

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\beta}[\sqrt{-g}\chi^{\alpha\beta\mu\nu}F_{\mu\nu}]=-\frac{4\pi}{c}J^\alpha, \tag{2}$$

$$\frac{\partial F_{\alpha\beta}}{\partial x^\nu}+\frac{\partial F_{\beta\nu}}{\partial x^\alpha}+\frac{\partial F_{\nu\alpha}}{\partial x^\beta}=0.$$

Any violation of the equivalence principle for photons in a freely falling laboratory, which contains mainly nucleons, in the gravitational field of the Sun, the Galaxy and the Universe should manifest itself in the form of anomalous elements in the Lagrangian of the free electromagnetic field. As a result, the tensor $\chi^{\alpha\beta\mu\nu}$ will slightly differ from the value

$$\chi^{\alpha\beta\mu\nu}=\frac{1}{2}[g^{\alpha\mu}g^{\beta\nu}-g^{\alpha\nu}g^{\beta\mu}],$$

which is prescribed to it in the case when the equivalence principle is strictly valid.

Therefore this tensor can be written as

$$\chi^{\alpha\beta\mu\nu}=\frac{1}{2}[g^{\alpha\mu}g^{\beta\nu}-g^{\alpha\nu}g^{\beta\mu}]+\Pi^{\alpha\beta\mu\nu}, \tag{3}$$

where $\Pi^{\alpha\beta\mu\nu}$ is the ''small'' tensor, which if nonzero causes the violation of the equivalence principle for photons.

Let us study the action of the tensor $\Pi^{\alpha\beta\mu\nu}$ on the propagation of plane electromagnetic waves in a freely falling laboratory. In this case $\sqrt{-g}=1$, $g_{00}=1$, $g_{nm}=-\delta_{nm}$, where δ_{jn} is the Kronecker symbol.

In the approximation of geometric optics, the system of equations (2) takes the form

$$\chi^{\alpha\beta\mu\nu}F_{\mu\nu}k_{\beta}=0,$$

$$F_{\alpha\beta}k_{\nu}+F_{\beta\nu}k_{\alpha}+F_{\nu\alpha}k_{\beta}=0,$$

where $k_{\nu}=\{k_0=\omega/c,-\vec{k}\}$-wave 4-vector, ω is the frequency and k_n are the three dimensional components of the electromagnetic wave vector.

However, as a result of the symmetric properties (1) of the tensor $\chi^{\alpha\beta\mu\nu}$ and the electromagnetic field tensor $F_{\alpha\beta}$, only following two equations in this system will be independent:

$$\chi^{j\beta\mu\nu}F_{\mu\nu}k_{\beta}=0, \tag{4}$$

$$\frac{\omega}{c}F_{mn}+F_{n0}k_m+F_{0m}k_n=0.$$

Multiplying the first equation of this system by ω/c, expanding the summation on indices μ and ν, and using the second equation of system (4), we bring the first equation to the form:

$$\chi^{j\beta\mu n}k_{\beta}k_{\mu}F_{0n}=0.$$

Denoting

$$E_n=F_{0n}, \quad h^{jn}=2\chi^{j\beta\mu n}k_{\beta}k_{\mu},$$

we rewrite this system of equations in the 3-dimensional form:

$$h^{jn}E_n=0. \tag{5}$$

Taking into account Eq. (3), we write h^{jn} as

$$h^{jn}=k^jk^n-\left[\frac{\omega^2}{c^2}-\vec{k}^2\right]g^{jn}+2\xi^{jn}, \tag{6}$$

where

$$\xi^{jn}=\Pi^{j\beta\mu n}k_{\beta}k_{\mu}.$$

In order to obtain nontrivial solutions of the equation system (5) it is necessary to satisfy the following condition:

$$\det||h_{jn}||=0.$$

In order to represent this equation in its explicit form, we shall use the tensor analysis formulas proved in [6]. Following [6,7], let us inductively define the S-th power $\Phi^{(S)}_{ik}(x)$ of the arbitrary second rank tensor $\Phi_{ik}(x)$ in the N-dimensional Riemannian space:

$$\Phi^{(0)}_{ik}=g_{ik} \quad \text{for} \quad S=0,$$

$$\Phi^{(S)}_{ik}=\Phi_{im}g^{mn}\Phi^{(S-1)}_{nk} \quad \text{for} \quad S\geqslant 1,$$

where g_{jn} is the metric tensor. The contraction of the indices in this expression will give an invariant with respect to the S-th power of this tensor:

$$\Phi_{(S)}=\Phi^{(S)}_{ml}(x)\cdot g^{ml}.$$

In accordance with this definition, when $S=0$ we get $\Phi_{(0)}=N$.

In the works [6,7] it has been shown that the N-th power of any tensor Φ_{ik} of second rank in the N-dimensional Euclidean space is a linear combination of the lower power of this tensor:

$$\Phi^{(N)}_{ml}(x)=-\sum_{S=1}^{N}\Phi^{(N-S)}_{ml}Y^{(S)},$$

where the coefficients $Y^{(S)}$ are defined by the recurrent equation

$$Y^{(S)}=-\frac{1}{S}\sum_{k=0}^{S-1}\Phi_{(S-k)}Y^{(k)}, \quad S=1,2,\ldots,N \tag{7}$$

and $Y^{(0)}=1$.

The determinant of the tensor $\Phi_{ik}(x)$ is directly associated with the coefficients $Y^{(N)}$:

$$\det||\Phi_{lm}||=(-1)^N\cdot Y^{(N)}. \tag{8}$$

For $N=3$, ratios (7) and (8) give

$$\det||\Phi_{lm}||=\frac{2\Phi_{(3)}-3\Phi_{(1)}\Phi_{(2)}+\Phi^3_{(1)}}{6}.$$

Using this formula, the condition $\det\|h_{lm}\|=0$ can be expressed in the following form:

$$2h_{(3)}-3h_{(1)}h_{(2)}+h^3_{(1)}=0. \tag{9}$$

Substituting the expression (6) in the relation (9), we get

$$3(k_{\nu}k^{\nu})^2[2\xi_{(1)}-k_{\alpha}k^{\alpha}+k_nk^n]+6k_{\nu}k^{\nu}[k_jk_n\xi^{jn}-k_nk^n\xi_{(1)}]$$
$$+6[k_{\nu}k^{\nu}-k_nk^n][\xi_{(2)}-\xi^2_{(1)}]+12k_jk_n\xi^{jn}_{(2)}$$
$$-12\xi_{(1)}k_jk_n\xi^{jn}+8\xi_{(3)}+4\xi^3_{(1)}-12\xi_{(1)}\xi_{(2)}=0.$$

Using the expression for the 3-dimensional tensor ξ^{jn}, we find the final form of the dispersion equation which connects the frequency ω with the wave vector $\vec{k}$:

$$3(k_\nu k^\nu)^2[2\Pi_n^{\cdot\alpha\beta n}k_\alpha k_\beta - k_\alpha k^\alpha + k_n k^n] + 6k_\nu k^\nu[\Pi^{j\alpha\beta n}k_\alpha k_\beta k_j k_n - k_n k^n \Pi_m^{\cdot\alpha\beta m}k_\alpha k_\beta] + 6[k_\nu k^\nu - k_n k^n]$$
$$\times[\Pi_j^{\cdot\alpha\beta m}\Pi_m^{\cdot\sigma\tau j}k_\alpha k_\beta k_\sigma k_\tau - (\Pi_m^{\cdot\alpha\beta m}k_\alpha k_\beta)^2] + 12\Pi^{j\alpha\beta m}\Pi_m^{\cdot\sigma\tau n}k_j k_n k_\alpha k_\beta k_\sigma k_\tau - 12\Pi_m^{\cdot\alpha\beta m}\Pi^{j\mu\nu n}k_j k_n k_\alpha k_\beta k_\mu k_\nu$$
$$+ 8\Pi_j^{\cdot\alpha\beta n}\Pi_n^{\cdot\mu\nu m}\Pi_m^{\cdot\sigma\tau j}k_\alpha k_\beta k_\mu k_\nu k_\sigma k_\tau + 4(\Pi_n^{\cdot\alpha\beta n}k_\alpha k_\beta)^3 - 12\Pi_n^{\cdot\alpha\beta n}\Pi_j^{\cdot\mu\nu m}\Pi_m^{\cdot\sigma\tau j}k_\alpha k_\beta k_\mu k_\nu k_\sigma k_\tau = 0.$$

This dispersion equation is an algebraic equation of sixth order with respect to the frequency and the components of the wave vector.

Since the expected violation of the equivalence principle will be extremely small (in no case will it exceed 10^{-12}, as in [1]), the components of $\Pi^{\alpha\beta\mu\nu}$ are small too, and we can solve this equation using the method of sequential approximation.

Assuming $\omega = ck[1+\eta]$, where η is a small term proportional to the small tensor $\Pi^{\alpha\beta\mu\nu}$, from the dispersion equation we obtain an equation which determines η. Restricting ourselves to a second order accuracy with respect to $\Pi^{\alpha\beta\mu\nu}$ and η we have

$$2\eta^2 k^4 - 2\eta k^2[A^{ij}k_i k_j + 2kN^i k_i] - B^{ijnm}k_i k_j k_n k_m - 4kC^{ijn}k_i k_j k_n = 0, \tag{10}$$

where we denote

$$A^{ij} = \Pi_\alpha^{\cdot ij\alpha} + \delta^{ij}\Pi_n^{\cdot 00n}, \quad N^i = \Pi_j^{\cdot 0ij}, \tag{11}$$

$$B^{ijnm} = \Pi_l^{\cdot ijp}\Pi_p^{\cdot nml} + 2\Pi^{0ijp}\Pi_p^{\cdot nm0} - 2\Pi_p^{\cdot ijp}\Pi^{0nm0} - \Pi_l^{\cdot ijl}\Pi_p^{\cdot nmp} + 2\delta^{ij}[\Pi_p^{\cdot 00l}\Pi_l^{\cdot nmp} + \Pi_p^{\cdot 0nl}\Pi_l^{\cdot 0mp} + \Pi_p^{\cdot 0nl}\Pi_l^{\cdot m0p}$$
$$- 2\Pi_p^{\cdot 0np}\Pi_l^{\cdot 0ml} - \Pi_l^{\cdot 00l}\Pi_p^{\cdot nmp} + \Pi^{0n0p}\Pi_p^{\cdot 0m0} - \Pi_p^{\cdot 00p}\Pi^{0nm0}] + \delta^{ij}\delta^{nm}[\Pi_p^{\cdot 00l}\Pi_l^{\cdot 00p} - (\Pi_p^{\cdot 00p})^2],$$

$$C^{ijn} = \delta^{ij}[\Pi_p^{\cdot 00l}\Pi_l^{\cdot 0np} - \Pi_p^{\cdot 00p}\Pi_l^{\cdot onl}] + \Pi_p^{\cdot 0il}\Pi_l^{\cdot njp} + \Pi^{0n0p}\Pi_p^{\cdot ij0} - \Pi_p^{\cdot 0np}\Pi^{0ij0} - \Pi_p^{\cdot 0np}\Pi_l^{\cdot ijl}.$$

By solving Eq. (10) we can express the frequency of the electromagnetic wave as a function of the wave vector:

$$\omega(\vec{k})_{1,2} = ck\left\{1 + \frac{1}{2k^2}[A^{ij}k_i k_j + 2kN^i k_i] \pm \pm \frac{1}{2k^2}\sqrt{[A^{ij}k_i k_j + 2kN^i k_i]^2 + 2(B^{ijnm}k_m + 4kC^{ijn})k_i k_j k_n}\right\}. \tag{12}$$

Thus, in the general case when the tensor $\Pi^{\alpha\beta\mu\nu}$ has an arbitrary structure and the equivalance principle is violated, it is possible to excite two plane electromagnetic waves having same wave vector but different frequencies in the freely falling laboratory. And, conversely, in this case two plane electromagnetic waves having equal frequencies and different in magnitudes wave vectors can be excited in vacuum.

When the principle of equivalence is strictly applicable, then $\Pi^{\alpha\beta\mu\nu} = 0$ and from Eqs. (11) and (12), as expected, we get $\omega = ck$.

In the general case, when the tensor $\Pi^{\alpha\beta\mu\nu} \neq 0$ has arbitrary structure, the frequency of generation of electromagnetic waves in the laser gyroscope depends also on the direction of propagation (substitution of $\vec{k}$ by $-\vec{k}$).

As a result, the relative difference in the frequencies of the two electromagnetic waves propagating in a laser gyroscope in mutually opposite directions will be

$$\frac{\Delta\omega_1}{\omega_1} = \frac{\omega_1(\vec{k}) - \omega_1(-\vec{k})}{\omega_1} = -\frac{2\vec{P}\cdot\vec{k}}{k}. \tag{13}$$

where the 3-dimensional vector $\vec{P}$ has the components

$$P^i = N^i + \frac{2[A^{ij}N^m + A^{jm}N^i + A^{mi}N^j + 2(C^{ijm} + C^{jmi} + C^{mij})]k_j k_m}{3[\sqrt{S+A} + \sqrt{S-A}]} \tag{14}$$

and for convenience the even and odd functions of the wave vector $\vec{k}$ are denoted as

$$S = (A^{ij}k_i k_j)^2 + 4k^2(N^i k_i)^2 + 2B^{ijnm}k_i k_j k_n k_m,$$

$$A = 4k[A^{ij}N^n + 2C^{ijn}]k_i k_j k_n.$$

Since in laser gyroscope $\vec{k} = k\vec{e}_\varphi$, the expression (13) takes the form

$$\frac{\Delta\omega_1}{\omega_1} = -2N^{\varphi} - \frac{4A^{\varphi\varphi}N^{\varphi} + 8C^{\varphi\varphi\varphi}}{\{\sqrt{[A^{\varphi\varphi}+2N^{\varphi}]^2+2B^{\varphi\varphi\varphi\varphi}+8C^{\varphi\varphi\varphi}} + \sqrt{[A^{\varphi\varphi}-2N^{\varphi}]^2+2B^{\varphi\varphi\varphi\varphi}-8C^{\varphi\varphi\varphi}}\}}. \tag{15}$$

Nowadays, laser gyroscopes having linear dimensions of ~ 10 cm interfaced with special microcomputers allow us to fix $\Delta\omega/\omega$ at the level 10^{-16}. Therefore, from Eqs. (14) and (15) it follows that by measuring $\Delta\omega/\omega$ at different orientations of the laser gyroscope with respect to the laboratory (with respect to the vector $\vec{P}$), we can measure the component P_{φ} of this vector with an accuracy of 10^{-16}.

Thus, the validity of the principle of equivalence using a laser gyroscope can be determined with an accuracy which exceeds that of the proposed by Gravity Probe-B experiments in two or three orders of magnitude.

In conclusion, we would like to point out that the rotation of the Earth and its complicated movement in the Universe gives rise to a large Sagnac effect. Therefore the laser gyroscope must be mounted on the top of the mechanical gyroscopic platform. Since the mechanical gyroscopic platform, on the whole, consist of nucleons, then the laser gyroscope can be regarded as mounted in a nonrotational frame of reference for the nucleons with high level of accuracy. If the equivalence principle with respect to rotation is violated on a certain level of accuracy, then this frame of reference will be a rotational one for photons moving inside the laser gyroscope and the laser gyroscope will register the presence of this relative rotation.

The maximum level of the accuracy 10^{-16} for the verification of the equivalence principle can be achieved if, while carrying out the measurements, the mechanical gyroscopic platform ensures stabilization of the laser gyroscope against the rotation at a level of 0.02 arcsecond per second. Mechanical gyroscopic platforms having such stabilization are widely used at present in the space research.

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