Moduli space dimensions of multipronged strings

Dongsu Bak,[1,*] Koji Hashimoto,[2,†] Bum-Hoon Lee,[3,‡] Hyunsoo Min,[1,§] and Naoki Sasakura[2,∥]

[1]*Department of Physics, University of Seoul, Seoul 130-743, Korea*
[2]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[3]*Department of Physics, Sogang University, Seoul 121-742, Korea*

The numbers of bosonic and fermionic zero modes of multipronged strings are counted in $\mathcal{N}=4$ super-Yang-Mills theory and compared with those of the type IIB string theory. We obtain nice agreement for the fermionic zero modes, while our result for the bosonic zero modes differs from that obtained in the type IIB string theory. The possible origin of the discrepancy is discussed. [S0556-2821(99)08012-1]

I. INTRODUCTION

The recent development of nonperturbative string theories has provided new powerful tools to understand supersymmetric gauge theories. The low energy dynamics of the D-branes is described by supersymmetric gauge theories. The Bogomol'nyi-Prasad-Sommerfield (BPS) spectrum of the supersymmetric theory will then correspond to the BPS configurations of strings and branes ending on the background brane configurations.

Various known properties of the $\mathcal{N}=4$ SU(N) supersymmetric Yang-Mills theory have been studied based on N parallel D3-branes. The BPS state spectra of the massive gauge bosons, monopoles, and dyons preserving half of the supersymmetry are identified with the (p,q) strings connecting two separated D3-branes. With more than two D3-branes, we can have string junction configurations [1] that preserve only 1/4 of the supersymmetries [2,3]. The condition for the string junction configurations gives the set of field equations describing the corresponding BPS states of the gauge theory [3–8]. In addition to the first order differential equations describing the 1/2 BPS states of monopoles, the string junction needs a second order equation of the Gauss law. The field-theoretic solutions corresponding to multipronged strings were explicitly constructed for SU(3) theory [5,6] and generalized to SU(N) theory [7,8].

To study the quantum properties of this string junction, one needs to understand the zero modes around the classical configurations. As in the case of monopoles [9,10], the bosonic zero modes will correspond to the collective coordinates of the moduli space, while the fermionic zero modes correspond to the spin structures of the supermultiplets. The number of zero modes of monopoles with an arbitrary gauge group is well known [11].

In this paper, we will count the number of bosonic and fermionic zero modes of the multipronged strings in the SU(N) field theory and compare it with that of the type IIB

string picture [12]. The number of fermionic zero modes was already discussed for a specific SU(3) solution in Ref. [6].

In Sec. II, we briefly describe the BPS equations describing the string junction and the zero-mode equations. In Sec. III, we count the zero modes. The equations of the bosonic zero modes consist of those for the magnetic monopole and one more second order equation. Usually, the index, i.e., the number of zero modes, is evaluated by asymptotic expansion of the field configurations [11]. However, for the string junctions, the method of evaluating the index by the expansion of the electric fields fails. Instead, we will count the bosonic and fermionic zero modes based on considering the constraints imposed on the zero modes of multimonopoles. The details of the mathematical arguments are given in the Appendix. In Sec. IV, this counting is shown to be different from that based on the type II string theory. In Sec. V, we summarize our main results and indicate future directions.

II. MULTIPRONGED STRINGS AND THEIR MODULI SPACE

We begin by recapitulating the basic properties of 1/4 BPS states in the $\mathcal{N}=4$ SU(N) super-Yang-Mills theory, whose Lagrangian, for the bosonic part only, reads

$$\mathcal{L}=-\frac{1}{4g_{YM}^2}\text{Tr}\left[F^{\mu\nu}F_{\mu\nu}+2D_\mu\phi^I D^\mu\phi^I-2\sum_{I<J}[\phi^I,\phi^J]^2\right],$$
(1)

where $F_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu-i[A_\mu,A_\nu]$, $D_\mu\phi^I\equiv\partial_\mu\phi^I-i[A_\mu,\phi^I]$, and the indices I and J run from 1 to 6. It is well known that, in this theory, the monopoles, dyons, and W particles preserve half of 16 supersymmetries of the theory. It was shown recently that there may be also 1/4 BPS states that preserve a quarter of the total supersymmetry. As shown in Refs. [3,12], these states describe multipronged strings connecting N D3-branes in the type IIB string picture. Examples of closed-form solutions corresponding to the pronged strings were found in the $\mathcal{N}=4$ SU(N) super-Yang-Mills theory [5–8]. We shall investigate the number of both bosonic and fermionic zero modes of the the multipronged strings in the $\mathcal{N}=4$ super-Yang-Mills theory for the gauge group SU(N). This will be the first step to extract the structure or geometry of the moduli space involved with the mul-

*Email address: dsbak@mach.uos.ac.kr
†Email address: hasshan@gauge.scphys.kyoto-u.ac.jp
‡Email address: bhl@ccs.sogang.ac.kr
§Email address: hsmin@dirac.uos.ac.kr
∥Email address: sasakura@gauge.scphys.kyoto-u.ac.jp

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tipronged strings. Thereby, one is ultimately interested in obtaining the low-energy effective dynamics of the multi-pronged strings.

The Bogomol'nyi bound for the 1/4 BPS states can be found by considering the energy functional of the $\mathcal{N}=4$ system:

$$
M = \frac{1}{2g_{\mathrm{YM}}^2} \int d^3x \, \mathrm{Tr}\left[\mathbf{E}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{B} + \mathbf{D}\phi^I\cdot\mathbf{D}\phi^I + D_0\phi^I D_0\phi^I \right.
$$
$$
\left. - \sum_{I<J} [\phi^I,\phi^J]^2 \right]. \tag{2}
$$

Introducing two orthonormal six-vectors e^I and b^I, we present the energy equivalently by

$$
M = \frac{1}{2g_{\mathrm{YM}}^2} \int d^3x \, \mathrm{Tr}\left[|e^I\mathbf{E} + b^I\mathbf{B} - \mathbf{D}\phi^I|^2 + |D_0\phi^I e^I|^2 \right.
$$
$$
+ |D_0\phi^I b^I + i[\phi^I e^I, \phi^J b^J]|^2 + |D_0\phi_\perp^I + i[\phi^J e^J, \phi_\perp^I]|^2
$$
$$
- \sum_I [\phi^J b^J, \phi_\perp^I]^2 - \sum_{I<J} [\phi_\perp^I, \phi_\perp^J]^2 + 2\nabla\cdot(\mathbf{E}\,\phi^I e^I)
$$
$$
\left. + 2\nabla\cdot(\mathbf{B}\,\phi^I b^I) \right], \tag{3}
$$

where $\phi_\perp^I$ refers to components of the six-vector perpendicular to the unit vectors, e^I and b^I, and we have used the Gauss law constraint

$$
\mathbf{D}\cdot\mathbf{E} + i[\phi^I, D_0\phi^I] = 0, \tag{4}
$$

and the Bianchi identity $\mathbf{D}\cdot\mathbf{B}=0$ to perform the integration by part. This, then, implies that the energy is bounded from below by

$$
g_{\mathrm{YM}}^2 M \geq Q_E^I e^I + Q_M^I b^I, \tag{5}
$$

where we define the charge six-vectors as

$$
Q_E^I \equiv \int d^3x \, \nabla\cdot\mathrm{Tr}\,\mathbf{E}\phi^I = Q_E^p h^{Ip}, \tag{}
$$

$$
Q_M^I \equiv \int d^3x \, \nabla\cdot\mathrm{Tr}\,\mathbf{B}\phi^I = Q_M^p h^{Ip}. \tag{6}
$$

For the equalities, we used the asymptotic condition

$$
\langle\phi^I\rangle = h^{Ip} H_p, \tag{7}
$$

where H_p is the $N-1$ mutually commuting operators that span the Cartan subalgebra. The raising and lowering generators E_α,

$$
[H_p, E_\alpha] = \alpha_p E_\alpha, \tag{8}
$$

are normalized by

$$
[E_\alpha, E_{-\alpha}] = \alpha_p H_p, \tag{9}
$$

where α_p are the roots. We will choose the simple roots β_p by requiring $h^{Ip} b^I \beta_p > 0$ for the maximal symmetry breaking[1] case along $\phi^I b^I$.

The saturation of the bound occurs if [4–8]

$$
\mathbf{D}\phi^I = e^I\mathbf{E} + b^I\mathbf{B}, \quad D_0\phi^I e^I = 0, \quad D_0\phi^I b^I + i[\phi^I e^I, \phi^J b^J] = 0 \tag{10}
$$

and

$$
[\phi_\perp^I, \phi_\perp^J] = 0, \quad [\phi^J b^J, \phi_\perp^I] = 0, \quad D_0\phi_\perp^I + i[\phi^J e^J, \phi_\perp^I] = 0. \tag{11}
$$

There are two types of BPS states that may be classified by considering two $\mathcal{N}=4$ central charges given by [4]

$$
Z_\pm = \sqrt{\|Q_E^I\|^2 + \|Q_M^I\|^2 \pm \|Q_E^I\|\|Q_M^I\|\sin\chi} \quad (\chi\in[0,\pi)), \tag{12}
$$

where χ denotes the angle between the charges vectors. For $\chi=0$, the state preserves the eight supersymmetries and these are described field theoretically by the monopole, dyons, and W particles. With nonvanishing χ, the two charge vectors are no longer parallel, and the state preserves only 1/4 of the supersymmetry. We will be concerned on these 1/4 BPS states.

Owing to the tracelessness of the charges, this, in fact, guarantees the balance of tension of the corresponding multipronged junction. Moreover, one finds a restriction on the electric and the magnetic charges:

$$
Q_E^p h^{Ip} b^I - Q_M^p h^{Ip} e^I = 0, \tag{13}
$$

which turns out to be the balance condition of the torque applied on the associated D3-branes by the pronged string.[2] This restriction follows from

$$
Q_E^p h^{Ip} b^I = Q_E^I b^I = \int d^3x \, \mathrm{Tr}\,\mathbf{E}\cdot\mathbf{D}\phi^I b^I = \int d^3x \, \mathrm{Tr}\,\mathbf{E}\cdot\mathbf{B}
$$
$$
= \int d^3x \, \mathrm{Tr}\,\mathbf{B}\cdot\mathbf{D}\phi^I e^I = Q_M^p h^{Ip} e^I, \tag{14}
$$

where we used the Bogomol'nyi equation and the definitions of charges.

The geometric shape of the junction and the meaning of the charges may be clearly found by fixing our six-coordinate system. For later purpose, we shall denote $\phi^I b^I \equiv A_4$, and $\phi^I e^I \equiv X$ and further set $\phi_\perp^I = 0$ without loss of generality.

The BPS equations (10) are rewritten as

$$\mathbf{B} = \mathbf{D}A_4, \tag{15}$$

$$\mathbf{E} = \mathbf{D}X, \tag{16}$$

$$D_0 X = 0, \quad D_0 A_4 + i[X, A_4] = 0, \tag{17}$$

with the Gauss law

$$\mathbf{D} \cdot \mathbf{D}X - [A_4, [A_4, X]] = 0. \tag{18}$$

Let us now choose a gauge $A_0 = X$. Equations (16) and (17) then lead to a relation $\dot{A}_m = \dot{X} = 0$ ($m = 1,2,3,4$), which implies that any solutions of the BPS equation are static with this gauge choice.

Equation (15) is the usual BPS monopole equation. Hence the junction BPS state is a kind of monopole surrounded by W-boson cloud, which is determined by Eq. (18). Topological argument leads to the quantization of the magnetic charge by

$$Q_M = 4\pi \sum_{a=1}^{N-1} m_a \beta_p^a h^{Ip} b^I, \tag{19}$$

where the integer m_a counts the number of each fundamental monopole.

The moduli space of a given multipronged string with fixed D-brane positions is defined by the solution space of the above BPS equation modulo gauge transformation with fixed vacuum expectation values of scalars and electric-magnetic charges.

The tangent vectors of the moduli space with the gauge $A_0 = X$ will satisfy the zero-mode equations

$$\epsilon^{ijk} D_j \delta A_k + D_4 \delta A^i - D^i \delta A_4 \equiv \eta_{mn}^i D_m \delta A_n = 0, \tag{20}$$

$$D_m D_m \delta A_0 + 2i[D_m A_0, \delta A_m] = 0, \tag{21}$$

$$D_m \delta A_m = 0, \tag{22}$$

where we introduced a notation $-i[A_4, (\cdot)] \equiv D_4(\cdot)$ and the 't Hooft symbol

$$\eta_{mn}^a = \begin{cases} \varepsilon_{amn}, & m,n = 1,2,3, \\ +\delta_{an}, & m = 4, \\ -\delta_{am}, & n = 4, \\ 0, & m = n = 4, \end{cases} \tag{23}$$

and Eq. (22) is the gauge condition. The zero modes are then normalizable solutions of the above equations with a norm:

$$||\delta A||^2 = \int d^3x \, (\delta A_0 \delta A_0 + \delta A_m \delta A_m). \tag{24}$$

The number of the zero modes will agree with the dimensions of the moduli space or the tangent space at a given point of the moduli space. In dealing with the above zero-mode equations, we note that a multipronged string of $SU(N)$ can be always embedded into the $SU(\bar{N})$ theory with a larger

$\bar{N}$. Then, in the $SU(\bar{N})$ theory, the field components of the multipronged solution other than the $SU(N)$ component are zero by construction. Then, with this background solution, the fluctuations of components other than the $SU(N)$ component will be dynamically decoupled in the above zero-mode equations and, hence, can be trivially set to vanish from the beginning. Hereafter, we shall choose such an $SU(N)$ subgroup with the minimum rank, where one can embed the pronged string, and work within such $SU(N)$. This N is then the number of D-branes where the prongs end, which will be denoted by $\tilde{N}$.

To expose the structure of the low-energy effective Lagrangian resulting from the moduli-space approximation, we begin by specifying the moduli-space element that satisfies the static Bogomol'nyi equations as $A(\mathbf{r}; \xi)$ where ξ^s ($s = 1,2,\ldots$, [No. of zero modes]) is the coordinate of the moduli space. In evaluating the effective Lagrangian with time-dependent moduli coordinates, we shall work in a gauge $A_0(\mathbf{r}; \xi(t)) = X(\mathbf{r}; \xi(t))$, which is achieved by an appropriate gauge transformation of the fields. The Gauss constraint in Eq. (4), then, takes the form $\dot{\xi}^s(D_i \delta_s A_i - i[A_0, \delta_s A_0] - i[A_4, \delta_s A_4]) = 0$, where $\delta_s A \equiv \partial_s A - D\epsilon_s$ with ϵ_s being the gauge function required to achieve the gauge choice. Inserting $A(\mathbf{r}; \xi(t))$ into the Lagrangian (1), performing the gauge transformation, and then using the gauge condition and the above form of the Gauss constraint, one finds that

$$L_{\text{eff}} = \frac{1}{2g_{\text{YM}}^2} \int d^3x \, \text{Tr}[\delta_t A_0 \delta_t A_0 + \delta_t A_m \delta_t A_m - 2\partial_i(A_0 \delta_t A^i)], \tag{25}$$

where $\delta_t A^i$ denotes $\dot{\xi}^s \delta_s A$, and we have dropped a constant term. The effective Lagrangian may be rewritten, to quadratic order in velocities, as

$$L_{\text{eff}} = \frac{1}{2} g_{ss'}(\xi) \dot{\xi}^s \dot{\xi}^{s'} - \mathcal{A}_s(\xi) \dot{\xi}^s. \tag{26}$$

The metric and the vector potential can be expressed, in terms of the zero mode, as

$$g_{ss'}(\xi) = \frac{1}{g_{\text{YM}}^2} \int d^3x \, \text{Tr}[\delta_s A_0 \delta_{s'} A_0 + \delta_s A_m \delta_{s'} A_m], \tag{27}$$

$$\mathcal{A}_s(\xi) = \frac{1}{g_{\text{YM}}^2} \int_{r=\infty} dS^i \, \text{Tr} A_0 \delta_s A_i$$

$$= \int d\Omega \, h_p^I e^I \lim_{r\to\infty} [r^2 \text{Tr}(\hat{r}^i \delta_s A^i H_p)], \tag{28}$$

where, in the last equation, we have used the normalizability condition $\delta_s A_i = O(1/r^2)$ for a large r. The vector potential term in Eq. (26) may be a total time derivative term. Although a total time derivative term does not affect the clas-

sical dynamics of the moduli space, it may be relevant in its quantum version especially when some of the directions of the moduli space are compact. The motion in these compact directions is expected to be involved with the quantization of the electric charges.

III. COUNTING THE NUMBER OF ZERO MODES

The equations of the bosonic zero modes and the gauge fixing condition are given by Eqs. (20), (21), and (22). In this section we shall analyze the number of normalizable solutions of these equations. Here and below, all vector potentials and the covariant derivative denote, respectively, the background solutions of pronged strings and the covariant derivative with respect to the background. The normalizable solutions of Eqs. (20) and (22) are the zero modes of a BPS monopole. The problem here is whether Eq. (21) gives a normalizable solution for δA_0 or not. As discussed in the Appendix, the condition that the solution of Eq. (21) be normalizable is given by

$$\int d^3x \, \mathrm{Tr}(\Lambda_a[D_m A_0, \delta A_m]) = 0, \quad (29)$$

where the trace is over the color indices and Λ_a ($a = 1, \ldots, \tilde{N}-1$) are the zero modes of the operator $-D_m D_m$. From the junction BPS equation, $D_m D_m A_0 = D_m D_m A_4 = 0$ holds. They are independent for a 1/4 BPS state, and hence they are two of Λ_a. As shown in the Appendix, the condition (29) is equivalent to the condition that the electric charges should not change under infinitesimal changes of the configuration.

Apparently these conditions seem to give $\tilde{N}-1$ conditions on the tangent moduli space of the monopole, but this is not so. In fact, one of the conditions

$$\int d^3x \, \mathrm{Tr}(A_4[D_m A_0, \delta A_m]) = 0 \quad (30)$$

is satisfied for any monopole zero mode δA_m. To show this, we can use an identity resulting from the simple fact that the magnetic charges do not change under the infinitesimal change. As shown in the Appendix, this implies that

$$\int d^3x \, \mathrm{Tr}(A_0[D_m A_4, \delta A_m]) = 0. \quad (31)$$

One can easily show that the partial integration of this equation gives Eq. (30). (Alternatively, one may understand Eq. (30) in terms of the torque balance identity (13) by $\delta \mathrm{Tr}[A_4(r=\infty) Q_E] = \delta \mathrm{Tr}[A_0(r=\infty) Q_M]$ and $\delta Q_M = 0$.) Hence the number of constraints is in fact $\tilde{N}-2$. The number of bosonic zero modes of the monopole BPS equations is given by [No. of monopoles] $\times 4$ [11], where ''No. of monopoles'' is the total number of the fundamental monopoles, $\Sigma_a m_a$ [see Eq. (19)]. Therefore, taking into account the constraints, the total number of the bosonic zero modes (BZM's) of the junction solution is given by

$$[\text{No. of BZMs}] = [\text{No. of monopoles}] \times 4 - \tilde{N} + 2. \quad (32)$$

Let us now discuss the number of the fermionic zero modes of the junction solution. The equations for the fermionic zero modes are given by

$$\mathcal{D}^{(-)} \psi^{(-)} = \begin{pmatrix} -2D_0 & \tau_m^- D_m \\ \tau_m^+ D_m & 0 \end{pmatrix} \psi^{(-)} = 0, \quad (33)$$

$$\mathcal{D}^{(+)} \psi^{(+)} = \begin{pmatrix} 0 & \tau_m^- D_m \\ \tau_m^+ D_m & -2D_0 \end{pmatrix} \psi^{(+)} = 0, \quad (34)$$

where D_0 is understood as $D_0(\cdot) = -i[A_0, (\cdot)]$ and the 2 $\times 2$ matrices are defined by $\tau_m^- = (\sigma^i, -i)$ and $\tau_m^+ = (\sigma^i, i)$ with the Pauli matrices σ^i. To first analyze Eq. (34), we decompose it by

$$\psi^{(+)} = \begin{pmatrix} \psi_1^{(+)} \\ \psi_2^{(+)} \end{pmatrix}. \quad (35)$$

Then Eq. (34) is

$$\tau_m^- D_m \psi_2^{(+)} = 0, \quad (36)$$

$$\tau_m^+ D_m \psi_1^{(+)} - 2D_0 \psi_2^{(+)} = 0. \quad (37)$$

Since the operator $\tau_m^- D_m$ has no zero modes, the solution is given by $\psi_2^{(+)} = 0$. Hence $\tau_m^+ D_m \psi_1^{(+)} = 0$, and $\psi_1^{(+)}$ is just the fermionic zero modes of a BPS monopole. Thus the number[3] of the fermionic zero modes resulting from Eq. (34) is given by [No. of monopoles] $\times 4$.

With a similar decomposition to two-component spinors, Eq. (33) can be rewritten as

$$\tau_m^+ D_m \psi_1^{(-)} = 0, \quad (38)$$

$$\tau_m^- D_m \psi_2^{(-)} - 2D_0 \psi_1^{(-)} = 0. \quad (39)$$

Equation (39) just gives the fermionic zero modes of a BPS monopole and the number of the solutions is $4 \times$ [No. of monopoles]. Let us discuss the normalizability of $\psi_2^{(-)}$ resulting from Eq. (39). First act an operator $\tau_m^+ D_m$ on Eq. (39).[4] Using the BPS equation, we obtain

$$D_m D_m \psi_2^{(-)} - 2\tau_m^+ D_m D_0 \psi_1^{(-)} = 0. \quad (40)$$

This equation looks very similar to the bosonic one (21). To obtain a normalizable solution of $\psi_2^{(-)}$, the following constraints on $\psi_1^{(-)}$ must be satisfied:

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[3]We count the number in real components. Hence ψ and $i\psi$ are counted as distinct solutions.

[4]The following discussions are obscured by the existence of the zero modes of the operator $\tau_m^+ D_m$. However, using the explicit known relations between the fermionic and bosonic zero modes of a monopole, one can show the same result given below. N.S. would like to thank S. Imai for clarifying this point.
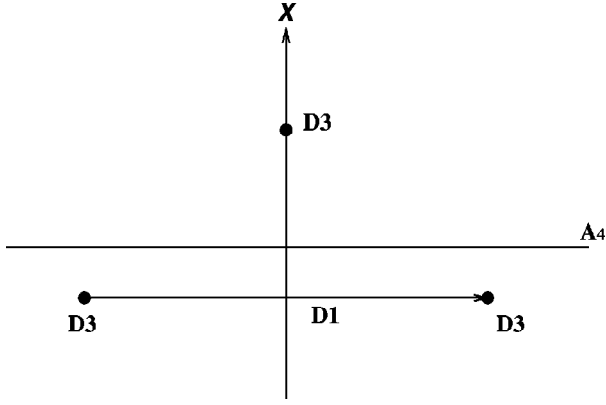
FIG. 1. The configuration of D-string ending on D3-branes is depicted. The figure is for magnetic charges $(m_1, m_2) = (1,1)$ in the SU(3) case. The moduli space is four dimensional.

$$\int d^3x \, \text{Tr}(\Lambda_a \tau_m^+ D_m D_0 \psi_1^{(-)}) = 0, \qquad (41)$$

where the trace is over the color indices. As similar to the bosonic case, one of the constraints,

$$\int d^3x \, \text{Tr}(A_4 \tau_m^+ D_m D_0 \psi_1^{(-)}) = 0, \qquad (42)$$

is identically satisfied by the monopole fermionic zero modes.

To show this, let us start with Eq. (39). Applying the operator $\tau_m^- D_m$ on it, one obtains $(D_m D_m + i \bar{\eta}_{mn}^a \sigma_a D_m D_n) \psi_1^{(-)} = 0$. [The symbols $\bar{\eta}_{mn}^a$ differ from η by a change in the sign of δ in the definition (23).] Using further the BPS equations, we obtain

$$(D_m D_m - 2i \tau_m^+ D_m D_4) \psi_1^{(-)} = 0. \qquad (43)$$

Hence the normalizability of the fermionic zero mode $\psi_1^{(-)}$ reads

$$\int d^3x \, \text{Tr}(\Lambda_a \tau_m^+ D_m D_4 \psi_1^{(-)}) = 0. \qquad (44)$$

By the partial integration of Eq. (44) with the substitution $\Lambda_a = A_0$, we obtain Eq. (42).

Thus the constraints (41) give $4(\tilde{N} - 2)$ constraints on the fermionic BPS zero modes. Thus the total number of the fermionic zero modes (FZMs) is given by

$$[\text{No. of FZMs}] = [\text{No. of monopoles}] \times 8 - 4(\tilde{N} - 2). \qquad (45)$$

In the next section, we will compare the numbers of the zero modes, Eqs. (32) and (45), with those derived from the type IIB description.

Finally we mention the reason why we should take $\tilde{N}$ instead of N in our formulas for the numbers of zero modes. Let us consider a D-string of SU(3) theory in Fig. 1. The zero-mode equations take the same form as Eqs. (20), (21), and (22) with $D_m A_0 = 0$. Hence the solution of Eq. (21) is

simply $\delta A_0 = 0$. The others possess 2×4 zero modes because the D-string is regarded as two fundamental monopoles in Eqs. (20) and (22). This is incorrect because we know from the beginning that there are only four moduli degrees of freedom around this configuration. This failure of the zero-mode analysis may be understood as follows. Around the D-string configuration, the field-theoretic potential is too flat to capture the correct number of zero modes by just considering the linearized fluctuations of Eqs. (20), (21), and (22). The higher-order analysis will show that the relative motions of the two fundamental monopoles drop out of the moduli space, which leaves just four overall translation degrees.

However, our formulas, Eqs. (32) and (45), for the zero modes are still valid for the D-string because we are using the minimal embedding of solutions and, hence, $\tilde{N} = 2$.

IV. COMPARISON WITH TYPE IIB STRING THEORY

Our result of the number of the bosonic zero modes does not agree with that from the type IIB string theory. To illustrate, let us consider the simplest case of a tree three-pronged string with two-form charges $(1,0)$, $(0,1)$, and $(-1,-1)$. The magnetic charge of the corresponding field theory solution in SU(3) is $(m_1, m_2) = (1,1)$. Thus the magnetic part has two fundamental monopoles, and the result (32) tells us that there are seven bosonic zero modes. This number looks quite odd, and moreover this does not agree with the type IIB result that is just three for the present case [12]. However, this is a very natural result from the explicit junction solution discussed in Ref. [6]. Their solution is composed of two monopole cores which are surrounded by clouds of W bosons, i.e., electric charges. The monopole part has eight bosonic zero modes which are composed of the zero modes associated with three translations, two gauge, two relative orientations, and one relative distance. The electric part is determined by the monopole part when the vacuum expectation values of the scalar fields take given fixed values. Especially, the electric charges of the junction solution are determined by the relative distance, while the magnetic charges are just topological numbers and stable. Since the electric charges appear in the asymptotic $1/r$ behavior of the junction solution, the change of the relative distance is not normalizable. Thus we should keep the relative distance fixed. The other monopole zero modes are naturally expected to be normalizable, and hence there are seven bosonic zero modes in the junction solution.

On the other hand, we find nice agreement for the fermionic part. Let us consider a monopole configuration with magnetic charges $(m_1, m_2, \ldots, m_{\tilde{N}-1})$. The corresponding string configuration is such that $\tilde{N}$ D3-branes are aligned on a line and that the ath and the $(a+1)$th D3-branes are connected by m_a D-strings $(a = 1, \ldots, \tilde{N}-1)$. This configuration may be regarded as the string configuration corresponding to a junction solution in the limit of vanishing electric charges. To recover from the limit, let us now add small Neveu-Schwarz–Neveu-Schwarz (NS-NS) charges on each D-string. We do not take care of the quantization of the NS-NS charges, since our treatment in the field theory is just

FIG. 2. The configuration of pronged strings ending on D3-branes which corresponds to our field theory analysis. The long strings have Ramond-Ramond (RR) charge one and small NS-NS charges. The figure is for magnetic charges $(m_1, m_2, m_3) = (1,3,2)$ in the SU(4) case.

classical and does not care about the quantization of the electric charges. The rule is as follows. We add distinct small NS-NS charges to each D-string, and connect the ends of them with strings with appropriate (p,q) charges to form a string network ending on the D3-branes. For the case with $(m_1, m_2, m_3) = (1,3,2)$ as an example, the configuration of the string is deformed into the one in Fig. 2. There are $m_a - 1$ internal loops between the ath and $(a+1)$th D3-branes in a general case. Although the diagram changes if the assignment of the small NS-NS charges on each D-string is changed, the number of loops does not change. It is also possible to add more strings, but we take the configuration with a minimal number of strings to be able to form a string network.[5] The total number of the loops in the diagram is given by

$$\sum_{a=1}^{\tilde{N}-1} (m_a - 1) = [\text{No. of monopoles}] - (\tilde{N}-1). \quad (46)$$

The zero-mode analysis in the type IIB framework was done in Ref. [12]. The result is

$$[\text{No. of FZMs (type IIB)}] = 8F_{\text{int}} + 4E_{\text{ext}}, \quad (47)$$

where F_{int} denotes the number of internal loops (faces) of the string diagram and E_{ext} is the number of the external strings. Thus, applying to the present case, we obtain

$[\text{No. of FZMs (type IIB)}]$

$$= 8[[\text{No. of monopoles}] - (\tilde{N}-1)] + 4\tilde{N}$$

$$= [\text{No. of monopoles}] \times 8 - 4\tilde{N} + 8, \quad (48)$$

which agrees with Eq. (45).

Finally, the discrepancy of the bosonic zero modes from the type IIB string picture might be understood as follows. Expression (32) of the bosonic zero modes can be written as

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[5]Although the possible number of the loops depends on the NS-NS charges of the external strings ending on D3-branes in type IIB theory, we take a special configuration derived by the above rule without taking into account the quantization of the NS-NS charges.

$$[\text{No. of BZMs}] = [2([\text{No. of monopoles}] - 1)$$

$$+ [\text{No. of monopoles}]]$$

$$+ [\text{No. of BZMs (type IIB)}], \quad (49)$$

where the last term is the type IIB result [12],

$$[\text{No. of BZMs (type IIB)}] = F_{\text{int}} + 3. \quad (50)$$

It is intriguing to note that the discrepancy (49) agrees with the number of the compact directions of the monopole moduli space; i.e., there are $2([\text{No. of monopoles}] - 1)$ relative spatial orientations among the fundamental monopoles and one U(1) gauge direction per each fundamental monopole. It might be expected that, when the junction BPS state is treated quantum mechanically, the wave function prevails the compact directions and these directions do not appear as the moduli of the state.

V. CONCLUSION

In this paper, we nonperturbatively identified the numbers of the bosonic and fermionic zero modes of the multipronged strings in the context of the $\mathcal{N}=4$ super-Yang-Mills theory. The bosonic zero modes differ from the type IIB string picture, but the fermionic zero modes are matching with those in the type IIB string picture.

The discrepancy is due to the softness of the field-theoretic configurations. Namely, the monopoles of the multipronged strings in the field theory can take a relative motion in the parallel space of the D3-branes, whereas the corresponding degrees in the type IIB picture cannot be permitted. In the case of the minimal three-pronged strings, the number of bosonic zero modes is 7 while there are 12 fermionic zero modes. On the grounds of the remaining supersymmetries of the system, the natural number of bosonic degrees would be even due to the complex structure of the remaining supersymmetry. We expect that the analysis of detailed moduli dynamics may be helpful in resolving this issue. The comparison with the M-theory result [13] or the D-string world sheet approach [14] would also be interesting.

The dynamics of the moduli space is in itself of importance, especially in relation to the quantizations of the electric charges. The supersymmetric quantum mechanics of the moduli space has been constructed in case of monopoles [15,16]. Our work can be used in the identification of the supersymmetric quantum mechanics for the multipronged strings. As is also done for monopoles and dyons [17], the response analysis of the multipronged strings to the excitations of unbroken gauge fields will clarify most of leading physical processes around the multipronged strings. These require further studies.

APPENDIX: THE NORMALIZABILITY CONDITION

In this appendix, we will show that the condition of getting a normalizable δA_0 from Eq. (21) is given by Eq. (29).

To simplify the expressions, we take a gauge where the vacuum expectation value of A_4 is expressed in a diagonal form.[6] We assume also that the diagonal entries take general distinct values. Then, since the massless fields are associated only with the diagonal entries, the asymptotic behavior of the solutions to the equation $D_m D_m \Lambda_a = 0$ should take the form

$$\Lambda_a = h_a^{(0)} + \frac{h_a^{(1)}}{r} + O\left(\frac{1}{r^2}\right), \tag{A1}$$

where $h_a^{(0,1)}$ are diagonal matrices. We assume that there are $\tilde{N} - 1$ solutions to this equation ($a = 1, \ldots, \tilde{N} - 1$), and that the $h_a^{(0)}$ span the Cartan subalgebra.[7] Note that Λ_a includes A_0 and A_4, and we denote $\Lambda_1 = A_0$ and $\Lambda_2 = A_4$. The diagonal entries of the $h_1^{(1)}$ are the electric charges of the junction solution, while those of $h_2^{(1)}$ are the magnetic charges. Since the vacuum expectation values are fixed, the infinitesimal changes caused by the changes of the moduli parameters of a monopole should be in the form

$$\delta A_0 = \frac{\delta h_1^{(1)}}{r} + O\left(\frac{1}{r^2}\right), \quad \delta A_4 = \frac{\delta h_2^{(1)}}{r} + O\left(\frac{1}{r^2}\right). \tag{A2}$$

In case δA_0 with a nonvanishing constant part is obtained from Eq. (21), the constant part can always be removed by adding an appropriate homogeneous solution (a linear combination of Λ_a) of Eq. (21). This is possible because the constant part of Λ_a that spans the Cartan subalgebra is used to cancel the constant part of δA_0. Since the magnetic charges are topological and do not change, $\delta A_m = O(1/r^2)$. We define generalized electric central charges by

$$Q_{\Lambda_a}^E \equiv \int_{r=\infty} dS_i \operatorname{Tr}(\Lambda_a E_i) = \int_{r=\infty} dS_i \operatorname{Tr}(\Lambda_a D_i A_0)$$
$$= \operatorname{Tr}(h_a^{(0)} h_1^{(1)}). \tag{A3}$$

Under the assumption that the $h_a^{(0)}$ span the Cartan subalgebra, the invariance of the electric charges is equivalent to the invariance of these central charges. Since $h_a^{(0)}$ is fixed in the zero-mode analysis, the deformation of these central charges by the presence of the zero modes is

$$\delta Q_{\Lambda_a}^E = \int_{r=\infty} dS_i \operatorname{Tr}[\Lambda_a \delta(D_i A_0)]$$
$$= \int d^3x \operatorname{Tr}[D_m \Lambda_a \delta(D_m A_0) + \Lambda_a D_m \delta(D_m A_0)]. \tag{A4}$$

Using the above asymptotic behaviors together with a little further manipulation of Eq. (A4), one obtains

$$\delta Q_{\Lambda_a}^E = \int d^3x \operatorname{Tr}[\Lambda_a D_m D_m \delta A_0]$$
$$= -2i \int d^3x \operatorname{Tr}(\Lambda_a [D_m A_0, \delta A_m]), \tag{A5}$$

where we have used Eq. (21) for the last equality. Similarly, defining

$$Q_{\Lambda_a}^M \equiv \int_{r=\infty} dS_i \operatorname{Tr}(\Lambda_a B_i) = \int_{r=\infty} dS_i \operatorname{Tr}(\Lambda_a D_i A_4)$$
$$= \operatorname{Tr}(h_a^{(0)} h_2^{(1)}), \tag{A6}$$

the deformation by the zero mode can be expressed as

$$\delta Q_{\Lambda_a}^M = -2i \int d^3x \operatorname{Tr}(\Lambda_a [D_m A_4, \delta A_m]), \tag{A7}$$

which is in fact automatically vanishing due to $\delta Q_M = 0$ or $\delta h_2^{(1)} = 0$. Thus, recalling the fact that there are $\tilde{N} - 1$ degrees of freedom of the electric charges and from Eq. (21), Eq. (29) is equivalent to the condition that the electric charges do not change under the infinitesimally small changes of the monopole moduli. Since the electric charges appear in the asymptotic $1/r$ behavior of δA_0, this is a necessary condition for the infinitesimal change to be normalizable under the measure $\int d^3x \operatorname{Tr}(\delta A_0^2)$. This necessary condition becomes a sufficient condition if the order next to $1/r$ is $1/r^2$ as in the expansions (A1) and (A2). In the spherically symmetric solution discussed in Refs. [5,7,8], the next order is exponentially damping. In general nonspherical cases [6], the next order is expected to behave as $1/r^2$ from dipole contributions.

There is another way to see the condition (29). Since the operator $-D_m D_m$ is a semipositive-definite Hermitian operator, one may expand δA_0 in terms of the eigenfunctions. Equation (21) is now

$$PC(P;\Omega) - 2i \int d^3x \operatorname{Tr}(f(P;\Omega)^\dagger [D_m A_0, \delta A_m]) = 0, \tag{A8}$$

where $f(P;\Omega)$ denotes the eigenfunction with eigenvalue P with Ω parametrizing the degeneracies of the eigenfunctions with the same eigenvalue, and δA_0 is expanded as $\Sigma_{P,\Omega} C(P;\Omega) f(P;\Omega)$. If the second term of Eq. (A8) is nonzero at $P=0$, the $C(P;\Omega)$ will behave in $1/P$ near $P=0$. (We assume that the eigenvalues of the operator $-D_m D_m$

exist continuously around $P=0$.) This behavior may violate the normalizability. Therefore the condition that the second term vanish for $P\rightarrow 0$ is related to the normalizability. This is the condition (29). But to conclude this we need the measure near $P=0$ from more knowledge of the spectrum of the eigenvalues and the eigenvectors.

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