Finite cutoff on the string worldsheet?

Vipul Periwal and Øyvind Tafjord
Department of Physics, Princeton University, Princeton, New Jersey 08544
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D-brane backgrounds are specified in closed string theories by holes with appropriate mixed Dirichlet and Neumann boundary conditions on the string worldsheet. As presently stated, the prescription defining D-brane backgrounds is such that the Einstein equation is not equivalent to the condition for scale invariance on the string worldsheet. A modified D-brane prescription is found that leads to the desired equivalence, while preserving all known D-brane lore. A possible interpretation is that the worldsheet cutoff is finite. Possible connections to recent work of Maldacena and Strominger and Gopakumar and Vafa are suggested. [S0556-2821(99)05914-7]

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I. INTRODUCTION

One of the basic successes of the framework of string theory is the consistent way in which one can introduce background condensates of fields. Only certain configurations of background fields are allowed by conformal invariance on the string worldsheet. It is crucial for consistency of string propagation that conformal invariance on the worldsheet is maintained—it implies unitarity, for example. A primary reason for thinking that string theories have something to do with fundamental physics is that the conditions for scale invariance on the worldsheet turn out to be equivalent to physically interesting equations of motion for the spacetime metric and other fields [1,2].

For instance, consider bosonic closed string theory with a background metric $G_{\mu\nu}(X)$ and dilaton field $\Phi(X)$ (for simplicity defined with zero expectation value). This is described by a sigma model action (we use the notation and conventions as in Polchinski [3])

$$S=\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}[g^{ab}G_{\mu\nu}(X)\partial_a X^\mu\partial_b X^\nu+\alpha' R\Phi(X)]. \tag{1}$$

Conformal invariance at the quantum level corresponds to requiring the beta functions to vanish. At one loop this means

$$\beta_{\mu\nu}=\alpha' R_{\mu\nu}+2\alpha'\nabla_\mu\nabla_\nu\Phi=0,$$
$$\beta_\Phi=-\frac{\alpha'}{2}\nabla^2\Phi+\alpha'(\nabla\Phi)^2=0. \tag{2}$$

These equations are equivalent to the equations of motion following from the spacetime action

$$S_{\mathrm{eff}}^{\mathrm{closed}}=\frac{1}{2\kappa^2}\int d^Dx\sqrt{-G}e^{-2\Phi}[R+4(\nabla\Phi)^2], \tag{3}$$

where we are dropping terms that are zero in the relevant case of $D=26$. Similarly, if we add open strings and a gauge field background on the boundary, we obtain conditions for conformal invariance equivalent to the equations of motion for a spacetime Born-Infeld action including metric and dilaton contributions $S_{\mathrm{eff}}^{\mathrm{open}}$. Thus the total spacetime action is given by $S_{\mathrm{eff}}^{\mathrm{closed}}+cS_{\mathrm{eff}}^{\mathrm{open}}$. The constant c is determined by the coupling between gravity and gauge fields, and it appears, for instance, multiplying the gauge field energy-momentum tensor in the full Einstein equation.

The closed string beta functions (2), which give the vacuum Einstein equation, do not change if one adds a boundary to the worldsheet, but the full equation is nevertheless obtained by a generalized form of conformal invariance including the effects of worldsheets of different topologies. This all works due to the Fischler-Susskind mechanism [4] where the extra breaking of conformal invariance comes from shrinking fixtures on the worldsheets of higher genus. In the present case it arises when the disk degenerates to a sphere, i.e., we represent the disk as a sphere with a hole cut out, and upon integrating the size of the hole there will be a logarithmic divergence for small hole size. The coefficient of the divergence looks similar to an insertion of a local operator on the sphere. Identifying the cutoff on the hole size with the cutoff used for the counterterms in the sphere amplitude, we can demand cutoff independence of the combined amplitude. This yields the full Einstein equation, and thus determines the constant c [4,5].

The case of a D-brane background [6] is in many ways analogous to the gauge field background, in fact, the latter is a special case (a D25-brane in the bosonic case). A static flat Dp-brane is introduced simply by adding a boundary on the worldsheet with Neumann boundary conditions on $p+1$ of the directions and Dirichlet boundary conditions on the others. The sphere amplitude gives the same beta functions as above, Eq. (2), while the disk amplitude gives conditions corresponding to equations of motion following from a Dirac-Born-Infeld action [7] (which in general also have contributions from the background gauge field and antisymmetric tensor field)

$$S_{\mathrm{eff}}^{\mathrm{DBI}}=c\int d^{p+1}\xi e^{-\Phi}\sqrt{-\tilde{G}}, \tag{4}$$

where ξ^A, $A=0,\dots,p$ are coordinates on the D-brane world volume, and $\tilde{G}_{AB}$ is the induced metric.

The full spacetime effective action should therefore be given by $S_{\rm eff}^{\rm closed}+cS_{\rm eff}^{\rm DBI}$, and the corresponding Einstein equation must have a source term due to the D-brane. In analogy with the open string case above, it would be natural to expect this source term to appear from the worldsheet point of view as a Fischler-Susskind [4] type effect. Thus one expects that there is a logarithmic divergence in the disk amplitude, whose coefficient represents the effect of the D-brane in the Einstein equation [8]. We would then have to correct the sphere metric to one that solves these equations to compute consistent amplitudes. This contradicts the prescription for introducing D-branes into the theory, which says that one can either expand about background fields representing the (super)gravity solution of the soliton (and thus extract restricted long-distance physics), or one can use the remarkable prescription of adding boundaries with Dirichlet boundary conditions [6], using a flat background metric. We should not need to do both. Indeed, explicit calculations involving D-branes show no logarithmic divergence in the disk amplitude for localized $(p+1<D)$ branes [9,10]. Furthermore, Leigh [7] explicitly noted that the disk amplitude does not change the closed string beta functions. The aim of this paper is to resolve this puzzle.

The two ways of introducing D-branes mentioned above are analogous to two different interpretations of the spacetime effective action, where one can either expand supergravity about a vacuum corresponding to the D-brane supergravity solution, or couple supergravity to a world-volume action [the Dirac-Born-Infeld (DBI) action] representing the D-brane. These two descriptions are useful in somewhat complementary regimes, the latter for isolated D-branes, and the former for N superposed D-branes, with $Ng_{\rm st}$ not necessarily small. We also note that a fact that does separate the case of localized branes from the open string case is that the Einstein equation imply that the beta functions are set to zero everywhere except at the source.

II. THE BOSONIC CASE

Let us look at the disk amplitude in more detail. For concreteness we will consider the two graviton amplitude in the background of a Dp-brane at the origin, first computed in Ref. [9], but the discussion is general. To isolate the part of the amplitude associated with the small hole, we represent the disk as a sphere with a hole cut out and integrate also over the radius a and position z of the hole. This increases the symmetry of the amplitude from SL(2,**R**) to SL(2,**C**) as in the sphere amplitude. The amplitude can be written [11,12]

$$A_{\rm disk}=2\pi^2\tau_p\int 2\,\frac{da}{a^3}\int_{|z_\alpha-z|>a}d^2z\prod_\alpha d^2z_\alpha \times\frac{1}{V_{\rm CKG}}\left\langle\prod_\alpha V_\alpha(z_\alpha)\right\rangle_{D_2}. \tag{5}$$

Here $2\pi^2\tau_p$ is the correct normalization for the Dp-brane disk amplitude, with τ_p being the Dp-brane tension [3]. We have $V_\alpha(z_\alpha)=(\kappa/\pi\alpha')\epsilon^\alpha_{\mu\nu}\partial X^\mu\overline{\partial}X^\nu e^{ik_\alpha\cdot X}(z_\alpha)$ for gravitons, and $V_{\rm CKG}$ denote the volume of the conformal killing group which we need to divide out. The $X^\mu(z)$ satisfy boundary conditions appropriate for a Dp-brane

$$\partial_n X^I(z)|_{\partial\Sigma}=0,\quad I=0,\dots,p, \tag{6}$$

$$X^i(z)|_{\partial\Sigma}=0,\quad i=p+1,\dots,D-1. \tag{7}$$

The exact Green functions for this geometry are for Neumann and Dirichlet directions, respectively:

$$\langle X^J(z_1)X^I(z_2)\rangle=\frac{\alpha'}{2}\eta^{IJ}\left(-\ln|z_1-z_2|^2 -\ln\left|1-\frac{a^2}{(z_1-z)(\overline{z_2}-\overline{z})}\right|^2\right), \tag{8}$$

$$\langle X^j(z_1)X^i(z_2)\rangle=\frac{\alpha'}{2}\eta^{ij}\left(-\ln|z_1-z_2|^2 +\ln\left|1-\frac{a^2}{(z_1-z)(\overline{z_2}-\overline{z})}\right|^2 +\ln\left|\frac{(z_1-z)(z_2-z)}{a}\right|^2\right). \tag{9}$$

Note that the last term in Eq. (9) must be kept even if it factorizes, due to the lack of momentum conservation in the transverse directions. We can now perform the contractions in $A_{\rm disk}$ and expand the integrand for small a. We find

$$A_{\rm disk}=2\pi^2\tau_p\int 2\,\frac{da}{a^3}\int_{|z_\alpha-z|>a}d^2z\prod_\alpha d^2z_\alpha \times\frac{1}{V_{\rm CKG}}a^{1/2\,\alpha' k_\perp^2}\left[\left\langle T(-k_\perp,z)\prod_\alpha V_\alpha(z_\alpha)\right\rangle_{S_2} +a^2\left\langle D(-k_\perp,z)\prod_\alpha V_\alpha(z_\alpha)\right\rangle_{S_2}+\mathcal{O}(a^4)\right], \tag{10}$$

where $k_\perp^i$ denotes the total transverse momentum $\Sigma_\alpha k_\alpha^i$, and $T(k,z)=:e^{ik\cdot X}(z):$ and $D(k,z)=(2/\alpha')(\eta M)_{\mu\nu}:\partial X^\mu\overline{\partial}X^\nu e^{ik\cdot X}(z):$ are (off-shell) tachyon and graviton-dilaton vertex operators (not properly normalized for simplicity). The flat metric is $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,\dots,1)$ while $M_{\mu\nu}\equiv\mathrm{diag}(1,\dots,1,-1,\dots,-1)$ with signature $(p+1,D-p-1)$. We denote expectation values on the sphere and disk by $\langle\,\rangle_{S_2}$ and $\langle\,\rangle_{D_2}$, respectively. In the sphere amplitudes we have only kept the zero mode integration in the Neumann direction, giving rise to a factor $(2\pi)^{p+1}\delta^{p+1}(\Sigma k_\parallel)$.

For the special case of open strings (space-filling brane) we have $k_\perp=0$, and the second term above gives rise to a logarithmic divergence in the hole size cutoff, proportional to a zero-momentum dilaton. This is the Fischler-Susskind mechanism yielding a cosmological constant in the Einstein equation [4]. However, for localized D-branes there is an extra factor of $a^{\alpha' k_\perp^2/2}$, and integrating over a gives rise to poles in $k_\perp^2$, rather than divergences. In the case of graviton scattering off the D-brane, these are physical t-channel poles. Thus we reproduce the well-known result that the disk amplitude is finite, and there is no need to introduce a cutoff on the hole size. A spacetime effective action can be deduced by examining the poles in these well-defined disk amplitudes [10]. Thus, if we were not interested in seeing how the full Einstein equation is related to conformal invariance on the worldsheet, we would stop here and be content with this answer for the scattering amplitude.

Let us now examine the full equations of motion following from the spacetime effective action $S_{\rm eff}^{\rm closed}+cS_{\rm eff}^{\rm DBI}$. Consider a flat, static Dp-brane, then the metric and dilaton equations of motion can be written

$$\frac{e^{-2\Phi}}{2\kappa^2\alpha'}\beta_{\mu\nu}=\frac{c}{4}e^{-\Phi}(\eta M)_{\mu\nu}\delta^l(x_\perp), \tag{11}$$

$$\frac{e^{-2\Phi}}{2\kappa^2\alpha'}(8\beta_\Phi-2\beta^\mu_\mu)=ce^{-\Phi}\delta^l(x_\perp). \tag{12}$$

In these equations we have already expanded the left hand sides for a flat background metric. We have defined $l=D-p-1$, the number of transverse directions. To illustrate our point, let us for the moment concentrate on the case $p=11$ (and $D=26$) where it is consistent to choose $\Phi=0$. It is then easy to linearize by setting $G_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ and solve for the leading term in $h_{\mu\nu}$. In Hilbert gauge we have $R_{\mu\nu}=-\frac{1}{2}\partial^2 h_{\mu\nu}$, and we get (for $l=14$)

$$h_{\mu\nu}=\kappa^2 c(\eta M)_{\mu\nu}\int\frac{d^l q_\perp}{(2\pi)^l}\frac{1}{q_\perp^2}e^{iq_\perp\cdot x_\perp}. \tag{13}$$

We now investigate what we get when we insert this metric perturbation on the sphere. To leading order the effect is an insertion of $h_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu$ into the amplitude. We normal order this operator using $e^{iqx}=\epsilon^{\alpha' q^2/2}:e^{iqx}:$ where ϵ is the cutoff on the worldsheet. In addition, there are contractions between the exponential in $h_{\mu\nu}$ and $\partial X^\mu\bar{\partial}X^\nu$. The term with both derivatives contracted contributes to the tachyon pole, and will be neglected together with similar terms below. The terms with one derivative contracted give rise to a total derivative, while contracting the derivatives with each other gives worldsheet curvature terms important for the dilaton equation of motion, as discussed in Refs. [12,13]. We will also not worry about these for our purposes as we concentrate on the $\beta_{\mu\nu}$ equation of motion.

Taking this into account, the leading order effect of using the metric (13) in the sphere amplitude is given by

$$A^{(1)}_{\rm sphere}=-\frac{32\pi^3}{\alpha'\kappa^2}\frac{1}{2\pi\alpha'}\kappa^2 c\frac{\alpha'}{2}\frac{\epsilon^{1/2\,\alpha' k_\perp^2}}{k_\perp^2}\int d^2z\prod_\alpha d^2z_\alpha \times\left\langle D(-k_\perp,z)\prod_\alpha V_\alpha(z_\alpha)\right\rangle_{S_2}. \tag{14}$$

Here the two first terms come from the normalization of the sphere amplitude and the prefactor of the action, respectively. The second and third terms come from $h_{\mu\nu}$ and the normalization of $D(-k_\perp,z)$. The momentum conservation in the Dirichlet directions has been used to integrate over $q_\perp$ (so again only the zero modes in the Neumann directions remain). In the background of a nontrivial metric, we should also correct the vertex operators so that they have the correct conformal weights with respect to the new energy-momentum tensor. However, for the two-graviton amplitude we consider here, the effect to this order will look similar to a two-point sphere amplitude and thus vanish, so we can use the vertex operators for the flat metric.

Now we compare this to the disk amplitude (10). We neglect the tachyon pole and focus on the massless pole. One obvious contribution comes from using the second term in the a expansion while neglecting the higher order effects of the integration domain. We can then integrate over a, with a lower limit ϵ, and we find that the ϵ-dependent part exactly cancels the ϵ dependence of the sphere amplitude (14) provided we choose

$$c=-\tau_p, \tag{15}$$

precisely as expected. We only did this for $p=11$, but, as we will see below, it holds for all p. Alternatively, we can say that the contribution from the sphere, with cutoff ϵ, exactly compensates for the piece left out from the disk amplitude by cutting off the hole size at ϵ, provided we have ''synchronized'' the sphere and the disk by choosing the correct metric, i.e., solved the correct Einstein equation with source term and specific value of c. We can take ϵ to zero if we want, then the sphere contribution vanishes, and the disk gives the full answer. If we want to extract a sensible long distance answer from the sphere by itself, as we would expect to be able to do, we will have to fix a nonzero ϵ and look at processes of very low momentum transfer,

$$k_\perp^2\ll\frac{2}{\alpha}\frac{1}{|\ln\epsilon|}. \tag{16}$$

Then $\epsilon^{\alpha' k_\perp^2/2}=1+\frac{1}{2}\alpha' k_\perp^2\ln\epsilon+\ldots$, ϵ-independent amplitude which corresponds to the semiclassical approximation of the amplitude, coinciding with the amplitude found from Polchinski's prescription in the same limit.

There are also other contributions to the massless pole in the disk amplitude. One comes from the tachyon insertion, considering $\mathcal{O}(a^2)$ effects of the integration region. Another comes from certain parts of the $\mathcal{O}(a^4)$ term in the expansion, for which integration over z yields extra $1/a^2$ singularities. These two contributions should cancel as discussed in Ref. [12].

A more formal way of deriving the same results is to consider the sphere amplitude with an arbitrary metric. We then need to introduce a cutoff, and at one loop we must put in a counterterm of the type $\beta_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu\ln\epsilon$. This is formal because the metrics involved here are not so smooth as to allow the normal coordinate expansion employed in deriving this. Nevertheless we can consider the sum of the sphere and the disk amplitude and ask for cutoff independence in some sense. Neglecting the tachyon pole, we want to demand that $\epsilon\partial_\epsilon(A_{\text{sphere}}+A_{\text{disk}})$ vanishes to leading order in ϵ (note that away from $k_\perp=0$ this derivative always vanishes for the disk amplitude as we take ϵ to zero). This will precisely be true if $\beta_{\mu\nu}$ is given as in Eq. (11), with c given in Eq. (15), using the same normal ordering considerations as we did above (treating $\beta_{\mu\nu}$ as ϵ-independent when taking the derivative). This holds for any p. Thus we have a way of extracting the full Einstein equation—not by demanding conformal invariance, which we already have on the disk alone—but by introducing a cutoff anyway and demanding agreement between the contributions from the sphere and the disk, cancelling the *leading order* cutoff dependence (neglecting tachyon effects) even if the dependence vanishes as the cutoff goes to zero.

III. THE SUPERSTRING CASE

Let us now do the same calculation in the case of the type II superstring. In this case we want to reproduce the full supergravity equations of motion, including a source term for the Dp-brane. The source term should again arise from the disk-amplitude, while the rest of the equation is given in terms of the β functions on the sphere and involve the metric, the dilaton, and the appropriate Ramond-Ramond gauge field. These equations are well known, as are the Dp-brane solutions. For clarity, we again concentrate on the constant dilaton example, the D3-brane. For our purposes we simply need the long distance behavior of the metric, conveniently given by Garousi and Meyers [10] as

$$h_{\mu\nu}=\frac{\kappa^2\tau_3}{4\pi^3x_\perp^4}(\eta M)_{\mu\nu}=\kappa^2\tau_3(\eta M)_{\mu\nu}\int\frac{d^6q_\perp}{(2\pi)^6}\frac{1}{q_\perp^2}e^{iq_\perp\cdot x_\perp}. \tag{17}$$

When this solution is inserted in the sphere amplitude, there will be a term of the following form:

$$A_{\text{sphere}}\sim-\frac{32\pi^3}{\alpha'\kappa^2}\frac{1}{2\pi\alpha'}\kappa^2\tau_3\frac{\alpha'}{2}\frac{\epsilon^{1/2\,\alpha' k_\perp^2}}{k_\perp^2}\int d^2z\prod_\alpha d^2z_\alpha \times\left\langle D(-k_\perp,z)\prod_\alpha V_\alpha(z_\alpha)\right\rangle_{S_2}. \tag{18}$$

There will be other terms, from the worldsheet fermions and the Ramond-Ramond gauge field, but these will follow from this term by worldsheet and spacetime supersymmetry, since the Dp-brane is a Bogomoln'yi-Prasad-Sommerfeld (BPS) state. This amplitude has all the same features as the corresponding bosonic amplitude, and we now want to show that the disk amplitude reproduces this behavior, again in full analogy with the bosonic case. To evaluate the superstring disk amplitude, we find it convenient to use the boundary state formalism [14,15], where the Dp-brane is represented by a closed string boundary state:

$$|B_p\rangle=4\times2\pi^2\tau_p\delta^l(x_\perp)\times\exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}(\eta M)_{\mu\nu}\tilde\alpha^\mu_{-n}\alpha^\nu_{-n}-\sum_{r=1/2,3/2,\ldots}(\eta M)_{\mu\nu}\tilde\psi^\mu_{-r}\psi^\nu_{-r}\right]|0\rangle. \tag{19}$$

There is also a ghost piece in $|B_p\rangle$ which will not be important here. The normalization is chosen such that amplitudes calculated using $|B_p\rangle$ agree with the corresponding disk amplitudes. In closed string operator language, the disk amplitude then reads

$$A_{\text{disk}}=\langle V|V\Delta V\cdots\Delta|B_p\rangle, \tag{20}$$

where the propagator is

$$\Delta=\frac{1}{2}(L_0+\tilde L_0-1)^{-1}\to\frac{1}{4\pi}\int\frac{d^2z}{|z|^2}z^{L_0-1/2}\bar z^{\tilde L_0-1/2}, \tag{21}$$

where the last expression also includes a projection over states annihilated by $L_0-\tilde L_0$ (as is the case for physical states). For completeness, we give

$$L_0=\frac{\alpha'}{4}p^2+\sum_{n=1}^{\infty}\alpha^\mu_{-n}\alpha_{n\mu}+\sum_{r=1/2,3/2,\ldots}r\psi^\mu_{-r}\psi_{r\mu}, \tag{22}$$

with a similar expression for $\tilde L_0$. We can then derive the key result

$$\begin{aligned}\Delta|B_p\rangle&=4\times2\pi^2\tau_p\frac{1}{4\pi}\int\frac{d^2z}{|z|^2}|z|^{1/2\,\alpha' k_\perp^2}\times\prod_{n,r}\exp\left[-|z|^{2n}(\eta M)_{\mu\nu}\frac{1}{n}\tilde\alpha^\mu_{-n}\alpha^\nu_{-n}-|z|^{2r}(\eta M)_{\mu\nu}\tilde\psi^\mu_{-r}\psi^\nu_{-r}\right]\left|0;-\sum k_\perp\right\rangle\\&=4\pi^2\tau_p\int\frac{da}{a^2}a^{1/2\,\alpha' k_\perp^2}[1-a(\eta M)_{\mu\nu}\tilde\psi^\mu_{-1/2}\psi^\nu_{-1/2}+\mathcal{O}(a^2)]\left|0;-\sum k_\perp\right\rangle.\end{aligned} \tag{23}$$

Here $\Sigma k_\perp$ denotes the total transverse momentum of the other vertex operators in the amplitude. The quantity $a=|z|$ now plays the role of the hole radius in the bosonic discussion above. When $a\to0$, the boundary state moves to infinite distance, and we recover a sphere amplitude with insertions of closed string operators. Again there are poles in $k_\perp^2$ from the integration around $a\approx0$. The first term is associated with the type II tachyon and should never appear in appropriately GSO projected amplitudes. The next term is the interesting one as it has a massless pole. The corresponding closed string operator insertion is

$$(\eta M)_{\mu\nu}\tilde{\psi}^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\left|0;-\sum k_{\perp}\right\rangle, \tag{24}$$

which we recognize as part of the graviton vertex operator in the ghost number $(-1,-1)$ picture. Of course, the insertion in Eq. (18) was nothing but a part of the graviton vertex in the (0,0) picture, so we see again that there is a correspondence between the disk and the sphere, exactly as in the bosonic case. We can further confirm that the dependences on ϵ and $k_{\perp}^2$ are identical, and for complete agreement we also verify that the numerical coefficients are the same, since

$$\frac{32\pi^3}{\alpha'\kappa^2}\frac{1}{2\pi\alpha'}\frac{\alpha'}{2}\kappa^2\tau_3=4\pi^2\tau_3\frac{2}{\alpha'}. \tag{25}$$

We have thus seen that the analysis for the superstring case goes through exactly as the bosonic case, i.e., the full equation of motion is not reproduced simply by demanding conformal invariance for $\epsilon\to 0$.

IV. CONCLUSIONS AND SPECULATIONS

How should one interpret the prescription that we have presented above? To take a scale invariant prescription [6], and introduce a scale dependence in order to cancel it against another term, may appear to be reckless, but has an illustrious precedent in string theory—the Fradkin-Tseytlin [1,2] term in the sigma model that is required for obtaining consistent dilaton equations of motion. It is usual [5] to restate the sort of analysis we have gone through in terms of a normalization scale dependence, phrasing the argumentation in terms of anomalies in conformal invariance. This is *not* possible when one introduces a scale dependence in a *finite* amplitude. The ''virtual anomaly'' [a phrase due to Polchinski (private communication)] in scale invariance introduced by the finite cutoff is required for spacetime diffeomorphism invariance, much as finite counterterms in ordinary field theories are required in many instances for preserving symmetries in the renormalized theories. The difference here is that we are concerned with conformally invariant, hence finite, theories, and the introduction of a scale in such a theory requires that the scale have a physical interpretation. It is also of interest to recall that Banks and Martinec [16] first suggested that the Wilsonian renormalization group underlies string theory—of course, the Wilsonian renormalization group automatically comes with a finite cutoff.

If we do take the worldsheet cutoff seriously, we must attempt to reconcile it with spacetime physics. The following remarks are speculative, but we will attempt to err on the conservative side in our interpretation of the string phantasmagoria.

Clearly, ϵ must be very small in string perturbation theory for self-consistency, so it is natural to conjecture an inverse relation, $|\ln\epsilon|\sim g_{\mathrm{st}}^{-\alpha}$, with $\alpha>0$. This would imply $\epsilon\sim\exp(-C/g_{\mathrm{st}}^{\alpha})$, which would be consistent with perturbative string theory. With this kind of dependence in the cutoff, we suggest the interpretation that the vanishing of beta functions is just the requirement that the weak-coupling limit is nonsingular. The worldsheet cutoff scale does not seem to be directly related to Shenker's observations [17]. Since strings are related to other p-branes in various dual formulations, such cutoffs are presumably in the quantum field theories living on the world volumes of such objects as well. For example, the remarkable matrix discretization of membranes due to de Wit, Hoppe, and Nicolai [18] may be more directly relevant for M theory than the continuum limit. A finite worldsheet cutoff has a number of physical consequences: (1) it implies a cutoff on the number of propagating massive modes, (2) nonrenormalization theorems may receive corrections of order ϵ, and (3) the decoupling of the conformal mode needs to be reconsidered.

A rather more drastic cutoff, but still something difficult to see in string perturbation theory, is a possible connection with the ''stringy exclusion principle'' suggested by Maldacena and Strominger [19]; see also Ref. [20]. This principle suggests that the maximum allowed occupation numbers of bosonic BPS particle modes grows in inverse proportion to the coupling constant. If it applies in general [21], though it is not entirely clear why it should, then one would expect a cutoff on the worldsheet that behaves as $\epsilon\propto g_{\mathrm{st}}^{\alpha}$, which is considerably more drastic than the cutoff suggested above. Nevertheless, given the strong divergence of string perturbation theory [23], it is not clear that such dependence of the cutoff is immediately ruled out. The spacetime length scale corresponding to Eq. (16), $l\approx\sqrt{|\ln\epsilon|}$ still diverges, but as $\sqrt{|\ln g|}$. Further, even with this kind of coupling constant dependence of the cutoff, the interpretation of beta functions suggested above still holds.

Gopakumar and Vafa [22] have recently suggested a novel picture for the relation of the closed string sigma model description to the hole description of D-branes. They propose, in a topological sigma model description, that a phase separation mechanism might account for the appearance of holes, which would be regions on the worldsheet where the fields would be frozen. If this is indeed the case, then it might in fact be natural for the size of the holes to be functions of the coupling constant, though perhaps not in a topological string theory.

As we go to higher order in the string perturbation theory we would expect similar effects to take place, with additional complications due to regimes of colliding holes, etc. We emphasize that all the technical details of these calculations are similar to those involved in the usual Fischler-Susskind mechanism. While there is no mathematical proof that the Fischler-Susskind mechanism is consistent to all orders in perturbation theory, there has appeared no evidence for any inconsistency in the physics literature. Therefore, while we make no claims to mathematical rigor, we are confident that our scheme is on the same footing as the standard Fischler-Susskind mechanism in this regard.

Starting at the annulus level, there are real divergences in D0-brane amplitudes in the limit of shrinking handles. These divergences are related to the effects of the finite mass D0-brane recoiling [24], and need to be taken into account as well. It is interesting to note that through the current analysis we have related the sphere and the disk amplitude, and thus the formalism knows about the finite D0-brane mass. In particular, we can use the full equations of motion, derived here from a worldsheet point of view, to calculate semiclassical scattering amplitudes involving gravitons and D0-branes. The classical solutions for this scattering process will necessarily involve a recoiling D0-brane, recalling that the Einstein equation implies the geodesic equation in general.

In summary, we have shown how the case of localized D-branes differs from the open string case in that demanding conformal invariance on the worldsheet does not immediately imply the full spacetime equations of motion. Instead the correct background metric and other fields arise as a secondary effect from adding boundaries with Dirichlet boundary conditions. We can still infer the full spacetime equations of motion though, by introducing a cutoff by hand and demanding that the leading cutoff dependence cancel between the sphere and the disk amplitudes. This prescription is consistent with T duality. It should be noted that a finite cutoff violates open-closed string duality—this duality is, however, manifestly a perturbative phenomenon. There is *no* evidence that it continues to have a physical significance at finite string coupling. Indeed, at $g_{st}>0$, the very validity of a string picture is rather unclear, given the $2n!$ growth of string perturbation theory [23].

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