Is the dark matter a solid?

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A smooth unclustered dark matter component with negative pressure could reconcile a flat universe with the many observations that find a density in ordinary, clustered matter well below the critical density and also explain the recent high red-shift supernova data suggesting that the expansion of the universe is now accelerating. For a perfect fluid negative pressure leads to instabilities that are most severe on the shortest scales. However, if instead the dark matter is a solid, with an elastic resistance to pure shear deformations, an equation of state with negative pressure can avoid these short wavelength instabilities. Such a solid may arise as the result of different kinds of microphysics. Two possible candidates for a solid dark matter component are a frustrated network of non-Abelian cosmic strings or a frustrated network of domain walls. If these networks settle down to an equilibrium configuration that gets carried along and stretched by the Hubble flow, equations of state result with $w = -1/3$ and $w = -2/3$, respectively. One expects the sound speeds for the solid dark matter component to comprise an appreciable fraction of the speed of light. Therefore, the solid dark matter does not cluster, except on the very largest scales, accessible only through observing the large-angle CMB anisotropy. In this paper we develop a generally covariant, continuum description for the dynamics of a solid dark matter component. We derive the evolution equations for the cosmological perturbations in a flat universe with CDM+(solid) and compute the resulting large-angle CMB anisotropy. The formalism presented here applies to any generalized dark matter with negative pressure and a nondissipative resistance to shear. [S0556-2821(99)05110-3]

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I. INTRODUCTION

Most determinations of the cosmological density parameter $\Omega_0 = (\rho/\rho_{\rm crit})$, where $\rho_{\rm crit} = (3/8\pi G)H_0^2$, now indicate that $\Omega_m \approx 0.2 \pm 0.1$, a value well below the $\Omega_m = 1$ value suggested by flat cosmological models. (For a nice review of the current observations see Refs. [1].) Most of these techniques for determining Ω_m, however, are sensitive only to matter that is clustered gravitationally and do not rule out a smooth, unclustered component that could make up the difference between the observed value of Ω_m and unity.

The earliest proposal for a smooth component is the cosmological constant Λ, first introduced by Einstein [2], later denounced by him [3], and more recently resurrected to reconcile the observations with a flat universe [4]. The cosmological constant is somewhat of an embarrassment for theoretical physics because dimensional arguments would suggest $\Lambda \approx M_{pl}^4$, a value more than a hundred orders of magnitude too big [5]. Perhaps some not yet discovered symmetry makes Λ vanish exactly, but at this point we lack even the vaguest idea of what kind of symmetry could do the job. Supersymmetry somewhat mitigates the difficulty, making M_{SSB}^4 rather than M_{pl}^4 the naive guess for Λ, but even with supersymmetry if Λ does not vanish, a formidable fine tun-

ing problem persists. A large Ω_Λ overpredicts the number of gravitationally lensed quasars [6]. As an alternative to Λ, it has been proposed that there could exist a very light, extremely weakly coupled scalar field that could act as a temporary cosmological constant, even though the true value of the cosmological constant vanishes exactly [23]. But this requires a particle of implausibly small mass, somewhere in the neighborhood of 10^{-33} eV.

In this paper we discuss another possibility: a solid dark matter component with significant negative pressure. Here *significant* means that the negative pressure, or equivalently tension, of the solid matter component is comparable in magnitude to its energy density. An equation of state with large negative pressures can lead to sound speeds comparable to the speed of light, so that the Jeans length for this component is enormous, comparable to the size of our present horizon. Consequently, the solid dark matter component does not cluster except on extremely large scales. Because of this the low measurements of Ω can be reconciled with a spatially flat universe. The clustering of the solid dark matter component on very large scales is accessible to observation only through its effect on the large-angle cosmic microwave anisotropy (CMB) anisotropy.

A solid dark matter component can also help explain the the recent observations of distant type Ia supernovae that suggest that the universe is now accelerating [7–14]. For the expansion of the universe to accelerate some exotic form of matter with $w = (p/\rho) < -1/3$ is required. A perfect fluid with $w < 0$ is not possible because its sound speed would be

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imaginary, indicating instabilities whose growth rate diverges as the wavelength approaches zero. Such a fluid would clearly be unphysical. For a solid, however, real sound speeds are possible because a shear modulus of sufficient magnitude removes these instabilities.

In this paper we explore the dynamics of a solid dark matter component by developing a continuum description for such a component within the framework of general relativity and incorporating the solid dark matter component into the linearized theory for the evolution of cosmological perturbations. In particular we explore the consequences of such a component for the large angle CMB anisotropy.

A solid dark matter component could arise from a variety of different microphysics. Two known ways such a component could arise are from networks of frustrated cosmic strings [15–18] or domain walls [20–22,24–26]. While the simplest Abelian cosmic strings obey a scaling solution so that the number of strings per horizon volume remains constant, for non-Abelian cosmic strings topological obstructions prevent the intercommuting necessary for the breakup long strings that leads to scaling behavior. The nonunit elements of the fundamental group $\pi_1(G/H)$ classify the possible types of cosmic strings. When two strings whose windings or magnetic fluxes are described by non-commuting elements of $\pi_1(G/H)$ try to cross, the strings cannot pass through each other without forming a third string between them. This has the effect of preventing crossings because the tension of the intermediate strings tries to pull the two strings back toward their previous uncrossed positions. It is possible that these effects lead to a scaling solution albeit one with many more strings per horizon volume, but the simulations by Pen and Spergel suggest that the strings settle down to a stable configuration which subsequently gets carried along with the Hubble flow. In a forthcoming paper, we show that stable string configurations do exist which strengthens the case for a string dominated universe. Similar simulations of domain walls by Kubotani suggest domain walls in models with many types of domain walls exhibit similar behavior. A cellular foam type structure in equilibrium forms with several wall meeting at linelike junctions. A string-dominated universe gives $w = -1/3$, which in the absence of other dark matter would give a universe that is neither accelerating or decelerating. A symmetry breaking scale of a few TeV and a string separation today of a few A.U. would give Ω_{string} today in the interesting range between zero and one. (This estimate is subject to considerable uncertainty because a range of string density at formation is possible and the length of the transient before string dominated behavior takes over is uncertain and model dependent.) For domain walls carried along with the Hubble flow, $w = -2/3$ and a symmetry breaking scale of a few MeV and a mean domain wall separation of some tens of parsecs are suggested (subject to the same uncertainties). The fact that new physics occurs at larger energy scales than for *quintessence* or Λ is a positive feature of these scenarios. It should be stressed that the formalism in the paper applies equally well to a solid component with the same continuum description but of completely different origin.

A slightly different type of solid dark matter has been proposed by Eichler to explain certain aspects of large-scale structure [30]. In this scenario a solid condenses and subsequently fractures when stretched beyond its breaking point by expansion of the universe. The dark matter contemplated in this paper does not fracture. It can experience unlimited stretching without becoming in any sense weakened. For a *solid* composed of frustrated topological defects it is easy to see why ruptures or fractures do not occur. The constituent defects lack a preferred size. Upon stretching or shrinking, the transverse structure of the defects remains unchanged. This is quite unlike an ordinary solid composed of atoms, for which quantum mechanics establishes a preferred length for the chemical bonds.

The organization of this paper is as follows. In Sec. II of this paper we develop a generally covariant description of the dynamics of a continuous medium (such as the string network) in curved space. For the spacetime with the metric $G_{\mu\nu} = a^2(\eta)[\eta_{\mu\nu} + h_{\mu\nu}]$ where $h_{\mu\nu}$ is regarded as small, we expand the action to quadratic order and compute the equations of motion and the resulting stress-energy for the solid dark matter component. In Sec. III we combine the results of Sec. II with the linearized theory of cosmological perturbations using Newtonian gauge and derive the equations of motion for a spatially flat universe with cold dark matter (CDM) and a solid dark matter (SDM) component. In Sec. IV compute the large-angle CMB anisotropy for models with SDM. Finally, in Sec. V we present some concluding remarks. In this paper we use the sign convention $\eta_{\mu\nu} = \text{diag}[-1, +1, +1, +1]$.

II. CONTINUUM DESCRIPTION

This section presents an action that describes the dynamics of a dissipationless elastic medium in curved space. Although developed to describe the response of a non-Abelian string network to metric cosmological perturbations, this formalism applies to other forms of solid dark matter and to a wide variety of situations involving continuous media in curved space. The problem of describing the dynamics of a solid within the framework of general relativity has been previously considered by Carter and Quintana in the study of the crusts of neutron stars [28] and others [29]. In this section we present a self contained treatment particularly suited to the consideration of linearized perturbations in an expanding universe.

A continuous medium is a kind of three-dimensional membrane but quite different from the now much discussed ''fundamental'' branes. A continuous medium has internal structure. As the medium moves, each constituent particle traces out its own world line. For continuous media the allowed reparametrizations are limited to reparametrizations that preserve these world lines. Geometrically a continuous medium may be regarded as a congruence of world lines. We use a three dimensional coordinate **y** to parametrize these world lines. The coordinate t is an arbitrary time coordinate parametrizing the direction along these world lines.

A metric h_{ab} is defined on **y**-space. The volume induced by this metric indicates the density of world lines and the additional internal metric structure provides a reference with

respect to which to characterize pure shear (i.e., volume preserving) deformations. In the classical exposition of elasticity theory (e.g., as in the book by Landau and Lifshitz [19]) $h_{ab} = \delta_{ab} = (\text{const})$. However, when a solid is formed in a warped (i.e., curved) spacetime, there generally does not exist any coordinatization of the world lines so that $h_{ab} = \delta_{ab}$. For the moment let us imagine ourselves in the instantaneous rest frame of a volume element of the medium, choosing t so that $\partial/\partial t$ is orthogonal to $\partial/\partial y^a$ ($a = 1, 2, 3$). The background spacetime metric $G_{\mu\nu}$ induces the following metric on **y**-space:

$$g_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}. \tag{2.1}$$

For an arbitrary time parametrization, where $\partial/\partial t$ is not necessarily orthogonal to $\partial/\partial y^a$, the induced metric may rewritten as

$$g_{ab} = G_{\mu\nu}^{(s)} \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}, \tag{2.2}$$

where $G_{\mu\nu}^{(s)} = U_\mu U_\nu + G_{\mu\nu}$ and $U^\mu = (\partial X^\mu/\partial t)/\sqrt{G_{\xi\eta}(\partial X^\xi/\partial t)(\partial X^\eta/\partial t)}$. $G_{\mu\nu}^{(s)}$ projects out displacements along $\partial/\partial t$.

The local deformation state of the medium is determined by comparing g_{ab} to h_{ab}—by the three principal values $\lambda_1, \lambda_2, \lambda_3$ of g_{ab} with respect to h_{ab}. (We assume that the medium is isotropic, for otherwise more structure than h_{ab} alone is required to characterize the deformation state of the medium.) In ordinary elasticity theory, $h_{ab} = \delta_{ab}$, g_{ab} is the strain tensor, and $\lambda_1, \lambda_2, \lambda_3$ are its principal values. The scalar invariants $g_{(1)} = h^{ab}g_{ab}$, $g_{(2)} = h^{ab}g_{bc}h^{cd}g_{da}$, $g_{(3)} = h^{ab}g_{bc}h^{cd}g_{de}h^{ef}g_{fa}$ suffice to characterize completely the principal values, and the local density in the local instantaneous rest frame with respect to the h volume element may be expressed as $\rho_{(h)} = \rho_{(h)}(g_{(1)}, g_{(2)}, g_{(3)})$. In terms of the principal values $g_{(1)} = \lambda_1 + \lambda_2 + \lambda_3$, $g_{(2)} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$, and $g_{(3)} = \lambda_1^3 + \lambda_2^3 + \lambda_3^3$.

It follows that the action is

$$S = -\int dt \int d^3y \sqrt{h} \, \rho(g_{(1)}, g_{(2)}, g_{(3)}) \sqrt{-G_{\mu\nu} \frac{\partial X^\mu}{\partial t} \frac{\partial X^\nu}{\partial t}}. \tag{2.3}$$

One may view Eq. (2.3) as a generalization of the free particle action $S = -m\int d\tau$. If $\rho(g_{(1)}, g_{(2)}, g_{(3)}) = (\text{const})$, the action (2.3) merely describes a congruence of noninteracting, freely falling particles. However, in the general case the comoving density with respect to the internal coordinates varies as deformations of the medium alter its internal energy. The potential energy of the medium resides in the function $\rho(g_{(1)}, g_{(2)}, g_{(3)})$.

We now recast the action (2.3) into a more familiar form by considering an elastic medium in almost flat space. We assume a spacetime metric $G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, an internal

metric $h_{ab} = \delta_{ab} + b_{ab}$, and $X^i = y^i + \xi^i(\mathbf{y}, t)$ where $h_{\mu\nu}$, b_{ab}, and $\xi^i(\mathbf{y}, t)$ are regarded as small. We also set $X^0 = t$.

We expand

$$\rho = \rho_s + \tau_s \left(\frac{\delta V}{V}\right) + K_s \left(\frac{\delta V}{V}\right)^2 + \mu_s S_{(5)ab} S_{(5)}^{ab}. \tag{2.4}$$

Here τ_s is the tension and K_s and μ_s are the compressional and shear moduli, respectively, and the tensor $S_{(5)ab}$ is the pure shear component of the strain tensor. From the relation (valid to quadratic order)

$$\left(1 + \frac{\delta V}{V}\right) = \frac{\sqrt{|g_{ab}|}}{\sqrt{|h_{ab}|}} = \frac{\sqrt{|\delta_{ab} + s_{ab}|}}{\sqrt{|\delta_{ab} + b_{ab}|}} = \frac{1 + \dfrac{s}{2} + \dfrac{s^2}{8} - \dfrac{s_{ij}s^{ij}}{4}}{1 + \dfrac{b}{2} + \dfrac{b^2}{8} - \dfrac{b_{ij}b^{ij}}{4}}, \tag{2.5}$$

it follows that

$$\frac{\delta V}{V} = \frac{s}{2} + \frac{s^2}{8} - \frac{s_{ij}s^{ij}}{4} - \frac{b}{2} + \frac{b_{ij}b^{ij}}{4} + \frac{b^2}{8} - \frac{bs}{4}. \tag{2.6}$$

Here to quadratic order

$$\begin{aligned}
s_{ab} &= \{\delta^i_{\ a} + \xi^i_{\ ,a}(\mathbf{y}, t)\}\{\delta_{ij} + h_{ij}(X) \\
&\quad + (\xi_i + h_{0i})(\xi_j + h_{0j})\}\{\delta^j_{\ b} + \xi^j_{\ ,b}(\mathbf{y}, t)\} - \delta_{ab} \\
&= \xi_{a,b} + \xi_{b,a} + h_{ab} + \xi^k\nabla_k h_{ab} + \xi_{i,a}\xi_{i,b} + h_{ai}\xi_{i,b} + h_{bi}\xi_{i,a} \\
&\quad + \dot{\xi}_a\dot{\xi}_b + h_{0a}\dot{\xi}_b + h_{0b}\dot{\xi}_a + h_{0a}h_{0b},
\end{aligned} \tag{2.7}$$

and to quadratic order

$$\begin{aligned}
\frac{\delta V}{V} &= \xi_{a,a} + \frac{1}{2}h_{aa} + \frac{1}{2}\xi^c\nabla_c h_{aa} - \frac{1}{2}\xi_{a,b}\xi_{b,a} \\
&\quad + \frac{1}{2}\xi_{a,a}\xi_{b,b} - \frac{1}{2}\xi_{a,a}b_{bb} - \frac{1}{4}h_{aa}b_{bb} \\
&\quad + \frac{1}{8}h_{aa}h_{bb} - \frac{1}{4}h_{ab}h_{ab} + \frac{1}{2}\xi_{a,a}h_{bb} - \frac{1}{2}b_{aa} + \frac{1}{4}b_{ab}b_{ab} \\
&\quad + \frac{1}{8}b_{aa}b_{bb} + \frac{1}{2}\dot{\xi}_a\dot{\xi}_a + h_{0a}\dot{\xi}_a + \frac{1}{2}h_{0a}h_{0b}.
\end{aligned} \tag{2.8}$$

To linear order (which is sufficient here), the pure shear part of the strain is

$$S_{ab}^{(5)} = \xi_{a,b} + \xi_{b,a} + h_{ab} - b_{ab} - \frac{1}{3}\delta_{ab}(2\xi_{c,c} + h_{cc} - b_{cc}). \tag{2.9}$$

Combining and expanding to quadratic order, we obtain

$$S = \int dt \int d^3 y \left[1 + \frac{b}{2} + \frac{b^2}{8} - \frac{b_{ab}b^{ab}}{4} \right]$$

$$\times \left[\rho_s + \tau_s \left\{ \xi_{a,a} + \frac{1}{2}\mathrm{h}_{aa} + \frac{1}{2}\xi^c \nabla_c \mathrm{h}_{aa} - \frac{1}{2}\xi_{a,b}\xi_{b,a} + \frac{1}{2}\xi_{a,a}\xi_{b,b} - \frac{1}{2}\xi_{a,a}b_{bb} \right. \right.$$

$$- \frac{1}{4}\mathrm{h}_{aa}b_{bb} + \frac{1}{8}\mathrm{h}_{aa}\mathrm{h}_{bb} - \frac{1}{4}\mathrm{h}_{ab}\mathrm{h}_{ab} + \frac{1}{2}\xi_{a,a}\mathrm{h}_{bb} - \frac{1}{2}b_{aa} + \frac{1}{4}b_{ab}b_{ab} + \frac{1}{8}b_{aa}b_{bb}$$

$$\left. + \frac{1}{2}\dot{\xi}_a\dot{\xi}_a + \mathrm{h}_{0a}\dot{\xi}_a + \frac{1}{2}\mathrm{h}_{0a}\mathrm{h}_{0b} \right\} + K_s \left\{ \xi^i{}_{,i} + \frac{1}{2}\mathrm{h}^i{}_i - \frac{1}{2}b^i{}_i \right\}^2$$

$$\left. + \mu_s \left\{ \xi_{a,b} + \xi_{b,a} + \mathrm{h}_{ab} - b_{ab} - \frac{1}{3}\delta_{ab}(2\xi_{c,c} + \mathrm{h}_{cc} - b_{cc}) \right\}^2 \right]$$

$$\times \left[\frac{\mathrm{h}_{00}}{2} + \frac{\mathrm{h}_{00}^2}{8} + \frac{1}{2}\xi^i \nabla_i \mathrm{h}_{00} + \mathrm{h}_{0i}\dot{\xi}^i + \frac{1}{2}\dot{\xi}^{i\,2} - 1 \right], \tag{2.10}$$

and expanding out to quadratic order and omitting terms that do not contribute to the equations of motion or to the stress-energy $T^{\mu\nu}$, we obtain

$$S = \int dt \int d^3 y \left[\left(\frac{\rho_s - \tau_s}{2} \right)\dot{\xi}^2 + (\rho_s - \tau_s)\mathrm{h}_{0i}\dot{\xi}^i \right.$$

$$- \rho_s \left\{ -\frac{\mathrm{h}_{00}}{2} - \frac{b_{aa}\mathrm{h}_{00}}{4} - \frac{\mathrm{h}_{00}^2}{8} - \frac{1}{2}(\xi^c \nabla_c)\mathrm{h}_{00} \right\}$$

$$- \tau_s \left\{ \xi_{a,a} + \frac{1}{2}\mathrm{h}_{aa} + \frac{1}{2}(\xi^c \nabla_c)\mathrm{h}_{aa} - \frac{1}{2}\xi_{a,b}\xi_{b,a} + \frac{1}{2}\xi_{a,a}\xi_{b,b} - \frac{1}{2}\xi_{a,a}\mathrm{h}_{00} \right.$$

$$\left. + \frac{1}{2}\xi_{a,a}\mathrm{h}_{bb} + \frac{1}{4}\mathrm{h}_{00}b_{aa} - \frac{1}{4}\mathrm{h}_{00}\mathrm{h}_{aa} + \frac{1}{8}\mathrm{h}_{aa}\mathrm{h}_{bb} - \frac{1}{4}\mathrm{h}_{ab}\mathrm{h}_{ab} + \frac{1}{2}\mathrm{h}_{0a}\mathrm{h}_{0a} \right\}$$

$$\left. - K_s \left\{ \xi^i{}_{,i} + \frac{1}{2}\mathrm{h}^i{}_i - \frac{1}{2}b^i{}_i \right\}^2 - \mu_s \left\{ \xi_{a,b} + \xi_{b,a} + \mathrm{h}_{ab} - b_{ab} - \frac{1}{3}\delta_{ab}(2\xi_{c,c} + \mathrm{h}_{cc} - b_{cc}) \right\}^2 \right]. \tag{2.11}$$

It follows that the equation of motion for $\boldsymbol{\xi}$ is

$$(\rho_s - \tau_s)\ddot{\xi}_i + (\rho_s - \tau_s)\dot{\mathrm{h}}_{0i} - \frac{1}{2}(\rho_s - \tau_s)\nabla_i \mathrm{h}_{00}$$

$$- K_s[2\xi_{j,ji} + \mathrm{h}_{jj,i} - b_{jj,i}] - \mu_s \left[4\xi_{i,jj} + \frac{4}{3}\xi_{j,ij} \right.$$

$$\left. - 4b_{ij,j} + \frac{4}{3}b_{jj,i} + 4\mathrm{h}_{ij,j} - \frac{4}{3}\mathrm{h}_{jj,i} \right] = 0. \tag{2.12}$$

With the coupling to gravity ignored, which is a reasonable approximation for short wavelengths, the sound speeds

$$c_S^2 = \frac{\frac{16}{3}\mu_s + 2K_s}{(\rho_s - \tau_s)}, \quad c_V^2 = \frac{4\mu_s}{(\rho_s - \tau_s)} \tag{2.13}$$

follow for the longitudinal (scalar) mode and the two transverse (vector) modes, respectively.

We next compute the stress-energy, defined by the relation

$$\delta S = \frac{1}{2}\int d^4 x \sqrt{-G}\, T^{\mu\nu}(\delta G_{\mu\nu}). \tag{2.14}$$

We use the expansion $1/\sqrt{-|G_{\mu\nu}|} = [1 + \mathrm{h}_{00}/2 - \mathrm{h}_{ii}/2 + O(\mathrm{h}^2)]$ and the relation $A w_{aa}^2 + B(w_{ab} - \frac{1}{3}\delta_{ab}w_{cc})^2 = (A - \frac{1}{3}B)w_{aa}^2 + B w_{ab}w_{ab}$. It follows that (to linear order)

$$T_0{}^0 = -\rho_s - \frac{1}{2}(\rho_s - \tau_s)(b_{ii} - \mathrm{h}_{ii} - 2\xi_{i,i}),$$

$$T_i{}^0 = (\rho_s - \tau_s)(\dot{\xi}^i + \mathrm{h}_{0i}),$$

$$T_i{}^j = -\tau_s \delta_{ij} - 2\left(K_s - \frac{4}{3}\mu_s \right)\delta_{ij}\left(\xi_{a,a} + \frac{1}{2}\mathrm{h}_{aa} - \frac{1}{2}b_{aa} \right)$$

$$- 4\mu_s (\xi_{i,j} + \xi_{j,i} + \mathrm{h}_{ij} - b_{ij}). \tag{2.15}$$

We now turn to the continuum description of the solid dark matter component with the expansion of the universe included using conformal time η so that the background metric becomes

$$G_{\mu\nu} = a^2(\eta) \cdot [\eta_{\mu\nu} + h_{\mu\nu}]. \tag{2.16}$$

We modify the notation for the expansion of the comoving density as follows:

$$\rho = \rho_s a^{\lambda}(\eta) \left\{ 1 + \tilde{\tau}_s \left(\frac{\delta V}{V} \right) + \tilde{K}_s \left(\frac{\delta V}{V} \right)^2 + \tilde{\mu}_s S_{(5)ab} S_{(5)}{}^{ab} \right\}, \tag{2.17}$$

so that $\tilde{\tau}_s$, $\tilde{K}_s$, and $\tilde{\mu}_s$ are dimensionless. We take the exponent λ to be constant, although the generalization of this is straightforward. The physical density scales as $\rho_{phys} = \rho_{cm}/a^3(\eta) = \rho_s a^{\lambda-3}(\eta)$. Given λ, the dimensionless parameters

$$\tilde{\tau}_s = \frac{\lambda}{3}, \quad \tilde{K}_s = \frac{\lambda(\lambda-3)}{18} \tag{2.18}$$

are fixed, and $\tilde{\mu}_s$ is variable only within the range

$$\max\left[0, \frac{-3}{8} \tilde{K}_s \right] \leq \tilde{\mu}_s \leq \frac{(1-\tilde{\tau}_s)}{4}, \tag{2.19}$$

obtained by requiring stability on short wavelengths and subluminal longitudinal and transverse sound speeds. For frustrated non-Abelian strings, $\tilde{\tau}_s = 1/3$ and $\tilde{K}_s = -1/9$; for a frustrated network of domain walls, $\tilde{\tau}_s = 2/3$ and $\tilde{K}_s = -1/9$.

For the expanding universe, the action (2.10) is modified as follows:

$$S = \int d\eta \int d^3 y \rho_s a^{\lambda+1}(\eta) \left[1 + \frac{b}{2} + \frac{b^2}{8} - \frac{b_{ab} b^{ab}}{4} \right]$$

$$\times \left[1 + \tilde{\tau}_s \left\{ \xi_{a,a} + \frac{1}{2} h_{aa} + \frac{1}{2} \xi^c \nabla_c h_{aa} - \frac{1}{2} \xi_{a,b} \xi_{b,a} + \frac{1}{2} \xi_{a,a} \xi_{b,b} - \frac{1}{2} \xi_{a,a} b_{bb} \right. \right.$$

$$\left. - \frac{1}{4} h_{aa} b_{bb} + \frac{1}{8} h_{aa} h_{bb} - \frac{1}{4} h_{ab} h_{ab} + \frac{1}{2} \xi_{a,a} h_{bb} - \frac{1}{2} b_{aa} + \frac{1}{4} b_{ab} b_{ab} + \frac{1}{8} b_{aa} b_{bb} + \frac{1}{2} \dot{\xi}_a \dot{\xi}_a + h_{0a} \dot{\xi}_a + \frac{1}{2} h_{0a} h_{0b} \right\}$$

$$\left. + \tilde{K}_s \left\{ \xi^i{}_{,i} + \frac{1}{2} h^i{}_i - \frac{1}{2} b^i{}_i \right\}^2 + \tilde{\mu}_s \left\{ \xi_{a,b} + \xi_{b,a} + h_{ab} - b_{ab} - \frac{1}{3} \delta_{ab} (2\xi_{c,c} + h_{cc} - b_{cc}) \right\}^2 \right]$$

$$\times \left[\frac{h_{00}}{2} + \frac{h_{00}^2}{8} + \frac{1}{2} \xi^i \nabla_i h_{00} + h_{0i} \dot{\xi}^i + \frac{1}{2} \dot{\xi}^{i\,2} - 1 \right]. \tag{2.20}$$

The equations of motion are modified to

$$(1-\tilde{\tau}_s) \left[\ddot{\xi}_i + (\lambda+1) \frac{\dot{a}}{a} \dot{\xi}_i \right] + (1-\tilde{\tau}_s) \left[\dot{h}_{0i} + (\lambda+1) \frac{\dot{a}}{a} h_{0i} \right]$$

$$- \frac{1}{2} (1-\tilde{\tau}_s) \nabla_i h_{00} - \tilde{K}_s [2\xi_{j,ji} + h_{jj,i} - b_{jj,i}]$$

$$- \tilde{\mu}_s \left[4\xi_{i,jj} + \frac{4}{3} \xi_{j,ij} - 4b_{ij,j} + \frac{4}{3} b_{jj,i} + 4h_{ij,j} - \frac{4}{3} h_{jj,i} \right]$$

$$= 0. \tag{2.21}$$

In the expanding universe, the stress-energy (to linear order) is

$$T_0{}^0 = a^{(\lambda-3)} \rho_s \left[-1 - \frac{1}{2} (1-\tilde{\tau}_s)(b_{ii} - h_{ii} - 2\xi_{i,i}) \right]$$

$$T_i{}^0 = a^{(\lambda-3)} \rho_s [(1-\tilde{\tau}_s)(\dot{\xi}^i + h_{0i})],$$

$$T_i{}^j = a^{(\lambda-3)} \rho_s \left[-\tilde{\tau}_s \delta_{ij} \right.$$

$$-2 \left(\tilde{K}_s - \frac{4}{3} \tilde{\mu}_s \right) \delta^{ij} \left(\xi_{a,a} + \frac{1}{2} h_{aa} - \frac{1}{2} b_{aa} \right)$$

$$\left. -4\tilde{\mu}_s (\xi_{i,j} + \xi_{j,i} + h_{ij} - b_{ij}) \right]. \tag{2.22}$$

III. COUPLING TO GRAVITY

We choose Newtonian gauge (which is equivalent to the gauge invariant formalism of Bardeen), so that the metric takes the form

$$ds^2 = a^2(\eta)[-d\eta^2(1+2\phi)$$

$$+ dx^i dx^j \{\delta_{ij}(1-2\psi) + h_{ij}^{(V)} + h_{ij}^{(T)}\}], \tag{3.1}$$

where $h_{ij}^{(V)}$ and $h_{ij}^{(T)}$ are pure *vector* and *tensor* parts of the spatial-spatial metric perturbation, respectively. In dealing with cosmological perturbations it is convenient to define any vector or tensor that can be expressed by taking derivatives of a scalar quantity as *scalar*. Likewise, a tensor that can be expressed as a derivative of a *vector* is regarded as *vector*. With these definitions the linearized equations separate into independent *scalar, vector*, and *tensor* blocks. We assume a flat universe filled with cold dark matter and a solid dark matter component.

We decompose the displacement field

$$\boldsymbol{\xi} = \boldsymbol{\xi}^{(S)} + \boldsymbol{\xi}^{(V)}, \tag{3.2}$$

and the internal metric of the solid dark matter component

$$b_{ij} = 2 b_{tr}^{(S)} \delta_{ij} + b_{ntr}^{(S)} \left(\frac{k_i k_j - \frac{1}{3} \delta_{ij} k^2}{k^2} \right) + b_{ij}^{(V)} + b_{ij}^{(T)}. \tag{3.3}$$

A. Scalar perturbations

We first consider the *scalar* perturbations. The linearized Einstein equations for the *scalar* sector are

$$\delta G_0^{\ 0} = \frac{-2}{a^2} [\nabla^2 \psi - 3\mathcal{H}(\dot{\psi} + \mathcal{H}\phi)]$$

$$= (8\pi G)[-\rho_c \delta + \Theta_0^{\ 0}], \tag{3.4a}$$

$$\delta G^{(S)0}_{\quad i} = \frac{-2}{a^2} [\dot{\psi} + \mathcal{H}\phi]_{|i}$$

$$= (8\pi G)[\rho_c v_{|i} + \Theta^{(S)0}_{\quad i}], \tag{3.4b}$$

$$\delta G^{(S-tr)j}_{\qquad i} = \frac{2}{a^2} \left[\ddot{\psi} + 2\mathcal{H}\dot{\psi} + \mathcal{H}\dot{\phi} + (2\dot{\mathcal{H}} + \mathcal{H}^2)\phi \right.$$

$$\left. + \frac{1}{3}\nabla^2(\phi - \psi) \right] \delta_i^{\ j}$$

$$= (8\pi G)\Theta^{(S-tr)j}_{\qquad i}, \tag{3.4c}$$

$$\delta G^{(S-ntr)j}_{\qquad i} = \frac{-1}{a^2} \left[\left(\nabla_i \nabla^j - \frac{1}{3} \delta_i^{\ j} \nabla^2 \right) (\phi - \psi) \right]$$

$$= (8\pi G)\Theta^{(S-ntr)j}_{\qquad i}, \tag{3.4d}$$

where *tr* and *ntr* denote the pure trace and traceless *scalar* parts of the spatial-spatial tensors, respectively. The dots represent derivatives with respect to conformal time, $\mathcal{H} = (\dot{a}/a)$, $\rho_c = \Omega_{cdm}(3/8\pi G)\mathcal{H}^2 a^{-2}$, and $v_i = v_{|i}$ (i.e., v is the potential for the *scalar* part of the velocity field). The covariantly divergenceless tensor $\Theta_{\mu\nu}$ is the perturbation in the stress-energy of the solid dark matter component.

Equations (3.4a) and (3.4b) may be combined to obtain

$$\nabla^2 \psi = \frac{3}{2} \mathcal{H}^2 \left[\Omega_{cdm}(\delta_{cdm} - 3\mathcal{H} v_{cdm}) \right.$$

$$\left. + \frac{1}{\rho_{crit}}(-\Theta_0^{\ 0} - 3\mathcal{H}\mathcal{V}) \right], \tag{3.5}$$

where $\Theta^0_{\ i}(string\text{-}scalar) = \mathcal{V}_{|i}$ (i.e., $\mathcal{V}$ is the potential used to represent the *scalar* component of the solid dark matter component momentum density). Similarly, Eq. (3.4d) may be recast as

$$\left(\nabla_i \nabla^j - \frac{1}{3} \delta_i^{\ j} \nabla^2 \right)(\psi - \phi) = 3\mathcal{H}^2 \frac{1}{\rho_{crit}} \Theta^{(S-ntr)j}_{\qquad i}. \tag{3.6}$$

The equations of motion for the cold dark matter (CDM) component are

$$\dot{\delta}_{cdm} = -(\nabla \cdot \mathbf{v}_{cdm}) + 3\dot{\psi},$$

$$\dot{\mathbf{v}}_{cdm} + \mathcal{H}\mathbf{v}_{cdm} = -\nabla\phi. \tag{3.7}$$

For the *scalar* mode of the solid dark matter component, the equation of motion is

$$(1 - \tilde{\tau}_s)\{\ddot{\boldsymbol{\xi}}^{(S)} + (\lambda + 1)\mathcal{H}\dot{\boldsymbol{\xi}}^{(S)} + \nabla\phi\}$$

$$= \left(2\tilde{K}_s + \frac{16\tilde{\mu}_s}{3} \right)\{\nabla^2 \boldsymbol{\xi}^{(S)} - 3\nabla(\psi + b_{tr}^{(S)})\}. \tag{3.8}$$

Finally, we have the equations

$$\Theta_0^{\ 0} = \frac{-(\rho_s - \tau_s)}{a^{(3-\lambda)}} \left[-\xi_{i,i} - \frac{1}{2} h_{ii} + \frac{1}{2} b_{ii} \right]$$

$$= \frac{-(\rho_s - \tau_s)}{a^{(3-\lambda)}} [-\xi_{i,i} + 3\psi + 3b_{tr}^{(S)}],$$

$$\Theta^{(S-ntr)j}_{\qquad i} = -4\tilde{\mu}_s \frac{\rho_s}{a^{(3-\lambda)}}$$

$$\times \left[\xi_{i,j} + \xi_{j,i} - \frac{2}{3}\delta_{ij}\xi_{k,k} - b_{ij}^{(S-ntr)} \right]. \tag{3.9}$$

With the equations of motion for the cosmological perturbations including the solid dark matter component derived, we now turn to initial conditions. For each wave number **k** there exist four modes: two growing modes and two decaying modes. This pair of perturbations corresponds to the two distinct ways in which the solid dark matter generates and alters the growth of perturbations. If prior to the phase transition that produced the solid dark matter, there were preexisting curvature fluctuations, then the presence of the solid dark matter alters the evolution of the fluctuations. This is similar to the way in which a cosmological constant, neutrinos, or quintessence alters the evolution of the fluctuations. In addition, the generation of the solid dark matter (e.g., string formation) at a phase transition can produce new fluctuations. These entropy fluctuations correspond to variations in the dark matter density at the surface $T = T_{pt}$, where T_{pt} is the phase transition temperature. These white noise entropy

fluctuations are likely to be small on scales large compared to the horizon scale at the phase transition.

We focus on the effect of the solid dark matter on the evolution of preexisting scalar, vector and tensor fluctuations. Inflation generates primarily scalar and tensor fluctuations; however, we include the vector term for completeness. On the surface $T = T_{pt}$ the solid dark matter component inherits as its intrinsic spatial metric the metric on this surface induced by the background spacetime metric. Specifically, for small perturbations this gives

$$b_{ab} = \left[-2\psi + \frac{2}{3} \frac{\delta_{rad}}{(1 + w_{rad})} \right] \delta_{ab}, \qquad (3.10)$$

or, equivalently,

$$b_{tr}^{(S)} = -\psi + \frac{1}{3} \frac{\delta_r}{(1 + w_r)}, \quad b_{ntr}^{(S)} = 0. \qquad (3.11)$$

Initially, we assume that $\boldsymbol{\xi} = \dot{\boldsymbol{\xi}} = 0$. The second term in Eq. (3.10) arises from the shift in time of the surface of constant density relative to the surfaces of constant cosmic time for Newtonian gauge. Since the wavelengths of interest at this point lie far beyond the Hubble length, we have ignored perturbations in the velocity of the radiation fluid. We assume that $(k\eta) \ll 1$ and that only the growing mode is relevant.

The perfect fluid analogue of the above is as follows. For temperatures $T > T_{pt}$, the universe is filled with a single perfect fluid, which at $T = T_{pt}$ instantaneously splits into several uncoupled perfect fluid components, labeled by ($A = 1, ..., N$). In this case the matching condition is $\delta/(1 + w) = \delta_A/(1 + w_A)$ for all A and all velocities may be neglected. While the Lagrangian formalism developed in this paper rather than the more familiar Eulerian formalism could be used to describe this perfect fluid situation, for non-Abelian strings, and similarly any solid with harmonic resistance to shear, the more general Lagrangian description is required.

We now consider the subsequent evolution given these initial conditions. In the situations of interest the solid dark matter component is formed well before radiation-matter equality and solid dark matter component contributes negligibly to the matter density of the universe compared to other components until well into the matter dominated epoch.

Assuming either complete matter or complete radiation domination gives $\delta = -2\phi = -2\psi$ on superhorizon scales. Consequently, $b_{tr}^{(S)} = -(3/2)\psi$ during radiation domination on the scales of interest. During the matter-radiation transition ψ drops by a factor of $(9/10)$ while $b_{tr}^{(S)}$ does not change; therefore, during matter domination on superhorizon scales $b_{tr}^{(S)} = -(5/3)\psi$.

To follow the evolution of the perturbations through the transition from matter to solid dark matter domination, it is convenient to define the variable $S = (\nabla \cdot \boldsymbol{\xi})$. The equation of motion becomes

$$(1 - \tilde{\tau}_s)\ddot{S} + (1 + \lambda)\mathcal{H}(1 - \tilde{\tau}_s)\dot{S} - \left(2\tilde{K}_s + \frac{16\tilde{\mu}_s}{3} \right) \nabla^2 S$$

$$= -\left[(1 - \tilde{\tau}_s)\nabla^2\phi + 3\left(2\tilde{K}_s + \frac{16\tilde{\mu}_s}{3} \right) \nabla^2(\psi + b_{tr}^{(S)}) \right],$$

$$(3.12)$$

and the sources become

$$\Theta_0{}^0 = \rho_{crit}\Omega_{sdm}(1 - \tilde{\tau}_s)[S - 3(\psi + b_{tr}^{(S)})],$$

$$\Theta^{(S-tr)}{}_i{}^j = \delta_i^j(-2\rho_{crit}\Omega_{sdm}\tilde{K}_s)[S - 3(\psi + b_{tr}^{(S)})],$$

$$\Theta^{(S-ntr)}{}_i{}^j = -8\tilde{\mu}_s\rho_{crit}\Omega_{sdm}\left(\frac{k_i k^j - \frac{1}{3}k^2\delta_i^j}{k^2} \right) S.$$

$$(3.13)$$

It follows that

$$-\nabla^2(\psi - \phi) = k^2(\psi - \phi) = 24\tilde{\mu}_s\Omega_{sdm}\mathcal{H}^2 S \qquad (3.14)$$

and

$$\ddot{\psi} + 2\mathcal{H}\dot{\psi} + \mathcal{H}\dot{\phi} + (2\dot{\mathcal{H}} + \mathcal{H}^2)\phi + \frac{1}{3}\nabla^2(\phi - \psi)$$

$$= -3\mathcal{H}^2\Omega_{sdm}\tilde{K}_s[S - 3(\psi + b_{tr}^{(S)})]. \qquad (3.15)$$

Initially, far outside the horizon, $S = \dot{S} = 0$. It follows that $S = O(1) \cdot (k\eta)^2 \cdot \psi$ on superhorizon scales.

The evolution of the gravitational potentials after the radiation epoch may be computed by solving Eqs. (3.12), (3.14), and (3.15) combined with the initial conditions $S = \dot{S} = 0$, $\psi = \psi_{init}$, $b_{tr}^{(S)} = -(5/3)\psi_{init}$, and $\dot{\psi} = 0$. For a non-Abelian string network stretched by the Hubble flow, the evolution of the scale factor is given by $a(\eta) = \bar{a}[\cosh \eta - 1]$ where $\Omega_m = \text{sech}^2[\eta/2]$, just as for a hyperbolic universe with CDM.

As a practical matter, it is better to use synchronous gauge to evolve the perturbations because synchronous gauge is better behaved on superhorizon scales. In Newtonian gauge the density and velocity perturbations on superhorizon scales are not small. This results from the warping of surfaces of constant cosmic time required to make the spatial part of the metric conformally flat. As a consequence using the constraint equations to determine the Newtonian potentials and their time derivatives involves delicate cancellations between large quantities, cancellations that become increasingly delicate as one passes to earlier times. Synchronous gauge, on the other hand, is more robust. For the adiabatic growing mode synchronous gauge (with the amplitude of the gauge modes set to zero) rapidly approaches a comoving, constant density gauge as one passes to superhorizon scales. With the convention (Ref. [26])

$$ds^2 = -d\eta^2 + a^2(\eta)\left[\delta_{ij} + h(\mathbf{k},\eta)\hat{k}_i\hat{k}_j\right.$$

$$\left. + 6\eta(\mathbf{k},\eta)\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)\right]dx^i dx^j, \quad (3.16)$$

at early times during the radiation epoch on superhorizon scales one has $\delta \sim O(k^2\eta^2)$ and $\theta \sim O(k^4\eta^3)$ for all components contributing to the stress-energy and $h \sim O(k^2\eta^2)$, and the only appreciable perturbation is $\eta \sim O(1)$. For the solid dark matter for initial conditions on superhorizon scales it is an adequate approximation to set $\boldsymbol{\xi} = \dot{\boldsymbol{\xi}} = 0$ and $b_{ij} = 6\eta(\mathbf{k},\eta)(\hat{k}_i\hat{k}_j - \frac{1}{3})\delta_{ij}$. The Newtonian potentials may be calculated from the synchronous variables according to

$$\phi = \frac{1}{2k^2}[(\ddot{h} + 6\ddot{\eta}) + \mathcal{H}(\dot{h} + 6\dot{\eta})],$$

$$\psi = \dot{\eta} - \frac{\mathcal{H}}{2k^2}(\dot{h} + 6\dot{\eta}). \quad (3.17)$$

B. Vector perturbations

For completeness in this subsection we give the evolution equations for the vector sector. Although for each wave number $\mathbf{k}$ the solid dark matter component has two dynamical vector degrees of freedom, for the usual inflationary models combined with a solid dark matter component these vector modes are not excited. As before, on the initial surface at $T = T_{pt}$ the solid dark matter component inherits as its intrinsic metric the metric on this surface induced by the spacetime metric, but if the $h_{ij}^{(V)} = 0$, it follows that $b_{ij}^{(V)} = 0$. Similarly, on this surface on superhorizon scales $\boldsymbol{\xi}^{(V)} = \dot{\boldsymbol{\xi}}^{(V)} = 0$. We also have the gauge condition $h_{0i}^{(V)} = 0$.

For the CDM the equation of motion for the *vector* modes is

$$\dot{\mathbf{v}}_{cdm}^{(V)} + \mathcal{H}\mathbf{v}_{cdm}^{(V)} = \frac{1}{2}\nabla \cdot \underline{h}^{(V)}. \quad (3.18)$$

Similarly, for the two *vector* modes of the solid dark matter component, the equations of motion are

$$(1-\tilde{\tau}_s)\{\ddot{\boldsymbol{\xi}}^{(V)} + (1+\lambda)\mathcal{H}\dot{\boldsymbol{\xi}}^{(V)}\}$$
$$= 2\tilde{\mu}\{\nabla^2\boldsymbol{\xi}^{(V)} + \nabla\cdot\underline{h}^{(V)} - \nabla\cdot\underline{b}^{(V)}\}. \quad (3.19)$$

The Einstein equations for the *vector* sector are

$$\frac{1}{a^2}\nabla_j \cdot \dot{h}_{ji}^{(V)} = (8\pi G)[\rho_{cdm}v_{cdm\,i}^{(V)} + \Theta_{0i}^{(V)}],$$

$$\frac{1}{a^2}[\ddot{h}_{ij}^{(V)} + 2\mathcal{H}\dot{h}_{ij}^{(V)}] = (8\pi G)\Theta_{ij}^{(V)}. \quad (3.20)$$

Finally, we have the equations

$$\Theta^{(V)}{}_i{}^0 = \frac{\rho_s}{a^{(3-\lambda)}}(1-\tilde{\tau}_s)\dot{\xi}_i^{(V)},$$

$$\Theta^{(V)}{}_i{}^j = \frac{\rho_s}{a^{(3-\lambda)}}(-4\tilde{\mu}_s)[\xi_{i,j}^{(V)} - \xi_{j,i}^{(V)} + h_{ij}^{(V)} - b_{ij}^{(V)}]. \quad (3.21)$$

C. Tensor perturbations

Primordial tensor perturbations, such as those generated during inflation, are influenced by the solid dark matter component.

The linearized Einstein equation for the *tensor* sector is

$$\frac{1}{a^2}[\ddot{h}_{ij}^{(T)} + 2\mathcal{H}\dot{h}_{ij}^{(T)} - \nabla^2 h_{ij}^{(T)}] = (8\pi G)\Theta_{ij}^{(T)}. \quad (3.22)$$

The *tensor* stress-energy from the solid dark matter component is given by

$$\Theta_i^{(T)j} = -4\tilde{\mu}_s\rho_{crit}\Omega_s h^T{}_i{}^j - h^T{}_i{}^j(n=0), \quad (3.23)$$

which may be inserted into Eq. (3.20) to obtain

$$\ddot{h}_{ij}^{(T)} + 2\mathcal{H}\dot{h}_{ij}^{(T)} - \nabla^2 h_{ij}^{(T)} + 12\tilde{\mu}_s\Omega_{sdm}\mathcal{H}^2 h^T{}_i{}^j - h^T{}_i{}^j(n=0)$$
$$= 0. \quad (3.24)$$

Physically, the response of the solid dark matter component contributes a mass term to the gravity waves.

IV. IMPLICATIONS FOR THE CMB ANISOTROPY

We now explore the consequences of a solid dark matter component for the predicted CMB anisotropy. The *scalar* contribution is given by the Sachs-Wolfe formula

$$\frac{\Delta T}{T}(\theta,\phi) = \frac{1}{3}\phi(r_{ls},\theta,\phi,\eta_{ls})$$

$$+ \int_{\eta_{ls}}^{\eta_0} d\eta\left(\frac{\partial\psi}{\partial\eta} + \frac{\partial\phi}{\partial\eta}\right)\bigg|_{r=\eta_0-\eta}. \quad (4.1)$$

Expanding the CMB anisotropy into spherical harmonics

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{lm} a_{lm}Y_{lm}(\theta,\phi), \quad (4.2)$$

one obtains the following expression for the expected multipole moment variance:

$$c_l = \langle|a_{lm}|^2\rangle = \int_0^\infty dk P(k)\left[\frac{1}{3}\Phi(\eta_{ls};k)j_l(kr_{ls})\right.$$

$$\left. + \int_{\eta_{ls}}^{\eta_0} d\eta j_l(kr)\{\dot{\Psi}(\eta,k) + \dot{\Phi}(\eta,k)\}\right]^2. \quad (4.3)$$

Here $P(k)$ is the primordial power spectrum, which we set to $P(k) \propto 1/k$, indicating a featureless, scale-invariant, Harrison-Zeldovich primordial spectrum. Here the functions $\Psi(\eta,k)$ and $\Phi(\eta,k)$ indicate the time dependence of the

growing modes of wave number k of ψ and ϕ, respectively, and are normalized to unity as $\eta \rightarrow 0$.

On small angular scales ($l \gtrsim 30$), the solid dark matter component does not play a significant role in determining the CMB anisotropy because on these scales the anisotropy is almost exclusively determined by what happens at the surface of last scatter when Ω_{solid} is negligible, and the contribution of the integrated Sachs-Wolfe term on these scales is negligible. On larger angular scales, however, the contribution through the integrated Sachs-Wolfe term of the decay of the gravitational potential contributes significantly to the low-l moments, and since the details of how the potential decays depend on the dynamics of the smooth component, one expects the dynamics of the solid dark matter component to play an important role in determining these CMB moments.

To illustrate the effect of a solid dark matter component, we compare the evolution of the gravitational potentials and the large-angle CMB moments for the following nine cosmological models, some with a solid dark matter component and others included for purposes of comparison.

(0). A flat $\Omega_m = 1$ universe. In this critical universe filled with cold dark matter the gravitational potentials ϕ and ψ remain constant at late times, so there is no integrated Sachs-Wolfe contribution.

(1). A hyperbolic $\Omega_m = 0.3$ universe. The scale factor $a(\eta)$ for this hyperbolic universe with no dark matter other than a subcritical cold dark matter component evolves exactly as the scale factor for the string dominated universe. However, the negative spatial curvature and differing decay of the gravitational potential at late times leads to a different shape for the low-l CMB moments.

(2). A flat $\Omega_m = 0.3$ string dominated universe. In this flat universe with a subcritical density of cold dark matter and the remainder in a network of frustrated non-Abelian strings, the physical density in the solid string network component scales as $\rho_s \propto a^{-2}$, becoming the dominant form of matter at late times. The behavior of the solid string component depends on the strength of the resistance to pure shear $\tilde{\mu}_s$. We consider the following three subcases.

(2a) $c_S = 0$, $c_V = 1/2$. ($\tilde{\mu}_s = 1/24$).
(2b) $c_S = 1/\sqrt{3}$, $c_V = 1/\sqrt{2}$. ($\tilde{\mu}_s = 1/12$).
(2c) $c_S = 1$, $c_V = 1$. ($\tilde{\mu}_s = 1/6$).

(3). A flat $\Omega_m = 0.3$ domain wall dominated universe. In this flat cosmological model a network of frustrated domain walls formed in a late-time phase transition gives a density that scales as $\rho_s = 1/a$. As for the string-dominated universe, we consider three subcases.

(3a) $c_S = 0$, $c_V = 1/\sqrt{2}$. ($\tilde{\mu}_s = 1/24$).
(3b) $c_S = 1/\sqrt{3}$, $c_V = \sqrt{3}/2$. ($\tilde{\mu}_s = 3/48$).
(3c) $c_S = \sqrt{2/3}$, $c_V = 1$. ($\tilde{\mu}_s = 1/12$).

(4). A flat $\Omega_m = 0.3$ Λ-dominated universe. This flat universe with a cosmological constant may be interpreted as a degenerate case of a solid dark matter component in which $\tau_s \rightarrow 1$.

Figures 1(a) and 1(b) indicate the evolution of the gravitational potentials ϕ and ψ, respectively, in the limit $k \rightarrow 0$ (i.e., on superhorizon scales) for the various models. The potentials have been normalized to unity at early times and
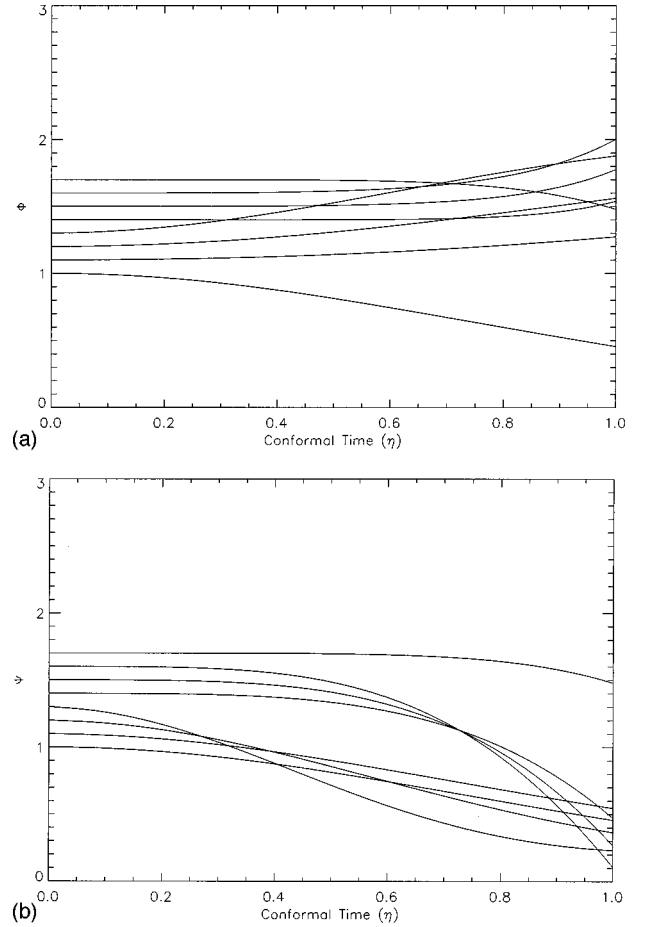

(a)

(b)

FIG. 1. Panels (a) and (b) show the evolution of the potentials ϕ and ψ, respectively for vanishing wave number as a function of conformal time for models (1), (2a)–(2c), (3a)–(3c), and (4). All the potentials are normalized to unity at vanishing conformal time. The curves for each successive model have been displaced upward by 0.1.

the horizontal axis indicates conformal time, normalized so that $\eta = 1$ today. Models (1), (2a)–(2c), (3a)–(3c), (4) are indicated, with no offset for model (1) and offsets increasing by 0.1 for each successive model, introduced to separate the curves in the plot. Although models (1) and (2a)–(2c) have the same evolution of the scale factor, the evolution of the potential is different at later times. The fact that ϕ and ψ evolve differently is due to the presence of large anisotropic stresses. In the domain wall dominated models the decay of the Newtonian potential ψ is much greater than in the hyperbolic or Λ models leading to a significantly larger integrated Sachs-Wolfe contribution to the CMB moments.

Figure 2 shows the CMB moments for these models from $l = 2$ through $l = 30$ normalized so that c_{30} agrees for all the models. The vertical axis is $c_l \cdot l(l+1)$ with offsets increasing by of 0.2. The shapes of the CMB moments were computed with a method that does not take into account the effects producing the beginning of the rise toward the Doppler peak toward increasing l. In other words, $l(l+1)c_l$ would be constant for a flat CDM universe when in fact there is a 40% rise in this quantity by $l = 30$. Therefore, only the
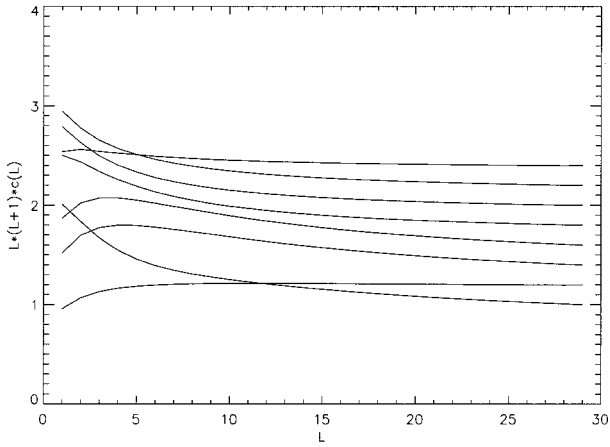
FIG. 2. *CMB multipole moments.* The CMB multipole moments are plotted for models (1), (2a)–(2c), (3a)–(3c), and (4) with successive curves displaced upward by $0.2.l(l+1)c_l$ is plotted and the curves are normalized to unity at $l=30$.

relative differences in shape are significant. A more detailed study of these models using a Boltzman code will be presented in a forthcoming paper.

V. DISCUSSION

In this paper we have developed a continuum formalism for describing the dynamics of a ''solid dark matter'' (SDM) component and shown how cosmological perturbations evolve with such a component included. The advantages of positing an SDM component are as follows. (1). It is possible to reconcile a spatially flat cosmology with the numerous measurements of Ω indicating a low value because most methods of measuring Ω are sensitive only to matter that is clustered (e.g., on scales comparable to the size of galaxy clusters or smaller) and the SDM remains unclustered, except on the very largest scales comparable in size to our present horizon. (2) SDM can provide the negative pressure suggested by the recent SNIa observations at high red-shift, thus explaining the lower than expected apparent luminosities of the distant supernovae. Supernova observations can potentially constrain the equation of state, thus distinguishing a SDM-dominated universe from a cosmological constant dominated universe. (3) With SDM it is not necessary to posit a new, surprisingly small energy scale. SDM from string or domain wall networks results from new physics at higher energy scales. SDM thus avoids the fine tuning difficulties of a straightout Λ term or of the ''quintessence'' models with an extremely slowly evolving scalar field that gives the same qualitative effect as Λ.

Introducing an equation of state with negative pressure is a delicate matter. If one wishes to consider perturbations to homogeneity and isotropy, considerations of general covariance and causality prohibit one from introducing a smooth background component that does not cluster in an *ad hoc* way. Since locally it is impossible to determine what the ''unperturbed'' background solution in the absence of perturbations would have been, a ''smooth component'' must be introduced in a manner that specifies how perturbations

evolve, and to do this more than merely specifying $w=(p/\rho)$ is required. For an equation of state with negative pressure, to posit a perfect fluid is not allowed, because the sound speed on short wavelengths would be imaginary, indicating instabilities whose growth rate diverges as the wavelength approaches zero. In SDM a sufficiently large shear modulus removes this instability. In the slowly rolling field models the instability is lacking for an entirely different reason: there is an inertia associated with changing the stress-energy.

Physically, how the instability is avoided in SDM and in quintessence is manifested by the following qualitative differences. (1) SDM, unlike quintessence and most types of dark matter, generates anisotropic stresses. (2) SDM has vector modes with nonvanishing sound speed. (3) The resistance to pure shear in SDM gives the graviton a mass, changing the gravity wave contribution to the CMB on large angular scales.

We finally offer the following more technical remarks comparing SDM to other possible sources of negative pressure discussed in the literature.

(1) A pure cosmological constant may be regarded as a degenerate case of the action (2.3) with $\rho(g_{(1)},g_{(2)},g_{(3)})$ set to $(\Lambda/8\pi)\sqrt{g_{ab}}$. This degenerate case greatly enlarges the reparametrization invariance of Eq. (2.3). Because $p=-\rho$ exactly, the smooth dark matter stress-energy no longer singles out a special time direction, and consequently for this special case general reparametrizations that mix **y** and t are allowed.

(2) If $\mu=0$, Eq. (2.3) becomes a Lagrangian description of a perfect fluid. In this special situation the Lagrangian description is much more cumbersome than the Eulerian description, especially with general relativity taken into account. When $\mu \neq 0$, however, an Eulerian description is no longer possible. If $\mu=0$ and $p=w\rho$ where $w<0$ and $w \neq -1$, the sound speed becomes imaginary, indicating an instability, most severe on the smallest scales. Without resistance against pure shear, the solid dark matter component would be similarly unstable. However, for the non-Abelian strings and domain wall networks μ is sufficiently large to stabilize the medium, as evidenced by all sound speeds being real.

(3) Quintessence differs in that the medium has an internal scalar degree of freedom. One could in fact write down a more general action for a low-energy description that combines quintessence with the continuous medium:

$$S = -\int dt \int d^3 y \sqrt{h} \rho(g_{(1)},g_{(2)},g_{(3)},\phi)$$

$$\times \sqrt{-G_{\mu\nu}\frac{\partial X^\mu}{\partial t}\frac{\partial X^\nu}{\partial t}} + \int d^4 x \sqrt{|G_{\mu\nu}|}\frac{1}{2}[(\mathcal{D}_{\hat{t}}\phi)^2$$

$$- c_s^2(\phi,g_{(1)},g_{(2)},g_{(3)})(\mathcal{D}_\mathbf{y}\phi)^2] - V[\phi]. \qquad (5.1)$$

Because the stress-energy of the medium breaks Lorentz in-

variance down to the rotation group, c_s need not equal the speed of light.

(4) Alexander Vilenkin [27] has pointed out that for the special case $w = -1/3$ (i.e., frustrated strings), due to a cancellation in Newtonian gauge the strings do not experience a gravitational force from nonrelativistic matter. In Eq. (2.21) this can be seen as a cancellation between the gradients of h_{00} and h_{ii} that occurs only for this special case under the assumption that the source of these potentials has no anisotropic stresses.

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