Direct signature of an evolving gravitational potential from the cosmic microwave background

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We show that a time dependent gravitational potential can be directly detected from cosmic microwave background (CMB) anisotropies. The signature can be measured by cross-correlating the CMB with the projected density field reconstructed from weak lensing distortions of the CMB itself. The cross-correlation gives a signal whenever there is a time dependent gravitational potential. This method traces dark matter directly and has a well defined redshift distribution of mass producing the lensing, thereby avoiding the problems plaguing other proposed cross-correlations. We show that both the Microwave Anisotropy Probe (MAP) and Planck satellites will be able to probe this effect for observationally relevant curvature and cosmological constant models, which will provide additional constraints on the cosmological parameters. [S0556-2821(99)00216-7]

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It is widely accepted that cosmic microwave background (CMB) anisotropies offer a unique enviroment to study cosmological models. The anisotropies were generated predominantly during recombination at redshift $z \sim 1100$, when the universe was still in a linear regime and the physics, at the eV energy scale, was simple. This allows one to make robust predictions for various cosmological models, which can be compared to an increasing number of observations. However, some degeneracies between cosmological parameters remain even for future satellite missions, and these are being further expanded as new parameters are being introduced [1]. The degeneracies are particularly severe between various components affecting the expansion of the universe, such as curvature, cosmological constant or any other term with a more general equation of state [2]. Other cosmological tests must therefore be used to break these degeneracies.

It has been pointed out that in a universe where the matter density does not equal the critical density the gravitational potential changes with time, which produces a significant component of the CMB on large scales [3]. This effect is generated at late times, and since the gravitational potential is related to the density field through Poisson's equation, the effect can also be looked for by cross-correlating the CMB with another tracer of the density field [4]. Unfortunately, no clean density map out to high redshift exists on large scales. The x-ray background has been suggested as a possible tracer of large scale structure out to $z \sim 3-4$, but the uncertainties associated with the redshift distribution of the sources, the relation between x-ray light and underlying mass, and the Poisson fluctuations from the nearby sources, make this test inconclusive [5,6].

Recently we developed a method to reconstruct the projected density field out to recombination directly from the CMB anisotropies [7]. The method is based on the gravitational lensing effect, which distorts the pattern of the CMB anisotropies [8]. Although the signal to noise for individual structures from such a reconstruction is small, averaging over independent patches of CMB reduces the noise and extracts the signal in a statistical sense. We were able to show that this allows one to extract the power spectrum of density perturbations with high accuracy over two orders of magnitude in scale [9].

In the present paper we use the reconstructed projected density field to cross-correlate with the CMB itself. If there is a component of the CMB from the time evolving gravitational potential, then it should correlate with the projected density field. Most of the signature comes from large angular scales, so we first generalize the method developed in [7] from the small scale limit to all sky. Because the small scale CMB anisotropies were generated uniquely during recombination, the weighting of density perturbations as a function of redshift in the projection is well defined. Moreover, the gravitational lensing effect depends on the dark matter distribution in the universe, so no assumption of how light traces mass is necessary. This avoids the shortcomings of cross-correlations with x-ray and other tracers of large scale structure mentioned in [4]. In addition, the projected density field is sensitive to the matter distribution out to a very high redshift and allows one to test the models where the time dependent potential is generating anisotropies at higher redshifts, such as in the curvature dominated models [6,10]. Here we compute the expected signal to noise of future CMB missions for cosmological constant and curvature dominated models, using both Microwave Anisotropy Probe (MAP) and Planck satellite characteristics. Although we limit ourselves to these two families of models, we note that other models, such as those with more general equation of state, would also produce a signature that one could look for.

To reconstruct the projected density field we consider the symmetric tensor of products of temperature derivatives transverse to direction $\hat{n}$ [7],

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$$\mathcal{H}_{ab}=\frac{1}{\sigma_{\mathcal{S}}}T_{:a}T_{:b}, \tag{1}$$

where $T_{:a}$, $T_{:b}$ are covariant derivatives of T with respect to the coordinate basis in the tangent space of direction $\hat{n}$, here defined with polar coordinates (θ,ϕ), and g_{ab} is the metric on the sphere. We define $\sigma_{\mathcal{S}}$ so that in the absence of lensing the average over the CMB $\langle\mathcal{H}_{ab}\rangle_{CMB}=\frac{1}{2}g_{ab}$. The tensor can be decomposed into the trace and traceless component as $\mathcal{H}_{ab}=\frac{1}{2}(1-\mathcal{S})g_{ab}-\widetilde{\mathcal{H}}_{ab}$. From the traceless tensor $\widetilde{\mathcal{H}}_{ab}$ one may define two rotationally invariant quantities

$$\mathcal{E}=\frac{1}{2}\nabla^{-2}\widetilde{\mathcal{H}}_{ab}^{:a:b}, \quad \mathcal{B}=\frac{1}{2}\nabla^{-2}\widetilde{\mathcal{H}}_{ab}^{:a:c}\epsilon_c^a, \tag{2}$$

where ϵ_c^a is the completely antisymmetric (Levi-Civita) tensor and ∇^{-2} is the inverse Laplacian on the sphere.

In the presence of lensing the average of $\mathcal{H}_{ab}$ becomes [7]

$$\langle\mathcal{H}_{ab}\rangle_{CMB}=\left(\frac{1}{2}-\kappa\right)g_{ab}-\gamma_{ab}, \tag{3}$$

where κ and γ_{ab} are the convergence and shear components of the symmetric shear tensor Φ_{ab}, defined as the covariant derivative of the displacement field on the sphere [12], which encodes the information on the gravitational lensing effect. All rotationally invariant quantities can be decomposed on a sphere into spherical harmonics, $X(\hat{n})=\Sigma_{lm}a_{X,lm}Y_{lm}(\hat{n})$, where X stands for T, κ, $\mathcal{S}$, $\mathcal{E}$ or $\mathcal{B}$. From Eq. (3) we find $\langle a_{\mathcal{S},lm}\rangle=M_{\mathcal{S}l}a_{\kappa,lm}$, with $M_{\mathcal{S}l}=2$. The multipole moments of the scalar field $\mathcal{E}$ average to $\langle a_{\mathcal{E},lm}\rangle=M_{\mathcal{E}l}a_{\kappa,lm}$, where $M_{\mathcal{E}l}=2(l+2)(l-1)/l(l+1)$ [12], while the average of the pseudoscalar field $\mathcal{B}$ identically vanishes in the large scale limit, $\langle a_{\mathcal{B},lm}\rangle=0$, because the gravitational potential from which shear is generated is invariant under the parity transformation. Convergence κ can thus be reconstructed in two independent ways from $\mathcal{S}$ and $\mathcal{E}$, while the third quantity $\mathcal{B}$ serves as a check for possible systematics. Note that since convergence is expressed as a quadratic quantity in T, its cross-correlation with T gives a nonvanishing third moment. This means it can also be looked for using the bispectrum, which is a method independently proposed by [11].

Convergence can be written as a projection of the gravitational potential [14] $\kappa=\int_0^{\chi_0}g(\chi,\chi_0)\nabla^2\phi(\chi)d\chi$. Here χ_0 is the comoving radial coordinate at recombination and g is the radial window, defined as $g(\chi,\chi_0)=r(\chi)r(\chi_0-\chi)/r(\chi_0)$, which is a bell shaped curve symmetric around $\chi/2$ and vanishing at 0 and χ_0. Here $r(\chi)$ is the comoving angular diameter distance, defined as $K^{-1/2}\sin K^{1/2}\chi$, χ, $(-K)^{-1/2}\sinh(-K)^{1/2}\chi$ for $K>0$, $K=0$, $K<0$, respectively, where K is the curvature. Curvature can be expressed using the present density parameter Ω_0 and the present Hubble parameter H_0 as $K=(\Omega_0-1)H_0^2$. In general Ω_0 consists of both matter contribution Ω_m and cosmological constant term Ω_λ.

The angular power spectra are defined as $C_l^{XX'}=1/(2l+1)\Sigma_m a_{X,lm}^* a_{X',lm}$. Their ensemble averages are given by [13]

$$C_l^{XX'}=(4\pi)^2\int\beta^2 d\beta P(\beta)\Delta_{Xl}(\beta,\tau_0)\Delta_{X'l}(\beta,\tau_0)$$
$$\Delta_{Xl}(\beta,\tau_0)=\int_0^{\tau_0}d\tau\Phi_\beta^l(\tau_0-\tau)S_X(\beta,\tau), \tag{4}$$

where $\Phi_\beta^l(x)$ are the ultraspherical Bessel functions and $P(\beta)$ is the primordial power spectrum. Equation (4) only applies to flat and open universes, whereas for the closed universe the eigenvalues of the Laplacian are discrete, so the integral over β is replaced with a sum over $K^{-1/2}\beta=3,4,5\ldots$. The source for temperature anisotropies S_T is a combination of several terms. These can be decomposed into effects generated during recombination, which consist of Sachs-Wolfe term, Doppler term, intrinsic anisotropy term and anisotropic stress term, and the late time effect generated by the time dependent gravitational potential [so-called integrated Sachs-Wolfe term (ISW)]. The latter term, while only important for low multipole moments, is the relevant one for our study. The full form of S_T can be found in [13]. The source for convergence is $S_\kappa=gk^2\phi$, where $k^2=\beta^2-K$. The key to our method is that the same large scale structures that generate ISW also contribute to κ. The two fields are thus correlated on large scales where ISW contributes. Note that most of the information on κ is coming from the small scale CMB, which is predominantly generated at $z\sim1100$. The fact that there may be a contribution to CMB from ISW at low z is limited to low l only and does not significantly affect the conclusions here, derived by ignoring this effect. It is also useful to define the correlation coefficient $\mathrm{Corr}_l^{T\kappa}=C_l^{T\kappa}/(C_l^{TT}C_l^{\kappa\kappa})^{1/2}$, which is the relevant quantity for the estimation of how strong the correlation is.

Using the above expressions one can compute C_l^{TT}, $C_l^{\kappa\kappa}$ and $C_l^{T\kappa}$ for any cosmological model. We performed numerical calculations using a modified version of the CMBFAST code [15]. We have verified that $C_l^{\kappa\kappa}$ agrees with previous calculations which were done in the small scale limit [14], as well as with the alternative all-sky expressions given in [12]. The results for $(\mathrm{Corr}_l^{T\kappa})^2$ are shown in Fig. 1 for a representative set of models. The correlation coefficient in a cosmological constant model is substantial only for very low l and rapidly drops on smaller scales. In a curvature dominated model with the same Ω_m the correlation coefficient is larger to start with and also drops less rapidly with l. This indicates that one will be able to set stronger limits on curvature than on a cosmological constant, which is confirmed below with a more detailed analysis. The reason is that in a cosmological constant model, gravitational potential changes only at late times ($z\gtrsim1$ for reasonable values of Ω_m), while in a curvature dominated model the potential changes also at higher redshift. This leads to two effects. First, in a cosmological constant model ISW is comparable to other terms only for the lowest multipoles, while in a curvature model ISW dominates up to higher l [10]. Secondly, the window g peaks at relatively high redshift $z\sim3$ and so is able to pick up correlations with ISW from an open universe better than that from a cosmological constant universe. We have also calculated the correlation coefficient for a flat model. It is small for all

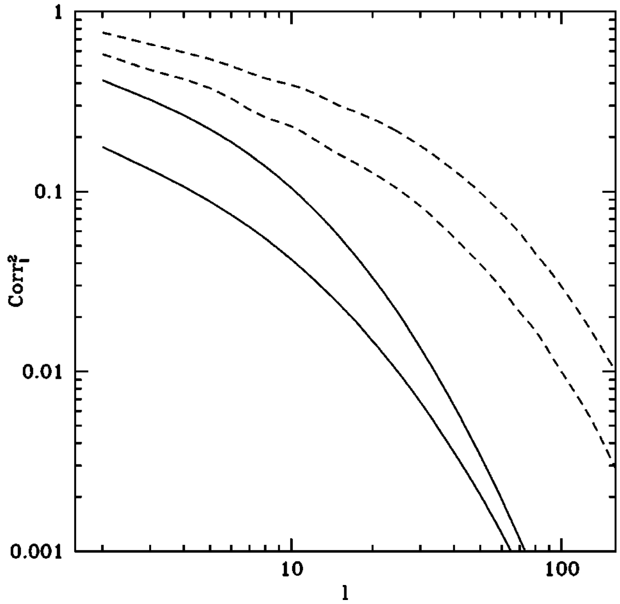

FIG. 1. Square of correlation coefficient $(\mathrm{Corr}_l^{T\kappa})^2$ as a function of l is shown for open $\Omega_m = 0.2$ and $\Omega_m = 0.4$ models (upper and lower dashed curves) and for flat cosmological constant $\Omega_m = 0.2$ and $\Omega_m = 0.4$ models (upper and lower solid curves).

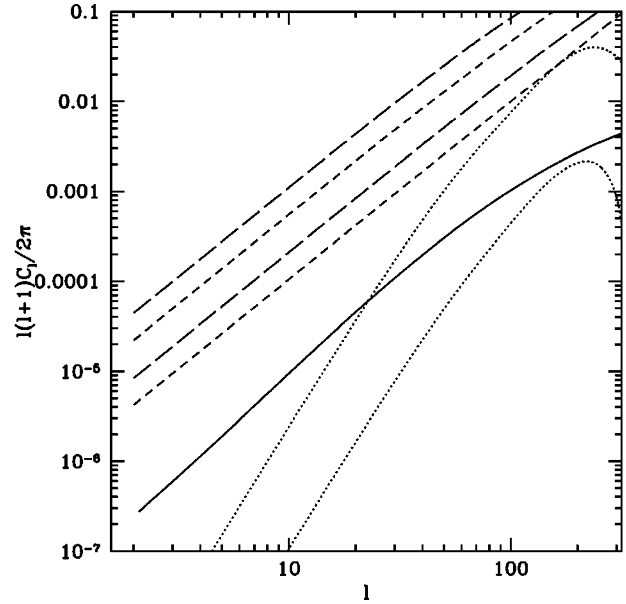

FIG. 2. Power spectra for noise $N_l^{\mathcal{SS}}$ (long dashed), $N_l^{\mathcal{EE}}$ (short dashed), $N_l^{\mathcal{ES}}$ (dotted), both for Planck (lower curves) and MAP (upper curves) for a cosmological constant model with $\Omega_m = 0.3$. Also shown is the power spectrum of convergence $4C_l^{\kappa\kappa}$ (solid) for the same model, normalized to $\sigma_8 = 1$.

l, $(\mathrm{Corr}_l^{T\kappa})^2 < 10^{-3}$, demonstrating that correlations with fluctuations generated at recombination are negligible and the cross-correlation is sensitive to the time dependent gravitational potential only.

We now address the question of signal detectability with future CMB missions. We continue to work in multipole moment space and assume we have an all-sky expansion, which allows us to decouple different m and l multipole moments. Given two random fields T and $\mathcal{W}$ (where $\mathcal{W}$ stands for $\mathcal{S}$ or $\mathcal{E}$), we want to develop a test that maximizes the signal in the presence of correlations against the null hypothesis that there are no correlations. The term that quantifies the correlations is the product between the two fields $X_{lm}^{\mathcal{W}} = T_{lm}^* \mathcal{W}_{lm}$ [here and below, average with the complex conjugate $(T_{lm}^* \mathcal{W}_{lm} + T_{lm} \mathcal{W}_{lm}^*)/2$ is implied]. Its expectation value under the null hypothesis of pure noise is $\langle X_{lm}^{\mathcal{W}} \rangle_0 = 0$, because the function entering this expression is a three-point function of T, which vanishes both for intrinsic fluctuations and for detector noise under the Gaussian assumption. The alternative hypothesis is that of pure signal which gives $\langle X_{lm}^{\mathcal{W}} \rangle_1 = \langle T_{lm}^* \mathcal{W}_{lm} \rangle = M_{\mathcal{W}l} C_l^{T\kappa}$. The variance under the null hypothesis is

$$\langle X_{lm}^{\mathcal{W}} X_{lm}^{\mathcal{W}'} \rangle_0 - \langle X_{lm}^{\mathcal{W}} \rangle_0 \langle X_{lm}^{\mathcal{W}'} \rangle_0 = (C_l^{TT} + N_l^{TT})(M_{\mathcal{W}l} M_{\mathcal{W}'l} C_l^{\kappa\kappa} + N_l^{\mathcal{W}\mathcal{W}'}), \quad (5)$$

where N_l^{TT} and $N_l^{\mathcal{W}\mathcal{W}'}$ are the noise power spectra for CMB anisotropies, $\mathcal{S}$, $\mathcal{E}$ or their cross-term, respectively.

Both $\mathcal{S}$ and $\mathcal{E}$ contribute information. If they are uncorrelated then the information contents can be added independently, otherwise the covariance matrix $\mathrm{Cov}(X_{lm}^{\mathcal{W}} X_{lm}^{\mathcal{W}'})$ has to be diagonalized first. The CMB term $C_l^{TT} + N_l^{TT}$ is the same for all matrix elements and can be computed using MAP and Planck noise characteristics. For these CMB missions detector noise on large scales will be negligible, hence $N_l^{TT} \ll C_l^{TT}$. The dominant sources of noise in $\mathcal{S}$ or $\mathcal{E}$ on large scales are the CMB anisotropies. The noise terms involve integrals over the CMB power spectrum and can be computed numerically using the expressions given in [7]. The results are shown in Fig. 2 both for MAP and Planck. They show that on large scales the CMB noise power spectrum has approximately white noise shape, $N_l^{\mathcal{W}\mathcal{W}} \sim \mathrm{const}$. At low l $N_l^{\mathcal{EE}} \sim N_l^{\mathcal{SS}}/2 \gg N_l^{\mathcal{ES}}, C_l^{\kappa\kappa}$. Therefore, noise dominates over the signal and the latter can only be extracted in a statistical sense by averaging over multipole moments. Because the off-diagonal term $\langle \mathcal{E}_{lm}^* \mathcal{S}_{lm} \rangle_0$ is much smaller than the two diagonal terms, the covariance matrix is nearly diagonal and the information from $\mathcal{S}$ and $\mathcal{E}$ can be added independently, with $\mathcal{E}$ contributing twice as much of information as $\mathcal{S}$. Note also that Planck has a factor of 5 better sensitivity than MAP.

We now want to combine the signal to noise from different multipole moments to maximize the overall signal to noise. To do this we add up the products weighted with some yet to be determined weights α_l, $X = \Sigma_{m,l} \alpha_l X_{lm}$. Since the moments are uncorrelated the expectation value and variance are

$$\langle X \rangle_1 = \sum_l (2l+1) \alpha_l M_{\mathcal{W},l} C_l^{T\kappa},$$
$$\langle X^2 \rangle_0 = \sum_l (2l+1) \alpha_l^2 (C_l^{TT} + N_l^{TT}) \times (M_{\mathcal{W}l}^2 C_l^{\kappa\kappa} + N_l^{\mathcal{W}\mathcal{W}}), \quad (6)$$

while the null hypothesis mean remains $\langle X\rangle_0=0$. We want to maximize $S/N=(\langle X\rangle_1-\langle X\rangle_0)/(\langle X^2\rangle_0)^{1/2}$ with respect to α_l. Taking derivatives with respect to α_l and setting the expression to 0 we find $\alpha_l=C_l^{T\kappa}/(C_l^{TT}+N_l^{TT})(M_{\mathcal{W}l}^2C_l^{\kappa\kappa}+N_l^{\mathcal{W}\mathcal{W}})$. The overall signal to noise is, combining the information from $\mathcal{S}$ and $\mathcal{E}$,

$$\frac{S}{N}=\left[f_{sky}\sum_l\sum_{\mathcal{W}=\mathcal{E},\mathcal{S}}\frac{(2l+1)(\mathrm{Corr}_l^{T\kappa})^2}{\left(1+\frac{N_l^{TT}}{C_l^{TT}}\right)\left(1+\frac{N_l^{\mathcal{W}\mathcal{W}}}{M_{\mathcal{W}l}^2C_l^{\kappa\kappa}}\right)}\right]^{1/2}, \tag{7}$$

where we have inserted f_{sky} to account for the fact that the effective number of multipoles will be smaller if only some fraction of the sky is measured. If the correlation is unity and noise is negligible, then each multipole moment contributes one degree of freedom and the signal to noise is $N_{\rm dof}^{1/2}$, where $N_{\rm dof}$ is the number of degrees of freedom. Decorrelation and/or noise decrease the effective number of degrees of freedom.

Using the above expressions we find $S/N=8$ for an $\Omega_m=0.4$ open model and $S/N=13$ for an $\Omega_m=0.2$ open model, both for Planck noise and beam properties using $f_{sky}=0.7$. This is a very strong signal indeed, corresponding to 8σ and 13σ, respectively. Corresponding numbers for MAP are 3.5 and 7. Both MAP and Planck will thus be able to usefully constrain open models with $\Omega_m<0.4$, which spans the range of currently favored values of Ω_m. If S/N is high as in the case of Planck satellite then one may combine the data to constrain more than one parameter. For example, instead of compressing the information into a single number with, e.g., $S/N=8$ one can estimate four independent bands of cross-correlation power spectrum with $S/N=4$ each, each of which still gives a strong detection. For cosmological constant models the numbers are somewhat lower, Planck giving $S/N=3$ and 6 for $\Omega_m=0.4$ and $\Omega_m=0.2$, respectively, while the corresponding MAP numbers are 1 and 2. A positive detection in these models can therefore only be obtained with Planck, unless Ω_m is very low. One can use the absence or presence of the cross-correlation to place constraints on the models. Any detection of the signal with MAP will, for example, be more easily explained in terms of curvature models than with cosmological constant models, while absence of the signal in Planck will certainly rule out all curvature models of interest, as well as putting strong constraints on cosmological constant models. Within the context of more specific models, such as the family of cold dark matter (CDM) models, one can use the cross-correlation to break the degeneracies present when only the CMB power spectrum constraints are used. The well-known degeneracy between curvature and cosmological constant can for example be broken using this cross-correlation. Note that the theoretical limit for signal to noise can be obtained by assuming κ is perfectly known, and is given by $S/N=\Sigma_l(2l+1)(\mathrm{Corr}_l^{T\kappa})^2$. This gives S/N about a factor of 2 higher than our results for Planck above.

Finally, the signal should be consistent with zero for the $\mathcal{B}$ field, in the large scale limit. Any evidence against that would be a sign of a systematic effect present in the data. This test provides a useful overall check of the method. Another useful test would be cross-correlating $\mathcal{E}$ and $\mathcal{S}$ with the polarization CMB map. Since ISW does not contribute to the latter the result should again be consistent with zero and any detected signal would likely be a sign of a systematic effect. The straightforward interpretation and many consistency checks make the method proposed here one of the more promising ways to determine cosmological parameters, and should provide further incentive for high sensitivity all-sky CMB experiments.

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