On the realization of assisted inflation

Panagiota Kanti* and Keith A. Olive†

Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

We consider conditions necessary for a successful implementation of the so-called assisted inflation. We generalize the applicability of assisted inflation beyond exponential potentials as originally proposed to include standard chaotic ($\lambda \phi^4$ or $m^2 \phi^2$) models as well. We also demonstrate that in a purely four-dimensional theory, unless the assisted sector is in fact decoupled, the additional fields of the assisted sector actually impede inflation. As a specific example of an assisted sector, we consider a five-dimensional Kaluza-Klein (KK) model for which the extra dimension may be somewhat or much larger than the inverse Planck scale. In this case, the assisted sector (coming from a KK compactification) eliminates the need for a fine-tuned quartic coupling to drive chaotic inflation. This is a general result of models with one or more ''large'' extra dimensions. [S0556-2821(99)05014-6]

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I. INTRODUCTION

One of the long standing problems in inflationary model building is the apparent necessity of some fine-tuned couplings or masses (see Ref. [1] for reviews). Barring an alternative to standard inflation, either a model predicting the presence of small couplings, or a more innovative model which does not require them must be constructed. Developments such as the pre-big-bang model [2] go a long way towards this goal, but issues such as the graceful exit still require resolution [3].

The simplest inflationary scenarios are by far the chaotic inflation models [4] involving a single scalar field. For example, a potential of the form $V(\phi) = \lambda \phi^4$ will produce sufficient inflation if the initial background field value is $\phi >$ few M_P. However, in order to obtain the correct magnitude for density fluctuations, one must require that the vacuum energy density during the last ~ 50 e-foldings of inflation be of order $V \sim (10^{16} \text{ GeV})^4$ or $\lambda \sim 10^{-12}$. Similarly chaotic models based on potentials of the form $V(\phi) = m^2 \phi^2$ require $m \sim 10^{-5} M_P$ in order to satisfy the Cosmic Background Explorer (COBE) constraint.

It is well known that power-law expansion [5] rather than exponential expansion may be sufficient to resolve the standard cosmological problems associated with inflation and that such solutions can be generated by exponential potentials [5,6]. For example, a potential of the form $V(\phi) = e^{-\lambda \phi}$, leads to power-law expansion with the cosmological scale factor growing as $R(t) \sim t^p$ with $p = 2/\lambda^2$. Furthermore, density fluctuations are no longer scale invariant but scale as $|(\delta \rho / \rho)(k)|^2 \sim k^{n-1}$ with $n = 1 - 2/(p-1)$. To obtain, $n \simeq 1$, one requires p to be large.

Recently, it was noticed [7] that a system of several scalar fields each with a potential

$$V_i = V_0 \exp\left(-\sqrt{\frac{2}{p_i}} \phi_i\right) \qquad (1)$$

could drive power-law inflation with a net power $\tilde{p} = \Sigma p_i$ sufficient to solve the cosmological problems, even if each of the fields ϕ_i alone are not capable of doing so. Furthermore, the spectral index of density fluctuations is also brought closer to the scale invariant spectrum if p is replaced by $\tilde{p}$. The dynamics of this type of ''assisted'' model was discussed in Ref. [8].

Here, we will show that the assisted paradigm can easily be extended to other types of inflationary models such as the chaotic models mentioned above. We will also show that the ansatz (1) of effectively decoupled scalar fields is absolutely necessary for assistance to work. For example, the case of N scalar fields each with a potential defined by $p_i = p$, would lead to $\tilde{p} = Np$ for self-coupled fields, while it would lead to $\tilde{p} = p/N$ for a system of fields which were cross coupled. Such a situation would undermine the benefits of assisted inflation.

Although the identity of these multiple fields was not specified in Ref. [7], one possible source for the necessary multiplicity is a theory with an extra compact dimension. The Kaluza-Klein reduction of a scalar field in five dimensions, will result in a spectrum of states with masses $\propto n^2/L^2$ where L is the size of the compact extra dimension. If $L \gg M_P^{-1}$, there may be many nearly massless ''copies'' of the original scalar field which may serve to assist inflation. We find that, although the resulting system of scalar fields produced from the Kaluza-Klein (KK) reduction may be heavily cross coupled, it can eliminate the usual fine-tuning associated with chaotic inflation driven by a quartic coupling and achieves the goals of assisted inflation.

II. ASSISTED INFLATION AND DECOUPLED FIELDS

Assisted inflation as described in Refs. [7,8] relies on the premise that there exist a set of N scalar fields each with potential given by Eq. (1). The Lagrangian for the system is given by

$$-\mathcal{L} = \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + V_i \right\}. \qquad (2)$$

*Email address: yiota@physics.umn.edu
†Email address: olive@mnhep.hep.umn.edu

Each field ϕ_i satisfies its equation of motion

$$\ddot{\phi}_i + 3H\dot{\phi}_i = -\frac{dV_i}{d\phi_i}, \tag{3}$$

where the Hubble parameter is given by

$$3H^2 = \sum_{i=1}^{N} \left\{ \frac{1}{2}\dot{\phi}_i^2 + V_i \right\}. \tag{4}$$

(We are working in units such that $8\pi/M_P^2 = 1$.) In Refs. [7,8] it was shown that this system has a late-time attractor solution described by a single rescaled scalar field $\tilde{\phi}^2 = (\tilde{p}/p_1)\phi_1^2$ with potential $\tilde{V} = (\tilde{p}/p_1)V_1$. The resulting power-law expansion of the Universe is simply $R(t) \sim t^{\tilde{p}}$ (provided that each of the $p_i > 1/3$).

As we will now demonstrate, the basic idea behind assisted inflation can be applied more generally than the case of exponential potentials. We can consider a general field theory of multiple, self-interacting scalar fields of the form

$$-\mathcal{L} = \sum_{i=1}^{N} \left\{ \frac{1}{2}(\partial\phi_i)^2 + \frac{m^2}{2}\phi_i^2 \right\} + \sum_{i=1}^{N} \left\{ \frac{\lambda_3}{3!}\phi_i^3 + \frac{\lambda_4}{4!}\phi_i^4 \right\}. \tag{5}$$

The equation of motion for each field ϕ_i derived from the variation of the above Lagrangian has the form

$$\nabla^2\phi_i = m^2\phi_i + \frac{\lambda_3}{2}\phi_i^2 + \frac{\lambda_4}{6}\phi_i^3. \tag{6}$$

From the above equation, it is obvious that the system consists of N completely decoupled scalar fields or equivalently of N copies of the same field. As a result, the Lagrangian can be written as

$$-\mathcal{L} = N\left\{ \frac{1}{2}(\partial\phi_1)^2 + \frac{m^2}{2}\phi_1^2 + \frac{\lambda_3}{3!}\phi_1^3 + \frac{\lambda_4}{4!}\phi_1^4 \right\}$$
$$= \frac{1}{2}(\partial\tilde{\phi})^2 + \frac{m^2}{2}\tilde{\phi}^2 + \frac{\tilde{\lambda}_3}{3!}\tilde{\phi}^3 + \frac{\tilde{\lambda}_4}{4!}\tilde{\phi}^4, \tag{7}$$

where

$$\tilde{\phi} = \sqrt{N}\,\phi_1, \quad \tilde{\lambda}_3 = \frac{\lambda_3}{\sqrt{N}}, \quad \tilde{\lambda}_4 = \frac{\lambda_4}{N}. \tag{8}$$

Notice that the above field redefinition (made to rewrite the Lagrangian in terms of a field with a canonical kinetic term) results in a scalar field with an unchanged mass. The resulting theory describes a single scalar field with the same type of self-interactions compared to the fields in the original theory. However, these self-interactions are considerably weaker since both of the coupling constants now scale with the number of scalar fields N. As a result, as the number of scalar fields that we include in the theory becomes larger, the effective coupling constants naturally become smaller and the corresponding fine-tuning becomes milder. Thus the

same basic idea expounded in Refs. [7,8] carries over very simply to chaotic inflation based on a quartic potential. While $\tilde{\lambda}_4$ must still be of order 10^{-12}, the fundamental coupling in the theory λ_4 can now be much larger if N is large. Note, however, that the additional scalar fields do not affect the quadratic version of chaotic inflation whatsoever.

III. GENERAL THEORIES WITH CROSS COUPLINGS

The success of the assisted paradigm demonstrated in the section above, is directly related to the absence of cross-coupling terms between different scalar fields. As soon as the multiple self-interacting scalar fields are substituted with cross-coupled fields, the assistance method ceases to work. To see that this is the case, it is reasonable to consider general field theories of multiple scalar fields of the form

$$-\mathcal{L} = \sum_{i=1}^{N} \left\{ \frac{1}{2}(\partial\phi_i)^2 + \frac{m^2}{2}\phi_i^2 \right\} + V_I, \tag{9}$$

where the potential may contain not only self-interaction terms, like the theory in Sec. II, but also cross-coupling terms between different fields. Specifically, we study the following three cases.

(A) We start by considering the following simple theory of coupled scalar fields with cubic and quartic interaction terms:

$$V_I = \frac{\lambda_3}{3!}\left(\sum_{i=1}^{N}\phi_i\right)^3 + \frac{\lambda_4}{4!}\left(\sum_{i=1}^{N}\phi_i\right)^4. \tag{10}$$

In this form, the invariance of the theory under the change $\phi_i \leftrightarrow \phi_j$ is obvious which leads to identical equations of motion for each of the different scalar fields

$$\nabla^2\phi_i = m^2\phi_i + \frac{\lambda_3}{2}\left(\sum_{k=1}^{N}\phi_k\right)^2 + \frac{\lambda_4}{6}\left(\sum_{k=1}^{N}\phi_k\right)^3. \tag{11}$$

By subtracting the equations of motion of two arbitrary fields ϕ_i and ϕ_j, we can easily see that the solution $\phi_i = \phi_j$ is the unique late-time attractor of the system. As a result, the Lagrangian can be written as

$$-\mathcal{L} = N\left[\frac{1}{2}(\partial\phi_1)^2 + \frac{m^2}{2}\phi_1^2\right] + \frac{\lambda_3}{3!}(N\phi_1)^3 + \frac{\lambda_4}{4!}(N\phi_1)^4$$
$$= \frac{1}{2}(\partial\tilde{\phi})^2 + \frac{m^2}{2}\tilde{\phi}^2 + \frac{\tilde{\lambda}_3}{3!}\tilde{\phi}^3 + \frac{\tilde{\lambda}_4}{4!}\tilde{\phi}^4, \tag{12}$$

where

$$\tilde{\phi} = \sqrt{N}\,\phi_1, \quad \tilde{\lambda}_3 = \lambda_3 N^{3/2}, \quad \tilde{\lambda}_4 = \lambda_4 N^2. \tag{13}$$

We notice that, when we allow cross-coupling terms between different fields to be present in the theory, we obtain a result for the effective potential which is radically different from the one we found in the case of self-interacting fields in Sec. II. The presence of these cross-coupling terms drives the effective potential, or the coupling constants, in the opposite

direction from that desired: the renormalized, single scalar field $\tilde{\phi}$ is more strongly coupled than the original scalar fields ϕ_i with the coupling parameters $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ increasing with the number of scalar fields that we include in the theory. As a result, the necessary fine-tuning of the coupling constants becomes now much more severe.

(B) A slightly different version of the above coupled scalar field theory can be formulated in the following way. Consider

$$-\mathcal{L} = \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + \frac{m^2}{2} \phi_i^2 \right\} + \sum_{i=1}^{N} \left\{ \frac{\lambda_3}{3!} \phi_i^3 + \frac{\lambda_4}{4!} \phi_i^4 \right\}$$
$$+ \sum_{i,j,k=1}^{N} \frac{\lambda_3 c_3}{3!} \phi_i \phi_j \phi_k + \sum_{i,j,k,l=1}^{N} \frac{\lambda_4 c_4}{4!} \phi_i \phi_j \phi_k \phi_l. \tag{14}$$

In the last two terms, the indices (i,j,k) and (i,j,k,l) are not allowed to all take on the same value and, as a result, these terms describe only cross couplings between different fields. The above formulation, i.e., the introduction of the parameters c_3 and c_4 in the cubic and quartic interaction terms, respectively, allows us to turn off the cross couplings between the scalar fields while keeping the self-interactions in the theory.

We can rewrite the above Lagrangian in the following way:

$$-\mathcal{L} = \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + \frac{m^2}{2} \phi_i^2 \right\} + \sum_{i=1}^{N} \left\{ \frac{\lambda_3}{3!} \phi_i^3 + \frac{\lambda_4}{4!} \phi_i^4 \right\}$$
$$+ \frac{\lambda_3 c_3}{3!} \left[\left(\sum_{i=1}^{N} \phi_i \right)^3 - \sum_{i=1}^{N} \phi_i^3 \right]$$
$$+ \frac{\lambda_4 c_4}{4!} \left[\left(\sum_{i=1}^{N} \phi_i \right)^4 - \sum_{i=1}^{N} \phi_i^4 \right]$$
$$= \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + \frac{m^2}{2} \phi_i^2 \right\}$$
$$+ \frac{\lambda_3}{3!} \left[(1-c_3) \sum_{i=1}^{N} \phi_i^3 + c_3 \left(\sum_{i=1}^{N} \phi_i \right)^3 \right]$$
$$+ \frac{\lambda_4}{4!} \left[(1-c_4) \sum_{i=1}^{N} \phi_i^4 + c_4 \left(\sum_{i=1}^{N} \phi_i \right)^4 \right] \tag{15}$$

and, then, the equation of motion for each field ϕ_j has the form

$$\nabla^2 \phi_j = m^2 \phi_j + \frac{\lambda_3}{2} \left[(1-c_3) \phi_j^2 + c_3 \left(\sum_{i=1}^{N} \phi_i \right)^2 \right]$$
$$+ \frac{\lambda_4}{6} \left[(1-c_4) \phi_j^3 + c_4 \left(\sum_{i=1}^{N} \phi_i \right)^3 \right]. \tag{16}$$

If we subtract the equations of motion of the fields ϕ_j and ϕ_k, we can easily conclude that, once again, the unique late-time attractor of the theory has all of the fields equal. By making use of this result, the Lagrangian may be written as

$$-\mathcal{L} = N \left[\frac{1}{2} (\partial \phi_1)^2 + \frac{m^2}{2} \phi_1^2 \right]$$
$$+ \frac{\lambda_3}{3!} [(1-c_3) N \phi_1^3 + c_3 (N \phi_1)^3]$$
$$+ \frac{\lambda_4}{4!} [(1-c_4) N \phi_1^4 + c_4 (N \phi_1)^4]$$
$$= \frac{1}{2} (\partial \tilde{\phi})^2 + \frac{m^2}{2} \tilde{\phi}^2$$
$$+ \frac{\tilde{\lambda}_3}{3!} \tilde{\phi}^3 [1 + c_3 (N^2 - 1)]$$
$$+ \frac{\tilde{\lambda}_4}{4!} \tilde{\phi}^4 [1 + c_4 (N^3 - 1)], \tag{17}$$

where

$$\tilde{\phi} = \sqrt{N} \phi_1, \quad \tilde{\lambda}_3 = \frac{\lambda_3}{\sqrt{N}}, \quad \tilde{\lambda}_4 = \frac{\lambda_4}{N}. \tag{18}$$

If we choose $c_3 = c_4 = 0$, then, we recover the theory of self-interacting scalar fields that was discussed in Sec. II and for which the assistance effect worked perfectly leading to an extremely weakly coupled scalar field theory. If, on the other hand, we choose $c_3 = c_4 = 1$, then, we go back to the case (A) studied above, where the potential increases rapidly with the number N of scalar fields. A third possibility arises when the parameters c_i adopt some intermediate values. For example, if, for large N, $c_3 \sim 1/N^2$ and $c_4 \sim 1/N^3$, the coefficients of the renormalized cubic and quartic terms that appear inside the brackets are of $\mathcal{O}(1)$ and the desired behavior (18) of the coupling parameters $\tilde{\lambda}_i$ is ensured. One could argue that the result of this analysis is to transfer the fine-tuning from the coupling constants to the parameters c_i. Indeed, it shows the degree to which the cross couplings must be fine-tuned for assistance to work.

(C) Finally, we consider a theory of N scalar fields coupled to each other through an exponential potential

$$-\mathcal{L} = \sum_{i=1}^{N} \frac{1}{2} (\partial \phi_i)^2 + V_0 \prod_{i=1}^{N} \exp \left(-\sqrt{\frac{2}{p}} \phi_i \right). \tag{19}$$

This is similar to the potential considered by Liddle *et al.* [7] with the sum of exponentials replaced by a product. In the case of the summation, the N scalar fields do not interact with each other and the unique late-time attractor has all fields equal. As discussed in the Introduction, this solution leads to a power law expansion which can solve the standard inflationary problems with a relatively flat spectrum of density fluctuations.

In our case, however, the scalar fields are coupled to each other. The equation of motion for the field ϕ_i takes the form

$$\nabla^2 \phi_i = -\sqrt{\frac{2}{p}} V_0 \exp\left(-\sqrt{\frac{2}{p}} \sum_{k=1}^{N} \phi_k\right). \qquad (20)$$

The right-hand-side of the above equation is the same for every field ϕ_i. As a result, the unique late-time attractor, which has all the fields equal, is still valid even in this case where the fields are coupled. Now, the Lagrangian can be written as

$$-\mathcal{L} = \frac{N}{2}(\partial\phi_1)^2 + V_0 \exp\left(-\sqrt{\frac{12}{p}} N\phi_1\right)$$

$$= \frac{1}{2}(\partial\tilde{\phi})^2 + V_0 \exp\left(-\sqrt{\frac{2}{\tilde{p}}}\tilde{\phi}\right), \qquad (21)$$

where now

$$\tilde{\phi} = \sqrt{N}\phi_1, \quad \tilde{p} = \frac{p}{N}. \qquad (22)$$

As a result, if p, in the original theory, was not large enough to support inflation, the situation is worsened since p is divided by the number of scalar fields that are present in the theory.

In each of the cases studied above, it is evident that the presence of interaction terms between the scalar fields of the theory undermines the benefits of assistance and impedes the successful implementation of inflation. While the cases we studied are certainly simplified, we expect the general result to hold, namely, in a theory with multiple scalar fields, assistance requires the absence (or near absence) of cross couplings between the scalar fields.

IV. FIELD THEORIES WITH MULTIPLE SCALAR FIELDS

Given the potential utility of having several or many fields which are in some sense copies of each other, we now look at a possible source for these fields in theories with extra spatial dimensions. It is well known that the Kaluza-Klein reduction of a theory leads to the existence of many new fields which appear as zero-modes in the final four-dimensional theory. For example, consider a simple five-dimensional gravitational action of the form

$$S_G = -\int d^5x \sqrt{G_5}\left\{\frac{M_5^3}{16\pi} R_5\right\}, \qquad (23)$$

where M_5 is the five-dimensional Planck mass. Upon compactification along dimension of circumference $2L$, we obtain

$$S_G = -\frac{1}{2}\int d^4x \sqrt{G_4} e^{\gamma}\left\{R_4 + e^{2\gamma}\frac{1}{4}F_{KK}^2\right\}, \qquad (24)$$

where $M_P^2 = 2LM_5^3$, γ is the scalar associated with the 5-5 component of the metric ($e^{2\gamma} = G_{55}$) and F_{KK} is the field strength of the Kaluza-Klein gauge field associated with

$G_{\mu 5}$. This action can be brought into the Einstein frame by the conformal transformation $G_{4\mu\nu} = e^{-\gamma}g_{\mu\nu}$ to give

$$S_G = -\frac{1}{2}\int d^4x \sqrt{g}\left\{R + \frac{3}{2}(\partial\gamma)^2 + e^{3\gamma}\frac{1}{4}F_{KK}^2\right\}. \qquad (25)$$

Let us further suppose that the original five-dimensional theory contains an additional massless scalar field $\hat{\phi}$ with action

$$S_\phi = -\int d^5x \sqrt{G_5}\{G_5^{AB}\partial_A\hat{\phi}\partial_B\hat{\phi}\}, \qquad (26)$$

where the indices $A,B = \{t,x_1,x_2,x_3,z\}$. We can Fourier expand $\hat{\phi}$ along z as

$$\hat{\phi}(x,z) = \hat{\phi}_0(x) + \hat{\phi}_z(x,z)$$

$$= \hat{\phi}_0(x) + \sum_{n=1}^{\infty}[\hat{\phi}_n(x)e^{i(n\pi/L)z} + \hat{\phi}_n^*(x)e^{-i(n\pi/L)z}], \qquad (27)$$

where $\hat{\phi}_0$ is the five-dimensional field that depends only on noncompact coordinates.

Upon reducing to four dimensions, and performing the same conformal transformation, the action (26) becomes

$$S_\phi = -\int d^4x \sqrt{g}\left\{\sum_{n=0}^{\infty}\left[\left|\left(\partial_\mu + i\frac{n\pi}{L}A_\mu\right)\phi_n\right|^2 \right.\right.$$

$$\left.\left. + \frac{n^2\pi^2}{L^2}e^{-3\gamma}|\phi_n|^2\right]\right\}, \qquad (28)$$

where we have defined the four-dimensional scalar field $\phi = \sqrt{2L}\hat{\phi}$. In what follows, we will assume that the dilaton-like field γ is fixed [9], and ignore the role of the KK gauge field A_μ. Although we have written the action in terms of an infinite sum, the momentum along z, p_z, should be limited by M_5. In that case, we should only consider fields up to $n = N \lesssim LM_5/\pi$. For $(\pi L^{-1} \ll M_5)$, there may be many fields which can in principle assist inflation. Such theories are, to say the least, of wide interest at the moment [10] (see also Ref. [11], and references therein).

Let us now consider the following five-dimensional scalar field, self-interacting through a quartic potential, as a concrete example:

$$-\mathcal{L}_{5D} = \frac{1}{2}\partial_A\hat{\phi}\partial^A\hat{\phi} + \frac{\hat{\lambda}}{4!M_5}\hat{\phi}^4. \qquad (29)$$

The kinetic term for the five-dimensional field $\hat{\phi}$ can be expanded as in Eq. (28). Similarly, the substitution of the expansion (27) in the potential gives rise to numerous interaction terms between the Kaluza-Klein scalar fields. Then, the four-dimensional Lagrangian can be written as

$$-\mathcal{L}_{4D} = \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 + \sum_{n=1}^{\infty}\left(\partial_\mu\phi_n\partial^\mu\phi_n^* + \frac{n^2\pi^2}{L^2}\phi_n\phi_n^*\right)$$

$$+ \frac{\lambda}{4!}\left[\phi_0^4 + 12\phi_0^2\sum_{n=1}^{\infty}\phi_n\phi_n^*\right.$$

$$+ 12\phi_0\sum_{n,k=1}^{\infty}(\phi_n\phi_k\phi_{n+k}^* + \phi_n^*\phi_k^*\phi_{n+k})$$

$$+ \sum_{n,k,l=1}^{\infty}(4\phi_n\phi_k\phi_l\phi_{n+k+l}^* + 4\phi_n^*\phi_k^*\phi_l^*\phi_{n+k+l}$$

$$\left.+ 6\phi_n\phi_k\phi_l^*\phi_{n+k-l(\geq1)}^*)\right], \qquad (30)$$

in terms of the four-dimensional field ϕ and the four-dimensional coupling $\lambda = \hat{\lambda}/(2LM_5)$. Note that in the last term in the last equation we include only the terms for which $n+k-l\geq1$. It is also important to note that the four-dimensional (dimensionless) coupling is now reduced relative to the original five-dimensional coupling $\hat{\lambda}$ by an amount $2LM_5\simeq N$.

It will be useful to begin the analysis of this system by first simplifying to a restricted set of fields. Thus, in the lowest order approximation, we may assume that, apart from the field ϕ_0, only ϕ_1 and ϕ_1^* are present in the theory and set all the other Kaluza-Klein fields equal to zero. By making use of the definitions

$$\phi_1 = \frac{X+iY}{\sqrt{2}}, \quad m^2 = \frac{\pi^2}{L^2}, \qquad (31)$$

the effective Lagrangian takes the form

$$-\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\phi_0)^2 + \frac{1}{2}(\partial X)^2 + \frac{1}{2}(\partial Y)^2 + \frac{m^2}{2}(X^2+Y^2)$$

$$+ \frac{\lambda}{4!}\phi_0^4 + \frac{\lambda}{4}\phi_0^2(X^2+Y^2) + \frac{\lambda}{16}(X^2+Y^2)^2. \qquad (32)$$

The variation of this Lagrangian with respect to ϕ_0, X and Y leads to the following equations of motion:

$$\nabla^2\phi_0 = \frac{\lambda}{6}\phi_0^3 + \frac{\lambda}{2}\phi_0(X^2+Y^2),$$

$$\nabla^2 X = m^2 X + \frac{\lambda}{2}\phi_0^2 X + \frac{\lambda}{4}X(X^2+Y^2),$$

$$\nabla^2 Y = m^2 Y + \frac{\lambda}{2}\phi_0^2 Y + \frac{\lambda}{4}Y(X^2+Y^2). \qquad (33)$$

Obviously, the latter two equations are the same, and so we can set

$$Y = \kappa X. \qquad (34)$$

Making this substitution for Y, the first two equations of motion reduce to

$$\nabla^2\phi_0 = \frac{\lambda}{6}\phi_0^3 + \frac{\lambda}{2}\phi_0 X^2(1+\kappa^2),$$

$$\nabla^2 X = m^2 X + \frac{\lambda}{2}\phi_0^2 X + \frac{\lambda}{4}X^3(1+\kappa^2). \qquad (35)$$

As long as the mass term is negligible compared to the cubic term, i.e., when $m^2 \ll \lambda\,\phi_0^2/2$,

$$\phi_0 = qX \qquad (36)$$

is also a solution, provided that

$$q^2 = \frac{3}{4}(1+\kappa^2). \qquad (37)$$

In this case, the kinetic part of the Lagrangian can be written as

$$-\mathcal{L}_{\text{kin}} = \frac{1}{2}(\partial\phi_0)^2 + \frac{1}{2}(\partial X)^2 + \frac{1}{2}(\partial Y)^2$$

$$= \frac{1}{2}(\partial\phi_0)^2\left(1 + \frac{1+\kappa^2}{q^2}\right) \qquad (38)$$

and by using the constraint (37), we obtain

$$-\mathcal{L}_{\text{kin}} = \frac{1}{2}\frac{7}{3}(\partial\phi_0)^2 = \frac{1}{2}(\partial\tilde{\phi})^2, \qquad (39)$$

where we have implemented the field redefinition

$$\tilde{\phi} = \sqrt{\frac{7}{3}}\,\phi_0 \qquad (40)$$

in order to map the system of the three real, scalar fields to a theory of a single scalar field. Next, we look at the quartic potential which now takes the form

$$V_{\text{eff}} = \frac{\lambda}{4!}\phi_0^4 + \frac{\lambda}{4}\phi_0^2(X^2+Y^2) + \frac{\lambda}{16}(X^2+Y^2)^2 = \frac{35}{3}\frac{\lambda}{4!}\phi_0^4$$

$$= \frac{15}{7}\tilde{V}, \qquad (41)$$

where $\tilde{V}$ is the quartic potential of the renormalized scalar field $\tilde{\phi}$. As a result, the presence of the two Kaluza-Klein fields X and Y have raised the value of the effective potential by a factor of 15/7 and so, the fine tuning of the coupling constant λ has worsened. Note that this result does not depend on the substitution (34) and holds even if we set $Y=0$.

Before we tackle the more general case, it will still be useful to consider the next-to-lowest order where we allow ϕ_2 and ϕ_2^*, apart from ϕ_0, ϕ_1, and ϕ_1^*, to be present in the theory. In a similar way, we set

$$\phi_1 = \frac{X_1 + iY_1}{\sqrt{2}}, \quad \phi_2 = \frac{X_2 + iY_2}{\sqrt{2}}, \quad (42)$$

and define

$$m_1^2 = \frac{\pi^2}{L^2}, \quad m_2^2 = \frac{4\pi^2}{L^2}. \quad (43)$$

Once again the system can be simplified. In this case, the choice $X_2 = 0$ corresponds to a special solution of the five field system as long as $X_1^2 = Y_1^2$. Then, the remaining equations of motion are given by

$$\nabla^2 \phi_0 = \frac{\lambda}{6} \phi_0^3 + \frac{\lambda}{2} \phi_0 (2X_1^2 + Y_1^2) + \frac{\lambda}{\sqrt{2}} X_1 Y_1 Y_2, \quad (44)$$

$$\nabla^2 Y_1 = m_1^2 Y_1 + \frac{\lambda}{2} \phi_0^2 Y_1 + \frac{\lambda}{\sqrt{2}} \phi_0 (X_1 Y_2)$$

$$+ \frac{\lambda}{2} Y_1 (X_1^2 + Y_2^2), \quad (45)$$

$$\nabla^2 Y_2 = m_2^2 Y_2 + \frac{\lambda}{2} \phi_0^2 Y_2 + \frac{\lambda}{\sqrt{2}} \phi_0 (X_1 Y_1)$$

$$+ \frac{\lambda}{4} Y_2 (4X_1^2 + Y_2^2). \quad (46)$$

If we further assume, as before, that the masses m_1^2 and m_2^2 are small compared to $\lambda \phi_0^2/2$ and neglect them, the ansatz

$$\phi_0 = qX_1, \quad Y_2 = pY_1 \quad (47)$$

is indeed a solution of the system (44)–(46). Rearranging equations (44),(45) and (45),(46), we obtain the constraints

$$3\sqrt{2} p(1 - q^2) - 2q^3 + 3q = 0, \quad (48)$$

$$2\sqrt{2} q(1 - p^2) - p^3 + 2p = 0, \quad (49)$$

respectively. The above system of algebraic equations can be solved numerically leading to the following pairs of values for the proportionality coefficients q and p:

$$(A): \quad q \to 2.22, \quad p \to -0.911, \quad (50)$$

$$(B): \quad q \to 1.07, \quad p \to 1.13, \quad (51)$$

$$(C): \quad q \to 0.956, \quad p \to -3.07, \quad (52)$$

$$(D): \quad q \to 0.722, \quad p \to -0.695. \quad (53)$$

The above set of solutions are supplemented by another set where the signs of q and p are opposite. But, as we will see, both the potential and the kinetic terms are invariant under the simultaneous change $q \to -q$ and $p \to -p$ and so, we may ignore the second set of solutions.

By setting $X_2 = 0$ and using the proportionality relations that hold between the remaining four scalar fields, the kinetic Lagrangian takes the form

$$-\mathcal{L}_{\text{kin}} = \frac{1}{2} (\partial \phi_0)^2 + \frac{1}{2} (\partial X_1)^2 + \frac{1}{2} (\partial Y_1)^2 + \frac{1}{2} (\partial Y_2)^2$$

$$= \frac{1}{2} (\partial \phi_0)^2 \left(1 + \frac{2 + p^2}{q^2} \right) = \frac{1}{2} (\partial \tilde{\phi})^2, \quad (54)$$

where

$$\tilde{\phi} = \phi_0 \sqrt{1 + \frac{2 + p^2}{q^2}}. \quad (55)$$

In the same way, the effective potential can be written as

$$V_{\text{eff}} = \frac{\lambda}{4!} \tilde{\phi}^4 \left\{ 1 + \frac{6(2 + p^2)}{q^2} + \frac{12\sqrt{2} p}{q^3} + \frac{3}{2q^4} (4 + 8p^2 + p^4) \right\}$$

$$\times \left(1 + \frac{2 + p^2}{q^2} \right)^{-2}. \quad (56)$$

Substituting the values of the parameters q and p from solutions (50)–(53), we obtain the final results for the value of the effective potential

$$(A): \quad V_{\text{eff}} = 1.51 \tilde{V}, \quad (57)$$

$$(B): \quad V_{\text{eff}} = 3.48 \tilde{V}, \quad (58)$$

$$(C): \quad V_{\text{eff}} = 1.75 \tilde{V}, \quad (59)$$

$$(D): \quad V_{\text{eff}} = 1.29 \tilde{V}, \quad (60)$$

where the renormalization of the scalar field (55) has been taken into account. According to the above results, there is one solution that multiplies the one-field-potential $\tilde{V}$ by a numerical coefficient 3.48, which can be compared to the lowest order solution where the potential was multiplied by $15/7 \simeq 2.14$. In this case the fine-tuning is further aggravated. However, there are three additional solutions that multiply the potential by coefficients which, although larger than unity, are smaller than the first one. Thus, there is the hope that as we add more and more extra fields these coefficients become smaller. Recall, that our goal is to ease the original fine-tuning problem associated with (in this case) a simple $\lambda \phi^4$ chaotic inflationary model. Furthermore, for $N \sim 2LM_5 \gg 1$, the four-dimensional coupling is $\lambda \sim \hat{\lambda}/N$ and will be small provided, $\hat{\lambda} \sim 1$ and $N \gtrsim 10^{12}$ or equivalently, $M_5 \lesssim 10^{-6} M_P$. Therefore, so long as the potential does not grow as N, there will be a viable solution for the assisted paradigm.

One could argue that the above results for the effective potential hold only for the special solution (47) supplemented by the relations $X_2 = 0$ and $X_1^2 = Y_1^2$ that we have considered. For this reason, we studied some additional special solutions of the equations of motion, namely,

$$\phi_0 = qX_1, \quad X_2 = pX_1, \quad Y_1 = Y_2 = 0, \tag{61}$$

$$\phi_0 = qY_1, \quad X_2 = pY_1, \quad X_1 = Y_2 = 0. \tag{62}$$

Under a numerical renormalization of the proportionality coefficients, i.e.,

$$q \rightarrow \pm \frac{\tilde{q}}{\sqrt{2}}, \quad p \rightarrow \frac{\tilde{p}}{\sqrt{2}}, \tag{63}$$

the above solutions, substituted in the equations of motion, lead to the same constraints (48),(49) for the coefficients $\tilde{q}$ and $\tilde{p}$ and the same results (57)–(60) for the effective potential. As in the lowest order, the effective potential seems to depend merely on the number of the real scalar fields included in the theory and not on their ''flavor,'' i.e., if they come from the real or imaginary parts of the complex Kaluza-Klein fields or from an arbitrary combination of them.

Indeed, the invariance of the effective potential under the selection of different special solutions exists, as long as these solutions are characterized by the same number of real fields and lead to the same number of constraints on the proportionality coefficients involved. As we mentioned above, the result is always independent of the origin of these real fields. We may, then, conclude, that the real and imaginary parts of N Kaluza-Klein complex fields, that come from the compactification of the fifth dimension, constitute equivalent degrees of freedom contributing equally to the final number of $2N$ degrees of freedom. This result allows us to substitute the N complex Kaluza-Klein fields with $2N$ real fields. Then, the Lagrangian (30) reduces to

$$-\mathcal{L}_{4D} = \frac{1}{2}(\partial\phi_0)^2 + \sum_{n=1}^{2N} \left\{ \frac{1}{2}(\partial\phi_n)^2 + \frac{n^2\pi^2}{2L^2}\phi_n^2 \right\} + \frac{\lambda}{4!}\phi_0^4$$

$$+ \frac{\lambda}{4}\phi_0^2\sum_{n=1}^{2N}\phi_n^2 + \frac{\lambda}{2\sqrt{2}}\phi_0\sum_{n,k=1}^{2N}\phi_n\phi_k\phi_{n+k}$$

$$+ \frac{\lambda}{12}\sum_{n,k,l=1}^{2N}\phi_n\phi_k\phi_l\left(\phi_{n+k+l} + \frac{3}{4}\phi_{n+k-l}\right) \tag{64}$$

with equations of motion given by

$$\nabla^2\phi_0 = \frac{\lambda}{6}\phi_0^3 + \frac{\lambda}{2}\phi_0\sum_{n=1}^{2N}\phi_n^2 + \frac{\lambda}{2\sqrt{2}}\sum_{n,k=1}^{2N}\phi_n\phi_k\phi_{n+k}, \tag{65}$$

$$\nabla^2\phi_n = m_n^2\phi_n + \frac{\lambda}{2}\phi_0^2\phi_n + \frac{\lambda}{2\sqrt{2}}\phi_0\sum_{k=1}^{2N}(2\phi_k\phi_{k+n}$$

$$+ \phi_k\phi_{n-k}) + \frac{\lambda}{4}\sum_{k,l=1}^{2N}\left\{\phi_k\phi_l\left(\phi_{k+l+n}\right.\right.$$

$$\left.\left. + \frac{1}{3}\phi_{n-k-l} + \frac{3}{4}\phi_{n+k-l} + \frac{1}{4}\phi_{k+l-n}\right)\right\}. \tag{66}$$

It is to be understood that the fields denoted by index combinations such as $n+k+l$ and $n+k-l$ are to be included only if they are ≥ 1 and $\leq 2N$.

Although the substitution of complex fields by an equivalent number of real fields simplifies the theory, it does not allow us to study the effect from the addition of a large number of extra scalar fields on the effective potential by using the method described above. The appearance of new cross-coupling terms as we increase the order of the theory makes the analytical formulation of the problem extremely tedious and the determination of the result for the effective potential impossible even through numerical methods.

As an alternative approach to the problem, we construct the function $\phi_{n+1} - \phi_n$ out of the difference of two consecutive Kaluza-Klein fields and it is easy to show that it satisfies the following equation:

$$\nabla^2(\phi_{n+1} - \phi_n) = \frac{\lambda}{2}\phi_0^2(\phi_{n+1} - \phi_n)$$

$$+ \frac{\lambda}{2\sqrt{2}}\phi_0\sum_{k=1}^{2N}\{2\phi_k(\phi_{k+n+1} - \phi_{k+n})$$

$$+ \phi_k(\phi_{n+1-k} - \phi_{n-k})\}$$

$$+ \frac{\lambda}{4}\sum_{k,l=1}^{2N}\left\{\phi_k\phi_l\left[(\phi_{k+l+n+1} - \phi_{k+l+n})\right.\right.$$

$$+ \frac{1}{3}(\phi_{n+1-k-l} - \phi_{n-k-l})$$

$$+ \frac{3}{4}(\phi_{n+1+k-l} - \phi_{n+k-l})$$

$$\left.\left. + \frac{1}{4}(\phi_{k+l-n-1} - \phi_{k+l-n})\right]\right\}. \tag{67}$$

The right-hand-side of the above equation, which is proportional to the first derivative of the effective potential with respect to the field $\phi_{n+1} - \phi_n$, has a minimum when ϕ_n approaches both ϕ_{n-1} and ϕ_{n+1} at late times. As a result, one of the possible late-time attractors of the theory has all of the extra fields equal. By setting $\phi_1 = \phi_2 = \cdots = \phi_{2N}$, the calculation of the effective potential in the presence of $2N$ extra scalar fields in the theory can be easily conducted.

However, the above argument suffers from two major loopholes: first, the attractor that has all of the Kaluza-Klein fields equal is only one of the possible late-time attractors and may be not the one chosen by the system and, second, the condition $\phi_{n-1} = \phi_n = \phi_{n+1}$ cannot be fulfilled for the ''boundary fields'' ϕ_1 and ϕ_{2N}. In the case $n=1$, the field ϕ_{n-1} does not exist by construction and the same holds for ϕ_{n+1} when $n=2N$. Both of the above problems can be eliminated by imposing the periodic condition $\phi_{2N+i} = \phi_i$ when $2N$ real Kaluza-Klein fields are present in the theory. Then, the ''boundaries'' are removed and we can define both ϕ_{n-1} and ϕ_{n+1} for every field ϕ_n. Moreover, we may prove that, after the imposition of the periodic condition, the attractor that has all of the fields equal is the unique late-time

attractor of the system. For this purpose, we are going to make use of the induction method. We start with the case with two real Kaluza-Klein fields for which the Lagrangian (64) becomes

$$-\mathcal{L}_{4D} = \frac{1}{2}(\partial\phi_0)^2 + \frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 + \frac{\lambda}{4!}\phi_0^4$$

$$+ \frac{\lambda}{4}\phi_0^2(\phi_1^2 + \phi_2^2)$$

$$+ \frac{\lambda}{2\sqrt{2}}\phi_0\,\phi_2(3\phi_1^2 + \phi_2^2) + \frac{7\lambda}{48}(\phi_1^4 + \phi_2^4 + 6\phi_1^2\phi_2^2).$$

(68)

Note that, strictly speaking, the above Lagrangian should follow from Eq. (30) in the next-to-lowest order considered above if we put $Y_1 = Y_2 = 0$. However, this is not exactly the case: due to the boundary condition imposed, there are *additional* terms present in the Lagrangian which modify the co-efficients of the cross-coupling terms while leaving their structure unchanged. The equations of motion of the fields ϕ_1 and ϕ_2, then, have the form

$$\nabla^2\phi_1 = \frac{\lambda}{2}\phi_0^2\phi_1 + \frac{3\lambda}{\sqrt{2}}\phi_0\phi_1\phi_2 + \frac{7\lambda}{12}\phi_1(\phi_1^2 + 3\phi_2^2),$$

(69)

$$\nabla^2\phi_2 = \frac{\lambda}{2}\phi_0^2\phi_2 + \frac{3\lambda}{2\sqrt{2}}\phi_0(\phi_1^2 + \phi_2^2)$$

$$+ \frac{7\lambda}{12}\phi_2(\phi_2^2 + 3\phi_1^2).$$

(70)

Subtracting the above equations, we obtain the result

$$\nabla^2(\phi_2 - \phi_1) = \frac{\lambda}{2}\phi_0^2(\phi_2 - \phi_1) + \frac{3\lambda}{2\sqrt{2}}\phi_0(\phi_2 - \phi_1)^2$$

$$+ \frac{7\lambda}{12}(\phi_2 - \phi_1)^3$$

$$= \frac{\lambda}{2}\psi\left(\phi_0^2 + \frac{3}{\sqrt{2}}\phi_0\psi + \frac{7}{6}\psi^2\right).$$

(71)

The right-hand-side of the above equation is the first deriva-tive of the effective potential with respect to the field $\psi = \phi_2 - \phi_1$. It is obvious that the choice $\psi = 0$ minimizes the potential. Actually, this is the only minimum of the potential since the expression inside the brackets does not have any real solutions.

Next, we assume that the only minimum of the effective potential, when $2N-1$ scalar fields are included in the theory, corresponds to $\phi_1 = \phi_2 = \cdots = \phi_{2N-1}$. We will show that if we add one more field ϕ_{2N} the aforementioned late-time attractor expands in order to include ϕ_{2N}, too. So, as-

suming that we have $2N-1$ equal scalar fields and the field ϕ_{2N}, the Lagrangian (64) takes the form

$$-\mathcal{L}_{4D} = \frac{1}{2}(\partial\phi_0)^2 + \frac{(2N-1)}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_{2N})^2 + \frac{\lambda}{4!}\phi_0^4$$

$$+ \frac{\lambda}{4}\phi_0^2[(2N-1)\phi_1^2 + \phi_{2N}^2]$$

$$+ \frac{\lambda}{2\sqrt{2}}\phi_0\{2(2N-1)(N-1)\phi_1^3$$

$$+ \phi_{2N}[3(2N-1)\phi_1^2 + \phi_{2N}^2]\}$$

$$+ \frac{7\lambda}{48}\{4(2N-1)[(N-1)\phi_1^2 + \phi_1\phi_{2N}]^2$$

$$+ [(2N-1)\phi_1^2 + \phi_{2N}^2]^2\}.$$

(72)

Now, the equations of motion of the fields ϕ_1 and ϕ_{2N} take the form

$$\nabla^2\phi_1 = \frac{\lambda}{2}\phi_0^2\phi_1 + \frac{3\lambda}{\sqrt{2}}\phi_0[(N-1)\phi_1^2 + \phi_1\phi_{2N}]$$

$$+ \frac{7\lambda}{12}\{[4(N-1)^2 + (2N-1)]\phi_1^3 + 6(N-1)\phi_1^2\phi_{2N}$$

$$+ 3\phi_1\phi_{2N}^2\},$$

(73)

$$\nabla^2\phi_{2N} = \frac{\lambda}{2}\phi_0^2\phi_{2N} + \frac{3\lambda}{2\sqrt{2}}\phi_0[(2N-1)\phi_1^2 + \phi_{2N}^2]$$

$$+ \frac{7\lambda}{12}[2(2N-1)(N-1)\phi_1^3 + 3(2N-1)$$

$$\times \phi_1^2\phi_{2N} + \phi_{2N}^3],$$

(74)

while the equation of motion of the field $\psi = \phi_{2N} - \phi_1$ is found to be

$$\nabla^2(\phi_{2N} - \phi_1) = \frac{\lambda}{2}\phi_0^2(\phi_{2N} - \phi_1) + \frac{3\lambda}{2\sqrt{2}}\phi_0(\phi_{2N} - \phi_1)^2$$

$$+ \frac{7\lambda}{12}(\phi_{2N} - \phi_1)^3.$$

(75)

The above equation is identical with Eq. (71) and, as a result, the effective potential has a unique minimum at $\psi = 0$. Ac-cording to the above result, the only late-time attractor for the system of $2N$ Kaluza-Klein scalar fields corresponds to $\phi_1 = \phi_2 = \cdots = \phi_{2N-1} = \phi_{2N}$.

Now, we proceed to calculate the kinetic term and the effective potential of the system. By using the late-time at-tractor of equal fields in the case of $2N$ scalar fields, the kinetic part of the Lagrangian (64) takes the form

$$-\mathcal{L}_{4D}=\frac{1}{2}(\partial\phi_0)^2+\frac{(2N)}{2}(\partial\phi_1)^2$$

$$=\frac{1}{2}(\partial\phi_0)^2\left(1+\frac{2N}{q^2}\right)=\frac{1}{2}(\partial\tilde{\phi})^2, \qquad (76)$$

where we have assumed the proportionality relation $\phi_0=q\,\phi_1$ and renormalized the scalar field ϕ_0. Then, the effective potential reduces to

$$V_{\mathrm{eff}}=\frac{\lambda}{4!}\,\phi_0^4+\frac{\lambda}{4}(2N)\,\phi_0^2\phi_1^2+\frac{\lambda}{2\sqrt{2}}(2N)^2\phi_0\phi_1^3$$

$$+\frac{7\lambda}{48}(2N)^3\phi_1^4$$

$$=\tilde{V}\left(1+\frac{12N}{q^2}+\frac{24\sqrt{2}N^2}{q^3}+\frac{28N^3}{q^4}\right)\left(1+\frac{2N}{q^2}\right)^{-2}, \qquad (77)$$

where $\tilde{V}$ is the quartic potential of the renormalized field $\tilde{\phi}$. According to the above result, the effective potential depends on two parameters: the number N of Kaluza-Klein fields that we include in the theory and the proportionality coefficient q. This coefficient, although a number, may itself depend on N changing radically the picture for the behavior of the effective potential. So, in order to draw consistent conclusions, we reconsider the equations of motion of the fields ϕ_0 and ϕ_1,

$$\nabla^2\phi_0=\frac{\lambda}{6}\,\phi_0^3+\frac{\lambda}{2}(2N)\,\phi_0\phi_1^2+\frac{\lambda}{2\sqrt{2}}(2N)^2\phi_1^3, \qquad (78)$$

$$\nabla^2\phi_1=\frac{\lambda}{2}\,\phi_0^2\phi_1+\frac{3\lambda}{2\sqrt{2}}(2N)\,\phi_0\phi_1^2+\frac{7\lambda}{12}(2N)^2\phi_1^3. \qquad (79)$$

By making use of the relation $\phi_0=q\,\phi_1$ and rearranging ac-

cordingly the above equations, we obtain the following constraint for the proportionality coefficient:

$$\frac{q^3}{3}+\frac{3\sqrt{2}}{2}Nq^2+\left(\frac{7N^2}{3}-N\right)q-\frac{2N^2}{\sqrt{2}}=0. \qquad (80)$$

This algebraic equation has the solutions

$$q_1=-N\sqrt{2}, \qquad q_{2,3}=\frac{-7N\pm\sqrt{49N^2+24N}}{2\sqrt{2}}. \qquad (81)$$

When each one of the above values is substituted in the expression (77), the potential exhibits a different behavior. Analytically we find the following.

(i) $q=q_1$. In this case, we obtain

$$V_{\mathrm{eff}}=\tilde{V}\left(\frac{N}{N+1}\right). \qquad (82)$$

For $N=2$, this gives $V_{\mathrm{eff}}=\frac{2}{3}\tilde{V}$. However, in the limit $N\to\infty$, the effective potential asymptotically tends to $\tilde{V}$. Note that the imposition of the periodic boundary condition demands the existence of two boundaries so $N\geq2$. When, at the next-to-lowest order, we studied the case $N=2$, we did not obtain any solution with the coefficient that multiplies $\tilde{V}$ being smaller that unity. This means that the above solution owes its existence to the imposition of the periodic condition on the Kaluza-Klein fields (for $N=2$) and it may not constitute a generic solution of the original theory. However, for large N, we expect this behavior to approximate the solution of the original Lagrangian. In particular, this is exactly the type of solution we were searching for. Namely, at large N, the potential of the late-time attractor fields does not depend on N relative to the original four-dimensional potential. Therefore, for large N, chaotic inflation is realized in four-dimensions with a quartic five-dimensional coupling $\hat{\lambda}\sim1$, and we have an explicit example of assisted inflation.

(ii) $q=q_2$. Then, we have

$$V_{\mathrm{eff}}=\tilde{V}\frac{2[360+1904N+2401N^2-(156+343N)\sqrt{49N^2+24N}]}{(20+49N-7\sqrt{49N^2+24N})^2}. \qquad (83)$$

Then,

$$\text{For } N=2: \quad V_{\mathrm{eff}}\simeq16.5\tilde{V}, \qquad (84)$$

$$\text{For } N\gg2: \quad V_{\mathrm{eff}}\simeq\tilde{V}\left\{7N+\frac{18}{7}+\mathcal{O}\left(\frac{1}{N}\right)\right\} \qquad (85)$$

which clearly shows that the potential tends to increase with

the number of scalar fields that we include in the theory. The above solution corresponds to the results (41) and (58) derived in the lowest $(N=1)$ and next-to-lowest order $(N=2)$, respectively. Both these solutions showed a tendency to increase with the number of Kaluza-Klein fields, a behavior which obviously survived after the imposition of the periodic condition. Of course this solution has exactly the N dependence that prohibits an assisted solution.

(iii) $q=q_3$. In this case,

$$V_{\text{eff}} = \tilde{V} \frac{2[3640 + 1904N + 2401N^2 + (156 + 343N)\sqrt{49N^2 + 24N}]}{(20 + 49N + 7\sqrt{49N^2 + 24N})^2}. \tag{86}$$

Now,

$$\text{For} \quad N = 2: \quad V_{\text{eff}} \simeq 1.04\tilde{V}, \tag{87}$$

$$\text{For} \quad N \gg 2: \quad V_{\text{eff}} \simeq \tilde{V} \left\{ 1 + \frac{32}{343N} - \frac{368}{16807N^2} + \mathcal{O}\left(\frac{1}{N}\right)^3 \right\}. \tag{88}$$

In this case, the largest value that the potential takes on corresponds to $N=2$ and, as we add more and more scalar fields, it asymptotically tends to $\tilde{V}$ with the multiplication coefficient being always larger than unity. This solution is the analogue of the solutions (57), (59), and (60) derived in the next-to-lowest order approximation. By making use of the periodic condition, we managed to include a large number of scalar fields in our model and found that these coefficients decrease with the number of fields, as we expected. As in the case with $q = q_1$, this class of solutions also allows for chaotic inflation with $\hat{\lambda} \sim 1$ through assistance.

Although the large number of fields has managed to remove the fine-tuning problems, it is necessary to verify that the initial conditions for inflation to occur are indeed fulfilled. In four dimensions, we normally assume $\tilde{V}(\tilde{\phi}) \sim M_P^4$, which for $\lambda \ll 1$, corresponds to $\tilde{\phi} \gg M_P$. If these conditions are translated into our five dimensional quantities, then we would find $\hat{\phi} \sim M_P^{1/2} M_5$ and $\hat{V} \sim M_P^2 M_5^3 \gg M_5^5$. Without a better understanding of the dynamics of the five-dimensional theory, we should instead insist that $\hat{V}(\hat{\phi}) \sim M_5^5$. This condition, then, becomes

$$\tilde{V}(\tilde{\phi}) = 2L\hat{V}(\hat{\phi}) \sim M_P^2 M_5^2 < M_P^4 \tag{89}$$

since $N = 2LM_5 = M_P^2/M_5^2 \gg 1$. The requirement that the four-dimensional coupling constant should be of $\mathcal{O}(10^{-12})$ imposes the following condition on the five-dimensional coupling and the four- and five-dimensional Planck mass

$$\hat{\lambda} \left(\frac{M_5}{M_P} \right)^2 \sim 10^{-12}. \tag{90}$$

By appropriately choosing the values of the above quantities, the required value of λ is naturally obtained. However, the initial condition for inflation $\tilde{\phi} \gg M_P$ puts a constraint on the smallest possible value of the ratio M_5/M_P: when the above condition is combined with the expression (89) for the four-dimensional potential, one finds $M_5 \gtrsim 10^{-6} M_P$. Then, even if $\hat{\lambda}$ is as large as of $\mathcal{O}(1)$, we can still obtain $\lambda \sim 10^{-12}$. The problems encountered when $M_5 \ll M_P$ have recently been discussed [12,13]. However, for $M_5 \sim 10^{-3} M_P$ as in many models of string unification [14], we would have $N \sim 10^6$,

and an initial value of $\hat{\lambda} \sim 10^{-6}$ would be brought down to the correct four dimensional coupling.

There is one more issue which must be addressed. In this section, we have discussed the conditions leading to inflation, and the attractor solution of the equations of motion. In doing so, we have neglected the KK mass terms, which is valid so long as $m^2 < \lambda \phi_0^2$. At the onset of inflation, this condition is obeyed by all of the KK fields only if the maximum mass we are considering (which corresponds to the Nth state and has mass $\sim M_5$) satisfies $M_5^2 < \lambda \phi_0^2 \sim \lambda^{1/2} M_P M_5$, or $M_5 \lesssim 10^{-6} M_P$. This means that only for the marginal value of $M_5 \simeq 10^{-6} M_P$ all of the KK fields can be considered effectively massless while for $M_5 \simeq 10^{-3} M_P$ we can ignore the masses only for those fields with $m^2 < 10^{-3} M_5^2$. Moreover, as the field $\tilde{\phi}$ moves toward the minimum of the potential, ϕ_0 becomes smaller as well and, gradually, more and more fields cease to satisfy our assumption on the masses of the fields ϕ_i. The equations of motion of these massive fields are dominated by their mass terms with the only late-time attractor being the trivial one. As a result, these fields get decoupled from the rest of the system with a time scale inversely proportional to their mass: the more massive they are, the faster they decouple. At the end of the day, when $\tilde{\phi}$ finally reaches the minimum of the potential, all of the massive KK fields have decoupled and only the massless (by construction) KK zero-mode, ϕ_0, has remained playing the role of the inflaton $\tilde{\phi}$. However, this behavior does not affect the resolution of the fine-tuning problem in the least. As the number of KK fields, that can be considered massless, decreases, the solution $q = q_1$ gradually disappears while the other two solutions, $q = q_2$ and $q = q_3$, tend to become identical resulting in an effective potential which is again independent of the number N. As a result, the resolution of the fine-tuning problem holds at all times: from the onset of inflation, when all or part of the KK fields can be considered massless and contribute to the inflaton, until its final stages, when all the KK fields have decoupled.

It appears that the compactification of a large extra dimension can lead to assistance effects enhancing the probabilities for inflation not only in the case of power-law potentials but in the case of exponential potentials as well. As an illuminating example, we consider the following five-dimensional scalar field theory:

$$-\mathcal{L}_{5D} = \frac{1}{2} \partial_A \hat{\phi} \partial^A \hat{\phi} + \hat{V}_0 \exp\left(-\sqrt{\frac{2}{\hat{p}}} \frac{\hat{\phi}}{M_5^{3/2}} \right), \tag{91}$$

where $\hat{V}_0$ and $\hat{p}$ are constants. As in the case of the quartic potential, the five-dimensional field $\hat{\phi}$ (which is perhaps a modulus field from the compactification of additional dimensions in the theory) can be Fourier expanded along the com-

pact coordinate z. When the expansion (27) is substituted in the above Lagrangian, we obtain a scalar field theory described by Eq. (19), interacting, through an exponential potential. Even in terms of five-dimensional quantities, this theory is ladened with interactions since the potential of every field multiplies the potential of every other field. When the integration over the compact coordinate is conducted, we expect an effective, heavily interacting, four-dimensional scalar theory to arise. However, the form of the potential makes the integration over z extremely difficult. Nevertheless, we can still make some qualitative arguments on the assistance effect that follows from compactification. In terms of four-dimensional quantities, the above Lagrangian can be written as

$$-\mathcal{L}_{\text{eff}} = \frac{1}{2} \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \tilde{V}_0 \exp\left(-\sqrt{\frac{2}{\tilde{p}}}\, \tilde{\phi}\right), \quad (92)$$

where now

$$\tilde{\phi} = \sqrt{2L}\,\hat{\phi} = \sqrt{N}\,\frac{\hat{\phi}}{M_5}, \quad \tilde{V}_0 = 2L\hat{V}_0 = N\frac{\hat{V}_0}{M_5}, \quad \tilde{p} = N\hat{p}. \quad (93)$$

As it is well known, a four-dimensional theory of the form given above leads to a power-law expansion of the Universe: $R(t) \sim t^{\tilde{p}}$. After compactification, the parameter $\hat{p}$ has been multiplied by the number of massive KK fields that are present in the theory and, as a result, for sufficiently large N, the four-dimensional theory will produce inflation even if the five-dimensional theory with the parameter $\hat{p}$ was not able to. Finally, it is worth noting that the above dependence of the field $\tilde{\phi}$ and the parameters $\tilde{V}_0$ and $\tilde{p}$ on the number of multiple fields N was also derived in Ref. [7] although the origin of the fields was not specified. Here, we argue that the compactification of a five-dimensional theory with an exponential potential could provide us with both the necessary multiplicity of scalar fields and the desired dependence of the parameters of the theory on the number of fields.

We summarize the results of this section: The Kaluza-Klein compactification of the fifth dimension of a five-dimensional theory of a single, self-interacting scalar field leads to the appearance of a large number of Kaluza-Klein scalar fields in the four-dimensional effective theory. A feature of this effective theory is the presence of a complicated web of interaction terms between the scalar fields of the theory. Once the late-time attractor of the system is determined, this field theory of multiple scalar fields can be mapped to a theory of a single, self-interacting scalar field $\tilde{\phi}$. The presence of the interaction terms drives the effective potential towards two different directions: in one case, it increases with the number of extra scalar fields that are present in the theory while, in the second case, is starts with a value slightly smaller or larger than the value of the one-field-self-interaction potential $\tilde{V}$ but asymptotically tends back to $\tilde{V}$. At the end of the analysis, the renormalized scalar field $\tilde{\phi}$ turns out to be much more ($q=q_2$) or equally strongly coupled ($q=q_{1,3}$) compared to the initial four-dimensional

Kaluza-Klein fields. As a result, the renormalized coupling $\tilde{\lambda}$, defined as λ multiplied by the expressions in brackets in Eqs. (82), (85), and (88), takes on a value which is much larger ($q=q_2$) or almost the same ($q=q_{1,3}$) compared to the value of the initial four-dimensional coupling λ. However, in the latter cases, $\tilde{\lambda}$ is suppressed by the number of scalar fields relative to the original coupling $\hat{\lambda}$ of the five-dimensional scalar field. This a concrete example of assisted inflation.

V. CONCLUSIONS

In this paper, we have dealt with the problem of fine-tuned coupling constants in the framework of field theories that involve self-coupled or interacting scalar fields. This problem inevitably arises when we consider the possibility of the creation of an inflationary epoch in our universe and demand an agreement between the theoretical predictions and the experimental (COBE) data on density fluctuations.

We have demonstrated by considering some general field theories of multiple scalar fields in four dimensions that the idea of assisted inflation based on exponential potentials [7] can be easily extended in the case of power-law potentials. In this case, the presence of multiple scalar fields leads to a renormalized theory of a single scalar field which is considerably less strongly coupled than the original fields of the theory. The renormalized coupling constants scale with the number of fields N which permits the creation of an inflationary period without severe fine-tuning. However, the effectiveness of assistance depends strongly on the interactions between the scalar fields of the theory. If the multiple scalar fields are assumed to be only self-coupled, both power-law inflation based on exponential potentials as well as chaotic inflation works well with only mild or no fine-tuning at all depending on the number of fields N that we include in the theory. If, on the other hand, we allow cross-coupling terms between different scalar fields, the assistance method breaks down leading to a much more strongly coupled theory.

As a concrete example of a field theory with multiple scalar fields, we considered a single, self-interacting scalar field living in five dimensions with a quartic potential. (Other recent constructions for inflationary models involving a large extra dimension can be found in Refs. [11–13,15–17].) Assuming that the fifth dimension is compactified along a circle and applying a Kaluza-Klein reduction of the five-dimensional field, we obtained a four-dimensional, effective theory with the necessary multiplicity of scalar fields fulfilled by the presence of the Kaluza-Klein modes. The resulting potential contains a complex network of cross-coupling terms. As suggested by our previous results, these interaction terms are expended to hinder inflation. In terms of four-dimensional quantities, this is indeed the case. Once the theory of multiple Kaluza-Klein fields is mapped to a theory of a single, renormalized scalar field, we found three different solutions for the corresponding effective potential: the first one follows a behavior similar to the one derived in the purely four-dimensional case and drives the potential, and thus the renormalized coupling constant, to large values in-

creasing with the number of scalar fields; the other two solutions start with a value for the effective potential which is slightly smaller or larger than the value of the one-field-potential $\tilde{V}$ but asymptotically tends to $\tilde{V}$ as we increase the number of fields. In both cases, the desired behavior of the effective potential is not achieved and the renormalized scalar field is either more or equally strongly coupled than the original four-dimensional Kaluza-Klein fields. Consequently, the necessary fine-tuning of the renormalized quartic coupling constant $\tilde{\lambda}$ becomes more severe or at best remains the same compared to that of λ, a result which is attributed to the presence of interaction terms between the Kaluza-Klein fields of the theory. However, the theory of the renormalized, scalar field does indeed get assisted although via a different path. The four-dimensional coupling constant λ of the Kaluza-Klein fields is determined by the five-dimensional one, $\hat{\lambda}$, divided by the number of the Kaluza-Klein modes. As a result, λ and thus $\tilde{\lambda}$, in the case of the latter two solutions, is suppressed by the number of scalar fields that are present in the theory relative to the original coupling constant $\hat{\lambda}$ of the five-dimensional theory. By choosing appropriate values of the five-dimensional Planck mass M_5 and the five-dimensional coupling constant $\hat{\lambda}$, we are able to naturally obtain a four-dimensional, self-interacted scalar theory with $\lambda \sim \mathcal{O}(10^{-12})$ (in agreement with COBE data) without the need of any fine-tuning. Moreover, our results do not depend on the number of massive KK fields that contribute to the inflaton field and, as a result, the resolution of the fine-tuning problem holds from the onset of inflation until its final stages.

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