AdS-CFT string duality and conformal gauge theories

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Compactification of type IIB superstring on an $AdS_5 \times S^5/\Gamma$ background leads to $SU(N)$ gauge field theories with prescribed matter representations. In the 't Hooft limit of large N such theories are conformally finite. For finite N and broken supersymmetry ($\mathcal{N}=0$) I derive the constraints to be two-loop conformal and examine the consequences for a wide choice of Γ and its embedding $\Gamma \subset \mathcal{C}^3 (\supset S^5)$. [S0556-2821(99)50214-2]

INTRODUCTION

Recently the relationship of string theory to gauge theory received stimulus from the conjecture by Maldacena [1] (related earlier papers are [2–5]) stemming from string duality which makes in its strongest form the assertion that the information contained in superstring theory is encoded in a four-dimensional gauge field theory including its non-perturbative sector. This has been vigorously pursued by many authors, especially Witten [6–8]. A brief review is in [9].

This relationship appears ironic when one recalls that the earliest string theories, the dual resonance models for strong interactions, were abandoned in favor of an $SU(3)$ gauge theory 25 years ago. String theory has generally been regarded as much more general than gauge field theory because of its far richer structure; however, that perception was based on perturbative arguments, and the new developments of Maldacena *et al.* are essentially non-perturbative.

The idea is to consider N coincident D3-branes with four-dimensional world volume theories having superconformal symmetry. This is conjectured [1] to be dual (weak coupling related to strong coupling) to type IIB superstring theory in a spacetime with geometry $AdS_5 \times S^5$. The world volume theory is in this case an $\mathcal{N}=4$ supersymmetric Yang-Mills theory with gauge group $SU(N)$. Originally it is $U(N)$ but this is broken to $SU(N)$.

The radii of the AdS_5 and S^5 are equal and both given by $R = \lambda^{1/4} l_s$ where λ is the 't Hooft parameter [10] $\lambda = g_{YM}^2 N$ ($g_{YM}^2 = g_S$, the string coupling constant) and $l_S^2 = \alpha'$ the universal Regge slope. The string tension is $T = (2\pi\alpha')^{-1}$.

The $\mathcal{N}=4$ $SU(N)$ gauge theory has been known to be ultraviolet finite for many years [11]. This is true not only for $N \rightarrow \infty$, the conformal limit of Maldacena, but also for finite N.

BREAKING SUPERSYMMETRIES

By factoring out a discrete group Γ in S^5/Γ it is possible to break some or all of the $\mathcal{N}=4$ supersymmetries. The isometry of S^5 is $SO(6) \sim SU(4)$ which may be identified with the R-parity of the $\mathcal{N}=4$ conformal gauge theory. The spinors are in the **4** and the scalars are in the **6** of this $SU(4)$. I shall here consider only Abelian groups $\Gamma = Z_p$, although non-Abelian Γ are worth further study (see e.g. [12,13]). I am considering only $AdS_5 \times S^5/\Gamma$, although the second five-dimensional orbifold can be more general e.g. the $T^{p,q}$ spaces considered in [14].

The number of unbroken symmetries has been studied in e.g. [15,16] with the result that if $\Gamma \subset SU(2)$ there remains $\mathcal{N}=2$ supersymmetry; if that is not satisfied but $\Gamma \subset SU(3)$ there remains $\mathcal{N}=1$ supersymmetry; finally if even that is not satisfied one is left with $\mathcal{N}=0$ or no supersymmetry. This last case is of most interest here.

It has been demonstrated that the large N limit of the resultant gauge theory coincides with that of the $\mathcal{N}=4$ case. Such arguments have been made both using string theory [17] and directly at the field theory level [18]. In the latter case the proof involves a monodromy of the representation for the group Γ.

For finite N, however, there is no argument that the resultant gauge theory is conformal, especially for $\mathcal{N}=0$ where there are no non-renormalization theorems.

Nevertheless, if there does exist a conformal gauge theory in four dimensions with $\mathcal{N}=0$, it would be so tightly constrained as to be possibly unique and would be of interest especially if it could contain the standard $SU(3) \times SU(2) \times U(1)$ model with its peculiar representations for the quarks and leptons.

The representations which occur in the resultant $\mathcal{N} \leq 2$ gauge theories from the orbifold construction have been studied using quiver diagrams [15]. I will find that these diagrams, while convenient for the cases $\mathcal{N} \geq 1$ need augmentation for the case $\mathcal{N}=0$.

To specify the potentially conformal gauge theory I need to state how the group Γ is embedded in $\mathcal{C}^3$. Let the three complex coordinates of $\mathcal{C}^3$ be denoted by $\underline{X} = (X_1, X_2, X_3)$. The action of Z_p is the specified by

$$\underline{X} \rightarrow (\alpha^{a_1} X_1, \alpha^{a_2} X_2, \alpha^{a_3} X_3) \tag{1}$$

where $\alpha = \exp(2\pi i/p)$ and the three integers $a_\mu = (a_1, a_2, a_3)$ specify the embedding.

In order to ensure an $\mathcal{N}=0$ result, I must insist that Γ is not contained in $SU(3)$ by the requirement that

$$a_1 \pm a_2 \pm a_3 \neq 0 \quad (mod \quad p). \tag{2}$$

For any given p, there is a finite $\nu(p)$ number satisfying Eq. (2). We shall indicate later how to enumerate these $\nu(p)$.

MATTER REPRESENTATIONS

Because the discrete group Z_p leads to the identification of p points in $\mathcal{C}^3$ and the N coinciding D3-branes converge on all p copies, the gauge group becomes $SU(N)^p$. The surviving states are invariant under the product of a gauge transformation and a Z_p transformation defined as in Eq. (1) above.

For the scalars, it then follows that the scalars fall into the representations

$$\sum_{\mu} (N_i, \bar{N}_{i \pm a_\mu}). \tag{3}$$

For $a_\mu \neq 0$ these are bi-fundamentals and for $a_\mu = 0$ complex adjoints. If we focus on one $SU(N)$ the only non-singlet representations (the same will be true for the fermions) are fundamentals, anti-fundamentals and adjoints. These representations also follow from the Douglas-Moore quiver diagram.

For the fermions we must consider the transformation of a 4-spinor by making four combinations $A_\lambda (1 \leq \lambda \leq 4)$ of the a_μ:

$$A_1 = (a_1 + a_2 + a_3)/2 \tag{4}$$

$$A_2 = (a_1 - a_2 - a_3)/2 \tag{5}$$

$$A_3 = (-a_1 + a_2 - a_3)/2 \tag{6}$$

$$A_4 = (-a_1 - a_2 + a_3)/2. \tag{7}$$

Again the surviving states are invariant under a product of the Z_p and gauge transformations. This leads to the fermion representation

$$\sum_{\lambda} (N_i, \bar{N}_{i + A_\lambda}) \tag{8}$$

which can, if required, be deduced from a (different) quiver diagram.

TWO-LOOP β-FUNCTIONS

I may take the detailed formula for the gauge coupling β-function β_g from [19]. The two leading orders are

$$\beta_g = \beta_g^{(1)} + \beta_g^{(2)} \tag{9}$$

with

$$\beta_g^{(1)} = -\frac{g^3}{(4\pi)^2} \left[\frac{11}{3} C_2(G) - \frac{4}{3} \kappa S_2(F) - \frac{1}{6} S_2(S) \right] \tag{10}$$

and

$$\beta_g^{(2)} = -\frac{g^5}{(4\pi)^4} \left[\frac{34}{3} (C_2(G))^2 \right.$$

$$- \kappa \left[4 C_2(F) + \frac{20}{3} C_2(G) \right] S_2(F)$$

$$\left. - \left[2 C_2(S) + \frac{1}{3} C_2(G) \right] S_2(S) + \frac{2 \kappa Y_4(F)}{g^2} \right]. \tag{11}$$

Here C_2, S_2 are the quadratic Casimir and Dynkin index respectively for the representations indicated, κ is 1/2, 1 for Weyl and Dirac fermions respectively, products like $C_2(R) S_2(R)$ imply a sum over irreducible representations and finally the Yukawa term is included naturally in the two-loop term (unlike in [19]) because here the Yukawa couplings are proportional to the gauge coupling. The crucial quantity $Y_4(F)$ is defined in terms of the Yukawa matrix $Y_{ij}^a \psi_i \zeta \psi_j \phi^a$ by

$$Y_4(F) = \text{Tr}(C_2(F) Y^a Y^{\dagger a}). \tag{12}$$

Looking first at $\mathcal{N}=4$, the values are easily seen to $C_2(G) = N, S_2(F) = 4N, S_2(S) = 6N$ while $C_2(F) S_2(F) = 4N^2$ and $C_2(S) S_2(S) = 6N^2$. Finally $Y_4(F) = 24 g^2 N^2$. It follows from Eq. (10) and Eq. (11) the $\beta_g = 0$ for $\mathcal{N}=4$ at two loops, as is well known [11]. However, the situation for $\mathcal{N}=0$ is much more complicated.

At one-loop level for $\mathcal{N}=0$ the evaluation of $\beta_g^{(1)}$ is the same term-by-term as for $\mathcal{N}=4$. This is already in [20–22] for the one-loop level and since the one-loop β-function is purely leading-order in N it conforms to the general arguments of [17,18].

At two-loop order I must examine the non-leading terms in $1/N$ in Eq. (11). The first, third and fifth terms are always the same for $\mathcal{N}=0$ as for $\mathcal{N}=4$, respectively $34N^2/3 - 40N^2/3 - 2N^2 = -4N^2$.

To evaluate the second, fourth and sixth terms I find it necessary to distinguish special cases. Substituting in Eq. (11), I find that $\beta_g^{(2)}$ is non-vanishing except in a subset of cases. Explicit examples are provided in subsequent article [23].

DIRECTIONS

A subsequent question to be addressed is what happens at three-loop and even higher orders. Also one must consider the running of the Yukawa and quartic Higgs self-couplings due to a possible non-vanishing of their β-functions β_Y and β_H. It is planned to publish a more complete analysis elsewhere; I conclude this proposal with comments and possible future directions.

Often low-energy supersymmetry is adopted in order to solve the hierarchy problem of the Planck or grand unified theory (GUT) scale to the weak scale. This hierarchy is theory-generated and one may instead be agnostic about physics at $\gtrsim 1000$ TeV scale where there is no real information. For example, recent ideas about extra Kaluza-Klein dimensions at reduced scales e.g. [24–27] avoid the hierarchy

altogether and remove the main motivation for low-energy supersymmetry. In the present case of a conformal theory, the couplings become constant beyond the onset of conformality. The GUT scale and its concomitant hierarchy are thereby obviated.

The possible role of an $\mathcal{N}=0$, $d=4$ conformal gauge theory may be put in context by imagining the level of skepticism to infinite renormalization of QED in 1948 (and later of the standard model) if the example of [11] had been found four decades earlier.

The exciting possibility is that the standard model is part of such an $\mathcal{N}=0$ conformal gauge theory. The mass scales Λ_{QCD} and M_W would arise from necessarily nonperturbative effects, and gravity would be accommodated through the holographic principle [28,6]. Using AdS/conformal field theory (CFT) duality could help identify the relevant conformal theory. If so, this could shed light on the outstanding questions (families, CP violation, etc.) posed by the standard model.

We are hoping for a fixed manifold (line, plane, ...) in which $\beta_{g,Y,H}$ vanishes for a continuous range of couplings but only future calculations will distinguish such a fixed manifold from a fixed point at which conformality is valid. The existence of a fixed point would, in any case, be sufficient to apply our conformality constraints.

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