Spectrum of softly broken $N=1$ supersymmetric Yang-Mills theory

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We study the spectrum of the lowest-spin bound states of the softly broken $N=1$ SUSY Yang-Mills theory in a certain approximation. Two dual formulations of the effective action for the model are given. The spurion method is used for the soft SUSY breaking. Masses of the bound states are calculated approximately and mixing patterns between the states with different parity and spin-orbital quantum numbers are discussed. Mass splittings of pure gluonic states are consistent with predictions of conventional Yang-Mills theory. The results can be tested or used in lattice simulations of the SUSY Yang-Mills model. [S0556-2821(99)02615-6]

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I. INTRODUCTION

Some time ago great progress was made in understanding the ground state structure of many supersymmetric (SUSY) gauge theories [1,2]. There is a possibility that these models can be simulated on the lattice. Some preliminary work toward this complicated task has already been performed (see Refs. [3,4]).

The lattice regularization violates supersymmetry [5]. Thus, some special fine-tuning is required to recover the SUSY limit on the lattice. Away from the SUSY point, the continuum limit of the lattice theory is described by a model with explicit SUSY breaking terms. In some cases those terms may trigger only soft SUSY breaking [6], although this is not guaranteed in general.

Softly broken SUSY models can be studied using the spurion technique [7]. Some ''exact'' results were obtained within this approach [8–10]. In this paper we consider softly broken supersymmetric Yang-Mills theory, the model which is relevant for lattice simulations. At the classical level supersymmetric Yang-Mills (SYM) is a theory with only one parameter, the gauge coupling constant. The lowest-dimensional renormalizable SUSY breaking term allowed by gauge invariance is the gaugino mass term. Therefore, we consider SYM with a gaugino mass term as a theory describing the continuum limit of the lattice regularized action.

In analogy with QCD, one expects that the spectrum of this model consists of colorless bound states of gluinos and gluons. Among those are pure gluonic bound states (glueballs), gluino-gluino mesons and gluon-gluino composites. These states fall into the lowest-spin representations of the $N=1$ SUSY algebra written in the basis of parity eigenstates [11]. The masses and interactions of these bound states can be given within the effective Lagrangian approach. The effective action for $N=1$ SYM was proposed by Veneziano and Yankielowicz (VY) [12].

Since there is a mass gap in the theory, there are no massless physical states for which one could write down a Wilsonian effective action and study vacuum properties as in $N=1$ SUSY QCD [1]. In that respect, it is not clear why the effective action in this case can be truncated so that it includes the lowest-mass states only. Thus, strictly speaking, the VY action cannot be used for precise calculations of the mass spectrum of the model. However, we would like to argue that the VY action (and any of its counterparts) can nevertheless describe certain crucial qualitative features of the low-energy spectrum of the theory. While presenting these arguments we will specify the approximations on which this approach is based. Indeed, one can regard the VY action as a generating functional for one-particle-irreducible (1PI) Green's functions. That is to say, one can read off corresponding zero-momentum Green's functions from the expansion of the respective 1PI effective action in powers of fields and derivatives (for details see discussions in [13]). Thus, in the quadratic approximation in physical fields and derivatives one finds the value of the corresponding zero-momentum two-point Green's function, in the cubic approximation, the zero-momentum three-point Green's function, and so on. Given the value of the zero-momentum two-point function of some composite operator one can try to extract the value of the corresponding lowest-mass state with the quantum numbers of the operator present in the correlation function. This is not an exact correspondence since there are a number of higher excited states which also contribute to the same two-point function. However, the effects of higher spin states are suppressed compared with the leading lowest-spin state by small couplings which emerge as one acts by a lowest-spin interpolating composite operator (present in the correlator) on a corresponding higher radial excitation. In conventional QCD these type of couplings are generically suppressed by a factor of 10 or so, at least in the case of mesons. Since these suppressions have nothing to do with SUSY, but are rather related to internal spin structure of hadrons, we expect the same to be happening in SYM too. In addition, these couplings are suppressed by extra powers of $1/N_c$ if one thinks of the large N_c expansion in the theory. Given these arguments, we expect that the accuracy of determination of the masses in this case is not as precise as one would like to see, however the results could be considered as reasonable estimates. Thus, usage of any results obtained by this method would crucially depend on how much qualitative information is stored in those results. In SUSY theories physical states form degenerate SUSY multiplets. Thus, what

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would matter in our case is not an exact value for the masses (which cannot be obtained analytically anyway), but rather some ratios of masses of different SUSY multiplets and mass splittings of the states with different parity and spin-orbital quantum numbers in the case when SUSY is softly broken. We aim to study these features in what follows.

As we mentioned above, the VY action [12] involves fields for gluino-gluino and gluino-gluon bound states. However, it does not include dynamical degrees of freedom which would correspond to pure gluonic composites (glueballs).

We argued in Ref. [11] that there are no physical reasons to expect glueballs to be heavier and decoupled from gluino-gluino and gluino-gluon bound states in $N=1$ SUSY YM theory. Moreover, there are SQCD sum rule based arguments indicating that the low-energy spectrum of SYM theory is not exhausted by the gluino containing bound states only [14]; glueball degrees of freedom should also be taken into account.

The generalization of the VY effective action that includes pure gluonic degrees of freedom was given in Ref. [11]. The generalized VY effective Lagrangian of Ref. [11] describes mixed states of glueballs, gluino-gluino and gluino-gluon bound states. The fundamental superfield upon which that construction of the generalized VY action is based [11] is a constrained tensor superfield [15]. The set of components of that superfield includes as a subset the VY chiral supermultiplet.

The aim of the present paper is twofold. First we propose a new representation of the generalized VY effective action of Ref. [11]. This action is equivalent to the previously proposed one [11], but it uses two different chiral supermultiplets instead of the tensor supermultiplet approach adopted in [11].

Then we introduce soft SUSY breaking terms in the generalized VY Lagrangian and study mass splittings and mixing patterns in the softly broken theory. These results can be directly tested in lattice calculations. Predictions for the masses of the gluino-gluino and gluon-gluino bound states and their splittings in the broken theory were made in Ref. [17] using the original VY effective action [12] and the spurion technique. We will see that the presence of the glueball degrees of freedom changes the vacuum state of the broken theory. As a result, the mass splittings are also modified.

The paper is organized as follows. In Sec. II we briefly review the generalized VY effective Lagrangian and recall some results obtained in Ref. [11]. In Sec. III we explain how one can reformulate the generalized VY Lagrangian in terms of two independent chiral superfields using the chiral-tensor superfield duality [15,16]. In Sec. IV we show how the effective action is modified when the gaugino mass term is introduced in SYM through the spurion method. Section V reports the masses and mixings for physical eigenstates of the broken theory.

II. THE GENERALIZED *VY* EFFECTIVE ACTION

The on-shell Lagrangian of SYM for an $SU(N_c)$ gauge group is

$$\mathcal{L}=\frac{1}{g^2}\left[-\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}+i\lambda^\dagger_{\dot\alpha}D^{\dot\alpha\beta}\lambda_\beta\right].$$

In terms of superfields the expression above can be written

$$\mathcal{L}=\int d^2\theta\frac{1}{8\pi}\mathrm{Im}\,\tau W^\alpha W_\alpha+\mathrm{H.c.},\tag{1}$$

where the gauge coupling is defined to be $\tau=(4\pi i/g^2)+(\theta_0/2\pi)$. For the purposes of this paper we set the theta term to be equal to zero, $\theta_0=0$.

The classical action of $N=1$ SYM theory is invariant under $U(1)_R$, scale and superconformal transformations. In the quantum theory these symmetries are broken by the chiral, scale and superconformal anomalies respectively. Composite operators that appear in the expressions for the anomalies can be thought of as component fields of a chiral supermultiplet S [18]:

$$S\equiv\frac{\beta(g)}{2g}W^\alpha W_\alpha\equiv A(y)+\sqrt{2}\theta\Psi(y)+\theta^2F(y),$$

where $\beta(g)$ is the SYM beta function for which the exact expression is known [19]. The lowest component of the S superfield is bilinear in gluino fields and has the quantum numbers of the scalar and pseudoscalar gluino bound states. The fermionic component in S describes the gluino-gluon composite and the F component of the chiral superfield includes operators corresponding to both the scalar and pseudoscalar glueballs ($G^2_{\mu\nu}$ and $G_{\mu\nu}\tilde{G}^{\mu\nu}$ respectively) [12].

Assuming that the effective action (more precisely, the generating functional for one-particle-irreducible (1PI) Green's functions [20]) of the model can be written in terms of the single superfield S, and requiring also that the effective action respects all the global continuous symmetries and reproduces the anomalies of the SYM theory, one derives the Veneziano-Yankielowicz effective action [12]. Let us mention that the actual variables, in terms of which the generating functional for the 1PI Green's functions (or effective action in our conventions) is written, are the vacuum expectation value (VEV's) of composite operators calculated at nonzero values of external sources [21]. In this paper, as well as in Ref. [11], we use a simpified notation where the VEV's are denoted by the corresponding composite operators.

It was noticed in Ref. [22] that the VY action does not respect the discrete Z_{2N_c} symmetry—the nonanomalous remnant of anomalous $U(1)_R$ transformations. The VY action was amended by an appropriate term which makes the action invariant under the discrete Z_{2N_c} group [22].[1]

However, as we mentioned above, the VY action does not include all possible lowest-spin bound states of SYM theory. Glueballs are missing in that description because they are only present in the auxilliary component of the S superfield

[1]The vacua with the broken chiral symmetry are labeled by an integer $n=0,\ldots,N_c-1$. In this work we study the spectrum of the model about the $n=0$ ground state.

and can be integrated out. In Ref. [11], in order to account for glueball degrees of freedom, we proposed to formulate the effective action in terms of a more general superfield, the real tensor superfield U [15]. The superfield U can be written in component form as follows:

$$U=B+i\theta\chi-i\overline{\theta\chi}+\frac{1}{16}\theta^2A^*+\frac{1}{16}\overline{\theta}^2A+\frac{1}{48}\theta\sigma^\mu\overline{\theta}\varepsilon_{\mu\nu\alpha\beta}C^{\nu\alpha\beta}+\frac{1}{2}\theta^2\overline{\theta}\left(\frac{\sqrt{2}}{8}\overline{\Psi}+\overline{\sigma}^\mu\partial_\mu\chi\right)+\frac{1}{2}\overline{\theta}^2\theta\left(\frac{\sqrt{2}}{8}\Psi-\sigma^\mu\partial_\mu\overline{\chi}\right)+\frac{1}{4}\theta^2\overline{\theta}^2\left(\frac{1}{4}\Sigma-\partial^2B\right). \quad (2)$$

It is straightforward to show that the real superfield U satisfies the relation[2]

$$S=-4\overline{D}^2U,$$

where the F term of the chiral supermultiplet S is related to the fields Σ and $C_{\mu\nu\alpha}$ in the following way:[3]

$$F=\Sigma+i\frac{1}{6}\varepsilon_{\mu\nu\alpha\beta}\partial^\mu C^{\nu\alpha\beta},$$

and A and Ψ are respectively the scalar and fermion components of the superfield S.

We argued [11] that the effective Lagrangian for the lowest-spin multiplets of the $N=1$ SYM theory can be written in terms of the U field only. That Lagrangian takes the following form [11]:

$$\mathcal{L}=\frac{1}{\alpha}(S^+S)^{1/3}|_D+\gamma\left[\left(S\log\frac{S}{\mu^3}-S\right)\Big|_F+\text{H.c.}\right]+\frac{1}{\delta}\left(-\frac{U^2}{(S^+S)^{1/3}}\right)_D, \quad (3)$$

where α and δ are arbitrary positive constants and $\gamma=-(N_cg/16\pi^2\beta(g))>0$. The F terms in this Lagrangian are fixed exactly by the anomalies [12]. However, the D terms cannot be determined explicitly by any symmetry considerations. Thus, generically, there are an infinite number of fields in this effective Lagrangian. The contribution which we keep here is the only quadratic D term in physical fields which is consistent with the $U(1)_R$, superconformal, and trace anomalies of the model. Thus, within the approximations discussed in the Introduction, this Lagrangian should define the qualitative properties of the spectrum of the lowest-spin states. Notice that the superfield S is not an independent variable in this Lagrangian. It is rather related to the U superfield through the formula

$$S-\langle S\rangle=-4\overline{D}^2U.$$

In the above equation we took into account that the S superfield has a nonzero VEV in the phase where chiral symmetry is broken, $\langle S\rangle\equiv\mu^3$. Thus, the only independent superfield in the Lagrangian (3) is the U field.

In this approach the following fields become dynamical [11]:

The B field propagates and it represents one massive real scalar degree of freedom (identified with the scalar glueball).

The three-form potential $C_{\mu\nu\alpha}$ describes one massive physical degree of freedom (identified with the pseudoscalar glueball).

The complex field A, being decomposed into parity eigenstates, describes the massive gluino-gluino scalar and pseudoscalar mesons.

χ and Ψ describe the massive gluino-gluon fermionic bound states.

Studying the potential of the model, we found that the physical eigenstates fall into two different mass "multiplets" (see Ref. [11] for details). Neither of them contain pure gluino-gluino, gluino-gluon or gluon-gluon bound states. Instead, the physical excitations are mixed states of these composites. The heavier set of states contains

A pseudoscalar meson, which without mixing reduces to the 0^{-+} gluino-gluino bound state (the analog of the QCD η' meson).

A scalar meson that without mixing is a 0^{++} $(l=1)$ gluino-gluino excitation.

A fermionic gluino-gluon bound state.

These heavier states form the chiral supermultiplet described by the VY action. That action is recovered in the $\delta\to\infty$ limit. The new states which appear as a result of our generalization form the lighter multiplet:

A scalar meson, which without mixing is a 0^{++} $(l=0)$ glueball.

A pseudoscalar state, which for zero mixing is identified as a 0^{-+} $(l=1)$ glueball.

A fermionic gluino-gluon bound state.

Notice, that although the physical states fall into multiplets whose J^P quantum numbers correspond to two chiral supermultiplets, the action was written in terms of the one real tensor supermultiplet U. In particular, the pseudoscalar glueball in this approach is described by the only physical component of the massive three-form potential $C_{\mu\nu\alpha}$. The field strength of that potential couples to the pseudoscalar

[2]Despite a seeming similarity, the tensor multiplet U should not be interpreted as a usual vector multiplet. The vector field which might be introduced in this approach as a Hodge dual of the three-form potential $C_{\mu\nu\alpha}$ would give mass terms with the wrong sign in our approach (see Ref. [11]), thus, the actual physical variable is the three-form potential $C_{\mu\nu\alpha}$ rather than its dual vector field (the Chern-Simons current).

[3]In this notation Σ is proportional to $G^2_{\mu\nu}$ and $\varepsilon_{\mu\nu\alpha\beta}\partial^\mu C^{\nu\alpha\beta}$ is proportional to $G_{\mu\nu}\tilde{G}^{\mu\nu}$ [11].

gluino-gluino bound state as it would couple to the η' meson in QCD [23].[4]

Since the physical spectrum of the mixed states fall into multiplets whose spin-parity quantum numbers correspond to two chiral supermultiplets, one might be wondering about the possibility of rewriting the whole action in terms of two different chiral superfields. If that is possible it would be crucial to study what peculiarities of the two-chiral-multiplet action allow it to be written in terms of only a real supermultiplet U, as was done in Ref. [11]. In the next section we address these questions.

III. THE TWO CHIRAL SUPERMULTIPLET ACTION

The relation between a real tensor and chiral supermultiplets (the so called chiral-linear duality) was established in Ref. [15]. For SYM theory the chiral-linear duality was used in Ref. [25] (see also discussions in Ref. [26]). Applied to our problem the results of Refs. [15,25,26] can be stated as follows. One introduces into the effective Lagrangian a new chiral superfield, let us denote it by χ:

$$\chi(y,\theta)\equiv\phi_\chi(y)+\sqrt{2}\,\theta\Psi_\chi(y)+\theta^2F_\chi(y). \tag{4}$$

One can find an effective Lagrangian written in terms of two chiral superfields, S and χ, which is equivalent to the expression given in Eq. (3). In our case

$$\mathcal{L}=\frac{1}{\alpha}(S^+S)^{1/3}|_D+\frac{\delta}{4}(S^+S)^{1/3}(\chi+\chi^+)^2|_D + \left[\gamma\left(S\log\frac{S}{\mu^3}-S\right)\Big|_F+\frac{1}{16}\chi(S-\mu^3)\Big|_F+\text{H.c.}\right]. \tag{5}$$

Comparing this expression to the VY Lagrangian one notices that both the Kähler potential and the superpotential are modified by new terms. The multiplets S and χ are independent.

We would like to relate this expression to the Lagrangian of the theory written in terms of the U field (3). If the U field is postulated as a fundamental degree of freedom, then the S field is a derivative superfield

$$S=\mu^3-4\bar{D}^2U. \tag{6}$$

Using this relation the Lagrangian (5) can be rewritten as

$$\mathcal{L}=\frac{1}{\alpha}(S^+S)^{1/3}|_D+\frac{\delta}{4}(S^+S)^{1/3}(\chi+\chi^+)^2|_D + \left[\gamma\left(S\log\frac{S}{\mu^3}-S\right)\Big|_F+\text{H.c.}\right]+U(\chi+\chi^+)|_D. \tag{7}$$

This expression depends on two superfields U and χ [S is expressed through U in accordance with Eq. (6)]. However, the dependence on the chiral superfield χ is trivial, the combination $\chi+\chi^+$ can be integrated out from the Lagrangian (7). As a result one derives

$$\chi+\chi^+=-\frac{2U}{\delta(S^+S)^{1/3}}. \tag{8}$$

Substituting this expression back into the Lagrangian (7) one arrives at the original expression (3) where the S field is a derivative field satisfying the relation (6).

Let us stress again that the descriptions in terms of the Lagrangian (3) and (5) are equivalent on the mass-shell. In the Lagrangian (3) the dynamical degrees of freedom are assigned to the only superfield U, while in the Lagrangian (5) the physical degrees of freedom are found as components of two chiral supermultiplets S and χ. The peculiarity of the expression (7) is that the chiral superfield χ enters only through the real combination $\chi+\chi^+$. That is why it was possible to formulate the action in terms of only the real superfield U. It is essential from a physical point of view since the component glueball field must be real.

Using the Lagrangian (5) one calculates the potential of the supersymmetric model. Integrating out the auxiliary fields of both chiral multiplets one finds

$$V_0=\frac{2}{\delta(16)^2}\frac{|\phi^3-\mu^3|^2}{|\phi|^2}+\frac{9\alpha|\phi|^4}{1-\dfrac{\alpha}{\delta}\dfrac{B^2}{|\phi|^4}}\cdot\left|\frac{\phi_\chi}{16}+3\gamma\log\frac{\phi}{\mu} + \frac{B(\phi^3-\mu^3)}{24\delta|\phi|^2\phi^3}\right|^2, \tag{9}$$

where the following notations are adopted:

$$\phi_\chi=\frac{1}{\sqrt{2}}(\sigma+i\pi),\qquad \sigma\equiv-\frac{\sqrt{2}B}{\delta|\phi|^2}.$$

The minimum of this potential is located at the point in field space where $\langle\phi\rangle=\mu$, $\langle B\rangle=\langle\phi_\chi\rangle=0$. The potential (9) is positive definite for field configurations satisfying $\alpha B^2<\delta|\phi|^4$. Since the VEV of the ϕ field is nonzero and the VEV of the B field is zero the positivity condition is satisfied for small oscillations about the SUSY minimum specified above. Notice that all SUSY field configurations are confined within a valley with infinite potential walls encountered at $\alpha B^2=\delta|\phi|^4$. Thus, the potential (9) and the Lagrangian (5) themselves describe only small oscillations about the SUSY minimum. In general, some higher order polynomials in the χ (or U) field could be present in the effective Lagrangian. In this work we are interested only in the mass spectrum of the model, so the approximation we used above is good enough for our goals.

In the next section we introduce soft SUSY breaking terms in the effective Lagrangian and study minima and the spectrum of the corresponding potential.

[4]The three-form potential proved to be useful for the description of the pseudoscalar glueball in conventional Yang-Mills theory [24].

IV. SOFT SUSY BREAKING

The gaugino mass term can be introduced in the Lagrangian (1) by means of the parameter τ. One regards τ as a chiral superfield [7]. A nonzero VEV of the F component of τ yields a SUSY breaking gaugino mass term in Eq. (1). Thus, one performs the following substitution in expression (1):

$$\tau \to \tau + F_\tau \theta\theta.$$

As a result, the following new term appears in the Lagrangian of SYM:

$$-\frac{1}{8\pi}\mathrm{Im}[F_\tau \lambda\lambda] + \mathrm{H.c.}$$

To make the gaugino mass canonically normalized one sets $F_\tau = i\,8\pi\, m_\lambda / g^2$. In the low-energy theory the τ parameter enters through the dynamically generated scale of the theory $\mu = \mu_0 \exp[-8\pi^2/\beta_0 g^2(\mu_0)] = \mu_0 \exp(i2\pi\tau/\beta_0)$. After the τ parameter is claimed to be a chiral superfield one should regard the μ parameter as a chiral superfield too. Thus, one also makes the following substitution in the low-energy effective Lagrangian of the model:

$$\mu \to \mu \exp\left(-\frac{16\pi^2 m_\lambda}{g^2\beta_0}\theta\theta\right),$$

where β_0 stands for the first coefficient of the beta function. Performing this redefinition of the μ parameter in the Lagrangian (5) one finds the following additional term in the scalar potential of the model:

$$\Delta V = -\tilde{m}_\lambda \mathrm{Re}\left(\frac{\mu^3}{16}\phi_\chi + \gamma\,\phi^3\right), \tag{10}$$

where $\tilde{m}_\lambda \equiv 32\pi^2/g^2 N_c\, m_\lambda$.

The expression (10) is the only correction to the effective potential to leading order in m_λ. All higher order corrections are suppressed by powers of m_λ/μ. Those corrections are neglected in this work.

V. THE MASS SPECTRUM

Having derived the potential of the broken theory one turns to the calculation of the mass spectrum. The potential consists of two parts, V_0 defined in Eq. (9) and the SUSY breaking term (10),

$$V = V_0 + \Delta V. \tag{11}$$

One calculates minima of the full scalar potential V. Explicit though tedious calculations yield the following results. The VEV of the ϕ field does not get shifted when the soft SUSY breaking terms are introduced. Thus, even in the broken theory $\langle\phi\rangle = \mu$. However, the ϕ_χ (and B) fields acquire nonzero VEV's in the broken case

$$\langle\phi_\chi\rangle = \frac{8}{9\alpha\mu}\tilde{m}_\lambda \quad \text{and} \quad \langle B\rangle = -\frac{8\delta}{9\alpha}\tilde{m}_\lambda\mu. \tag{12}$$

The shift of the vacuum energy causes the spectrum of the model to be also rearranged. Explicit calculations of the masses of all lowest-spin states yield the following results:

$$M^2_{scalar\,\pm} = M^2_\pm - \frac{3}{4}\alpha\gamma\mu\tilde{m}_\lambda(1 \pm \sqrt{1+x})\left(2 \pm \frac{1}{\sqrt{1+x}}\right), \tag{13}$$

$$M^2_{fermion\,\pm} = M^2_\pm - \frac{3}{4}\alpha\gamma\mu\tilde{m}_\lambda(1 \pm \sqrt{1+x})\left(3 \pm \frac{1}{\sqrt{1+x}}\right), \tag{14}$$

$$M^2_{p-scalar\,\pm} = M^2_\pm - \frac{3}{4}\alpha\gamma\mu\tilde{m}_\lambda(1 \pm \sqrt{1+x})\left(4 \pm \frac{1}{\sqrt{1+x}}\right), \tag{15}$$

where $M^2_\pm$ denote the masses in the theory with unbroken SUSY [11]:

$$M^2_\pm = \frac{18}{(16)^2}\frac{\alpha}{\delta}\mu^2 + \frac{81}{2}(\alpha\gamma)^2\mu^2[1 \pm \sqrt{1+x}] \quad \text{and}$$

$$x \equiv \frac{1}{288}\frac{\alpha}{\delta}\frac{1}{(\alpha\gamma)^2}. \tag{16}$$

In these expressions the plus sign refers to the heavier supermultiplet and the minus sign to the lighter set of states.[5] One can verify that these values satisfy the mass sum rule to leading order in $O(m_\lambda)$:

$$\sum_j (-1)^{2j+1}(2j+1)M_j^2 = 0,$$

where the summation goes over the spin j of particles in the supermultiplet.

Let us discuss the mass shifts given in Eqs. (13)–(15). Consider the light supermultiplet. In accordance with Eqs. (13)–(15), the masses in the light multiplet are increased in the broken theory. The biggest mass shift is found in the pseudoscalar channel. The smallest shift is observed in the scalar channel. The fermion mass falls in between these two meson states. Thus, the lightest state in the spectrum of the model is the particle which without mixing would have been the scalar glueball. There is a fermion state above that scalar. Finally, the pseudoscalar glueball is heavier than those two states.

[5] In Ref. [11] we used slightly different notation. Masses in the heavy supermultiplet were denoted by m_H and in the light supermultiplet by m_L, so $M^2_\pm \equiv m^2_{H,L}$. Note, that in the $\delta\to\infty$ limit the spectrum of our effective Lagrangian reduces to the VY spectrum. Naively, this seems not to be the case if one takes into consideration only Eqs. (13)–(15). However, one should recall that these masses are deduced for the fields rescaled by a factor δ [see Eq. (14) in [11] and the second expression in Eq. (12)]. A careful treatment of the limit $\delta\to\infty$ shows that all the glueballs are decoupled and one is left with the VY model [12].

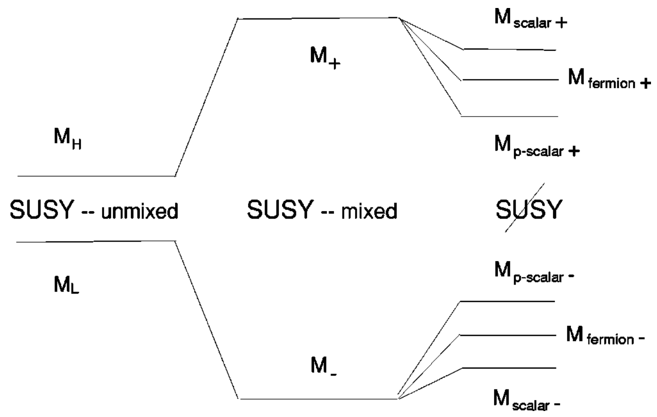

FIG. 1. Qualitative behavior of mass spectrum when passing from SYM to a softly broken model.

Let us now turn to the heavy supermultiplet. In the broken theory the masses in that multiplet get pulled down. However, all states of the heavy multiplet are still heavier than any state of the light multiplet in the domain of validity of our approximations. The ordering of the states in the heavy supermultiplet is just the opposite as in the light supermultiplet: the lightest state is the pseudoscalar meson, the heaviest is the scalar, and the fermion, as required, falls between them. The qualitative features of the spectrum are shown in Fig. 1.

It is not surprising that the lowest mass state obtained in Eqs. (13)–(15) is a scalar particle. This is in agreement with the result of Ref. [27] where it was shown that the mass of the lightest state which couples to the operator $G^2_{\mu\nu}$ is less than the mass of the lightest state that couples to $G\tilde{G}$, in pure Yang-Mills theory. As a result, the lightest glueball turns out to be the scalar glueball [27]. One can apply the method of Ref. [27] to the SYM theory as well. Due to the positivity of the gluino determinant (see Ref. [28]) one also deduces that the lightest state in softly broken SYM spectrum should be a scalar particle. The pseudoscalar of that multiplet is therefore heavier.

Our result that the multiplet containing glueballs is split in such a way that the scalar is lighter than the pseudoscalar, and vice versa for the multiplet containing gluino-gluino bound states, is consistent with expectations from quark-model lore. In ordinary mesons the $l=1$ states are heavier than their $l=0$ counterparts and the $l=0$ gluino-gluino bound state is a pseudoscalar, while an $l=0$ gluon-gluon bound state is a scalar. It is interesting that in SYM with massless gluinos the $l=0$ and $l=1$ bound states are degenerate, but when the gluino masses are turned on one recovers the expected ordering seen in $q\bar{q}$ states.

In the $\delta\to\infty$ limit one recovers the VY effective action. The spectrum of the softly broken VY Lagrangian was studied in [17]. In that limit only the heavy multiplet of the spectrum survives. It is interesting that in the limit $\delta\to\infty$, the ratio of the mass-shifts of the surviving states in Eqs. (13)–(15) is 5:4:3, which differs from the prediction of Ref. [17]. The seeming discrepancy is resolved because in the limit $\delta\to\infty$ the vacuum expectation value of the glueball field B tends to infinity. Thus, perturbing states about that vacuum is not a well defined procedure. The right way to obtain the $\delta\to\infty$ limit would be to decouple the ''glueball'' modes first, and then minimize the potential. This leads to a shift of the VEV of the ϕ field in the broken theory (as in Ref. [17]). As a result, the mass shifts calculated within this new vacuum state are in agreement with the values reported in the second work of Ref. [17]. We stress, however, that on physical grounds we do not expect SYM to realize the $\delta\to\infty$ limit of the general effective Lagrangian (5).

VI. SUMMARY AND DISCUSSION

In this paper we studied certain interesting features of the spectrum of the lowest-spin states of $N=1$ SUSY YM model applying an effective Lagrangian approach. Though this approach does not provide exact mass relations, nevertheless, it can be used to learn very important qualitative properties of the spectrum, such as the mass degeneracies in different multiplets in the exact SUSY limit and the corresponding mass splittings of states with different parity and spin-orbital quantum numbers for the broken SUSY case. These properties are crucial for recent lattice studies of the model [3].

In fact, we have shown that the generalized VY effective action can be written in two different ways. In one case the fundamental superfield upon which the action is constructed is the real tensor superfield U. In another approach all degrees of freedom of the model are described by two chiral superfields χ and S. In both cases the spectrum consists of two multiplets which are not degenerate in masses even when SUSY is unbroken. The spin-parity quantum numbers of these multiplets are identical to those of certain chiral supermultiplets.

The physical mass eigenstates are not pure gluon-gluon, gluon-gluino or gluino-gluino composites; rather, the physical particles are mixtures of them. The multiplet which without mixing would have been the glueball multiplet is lighter. As a result, those states cannot be decoupled from the effective Lagrangian. This means that comparisons of lattice results to analytic predictions based on the original VY action are not justified.

We introduced a soft SUSY breaking term in the Lagrangian of the $N=1$ SUSY Yang-Mills model. The spurion method was used to calculate the corresponding soft SUSY breaking terms in the generalized VY Lagrangian. These soft breaking terms cause a shift of the vacuum energy of the model. The physical eigenstates, which are degenerate in the SUSY limit, are split when SUSY breaking is introduced. We studied these mass splittings in detail. We have confirmed that the spectrum of the broken theory is in agreement with some low-energy theorems [27], namely the scalar glueball turns out to be lighter than the pseudoscalar one. The results of the present paper can be directly tested in lattice studies of $N=1$ supersymmetric Yang-Mills theory ([3]).

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