

Axion in an external electromagnetic field

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We investigate the effective interaction of a pseudoscalar particle with two photons in an external electromagnetic field in the general case of all external particles being off the mass shell. This interaction is used to study the radiative decay $a \rightarrow \gamma\gamma$ of the axion, a pseudoscalar particle associated with the Peccei-Quinn symmetry, and the crossing channel of photon splitting $\gamma \rightarrow \gamma a$. It is shown that the external field removes the suppression associated with the smallness of the axion mass and so leads to the strong catalysis of these processes. The field-induced axion emission by the photon is analyzed as one more possible source of energy loss by astrophysical objects. [S0556-2821(99)02011-1]

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I. INTRODUCTION

For quite a long time there has been interest in various extensions of the standard model (SM) with a nonstandard set of Higgs particles. In the simplest version of the SM one needs only one Higgs doublet. However, there are reasons to believe that if scalar bosons exist at all, their number may be significant. The Higgs sector can be so rich that a situation becomes possible when one or several Goldstone degrees of freedom are not absorbed by the Higgs mechanism and appear as physical massless particles. Such Goldstone (or pseudo Goldstone) bosons, whose existence is related to the breakdown of exact (approximate) symmetries, is also of interest from the experimental point of view: they could be observed in low-energy experiments. Such particles include, e.g., axions [1,2], Majorons [3], arions [4]. The most widely discussed pseudo Goldstone boson, the axion, associated with the Peccei-Quinn (PQ) symmetry continues to be an attractive solution to the problem of CP conservation in strong interactions [5]. Although the original axion, associated with the spontaneous breakdown of the global PQ symmetry at the weak scale, f_w , is excluded experimentally, modified versions of PQ models with their associated axions are of great interest. If the breaking scale of the PQ symmetry f_a is much larger than the electroweak scale $f_a \gg f_w$, the resulting ‘‘invisible axion’’ is very weakly interacting (coupling constant $\sim f_a^{-1}$), very light ($m_a \sim f_a^{-1}$) and very long lived. The axion lifetime in vacuum is gigantic:

$$\tau(a \rightarrow 2\gamma) \sim 6.3 \times 10^{48} \text{ s} \left(\frac{10^{-3} \text{ eV}}{m_a} \right)^6 \left(\frac{E_a}{1 \text{ MeV}} \right). \quad (1)$$

The allowed range for the axion mass is strongly constrained by astrophysical and cosmological considerations which leave a rather narrow window [6–9]:¹

$$10^{-5} \text{ eV} \lesssim m_a \lesssim 10^{-2} \text{ eV}, \quad (2)$$

¹However in Ref. [10] a possibility to solve the CP problem of QCD within a grand unified theory model with a heavy axion $m_a \lesssim 1 \text{ TeV}$ is considered.

where axions could exist and provide a significant fraction or all of the cosmic dark matter. A survey of various processes involving the production of weakly coupled particles and astrophysical methods for obtaining constraints on the parameters of axion models is given in Ref. [11].

The important role which axions play in elementary particle physics stimulates constant interest in theoretical and experimental studies. Along with laboratory searches [12,13], stars are a unique laboratory to investigate physical processes [11]. For example, the upper bound on m_a is obtained from the requirement that stars not lose too much energy by axions. In studies of processes occurring inside astrophysical objects one has to take into account that dense matter substantially influences particle properties. An intensive electromagnetic field also plays the role of an active medium modifying the dispersion relations of particles so that novel processes are not only opened kinematically, but become substantial as well. Photon splitting $\gamma \rightarrow \gamma\gamma$ [14] and the Cherenkov process $\nu \rightarrow \nu\gamma$ [15,16] are among them.

Actually, both components of the active medium, plasma and a magnetic field, are presented in most astrophysical objects. A situation is also possible when the magnetic component dominates. For example, in a supernova explosion or in a coalescence of neutron stars a region outside the neutrinosphere of order of several tens of kilometers with a strong magnetic field and a rather rarefied plasma could exist. The possible existence of astrophysical objects with $B \sim 10^{15} - 10^{17} \text{ G}$, significantly above the critical, Schwinger value $B_e = m_e^2/e \approx 4.41 \times 10^{13} \text{ G}$, was discussed both for toroidal [17] and for poloidal [18] fields.

In this paper we investigate a field-induced effective interaction of a pseudoscalar particle with two photons described by three-point loop diagrams. By now the detailed studies of the analogous three-point loop vertex of the photons’ interaction in external electromagnetic fields are known only. The history of these investigations goes back to the pioneer paper by Adler [14] and it is still in progress [19–22].

Note that amplitudes of processes in an external field depend not only on the kinematical invariants of type m^2 or p^2 , but also on field invariants $|e^2(FF)|^{1/2}$, $|e^2(\vec{F}\vec{F})|^{1/2}$, $|e^2(pFFp)|^{1/3}$ where m and p_μ are the mass and the four-

momentum of a particle, $F_{\mu\nu}$ is the external field tensor, and $(pFFp) = p^\mu F_{\mu\nu} F^{\nu\rho} p_\rho$. In the ultrarelativistic limit the field invariant $|e^2(pFFp)|^{1/3}$ can occur as the largest one. This is due to the fact that in the relativistic particle rest frame the field may turn out to be of order of the critical one or even higher, appearing very close to the constant crossed field ($\mathbf{E} \perp \mathbf{B}, E = B$), where $(FF) = (F\tilde{F}) = 0$. Thus, calculations in this field are the relativistic limit of calculations in an arbitrary relatively smooth field.

In Sec. II we present the result of our calculations of the $a\gamma\gamma$ vertex in the external crossed field in a general case when the external particles are off the mass shell. In Sec. III we discuss photon and axion dispersion relations in the crossed field and analyze the influence of the field-induced ‘‘effective masses’’ of the particles on kinematics. It is shown that a novel channel of the photon decay $\gamma \rightarrow \gamma + a$ is opened kinematically. In Sec. IV the effective $a\gamma\gamma$ vertex is used to study the axion decay into two photons $a \rightarrow \gamma + \gamma$. A catalyzing influence of the external field on this process is analyzed. In Sec. V we study the photon decay $\gamma \rightarrow \gamma + a$. The energy loss by a photon gas due to this process is investigated as an additional source of the axion luminosity in a supernova explosion. In the appendixes, the important details of the calculation of the $a\gamma\gamma$ vertex are collected.

II. $a\gamma\gamma$ VERTEX IN THE CROSSED FIELD

Invisible axion models are classified into two types, depending on whether or not axions couple to leptons (see, e.g., [5,11]). Here we investigate the field-induced $a\gamma\gamma$ interaction of ‘‘Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axions’’ [23] which couple with both quarks and leptons at the tree level.

The Lagrangian describing the axion-fermion interaction is

$$\mathcal{L}_{af} = -ig_{af}(\bar{f}\gamma_5 f)a, \quad (3)$$

where $g_{af} = C_f m_f / f_a$ is a dimensionless Yukawa coupling constant, C_f is a model-dependent factor [11], m_f is the fermion’s mass, γ_5 is the Dirac γ matrix, and a and f are the axion and fermion fields.

The two-photon axion interaction is investigated in a general case when the photons are not assumed to be on the mass shell. It means that the expression for the $a\gamma\gamma$ vertex can be used as an effective Lagrangian of axion-photon interaction in studies of processes involving axion or other pseudoscalar particles with a coupling of type (3). In the third order of the perturbation theory the matrix element of the effective $a\gamma\gamma$ interaction in an external field is described by two three-point loop diagrams (Fig. 1), where double lines imply that the influence of the external field in the propagators of virtual charged fermions is taken into account exactly.

The general form for the matrix element corresponding to the diagrams in Fig. 1 is

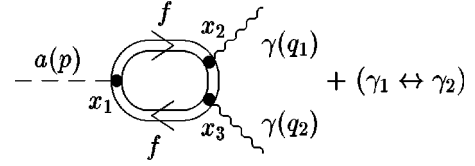


FIG. 1. The effective vertex of the interaction of a pseudoscalar particle with two photons. Double lines correspond to fermion propagators in an external electromagnetic field.

$$\begin{aligned} S = & \frac{4\pi\alpha Q_f^2 g_{af}}{\sqrt{2E_a V \times 2\omega_1 V \times 2\omega_2 V}} \\ & \times \int d^4x_1 d^4X d^4Y \exp\{i(x_1(q_1 + q_2 - p))\} \\ & \times \exp\left(-i\left[(q_1 X) + (q_2 Y) + \frac{eQ_f}{2}(XFY)\right]\right) \\ & \times \text{Sp}\{S(Y)(\varepsilon_2 \gamma)S(X - Y)(\varepsilon_1 \gamma)S(-X)\gamma_5\} \\ & + (\varepsilon_1, q_1 \leftrightarrow \varepsilon_2, q_2), \end{aligned} \quad (4)$$

where $\alpha = 1/137$ is the fine-structure constant; $e > 0$ is the elementary charge, Q_f is a relative fermion charge in the loop; p is the four-momentum of the decaying axion, $q_{1,2} = (\omega, \mathbf{q})_{1,2}$ and $\varepsilon_{1,2}$ are the four-momenta of the final photons and their polarization four-vectors, respectively; $(\varepsilon \gamma) = \varepsilon_\mu \gamma^\mu$, γ_μ are the Dirac γ matrices; $X = x_1 - x_2$, $Y = x_1 - x_3$; $S(X)$ is the translationally invariant part of the fermion propagator in the crossed field (Appendix A).

Further we will calculate a vertex function Λ connected with the matrix element Eq. (4) in the following way:

$$S = \frac{i(2\pi)^4 \delta^{(4)}(p - q_1 - q_2)}{\sqrt{2E_a V \times 2\omega_1 V \times 2\omega_2 V}} \Lambda. \quad (5)$$

The function Λ , thus defined, is the effective Lagrangian of the axion-photon interaction in the momentum representation. The calculation technique of the three-point vertex $a\gamma\gamma$ in the crossed field is given in Appendixes B and C. In the following, we will consider only the field-induced contribution $\Lambda^{(F)} = \Lambda - \Lambda^{(0)}$, where $\Lambda^{(0)}$ is the vacuum part of Λ , to the effective Lagrangian, corresponding to the transition $a(p) \rightarrow \gamma(q_1) + \gamma(q_2)$:

$$\begin{aligned} \Lambda^{(F)} = & \frac{\alpha}{\pi} \sum_f \frac{Q_f^2 g_{af}}{m_f} \left[(f_1 \tilde{f}_2) J_0 + (f_1 \tilde{f}_2) J_1 + (f_1 \tilde{f}_2) (f_2 \mathcal{F}) J_2 \right. \\ & + (f_2 \tilde{f}_1) (f_1 \mathcal{F}) \tilde{J}_2 + (f_1 \tilde{f}_2) \frac{(q_2 f_2 \mathcal{F} \mathcal{F} q_2)}{(q_2 \mathcal{F} \mathcal{F} q_2)} J_3 \\ & \left. + (f_2 \tilde{f}_1) \frac{(q_1 f_1 \mathcal{F} \mathcal{F} q_1)}{(q_1 \mathcal{F} \mathcal{F} q_1)} \tilde{J}_3 \right], \end{aligned} \quad (6)$$

$$\mathcal{F}_{\mu\nu} = \frac{F_{\mu\nu}}{F_f}, \quad \tilde{\mathcal{F}}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} \mathcal{F}_{\alpha\beta}, \quad F_f = \frac{m_f^2}{eQ_f}, \quad (7)$$

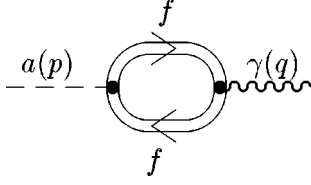


FIG. 2. The diagram describing the effective axion-photon interaction.

$$f_{i\alpha\beta} = q_{i\alpha}\varepsilon_{i\beta} - q_{i\beta}\varepsilon_{i\alpha}, \quad \tilde{f}_{i\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}f_{i\mu\nu}, \quad i=1,2,$$

$$J_0 = i \int_0^\infty dz \int_D dx dy [\cos \varphi_2 e^{-i\Phi} - e^{-iz\varphi_0}],$$

$$J_1 = 2 \int_0^\infty z dz \int_D dx dy \left[1 - \left(1 + \frac{\chi}{\chi_1} \right) x - \left(1 + \frac{\chi}{\chi_2} \right) y \right] \sin \varphi_2 e^{-i\Phi},$$

$$J_2 = \frac{i\chi}{\chi_2} \int_0^\infty z^2 dz \int_D dx dy y(1-2x-y) \cos \varphi_2 e^{-i\Phi},$$

$$J_3 = \int_0^\infty z dz \int_D dx dy (1-2x) \sin \varphi_2 e^{-i\Phi}, \quad (8)$$

$$\tilde{J}_i = J_i(q_1 \rightarrow q_2), \quad i=2,3,$$

$$\Phi = z\varphi_0 + \frac{z^3}{3}\varphi_1,$$

$$\varphi_0 = 1 - 2xy \frac{(q_1 q_2)}{m_f^2} - y(1-y) \frac{q_1^2}{m_f^2} - x(1-x) \frac{q_2^2}{m_f^2},$$

$$\begin{aligned} \varphi_1 &= [x(1-x)|\chi_2| - y(1-y)|\chi_1|]^2 + 2xy(1-x-y)^2 \\ &\times (|\chi_1\chi_2| - \chi_1\chi_2) + 2xy[xy + (x+y)(1-x-y)] \\ &\times (|\chi_1\chi_2| + \chi_1\chi_2) \geq 0, \end{aligned}$$

$$\varphi_2 = -2z^2xy(1-x-y) \frac{(q_1 \mathcal{F} q_2)}{m_f^2},$$

$$\chi^2 = \frac{(p\mathcal{F}p)}{m_f^2}, \quad \chi_i^2 = \frac{(q_i \mathcal{F} q_i)}{m_f^2}, \quad i=1,2.$$

The integration area D in the integrals J_i is $x, y \geq 0$, $x+y \leq 1$.

We note that the vertex $\Lambda^{(F)}$ (6) contains as a limit an amplitude of the field-induced effective $a\gamma$ interaction which can be used in studies of axion-photon oscillations. The amplitude of the $a(p) \rightarrow \gamma(q)$ transition (the diagram is shown in Fig. 2) is obtained from Eq. (6) by means of replacing the field tensor of one of the photons by the electromagnetic field tensor:

$$q_{1\alpha} \rightarrow q_\alpha, \quad f_{1\alpha\beta} \rightarrow f_{\alpha\beta}, \quad q_{2\alpha} \rightarrow 0, \quad f_{2\alpha\beta} \rightarrow -iF_{\alpha\beta}.$$

As a result, the amplitude of the $a \rightarrow \gamma$ transition can be presented in the form:²

$$M = -\frac{i\alpha}{2\pi} (f\tilde{F}) \sum_f \frac{Q_f^2 g_{af}}{m_f} [I - I(F=0)], \quad (9)$$

$$I = \left(\frac{4}{\chi} \right)^{2/3} \int_0^{\pi/2} f(\eta) \sin^{-1/3} \phi d\phi,$$

$$f(\eta) = i \int_0^\infty du \exp \left\{ -i \left(\eta u + \frac{u^3}{3} \right) \right\},$$

$$\eta = \left(1 - \frac{q^2 \sin^2 \phi}{4m_f^2} \right) \left(\frac{4}{\chi \sin^2 \phi} \right)^{2/3},$$

where $f(\eta)$ is the Hardy-Stokes function and χ is the dynamic parameter of the axion defined in Eq. (8).

III. FIELD-INDUCED ‘EFFECTIVE MASSES’ OF THE PARTICLES

The vertex $a\gamma\gamma$ Eq. (6) can be used as an effective axion-photon Lagrangian in studies of processes such as the radiative decay $a \rightarrow \gamma\gamma$, photon splitting $\gamma \rightarrow \gamma a$, and the Primakoff-type process for photoproduction of pseudoscalars on electrons $\gamma + e \rightarrow e + a$.

We note that in a homogeneous electromagnetic field the energy-momentum conservation law for axion-photon processes coincides formally with the vacuum one (5). However, in calculating the probabilities of the processes one has to integrate over the phase space of the final particles taking into account their nontrivial field-induced kinematics. This is due to the fact that an external field plays the role of a peculiar medium with dispersion and absorption. In our case the external crossed field plays the role of a homogeneous anisotropic medium. The photon ‘effective masses’ squared μ_λ^2 induced by the external field are defined as the eigenvalues of the photon polarization operator:

$$\Pi_{\mu\nu} = i \sum_{\lambda=1}^3 \mu_\lambda^2 \frac{b_\mu^{(\lambda)} b_\nu^{(\lambda)}}{b^{(\lambda)2}}, \quad b_\alpha^{(\lambda)} b_\alpha^{(\lambda')} = \delta_{\lambda\lambda'} (b_\alpha^{(\lambda)})^2. \quad (10)$$

Here $b_\mu^{(\lambda)}$ are the polarization operator eigenvectors described in Appendix D. We stress that only two eigenmodes of the photon propagation with polarization vectors

$$\varepsilon_\alpha^{(1)} = \frac{b_\alpha^{(1)}}{\sqrt{-b^{(1)2}}} = \frac{(qF)_\alpha}{\sqrt{(qFFq)}},$$

²For a numerical evaluation note that the external field tensor $F_{\mu\nu}$ is presented here in rationalized natural units.

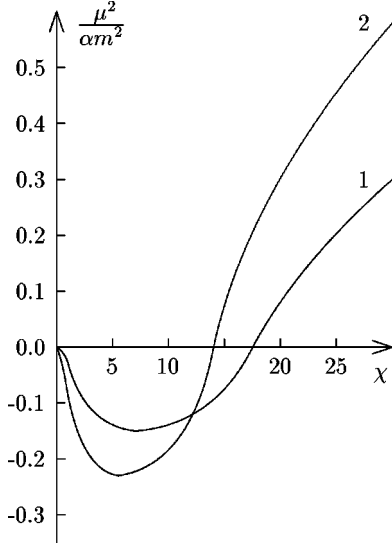


FIG. 3. The photon dispersion curves in the crossed field.

$$\varepsilon_\alpha^{(2)} = \frac{b_\alpha^{(2)}}{\sqrt{-b^{(2)2}}} = \frac{(q\tilde{F})_\alpha}{\sqrt{(q\tilde{F}\tilde{F}q)}}, \quad (11)$$

are realized in an external electromagnetic field. The analysis of the photon polarization operator $\Pi_{\mu\nu}$ in a one-loop approximation in the crossed field [24] shows that the dispersion curves corresponding to the above-mentioned photon eigenmodes (Fig. 3), though being alike in their qualitative behavior, are different quantitatively. It is seen from Fig. 3 that each dispersion curve has a negative minimum and changes its sign with increasing the dynamic parameter χ .

The fact that both of the photon eigenmodes have negative ‘‘squared masses’’ in the region of $\chi \lesssim 15\text{--}18$ means that two-photon decay is kinematically opened even for massless pseudoscalars (e.g., arions [4]). The difference in values of the field-induced ‘‘squared masses’’ of the first and second photon eigenmodes makes possible the process of the photon splitting $\gamma \rightarrow \gamma a$, where a is an arbitrary, relatively light pseudoscalar.

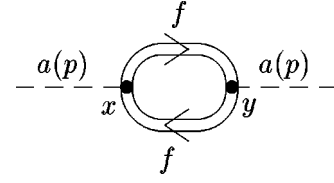
In general, it is necessary to analyze the influence of the external field on the axion mass m_a . The field-induced contribution to m_a is connected with the real part of the amplitude ΔM of $a \rightarrow f\bar{f} \rightarrow a$ transition via the fermion loop by the relation:

$$\delta m_a^2 = -\text{Re } \Delta M. \quad (12)$$

To obtain a correct result for ΔM one has to use the Lagrangian with the derivative as it was first emphasized by Raffelt and Seckel [25]:

$$\mathcal{L}_{af} = \frac{g_{af}}{2m_f} (\bar{f}\gamma_\mu\gamma_5 f) \partial_\mu a. \quad (13)$$

In the second order of the perturbation theory the amplitude $a \rightarrow f\bar{f} \rightarrow a$ is described by the diagram in Fig. 4 and can be presented in the form:


 FIG. 4. The diagram of the axion transition $a \rightarrow a$ in an external electromagnetic field.

$$\Delta M = -i \frac{g_{af}^2}{4m_f^2} \int d^4 Z e^{-i(qZ)} \text{Sp}[S(-Z)(q\gamma)\gamma_5 S(Z) \times (q\gamma)\gamma_5], \quad (14)$$

where $Z = x - y$. The integration with respect to the four-coordinate Z is reduced to the generalized Gaussian integrals of the type (C6) from Appendix C and results in

$$\Delta M = \frac{g_{af}^2}{16\pi^2} \left\{ 2q^2 \int_0^{\pi/2} \sin \phi d\phi \int_0^\infty (e^{-i\Phi} - e^{-i\Phi_0}) \frac{dt}{t} - m_f^2 \chi^2 \int_0^{\pi/2} \sin^3 \phi d\phi \int_0^\infty e^{-i\Phi} t dt \right\}, \quad (15)$$

$$\Phi = \Phi_0 + \frac{t^3 \chi^2 \sin^4 \phi}{48}, \quad \Phi_0 = t \left(1 - \frac{q^2 \sin^2 \phi}{4m_f^2} \right).$$

The main contribution to δm_a^2 has the following asymptotic behavior at large values of the dynamic parameter:

$$\delta m_a^2 \simeq \frac{g_{af}^2 m_f^2}{24\pi^{3/2}} \Gamma^2(2/3) (6\chi)^{2/3}, \quad (16)$$

and permits one to estimate the field-induced correction to the axion mass:

$$\frac{\delta m_a^2}{m_a^2} \simeq 10^{-9} \left(\frac{C_f m_f^2}{1 \text{ MeV}^2} \right)^2 \chi^{2/3}. \quad (17)$$

Here we used that m_a and g_{af} are connected with the PQ symmetry-breaking scale f_a by virtue of the relationships $m_a = 0.62 \text{ eV} \times (10^7 \text{ GeV}/f_a)$ and $g_{af} = C_f m_f/f_a$ [11]. Equation (17) shows that we can neglect the influence of an external electromagnetic field on m_a .

IV. THE AXION DECAY $a \rightarrow \gamma + \gamma$

A. The decay amplitude

The expression for the effective $a\gamma\gamma$ interaction (6) is used to study the ultrarelativistic ($E_a \gg m_a$) axion decay $a(p) \rightarrow \gamma(q_1) + \gamma(q_2)$. Because of the smallness of the axion mass ($m_a \lesssim 10^{-2} \text{ eV}$), even axions with the energy of several eV are ultrarelativistic. Assuming that the field-induced photon ‘‘masses’’ are also small in comparison with their energies, below we will consider photons as massless. This case corresponds to the collinear kinematics:

$$p_\mu \sim q_{1\mu} \sim q_{2\mu}.$$

Before starting to study the physically most interesting case of the ultrarelativistic axion decay let us analyze all tensor structures of the effective axion-photon vertex (6), which define the decay amplitude, in an attempt to reveal the dominating contribution. Note that for real photons the terms with J_3 and \tilde{J}_3 are suppressed by the smallness of the field-induced photon ‘‘masses’’ ($q_i f_i)_\alpha \sim \mu^2 \ll E_a^2$. It is natural to denote the other terms in Eq. (6) as $M^{(0)}$, $M^{(1)}$, and $M^{(2)}$ depending on the power of the external field tensor.

Due to the Lorentz invariance the amplitude can be analyzed in any frame. We will analyze it in the rest frame of the decaying axion with the components of the four-momenta p , q_1 , and q_2 being of order m_a . In this case it is sufficient to allow for the order of the dimensional quantities $M^{(0)}$, $M^{(1)}$, and $M^{(2)}$:

$$\begin{aligned} M^{(0)} &\sim \frac{1}{m_f} (f_1 \tilde{f}_2) \sim \frac{m_a^2}{m_f}, \\ M^{(1)} &\sim \frac{1}{m_f^3} (f_1 \tilde{F} f_2) \sim \frac{m_a^2}{m_f^3} F', \\ M^{(2)} &\sim \frac{1}{m_f^5} (f_1 \tilde{F})(f_2 F) \sim \frac{m_a^2}{m_f^5} F'^2, \end{aligned} \quad (18)$$

where F' stands for the strengths of the magnetic and electric fields in the axion rest frame. In this case the electromagnetic field in Eq. (18) is obtained by the Lorentz transformation from the laboratory frame, in which the external field F is given, to the rest frame of the decaying axion:

$$F' \sim \frac{E_a}{m_a} F \gg F. \quad (19)$$

In view of Eq. (19), the expressions (18) can be written in the form

$$\begin{aligned} M^{(0)} &\sim \frac{m_a^2}{m_f}, \\ M^{(1)} &\sim \frac{m_a E_a}{m_f^3} F, \\ M^{(2)} &\sim \frac{E_a^2}{m_f^5} F^2. \end{aligned} \quad (20)$$

The following is seen from Eqs. (20).

(1) The external electromagnetic field affects substantially the ultrarelativistic axion decay because the field reduces the suppression caused by the smallness of the axion mass. A similar catalyzing effect of the external electromagnetic fields of various configurations on the radiative decay of the massive neutrino was discovered in [26,27].

(2) The contribution to the amplitude bilinear in the external field dominates because it does not contain the above-mentioned suppression factor.

With regard to the analysis performed only terms bilinear in the external field should be kept in the amplitude of the ultrarelativistic axion decay:

$$M \simeq \frac{\alpha}{\pi} \sum_f \frac{Q_f^2 g_{af}}{m_f} \left\{ \frac{\chi}{\chi_2} (f_1 \tilde{F})(f_2 F) J + (\varepsilon_1, q_1 \leftrightarrow \varepsilon_2, q_2) \right\}, \quad (21)$$

$$\begin{aligned} J(\chi_1, \chi_2) &= \int_0^1 dx \int_0^{1-x} dy y(1-y-2x) \eta^3 [1 - \eta f(\eta)], \\ \eta &= \{ [x(1-x)\chi_2 - y(1-y)\chi_1]^2 + 4xy \\ &\quad \times [xy + (1-x-y)(x+y)] \chi_1 \chi_2 \}^{-1/3}, \end{aligned} \quad (22)$$

where $f(\eta)$ is the Hardy-Stokes function defined in Eq. (9), χ , χ_1 , and χ_2 are the dynamic parameters determined in Eq. (8). We have neglected terms of order m_a^2/E_a^2 , μ_i^2/E_a^2 in the argument of the function $f(\eta)$.

B. The decay probability

The probability of the field-induced decay of the axion has the following form:

$$\begin{aligned} W^{(F)} &= \frac{1}{2!} \frac{1}{32\pi^2 E_a} \int \frac{d^3 q_1 d^3 q_2}{\omega_1 \omega_2} \delta^{(4)}(p - q_1 - q_2) \sum_{\lambda, \lambda'} |M|^2 \\ &= \frac{1}{32\pi E_a} \int_0^1 dt \sum_{\lambda, \lambda'} |M|^2. \end{aligned} \quad (23)$$

The factor $1/2!$ in the integration over the final state takes into account the identity of two final photons. The process considered is two-body decay, but its amplitude is not a constant as in vacuum because it depends on the external electromagnetic field tensor. That is why the calculation of the decay probability is not reduced to multiplying the matrix element squared with the final phase space. Under the summation of the polarizations λ , λ' in Eq. (23) it is convenient to use the polarization vectors (11). The expression for the probability is

$$W^{(F)} = \frac{1}{\pi E_a} \left(\frac{\alpha}{\pi} \right)^2 \int_0^1 dt t^2 \left| \sum_f Q_f^2 g_{af} m_f \chi^2 J[t\chi, (1-t)\chi] \right|^2. \quad (24)$$

With the strong hierarchy of the fermion masses,

$$\frac{\chi_{f_1}^2}{\chi_{f_2}^2} = \left(\frac{m_{f_2}}{m_{f_1}} \right)^6 \gg 1,$$

the contribution from the fermion with the maximum value of the dynamic parameter $\chi^2 = e_f^2(pFFp)/m_f^6$ can dominate in Eq. (24):

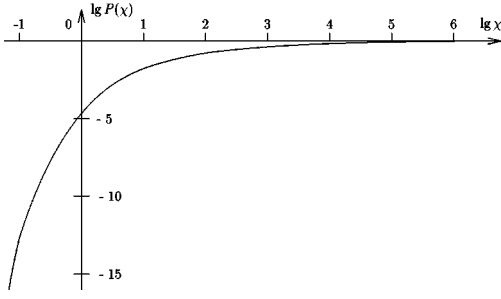


FIG. 5. The plot of the decay factor $P(\chi)$ as a function of the dynamic parameter χ .

$$W^{(F)} \approx 3.32 \left(\frac{\alpha}{\pi} \right)^2 \frac{(Q_f^2 g_{af} m_f)^2}{\pi E_a} P(\chi). \quad (25)$$

The plot of $P(\chi)$ is shown in Fig. 5. We also give the asymptotic behavior of the function $P(\chi)$ for both small and large values of the dynamic parameter χ :

$$P(\chi)|_{\chi \ll 1} \approx 2.31 \times 10^{-5} \chi^8 (1 + 9.5 \chi^2 + \dots),$$

$$P(\chi)|_{\chi^{1/3} \gg 1} \approx 1 - \frac{8.8}{\chi^{1/3}} + \frac{32.0}{\chi^{2/3}} - \dots.$$

As it is seen from the asymptotic behavior of the function $P(\chi)$ at large χ , the decay probability (25) does not depend on the dynamic parameter. Note that a similar behavior of the amplitude and probability at $\chi \gg 1$ takes place in the photon splitting $\gamma \rightarrow \gamma\gamma$ [19].

To illustrate the catalyzing influence of the external field on the ultrarelativistic axion decay, let us compare Eq. (25) with the well-known axion decay probability in vacuum [7,11]:

$$W_0 = \frac{g_{a\gamma}^2 m_a^4}{64 \pi E_a}. \quad (26)$$

Here $g_{a\gamma} = (\alpha/2\pi f_a) \times (E/N - 1.92 \pm 0.08)$ [11], where E and N are model-dependent coefficients of the electromagnetic and color anomalies. A comparison of Eqs. (25) and (26),

$$R_f = \frac{W^{(F)}}{W_0} \approx 2.12 \times 10^2 \left(\frac{\alpha Q_f g_{af} m_f}{\pi g_{a\gamma} m_a^2} \right)^2 P(\chi), \quad (27)$$

demonstrates a strong catalyzing influence of the external field on the ultrarelativistic axion decay ($E_a \gg m_a$), because $W^{(F)}$ has no suppression factor associated with the smallness of the axion mass, except the coupling constants ($g_{af}, g_{a\gamma} \sim f_a^{-1}$). Let us estimate in Eq. (27) the contribution from the electron in the loop because this fermion is the most sensitive to the external field:

$$R_e \approx 10^{37} \left(\frac{\cos^2 \beta}{E/N - 1.92} \right)^2 \left(\frac{10^{-3} \text{ eV}}{m_a} \right)^4 P(\chi). \quad (28)$$

Here $\cos^2 \beta$ determines the electron Yukawa coupling $g_{ae} = \frac{1}{3} \cos^2 \beta (m_e/f_a)$ in the DFSZ model [23]. Large values of the dynamic parameter $\chi \gg 1$, when $P(\chi) \sim 1$, can be realized, for example, in the case of the decaying axion energy ~ 10 MeV and the magnetic-field strength $B \sim B_e \approx 4.41 \times 10^{13}$ G.

V. PHOTON SPLITTING $\gamma \rightarrow \gamma + a$

As it was pointed in Sec. III the external field has a substantial influence on the photon dispersion relation so that the photon splitting $\gamma \rightarrow \gamma a$ is allowed kinematically due to the difference in the field-induced effective photon ‘‘masses.’’ This process can play a role as a possible additional mechanism of energy loss by astrophysical objects. As this process is a crossing channel of the axion decay $a(p) \rightarrow \gamma(q_1) + \gamma(q_2)$ and its amplitude can be easily obtained from the amplitude (21) using the replacement:

$$q_1 \rightarrow -q_1, \quad p \rightarrow -p, \quad (29)$$

which corresponds to

$$f_1 \rightarrow -f_1, \quad \chi \rightarrow -\chi, \quad \chi_1 \rightarrow -\chi_1. \quad (30)$$

The kinematics of this process is close to the collinear one as well as in the ultrarelativistic axion decay. Note that in studying this process in different ranges of the dynamic parameter one should take into account the specific dispersion behavior of the photon modes in the initial and final states.

A. The limit of small values of the dynamic parameter $\chi_1 \ll 1$

In the case of small values of the initial photon dynamic parameter χ_1 the splitting of the photon with the first polarization³ is allowed kinematically only due to the condition $\mu_1^2 > \mu_2^2$ (see Fig. 3):

$$\gamma^{(1)} \rightarrow \gamma^{(2)} + a. \quad (31)$$

With the photon polarization vectors (11) the amplitude can be presented in the form:

$$M \approx -\frac{4\alpha}{\pi} t(1-t) \sum_f Q_f^2 g_{af} m_f \chi_1^2 J(t\chi_1, \chi_1), \quad (32)$$

where $t = \omega_2/\omega_1$ is the relative energy of the final photon. The function J defined in Eq. (22) has the following asymptotic behavior at small values of its arguments:

$$J(t\chi_1, \chi_1)|_{\chi_1 \ll 1} \approx \frac{2}{63} \chi_1^2 (1-2t) + O(\chi_1^4).$$

The probability of the photon splitting is

³Numbering of the photon types is made by their polarizations in accordance with Eq. (11).

$$W^{(F)} = \frac{1}{16\pi\omega_1} \int_0^1 dt |M|^2$$

$$\approx 4.8 \times 10^{-6} \left(\frac{\alpha}{\pi} \right)^2 \frac{\left(\sum_f Q_f^2 g_{af} m_f \chi_1^4 \right)^2}{\pi\omega_1}. \quad (33)$$

To illustrate a possible application of the result obtained we estimate the contribution of this process to the axion emissivity Q_a of the photon gas:

$$Q_a = \int \frac{d^3 q_1}{(2\pi)^3} \omega_1 n_B(\omega_1) \int_0^1 dt \frac{dW^{(F)}}{dt} (1-t) [1 + n_B(\omega_1 t)], \quad (34)$$

where $n_B(\omega_1)$ and $n_B(\omega_1 t)$ are the Planck distribution functions of the initial and final photons at temperature T , respectively. In Eq. (34) we have taken into account that the photon of only one polarization (the first one in this case) splits. Note that the dynamic parameter χ_1 in $dW^{(F)}/dt$ depends on the photon energy ω_1 and the angle θ between the initial photon's momentum \mathbf{q}_1 and the magnetic-field strength \mathbf{B} :

$$\chi_1 = \frac{\omega_1}{m_f} \frac{B}{B_f} \sin \theta,$$

where $B_f = m_f^2/eQ_f$ is the critical value of the magnetic-field strength for the given fermion. Neglecting the photon ‘‘effective masses’’ squared ($\omega_1^2 = |\mathbf{q}_1|^2 + \mu_1^2 \approx |\mathbf{q}_1|^2$), the result of the calculation of the axion emissivity (34) can be presented in the form

$$Q_a \approx C \frac{\alpha^2}{\pi^5} \left(\sum_f \frac{Q_f^2 g_{af}}{m_f^3 B_f^4} \right)^2 T^{11} B^8. \quad (35)$$

The numerical factor C is expressed in terms of an infinite sum of the Hurwitz ζ functions $\zeta(k, z) = \sum_{n=0}^{\infty} 1/(n+z)^k$:

$$C = \frac{1}{63^2} \int_0^\pi \sin^9 \theta d\theta \int_0^1 dt t^2 (1-t)^3 (1-2t)^2$$

$$\times \int_0^\infty \frac{x^{10} dx}{(e^x - 1)(1 - e^{-xt})}$$

$$= \frac{2^{13}}{63^2} \sum_{n=1}^{\infty} \left(\frac{\zeta(3, n)}{n^8} - \frac{4\zeta(5, n)}{7n^6} + \frac{3\zeta(7, n)}{7n^4} \right)$$

$$= 2.15145,$$

where $x = \omega_1/T$ is the relative energy of the decaying photon.

Let us estimate the contribution of the photon splitting $\gamma \rightarrow \gamma a$ into the axion luminosity in a supernova explosion from a region of order of a hundred kilometers in size outside the neutrinosphere. In this region a rather rarefied

plasma with the temperature of order of MeV and the magnetic field of order 10^{13} G can exist. Under these conditions the axion luminosity is

$$L_a \approx 3 \times 10^{36} \frac{\text{erg}}{\text{s}} \left(\frac{g_{ae}}{10^{-13}} \right)^2 \left(\frac{T}{1 \text{ MeV}} \right)^{11}$$

$$\times \left(\frac{B}{10^{13} \text{ G}} \right)^8 \left(\frac{R}{10^3 \text{ km}} \right)^3. \quad (36)$$

The comparison of Eq. (36) with the total neutrino luminosity $L_\nu \sim 10^{52}$ erg/s from the neutrinosphere shows that the contribution of the photon splitting to the energy loss is negligibly small. Here we used the strongest restriction on the axion coupling constant with electrons $g_{ae} \sim 10^{-13}$ [8,11,28].

B. The limit of large values of the dynamic parameter $\chi_1 \gg 1$

At large values of the dynamic parameter χ_1 the field-induced ‘‘effective mass’’ squared of the photon of the second polarization becomes greater than that of the photon of the first polarization ($\mu_2^2 > \mu_1^2$, see Fig. 3). In this case the channel

$$\gamma^{(2)} \rightarrow \gamma^{(1)} + a \quad (37)$$

is kinematically opened only. The process amplitude at large χ_1 is

$$M \approx \frac{4\alpha}{\pi} (1-t) \sum_f Q_f^2 g_{af} m_f I(t), \quad (38)$$

$$I(t) = \int_D \frac{y(1-y-2x) dx dy}{[x(1-x)t - y(1-y)]^2 + 4xyt(1-x-y)^2},$$

where the integration is carried out over the area D : $x, y \geq 0$, $x+y \leq 1$. It is seen from Eq. (38) that the amplitude does not depend on the dynamic parameter.

With the amplitude (38) the splitting probability has the form:

$$W^{(F)} \approx 2.5 \left(\frac{\alpha}{\pi} \right)^2 \frac{\left(\sum_f Q_f^2 g_{af} m_f \right)^2}{\pi\omega_1}. \quad (39)$$

In real astrophysical objects with strong magnetic fields the temperature $T \geq 10$ MeV can exist in the central regions where from both components of the active medium, the magnetic field and a plasma, the plasma component substantially dominates. Thus, the expressions obtained are not applied for the estimation of the luminosity of astrophysical objects because we have taken into account the influence of the external field only.

VI. CONCLUSION

In this work we have investigated the effective $a\gamma\gamma$ interaction (a is a pseudoscalar particle) induced by an exter-

nal constant crossed field. Calculations of processes in this electromagnetic field configuration are very general, being a relativistic limit of calculations in an arbitrary weak smooth field. For the pseudoscalar particle we considered the axion, the most widely discussed particle corresponding to the spontaneous breaking of the PQ symmetry.

In the third order of the perturbation theory, the field-induced vertex $a\gamma\gamma$ (6) was obtained for the case when all the particles were off the mass shell. This vertex can be used as the effective Lagrangian of the $a\gamma\gamma$ interaction in the investigation of processes involving axion or other pseudoscalar particles with a coupling of the type (3).

As a specific example of the way this vertex can be used we considered the axion decay $a \rightarrow \gamma + \gamma$ in an external field. The dominant contribution to this process (not suppressed by the axion mass) is presented in Eq. (21). The decay probability (25) is calculated in the limits of both large and small values of the dynamical parameter χ . Comparison of Eq. (25) with the probability of the two-photon axion decay in vacuum shows a strong effect of the external field ($\sim 10^{37}$) on the decay of the ultrarelativistic axion ($E_a \gg m_a$).

As another example, we have studied the photon splitting $\gamma \rightarrow \gamma + a$ which could be of interest as an additional mechanism of energy losses by astrophysical objects. This forbidden in vacuum process becomes kinematically possible because photons of different polarizations obtain different field-induced effective ‘‘squared masses.’’ At the same time an external field influence on the axion mass is negligible. The axion luminosity estimated for supernova conditions turn out to be small in comparison with the neutrino luminosity.

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APPENDIX A: THE PROPAGATOR OF A CHARGED FERMION IN A CROSSED FIELD

A charged fermion propagator $S^{(F)}(x,y)$ in the external crossed field in the proper time formalism [29,30] can be presented in the relativistically invariant form [27]

$$S^{(F)}(x,y) = e^{i\Omega(x,y)} S(X), \quad (\text{A1})$$

$$\Omega(x,y) = -e_f \int_y^x d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi-y)^\nu \right],$$

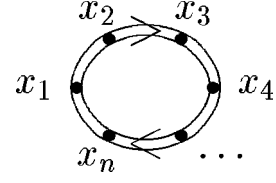


FIG. 6. n -point loop fermion diagram with cut legs in an external electromagnetic field.

$$\begin{aligned} S(X) = & -\frac{i}{16\pi^2} \int_0^\infty \frac{ds}{s^2} \left[\frac{1}{2s} (X\gamma) - \frac{ie_f}{2} (X\tilde{F}\gamma) \gamma_5 \right. \\ & \left. - \frac{se_f^2}{3} (XFF\gamma) + m_f + \frac{sm_f e_f}{2} (\gamma F\gamma) \right] \\ & \times \exp \left[-i \left(sm_f^2 + \frac{X^2}{4s} + \frac{se_f^2}{12} (XFFX) \right) \right], \end{aligned}$$

where $X_\mu = (x-y)_\mu$; m_f and e_f are the mass and charge of the virtual fermion ($e_f = eQ_f$, $e > 0$ is the elementary charge, Q_f is the relative fermion charge); A_μ , $F_{\mu\nu}$, and $\tilde{F}_{\mu\nu}$ are the four-potential, tensor, and dual tensor of the crossed field; $(X\gamma) = X_\mu \gamma^\mu$, γ^μ , γ_5 are the Dirac γ matrices.

The inclusion of translationally noninvariant factors $\Omega(x_i, x_{i+1})$ arising from the corresponding propagators $S^{(F)}(x_i, x_{i+1})$ in calculations of n -point loop diagrams is discussed in detail in Appendix B.

APPENDIX B: INCLUSION OF TRANSLATIONALLY NONINVARIANT FACTORS $\Omega(x,y)$ OF FERMION PROPAGATORS IN n -POINT LOOP DIAGRAM

In this Appendix we discuss the inclusion of the translationally noninvariant factor $\Omega(x,y)$ from the propagator $S^{(F)}(x,y)$ Eq. (A1) in the calculation of n -point loop diagrams. In particular, the diagrams in Fig. 1 describing the effective $a\gamma\gamma$ interaction are three-point loop diagrams.

The four-potential $A_\mu(x)$ of a constant external electromagnetic field can be written as

$$A_\mu = \frac{1}{2} (xF)_\mu = \frac{1}{2} x^\nu F_{\nu\mu}. \quad (\text{B1})$$

With Eq. (B1), the expression for $\Omega(x_1, x_2)$ is substantially simplified:

$$\begin{aligned} \Omega(x_1, x_2) = & -e_f \int_{x_2}^{x_1} d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi-x_2)^\nu \right] \\ = & \frac{e_f}{2} (x_1 F x_2). \end{aligned} \quad (\text{B2})$$

The contribution from the translationally noninvariant factors $\Omega(x_i, x_{i+1})$ Eq. (B2) to the amplitude of an n -point diagram in Fig. 6 is generally given by

$$\Omega_{\text{tot}} = \frac{e_f}{2} \sum_{i=1}^n (x_i F x_{i+1}) \Big|_{x_{n+1}=x_1} = \frac{e_f}{4} \sum_{i=1}^{n-1} (Z_i F Z_{i+1}),$$

$$Z_i = x_i - x_{i+1}, \quad Z_n = x_n - x_1, \quad \sum_{i=1}^n Z_i = 0. \quad (\text{B3})$$

We note that, while for each propagator $S^{(F)}(x_i, x_{i+1})$ the phase $\Omega(x_i, x_{i+1})$ is generally translationally and gauge non-invariant, the total phase of the n -point fermion loop Ω_{tot} Eq. (B3) only depends on the difference of the coordinates and satisfies the requirement of gauge and translational invariance.

The simplest situation is realized for the two-point loop diagram when the total phase is zero:

$$\Omega_{\text{tot}} = \Omega(x_1, x_2) + \Omega(x_2, x_1) = -\frac{e_f}{4}(XFX) = 0,$$

$$X = x_1 - x_2.$$

For the three-point loop diagram studied in this paper we have

$$\Omega_{\text{tot}} = \Omega(x_1, x_3) + \Omega(x_3, x_2) + \Omega(x_2, x_1) = -\frac{e_f}{2}(XFY),$$

$$X = x_1 - x_2, \quad Y = x_1 - x_3.$$

APPENDIX C: CALCULATION OF THE VERTEX FUNCTION Λ

In this Appendix we give the basic stages of the calculation of the three-point vertex function Λ describing the effective $a\gamma\gamma$ interaction (a is an arbitrary pseudoscalar particle) in the external crossed field. After integration with respect to the coordinate x_1 in Eq. (4) and separation of the delta-function $\delta^{(4)}(p - q_1 - q_2)$, the expression for Λ can be represented according to Eq. (5) as

$$\Lambda = -ie_f^2 g_{af} \int d^4X d^4Y \exp\left(-i\left\{(q_1X) + (q_2Y) + \frac{e_f}{2}(XFY)\right\}\right) \text{Sp}\{S(Y)(\varepsilon_2\gamma)S(X-Y)(\varepsilon_1\gamma)S(-X)\gamma_5\} + (\varepsilon_1, q_1 \leftrightarrow \varepsilon_2, q_2). \quad (\text{C1})$$

With the explicit expression for the propagators (A1) the vertex function is

$$\Lambda = \frac{\alpha}{\pi} \sum \frac{Q_f^2 g_{af} m_f}{8(4\pi)^4} \int_0^\infty \frac{ds dv d\tau}{s^2 v^2 \tau^2} e^{-im_f^2(s+v+\tau)} \int d^4X d^4Y \left\{ ((X-Y)R_1X) + (YR_2(Y-X)) + (YR_3X) - \frac{m_f^2}{4}R_0 \right\} \times \exp\left[-i\left\{(q_1X) + (q_2Y) + \frac{X^2}{4}\left(\frac{1}{v} + \frac{1}{\tau}\right) + \frac{Y^2}{4}\left(\frac{1}{s} + \frac{1}{v}\right) - \frac{(XY)}{2v} + \frac{e_f^2}{12}(v+\tau)(XFFX) + \frac{e_f^2}{12}(s+v)(YFFY) - \frac{e_f^2 v}{6}(XFFY) - \frac{e_f}{2}(XFY)\right\}\right] + (\varepsilon_1, q_1 \leftrightarrow \varepsilon_2, q_2), \quad (\text{C2})$$

where s , v , and τ are proper time variables defined by three propagators of the form (A1), $X = x_1 - x_2$, $Y = x_1 - x_3$. In Eq. (C2) all the dependence on four-coordinates X and Y is shown explicitly in the preexponential factor and in the exponent, so that R_0 and the tensors $(R_i)_{\alpha\beta}$ ($i=1,2,3$) are functions of proper time variables and the constant external field tensor:

$$R_0 = \text{Sp}\{\gamma_5 h_1(s)(\varepsilon_2\gamma)h_1(v)(\varepsilon_1\gamma)h_1(\tau)\},$$

$$(R_1)_{\alpha\beta} = \text{Sp}\{\gamma_5 h_1(s)(\varepsilon_2\gamma)h_{2\alpha}(v)(\varepsilon_1\gamma)h_{2\beta}(\tau)\},$$

$$(R_2)_{\alpha\beta} = \text{Sp}\{\gamma_5 h_1(\tau)h_{2\alpha}(s)(\varepsilon_2\gamma)h_{2\beta}(v)(\varepsilon_1\gamma)\},$$

$$(R_3)_{\alpha\beta} = \text{Sp}\{\gamma_5 h_1(v)(\varepsilon_1\gamma)h_{2\beta}(\tau)h_{2\alpha}(s)(\varepsilon_2\gamma)\},$$

$$h_1(s) = 2 + e_f s (\gamma F \gamma),$$

$$h_{2\alpha}(s) = \frac{1}{2s} \gamma_\alpha - \frac{e_f^2 s}{3} (\gamma F F)_\alpha + \frac{ie_f}{2} (\gamma \tilde{F})_\alpha \gamma_5.$$

The integration with respect to the four-coordinates X and Y in Eq. (C2) reduces to the calculation of the generalized Gaussian integrals of three types: scalar, vector, and tensor of rank two. The calculation method of such integrals which arise in the integration of the two-point loop diagrams is given in Ref. [27]. However the integration with respect to the coordinates is significantly simplified by formally introducing the eight-coordinate Z and the eight-momentum Q defined in terms of the four-coordinates X , Y and the four-momenta of photons q_1 , q_2 in the following way:

$$Z_\mu = \begin{pmatrix} X_\mu \\ Y_\mu \end{pmatrix}, \quad Q_\mu = \begin{pmatrix} q_{1\mu} \\ q_{2\mu} \end{pmatrix}. \quad (\text{C3})$$

In terms of the generalized coordinate Z_μ and the generalized momentum Q_μ the vertex function (C2) can be written in the most compact form:

$$\begin{aligned} \Lambda &= \frac{\alpha}{\pi} \sum_f \frac{Q_f^2 g_{af} m_f}{8(4\pi)^4} \int_0^\infty \frac{ds dv d\tau}{s^2 v^2 \tau^2} e^{-im_f^2(s+v+\tau)} \\ &\times \int d^8 Z e^{-i[1/4(ZGZ)+(QZ)]} [R_0 + (ZNZ)] \\ &+ (\varepsilon_1, q_1 \leftrightarrow \varepsilon_2, q_2). \end{aligned} \quad (C4)$$

The matrices G and N can be presented as the Kronecker product of (2×2) matrices and Lorentz tensors of rank two $[(4 \times 4)$ -matrices]:

$$\begin{aligned} G_{\alpha\beta} &= g_{\alpha\beta} G_1 + e_f F_{\alpha\beta} G_2 + e_f^2 (FF)_{\alpha\beta} G_3, \quad (C5) \\ G_1 &= \frac{1}{sv\tau} \begin{pmatrix} s(v+\tau) & -s\tau \\ -s\tau & \tau(s+v) \end{pmatrix}, \\ G_2 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad G_3 = \frac{1}{3} \begin{pmatrix} v+\tau & -v \\ -v & s+v \end{pmatrix}, \\ N_{\alpha\beta} &= N_1(R_1)_{\alpha\beta} + N_2(R_2)_{\alpha\beta} + N_3(R_3)_{\alpha\beta}, \\ N_1 &= \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}, \quad N_2 = \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}, \quad N_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \end{aligned}$$

Thus, the integration with respect to the coordinate Z_μ in Eq. (C4) has been reduced to the calculation of the generalized Gaussian integrals of two types—the scalar and tensor of rank two:

$$\begin{aligned} J &= \int d^8 Z e^{-i[1/4(ZGZ)+(QZ)]} \\ &= -(4\pi)^4 \frac{s^2 v^2 \tau^2}{(s+v+\tau)^2} e^{i(QG^{-1}Q)}, \quad (C6) \end{aligned}$$

$$\begin{aligned} T_{\mu\nu} &= \int d^8 Z Z_\mu Z_\nu e^{-i[1/4(ZGZ)+(QZ)]} = -\frac{\partial^2 J}{\partial Q^\mu \partial Q^\nu} \\ &= [4(QG^{-1})_\mu (G^{-1}Q)_\nu - 2iG_{\mu\nu}^{-1}]J, \end{aligned}$$

where G^{-1} is the inverse of the matrix (C5):

$$\begin{aligned} G_{\alpha\beta}^{-1} &= g_{\alpha\beta} \tilde{G}_1 + e_f F_{\alpha\beta} \tilde{G}_2 + e_f^2 (FF)_{\alpha\beta} \tilde{G}_3, \quad (C7) \\ \tilde{G}_1 &= \frac{1}{z} \begin{pmatrix} \tau(s+v) & s\tau \\ s\tau & s(v+\tau) \end{pmatrix}, \quad \tilde{G}_2 = \frac{sv\tau}{z} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \\ (\tilde{G}_3)_{11} &= \frac{\tau^2}{3z^2} [z(4sv - s^2 - v^2) - 6sv\tau], \\ (\tilde{G}_3)_{12} = (\tilde{G}_3)_{21} &= \frac{s\tau}{3z^2} [v^3 + s^3 + \tau^3 + 3s\tau v - z(s^2 + \tau^2)], \\ (\tilde{G}_3)_{22} &= \frac{s^2}{3z^2} [z(4v\tau - \tau^2 - v^2) - 6sv\tau]. \end{aligned}$$

Here the variable z is defined as $z = s + v + \tau$.

APPENDIX D: EXPANSION IN THE BASIS IN AN EXTERNAL ELECTROMAGNETIC FIELD

Calculations are significantly simplified by using a basis which is constructed of the photon four-momentum q_μ and the external electromagnetic field tensor $F_{\mu\nu}$ [19]:

$$b_\mu^{(1)} = (qF)_\mu, \quad b_\mu^{(2)} = (q\tilde{F})_\mu, \quad (D1)$$

$$b_\mu^{(3)} = q^2 (qFF)_\mu - q_\mu (qFFq), \quad b_\mu^{(4)} = q_\mu.$$

In the specified basis a polarization vector of an arbitrary photon is

$$\begin{aligned} \varepsilon_\mu &= \frac{(q\varepsilon)}{q^2} q_\mu - \frac{(Ff)}{2(qFFq)} (Fq)_\mu - \frac{(\tilde{F}f)}{2(qFFq)} (\tilde{F}q)_\mu \\ &- \frac{(qfFFq)}{q^2 (qFFq)^2} [q^2 (FFq)_\mu - (qFFq) q_\mu], \quad (D2) \end{aligned}$$

where $f_{\alpha\beta} = q_\alpha \varepsilon_\beta - q_\beta \varepsilon_\alpha$ is the photon field tensor. Note that the last term in Eq. (D2) is not equal to zero only for virtual photons.

Taking into account Eq. (D2), we can reduce all structures of the type $(l_1 f l_2)$ that appear in the expression for the vertex function Λ to the form:

$$\begin{aligned} (l_1 f l_2) &= (fF) \frac{(l_1 q)(qF l_2) - (l_2 q)(qF l_1)}{2(qFFq)} \\ &+ (f\tilde{F}) \frac{(l_1 q)(q\tilde{F} l_2) - (l_2 q)(q\tilde{F} l_1)}{2(qFFq)} \\ &+ \frac{(l_2 q)(qfFF l_1) - (l_1 q)(qfFF l_2)}{(qFFq)}, \quad (D3) \end{aligned}$$

where l_1 and l_2 are arbitrary four-vectors. Below we give some structures for which we used the representation (D3):

$$\begin{aligned} (q_1 f_2 FF q_1) &= \frac{(q_1 FF q_2)}{2(q_2 FF q_2)} [(f_2 F)(q_1 F q_2) + (f_2 \tilde{F}) \\ &\times (q_1 \tilde{F} q_2)], \quad (D4) \end{aligned}$$

$$(q_1 f_2 FF q_2) = \frac{1}{2} [(f_2 F)(q_1 F q_2) + (f_2 \tilde{F})(q_1 \tilde{F} q_2)],$$

$$(q_1 f_2 F q_2) = \frac{1}{2} (q_1 q_2) (f_2 F),$$

where $(f_i)_{\alpha\beta}$ is the electromagnetic field tensor of the i th photon ($i=1,2$).

Note that the following relationship:

$$(T_1 \tilde{T}_2)_{\mu\nu} + (T_2 \tilde{T}_1)_{\mu\nu} = \frac{1}{2} (T_1 \tilde{T}_2) g_{\mu\nu} \quad (\text{D5})$$

proves very helpful between two arbitrary antisymmetric tensors $(T_1)_{\mu\nu}$ and $(T_2)_{\mu\nu}$. If for tensors T_1 and T_2 we take the constant electromagnetic field tensor $T_1 = T_2 = F$, which

meets the condition $(F\tilde{F}) = 0$, then from Eq. (D5) the condition on the electromagnetic field tensor F follows automatically,

$$(F\tilde{F})_{\mu\nu} = 0.$$

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