Small-x F_2 structure function of a nucleus including multiple Pomeron exchanges

Yuri V. Kovchegov
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 13 January 1999; published 25 June 1999)

We derive an equation determining the small-x evolution of the F_2 structure function of a large nucleus which includes all multiple Pomeron exchanges in the leading logarithmic approximation using Mueller's dipole model. We show that in the double leading logarithmic limit this evolution equation reduces to the Gribov-Levin-Ryskin equation. [S0556-2821(99)04013-8]

PACS number(s): 12.38.Bx, 12.38.Cy, 24.85.+p

I. INTRODUCTION

The problem of understanding the large gluon density regime in high-energy scattering has always been one of the challenges of perturbative QCD (PQCD) [1–6]. Unitarity of the total cross section and saturation of the gluon distribution are among the most important issues related to the problem. The Balitskiĭ-Fadin-Kuraev-Lipatov (BFKL) equation [7,8] is the only well-established tool of PQCD which allows us to explore this high-density region by resumming the leading longitudinal logarithmic contribution to the scattering process. In BFKL evolution the small-x partons are produced overlapping each other in the transverse coordinate space [9], therefore creating high-density regions in the hadron's wave function (hot spots). The next-to-leading order correction to BFKL equation has been calculated recently [10,11]. Although the final conclusion one should draw from the calculation of [10,11] is still to be understood, there are some serious problems associated with the interpretation of the result [12–14]. However, we are not going to address these issues in this paper for the following reason. As was shown in [12,13] the effects of the second-order BFKL kernel become important in hadron-hadron scattering at the rapidities of the order of $Y_{\mathrm{NLO}} \sim 1/\alpha^{5/3}$, with α being the strong-coupling constant. At the same time the unitarity constraints, associated with the multiple (leading order) hard Pomeron exchanges are expected to be reached at $Y_U \sim (1/\alpha)\ln(1/\alpha)$ [15], which is parametrically smaller than Y_{NLO} for small coupling constant. Therefore, multiple pomeron exchanges become important at lower center-of-mass energies than the effects of subleading corrections, possibly leading to unitarization of the total hadron-hadron cross section. Hence the problem of resummation of the multiple pomeron exchanges seems to be more important for describing the recent experimental results, such as ZEUS 1995 data [16], which probably shows evidence of saturation of the F_2 structure function at low Q^2.

In this paper we are going to consider deep inelastic scattering (DIS) of a virtual photon on a large nucleus and will resum all multiple pomeron exchanges contributing to the F_2 structure function of the nucleus in the leading longitudinal logarithmic approximation in the large-N_c limit. The first step in that direction in PQCD was the derivation by Gribov, Levin, and Ryskin (GLR) of an equation describing the fusion of two pomeron ladders into one in the double logarithmic approximation [5]. The resulting equation with the coefficient in front of the quadratic term calculated by Mueller and Qiu [6] for a low density picture of a spherical proton of radius R reads

$$\frac{\partial^2 xG(x,Q^2)}{\partial \ln(1/x)\, \partial \ln(Q^2/\Lambda^2_{QCD})} = \frac{\alpha N_c}{\pi} xG(x,Q^2) - \frac{4\alpha^2 N_c}{3 C_F R^2} \frac{1}{Q^2} [xG(x,Q^2)]^2. \tag{1}$$

This equation sums up all multiple hard Pomeron exchanges in the gluon distribution function in the double logarithmic limit.

Since then there have been several attempts to generalize the GLR equation. Recently an equation has been proposed by Ayala, Gay Ducati, and Levin in [17,18], which tries to incorporate the Glauber-type multiple rescatterings of a probe on the nucleons in a nucleus (see Fig. 1). Using the results of Mueller in [19] for a pair of gluons multiply rescattering inside a nucleus, the authors of [17,18] obtained the following equation for the gluon distribution of the nucleus in the double logarithmic approximation:

$$\frac{\partial^2 xG_A(x,Q^2)}{\partial \ln(Q^2/\Lambda^2_{\mathrm{QCD}})\, \partial \ln(1/x)} = \frac{N_c C_F S_\perp}{\pi^3} Q^2 \times \left\{ 1 - \exp\left[- \frac{2\alpha\pi^2}{N_c S_\perp} \times \frac{1}{Q^2} xG_A(x,Q^2) \right] \right\}. \tag{2}$$

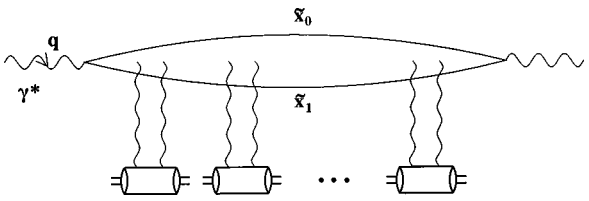

FIG. 1. Forward amplitude of DIS on a nucleus in the quasiclassical (Glauber) approximation.

If one expands the right-hand side of Eq. (2) to the second order in xG_A one recovers the GLR equation [Eq. (2) is written here for a cylindrical nucleus. Therefore the coefficients in the obtained GLR equation will not match those of Eq. (1)].

An extensive work on resumming the multiple Pomeron exchanges in the gluon distribution function in the leading $\ln(1/x)$ approximation (i.e., without taking the double logarithmic limit) has been pursued by Jalilian-Marian, Kovner, Leonidov, McLerran, Venugopalan, and Weigert [20–25]. Starting with a model of a large nucleus [20,26], which provides some effective action [20,21], they develop a renormalization-group procedure which integrates out harder longitudinal gluonic degrees of freedom in the nucleus and allows one to resum the leading $\ln(1/x)$ contribution to the gluon distribution function. The resulting equation, written in a functional form in [23] is supposed to resum these leading logarithms including also all powers of color charge density of the nucleus, which in a more traditional language corresponds to resummation of multiple Pomeron exchanges. However, even though that equation at the lowest (one Pomeron) level reduces to the expected BFKL equation [22], in general it is a very complicated functional differential equation which cannot be solved even numerically. Recently the double logarithmic limit of that equation was obtained [25], providing us with another equation for xG:

$$\frac{\partial^2 xG(x,Q^2)}{\partial \ln(Q^2/\Lambda^2_{\rm QCD})\,\partial \ln(1/x)} = \frac{N_c(N_c-1)}{2} S_\perp Q^2 \left[1 + \frac{\pi(N_c-1)Q^2 S_\perp}{2\alpha xG} \times \exp\left(\frac{\pi(N_c-1)Q^2 S_\perp}{2\alpha xG} \right) Ei\left(- \frac{\pi(N_c-1)Q^2 S_\perp}{2\alpha xG} \right) \right]. \tag{3}$$

Equation (3) is written here for a cylindrical nucleon with transverse area $S_\perp$. In the limit of small gluon density Eq. (3) reduces to GLR equation [25].

Our approach in this paper is pursuing the same goals as the authors of [20–25]. We will also write an equation which resums all multiple Pomeron exchanges on a nucleus in the leading logarithmic approximation. However our strategy is a bit different from [20–25]. We will consider the scattering of a virtual photon on a nucleus at rest, therefore putting all the QCD evolution in the wave function of the virtual photon. This is different from what was done by the authors of [20–25], since they were developing the QCD evolution inside the nucleus. The virtual photon's wave function including the leading logarithmic evolution was constructed in the large-N_c limit by Mueller in [1–4]. This so-called dipole wave function in fact contains all multiple pomeron exchanges, which in the large-N_c language can be pictured as multiple color ''cylinders.'' A numerical analysis of the unitarization of the total onium-onium cross section through multiple pomeron exchanges was carried out in the framework of the dipole model by Salam in Ref. [27]. Considering the scattering of a virtual photon (quarkonium state) on a nucleus simplifies the problem, allowing us to treat it analytically. In Sec. II we will use the dipole wave function to write down an equation which governs the evolution of the F_2 structure function of the nucleus [formula (15)]. Our equation is directly related to a physical observable (F_2) and, therefore, is free from all the problems and ambiguities associated with dealing with the gluon distribution function xG. It is a nonlinear integral equation, not a functional differential equation like in [22]. Therefore, one should be able to solve Eq. (15) at least numerically.

We will dedicate Sec. III to exploring the equation resulting from taking double logarithmic (large-Q^2) limit of the equation derived in Sec. II. We see that our equation reduces to the GLR equation, failing to reproduce Eqs. (2), (3). Finally, in Sec. IV we will conclude by discussing the limitations of the large-N_c approximation, as well as some advantages of our approach.

II. EVOLUTION EQUATION FROM THE DIPOLE MODEL

We start by considering a deep inelastic scattering process on a nucleus. As shown in Fig. 1, the incoming virtual photon with a large q_+ component of the momentum splits into a quark-antiquark pair which then interacts with the nucleus at rest. We model the interaction by no more than two gluon exchanges between each of the nucleons and the quark-antiquark pair. This is done in the spirit of the quasiclassical approximation used previously in [19,28,29]. The interactions are taken in the eikonal approximation. Then, as could be shown in general, i.e., including the leading logarithmic QCD evolution, the total cross section, and, therefore, the F_2 structure function of the nucleus can be rewritten as a product of the square of the virtual photon's wave function and the propagator of the quark-antiquark pair through the nucleus [30,31].The expression reads [30]

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\rm EM}} \int \frac{d^2\mathbf{x}_{01} dz}{2\pi} [\Phi_T(\mathbf{x}_{01},z) + \Phi_L(\mathbf{x}_{01},z)]\; d^2b_0\; N(\mathbf{x}_{01},\mathbf{b}_0,Y), \tag{4}$$

where the incoming photon with virtuality Q splits into a quark-antiquark pair with the transverse coordinates of the quark and antiquark being $\tilde{\mathbf{x}}_0$ and $\tilde{\mathbf{x}}_1$ correspondingly, such that $\mathbf{x}_{10}=\tilde{\mathbf{x}}_1-\tilde{\mathbf{x}}_0$. The coordinate of the center of the pair is given by $\mathbf{b}_0=\frac{1}{2}(\tilde{\mathbf{x}}_1+\tilde{\mathbf{x}}_0)$. Y is the rapidity variable $Y=\ln s/Q^2=\ln 1/x$. The square of the light cone wave function of $q\bar{q}$ fluctuations of a virtual photon is denoted by $\Phi_T(\mathbf{x}_{01},z)$ and $\Phi_L(\mathbf{x}_{01},z)$ for transverse and longitudinal photons correspondingly, with z being the fraction of the photon's longitudinal momentum carried by the quark. At the lowest order in electromagnetic coupling ($\alpha_{\rm EM}$) $\Phi_T(\mathbf{x}_{01},z)$ and $\Phi_L(\mathbf{x}_{01},z)$ are given by ([30,31], and references therein)

$$\Phi_T(\mathbf{x}_{01},z)=\frac{2N_c\alpha_{\rm EM}}{\pi}\{a^2\ K_1^2(x_{01}a)\ [z^2+(1-z)^2]\},\qquad (5a)$$

$$\Phi_L(\mathbf{x}_{01},z)=\frac{2N_c\alpha_{\rm EM}}{\pi}\ 4Q^2z^2(1-z)^2\ K_0^2(x_{01}a)\qquad (5b)$$

with $a^2=Q^2z(1-z)$. We consider massless quarks having only one flavor.

The quantity $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$ has the meaning of the forward scattering amplitude of the quark-antiquark pair on a nucleus [31]. At the lowest (classical) order not including the QCD evolution in rapidity it is given by

$$N(\mathbf{x}_{01},\mathbf{b}_0,0)=-\gamma(\mathbf{x}_{01},\mathbf{b}_0)\equiv\left\{1-\exp\left[-\frac{C_F}{N_c}\frac{\mathbf{x}_{01}^2\tilde{v}(x_{01})R}{2\lambda}\right]\right\},\qquad (6a)$$

with $\tilde{v}$ as defined in [28] and λ being the mean free path of a gluon in a nuclear medium, as defined in [28]. In the logarithmic approximation for large Q^2 (small x_{01}), Eq. (6a) can be rewritten as

$$N(\mathbf{x}_{01},\mathbf{b}_0,0)=-\gamma(\mathbf{x}_{01},\mathbf{b}_0)\approx\left\{1-\exp\left[-\frac{\alpha\pi^2}{2N_cS_\perp}\times\mathbf{x}_{01}^2AxG(x,1/\mathbf{x}_{01}^2)\right]\right\}.\qquad (6b)$$

$\gamma(\mathbf{x}_{01},\mathbf{b}_0)$ is the propagator of the $q\bar{q}$ pair through the nucleus. The propagator could be easily calculated, similarly to [28,30], giving the Glauber multiple rescattering formula (6). Here and throughout the paper we assume for simplicity that the nucleus is a cylinder, which appears as a circle of radius R in the transverse direction and has a constant length $2R$ along the longitudinal z direction. Therefore, its transverse cross-sectional area is $S_\perp=\pi R^2$. In formula (6) A is the atomic number of the nucleus, α is the strong-coupling constant and $xG(x,1/\mathbf{x}_{01}^2)$ is the gluon distribution in a nucleon in the nucleus, taken at the lowest order in α, similarly to [28].

Equation (6) resums all Glauber-type multiple rescatterings of a $q\bar{q}$ pair on a nucleus. As was mentioned before, since each interaction of the pair with a nucleon in the nucleus is restricted to the two gluon exchange, formula (6) effectively sums up all the powers of the parameter $\alpha^2A^{1/3}$. Or, looking at the power of the exponent in Eq. (6) we conclude that since $x_{01}\sim 1/Q$, it resums all the powers of $\alpha^2A^{1/3}/Q^2$. This is the definition of quasi-classical limit, a more detailed discussion of which could be found in [26].

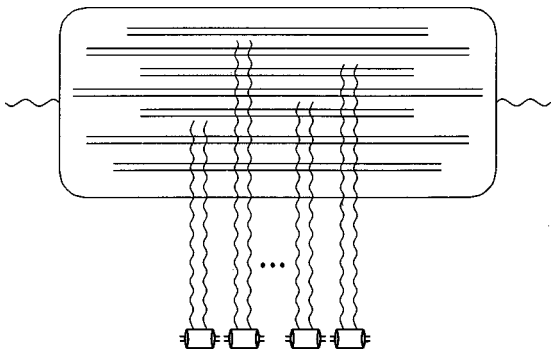

FIG. 2. DIS on a nucleus including the QCD evolution in the quark-antiquark pair in the form of dipole wave function. Each double line represents a gluon in the large-N_c limit.

Since the nucleus is at rest in order to include the QCD evolution of F_2 structure function, we have to develop the soft gluon wave function of the incoming virtual photon. In the leading longitudinal logarithmic approximation ($\ln 1/x$) the evolution of the wave function is realized through successive emissions of small-x gluons. The $q\bar{q}$ pair develops a cascade of gluons, which then scatter on the nucleus. In order to describe the soft gluon cascade we will take the limit of a large number of colors, $N_c\to\infty$. Then, this leading logarithmic soft gluon wave function will become equivalent to the dipole wave function, introduced by Mueller in [1–3]. The physical picture becomes straightforward. The $q\bar{q}$ pair develops a system of dipoles (dipole wave function), and each of the dipoles independently scatters on the nucleus, as shown in Fig. 2. Since the nucleus is large, we may approximate the interaction of a dipole (quark-antiquark pair) with the nucleus by $\gamma(\mathbf{x},\mathbf{b})$ given by Eq. (6), with $\mathbf{x}$ and $\mathbf{b}$ being the dipole's transverse separation and impact parameter. That means that each of the dipoles interacts with several nucleons (Glauber rescattering) in the nucleus independent of other dipoles. The interaction of each of the dipoles with the nucleus is the same as was shown in Fig. 1 for the initial $q\bar{q}$ pair.

To construct the dipole wave function we will heavily rely on the techniques developed in [1–4]. Following [1,3] we define the generating functional for dipoles $Z(\mathbf{b}_0,\mathbf{x}_{01},Y,u)$ [see formulas (16) and (17) in [1]]. The generating functional then obeys the equation [see Eq. (12) in [3]]

$$Z(\mathbf{b}_0,\mathbf{x}_{01},Y,u)=u(\mathbf{b}_0,\mathbf{x}_{01})\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)Y\right]+\frac{\alpha C_F}{\pi^2}\int_0^Y dy\,\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)(Y-y)\right]\times\int_\rho d^2\tilde{x}_2\frac{x_{01}^2}{x_{02}^2x_{12}^2}Z\left(\mathbf{b}_0+\frac{1}{2}\mathbf{x}_{12},\mathbf{x}_{02},y,u\right)Z\left(\mathbf{b}_0-\frac{1}{2}\mathbf{x}_{20},\mathbf{x}_{12},y,u\right),\qquad (7)$$

where $\mathbf{x}_{20}=\tilde{\mathbf{x}}_0-\tilde{\mathbf{x}}_2$, $\mathbf{x}_{21}=\tilde{\mathbf{x}}_1-\tilde{\mathbf{x}}_2$ and the integration over $\tilde{x}_2$ is performed over the region where $x_{02}\geqslant\rho$ and $x_{12}\geqslant\rho$. This ρ

serves as an ultraviolet cutoff in the equation and disappears in the physical quantities. $\mathbf{b}_0=\frac{1}{2}(\mathbf{x}_0+\mathbf{x}_1)$ is the position of the center of the initial dipole in the transverse plane [3]. $C_F=N_c/2$ in the large-N_c limit. The generating functional is defined such that $Z(\mathbf{b}_0,\mathbf{x}_{01},Y,u=1)=1$ (see [1]).

Analogous to [2,3] we now define the dipole number density by

$$\frac{1}{2\pi x^2}n_1(x_{01},Y,|\mathbf{b}-\mathbf{b}_0|,x)=\frac{\delta}{\delta u(\mathbf{b},\mathbf{x})}Z(\mathbf{b}_0,\mathbf{x}_{01},Y,u)|_{u=1}. \tag{8}$$

$n_1(x_{01},Y,|\mathbf{b}-\mathbf{b}_0|,x)$ convoluted with the virtual photon's wave function gives the number of dipoles of transverse size x at the impact parameter $|\mathbf{b}-\mathbf{b}_0|$ with the smallest light cone momentum in the pair greater or equal to $e^{-Y}q_+$. Similarly to the dipole number density we can introduce dipole pair density [2,3] for a pair of dipoles of sizes x_1 and x_2 at the impact parameters $|\mathbf{b}_1-\mathbf{b}_0|$ and $|\mathbf{b}_2-\mathbf{b}_0|$ by

$$\frac{1}{2\pi x_1^2}\frac{1}{2\pi x_2^2}n_2(x_{01},Y,|\mathbf{b}_1-\mathbf{b}_0|,x_1,|\mathbf{b}_2-\mathbf{b}_0|,x_2)=\frac{1}{2!}\frac{\delta}{\delta u(\mathbf{b}_1,\mathbf{x}_1)}\frac{\delta}{\delta u(\mathbf{b}_2,\mathbf{x}_2)}Z(\mathbf{b}_0,\mathbf{x}_{01},Y,u)|_{u=1}. \tag{9}$$

Our notation is different from the conventional approach of [2,3] by the factor of a factorial, for reasons which will become obvious later. Generalizing the definition (9) to k dipoles of sizes $x_1,\ldots,x_n$ situated at the impact parameters $|\mathbf{b}_1-\mathbf{b}_0|,\ldots,|\mathbf{b}_k-\mathbf{b}_0|$, we easily obtain

$$\prod_{i=1}^{k}\frac{1}{2\pi x_i^2}n_k(x_{01},Y,|\mathbf{b}_1-\mathbf{b}_0|,x_1,\ldots,|\mathbf{b}_k-\mathbf{b}_0|,x_k)=\frac{1}{k!}\prod_{i=1}^{k}\frac{\delta}{\delta u(\mathbf{b}_i,\mathbf{x}_i)}Z(\mathbf{b}_0,\mathbf{x}_{01},Y,u)|_{u=1}. \tag{10}$$

One can now see that in order to include all the multiple Pomeron exchanges one has to sum up the contributions of different numbers of dipoles interacting with the nucleus. Namely, we should take the dipole number density $n_1(x_{01},Y,\mathbf{b},x)$ and convolute it with the propagator of this one dipole in the nucleus $\gamma(\mathbf{x},\mathbf{b})$. Then we should take the dipole pair density $n_2(x_{01},Y,\mathbf{b}_1,x_1,\mathbf{b}_2,x_2)$ and convolute it with two propagators $\gamma(\mathbf{x}_1,\mathbf{b}_1)$ and $\gamma(\mathbf{x}_2,\mathbf{b}_2)$, etc. That way we obtain an expression for $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$

$$\begin{aligned}-N(\mathbf{x}_{01},\mathbf{b}_0,Y)&=\int n_1(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1)\left(\gamma(\mathbf{x}_1,\mathbf{b}_1)\frac{d^2x_1}{2\pi x_1^2}d^2b_1\right)\\&\quad+\int n_2(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1,\mathbf{b}_2,\mathbf{x}_2)\left(\gamma(\mathbf{x}_1,\mathbf{b}_1)\frac{d^2x_1}{2\pi x_1^2}d^2b_1\right)\left(\gamma(\mathbf{x}_2,\mathbf{b}_2)\frac{d^2x_2}{2\pi x_2^2}d^2b_2\right)+\ldots\\&=\sum_{i=1}^{\infty}\int n_i(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1,\ldots,\mathbf{b}_i,\mathbf{x}_i)\left(\gamma(\mathbf{x}_1,\mathbf{b}_1)\frac{d^2x_1}{2\pi x_1^2}d^2b_1\right)\ldots\left(\gamma(\mathbf{x}_i,\mathbf{b}_i)\frac{d^2x_i}{2\pi x_i^2}d^2b_i\right),\end{aligned} \tag{11}$$

where we put the minus sign in front of N to make it positive, since γ is negative. Equation (11) clarifies the physical meaning of N as a total cross section of a $q\bar{q}$ pair interacting with a nucleus. One can understand now the factorials in the definitions of the dipole number densities (8), (9), and (10): once the convolutions with the propagators γ are done then the dipoles become ''identical'' and we have to include the symmetry factors.

In order to write down an equation for $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$ we have to find the equations for n_i's first. Following the techniques introduced in [1–4] we have to differentiate the equation for the generating functional (7) with respect to $u(\mathbf{x},\mathbf{b})$ setting $u=1$ at the end, keeping in mind that $Z(\mathbf{b}_0,\mathbf{x}_{01},Y,u=1)=1$. Differentiating formula (7) once we obtain an equation for $n_1(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1)$:

$$\begin{aligned}n_1(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1)&=\delta^2(\mathbf{x}_{01}-\mathbf{x}_1)\,2\pi\mathbf{x}_1^2\,\delta^2(\mathbf{b}_1)\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)Y\right]\\&\quad+\frac{\alpha C_F}{\pi^2}\int_0^Y dy\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)(Y-y)\right]\int_\rho d^2\tilde{x}_2\frac{x_{01}^2}{x_{02}^2x_{12}^2}2\,n_1(x_{02},y,\overline{\mathbf{b}}_1,\mathbf{x}_1),\end{aligned} \tag{12}$$

where, following [3], we have defined $\overline{\mathbf{b}}_i=\mathbf{b}_i-\mathbf{b}_0-\frac{1}{2}\mathbf{x}_{12}$.

Differentiating Eq. (7) twice we obtain an equation for $n_2(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1,\mathbf{b}_2,\mathbf{x}_2)$:

$$n_2(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1,\mathbf{b}_2,\mathbf{x}_2)=\frac{\alpha C_F}{\pi^2}\int_0^Y dy\,\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)(Y-y)\right]$$

$$\times\int_\rho d^2\tilde{x}_2\,\frac{x_{01}^2}{x_{02}^2x_{12}^2}\left[2\,n_2(x_{02},y,\overline{\mathbf{b}}_1,\mathbf{x}_1,\overline{\mathbf{b}}_2,\mathbf{x}_2)+n_1(x_{02},y,\overline{\mathbf{b}}_1,\mathbf{x}_1)\,n_1(x_{12},y,\tilde{\mathbf{b}}_2,\mathbf{x}_2)\right],\tag{13}$$

where $\tilde{\mathbf{b}}_i=\mathbf{b}_i-\mathbf{b}_0+\frac{1}{2}\mathbf{x}_{20}$. Now higher-order differentiation of Eq. (7) becomes apparent, and could be easily done yielding the following equation for the number density of i dipoles:

$$n_i(x_{01},Y,\mathbf{b}_1,\mathbf{x}_1,\ldots,\mathbf{b}_i,\mathbf{x}_i)=\frac{\alpha C_F}{\pi^2}\int_0^Y dy\,\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)(Y-y)\right]\int_\rho d^2\tilde{x}_2\,\frac{x_{01}^2}{x_{02}^2x_{12}^2}\left[2\,n_i(x_{02},y,\overline{\mathbf{b}}_1,\mathbf{x}_1,\ldots,\overline{\mathbf{b}}_i,\mathbf{x}_i)\right.$$

$$\left.+\sum_{j+k=i}n_j(x_{02},y,\overline{\mathbf{b}}_1,\mathbf{x}_1,\ldots,\overline{\mathbf{b}}_j,\mathbf{x}_j)\,n_k(x_{12},y,\tilde{\mathbf{b}}_{j+1},\mathbf{x}_{j+1},\ldots,\tilde{\mathbf{b}}_i,\mathbf{x}_i)\right],\tag{14}$$

where we anticipate the integration over the dipole sizes and treat the dipoles as identical objects. In principle, Eq. (14) should contain the permutations of the arguments of the gluon densities in the product on the right-hand side, but for the above-mentioned reason we do not write these terms explicitly.

Multiplying formula (14) by

$$\left(\gamma(\mathbf{x}_1,\mathbf{b}_1)\frac{d^2x_1}{2\pi x_1^2}d^2b_1\right)\ldots\left(\gamma(\mathbf{x}_i,\mathbf{b}_i)\frac{d^2x_i}{2\pi x_i^2}d^2b_i\right),$$

integrating over the dipole sizes and impact parameters, and summing all such equations, i.e., summing over i from 1 to ∞ in Eq. (14) one obtains the equation for $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$

$$N(\mathbf{x}_{01},\mathbf{b}_0,Y)=-\gamma(\mathbf{x}_{01},\mathbf{b}_0)\,\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)Y\right]+\frac{\alpha C_F}{\pi^2}\int_0^Y dy\,\exp\left[-\frac{4\alpha C_F}{\pi}\ln\left(\frac{x_{01}}{\rho}\right)(Y-y)\right]$$

$$\times\int_\rho d^2\tilde{x}_2\,\frac{x_{01}^2}{x_{02}^2x_{12}^2}\left[2\,N\left(\mathbf{x}_{02},\mathbf{b}_0+\frac{1}{2}\mathbf{x}_{12},y\right)-N\left(\mathbf{x}_{02},\mathbf{b}_0+\frac{1}{2}\mathbf{x}_{12},y\right)N\left(\mathbf{x}_{12},\mathbf{b}_0-\frac{1}{2}\mathbf{x}_{20},y\right)\right].\tag{15}$$

Equation (15), together with Eqs. (4) and (5), provide us with the leading logarithmic evolution of the F_2 structure function of a nucleus including all multiple Pomeron exchanges in the large-N_c limit.

Throughout the preceding calculations we never made an assumption that Q^2 is large. Of course, it should be large enough for the perturbation theory to be applicable. The only assumption about the incoming photon's momentum that we made was that its light-cone component q_+ is large, therefore we could neglect the inverse powers of q_+. This is eikonal approximation, which is natural for leading $\ln(1/x)$ calculation. However, if the inverse power of q_+ comes with an inverse power of q_-, forming something like $1/2q_+q_-\sim 1/Q^2$ we do not neglect these terms, therefore resumming all the inverse powers of Q^2 (''higher twist terms''). That way we proceed to conclude that Eq. (15) sums up in the leading logarithmic approximation all diagrams that include the effects of multiple Pomeron exchanges, with Pomeron ladders together with Pomeron splitting vertices being incorporated in the dipole wave function. In terms of conventional (not ''wave functional'') language Eq. (15) resums the so-called ''fan'' diagrams (see Fig. 3) which were summed up by conventional GLR equation [5,6]. The difference between our equation and GLR is that Eq. (15) does not assume leading transverse logarithmic (large Q^2) approximation.

III. DOUBLE LOGARITHMIC LIMIT

In order to reconcile our approach with traditional results in this section we will take the large-Q^2 limit of Eq. (15) and show that in this double logarithmic approximation Eq. (15) reduces to the GLR equation [5,6]. We consider a scattering of a virtual photon, characterized by large momentum scale Q, on a nucleus at rest characterized by the scale $\Lambda_{\rm QCD}$. The $Q^2\gg\Lambda_{\rm QCD}^2$ limit implies that the dipoles produced at each step of the evolution in the dipole wave function must be of much greater transverse dimensions than the dipoles off which they were produced. Basically, since in the double logarithmic approximation the transverse momentum of the gluons in the dipole wave function should evolve from the large scale Q to the small scale $\Lambda_{\rm QCD}$, than the transverse sizes of the dipoles should evolve from the small scale $1/Q$ to the large scale $1/\Lambda_{\rm QCD}$.

In the limit when the produced dipoles are much larger

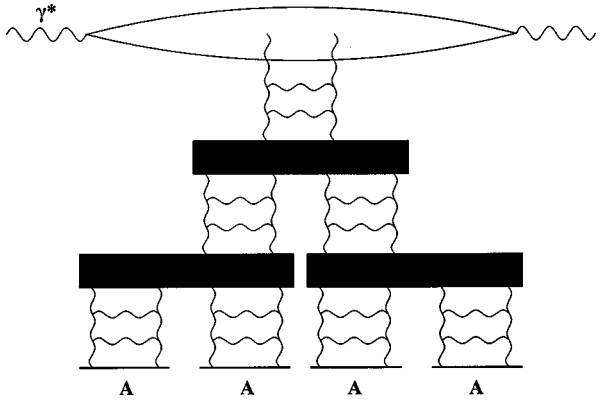

FIG. 3. Multiple Pomeron exchanges and splittings resummed by Eq. (15). Each Pomeron ladder interacts with a nucleus, which is symbolically denoted by *A*.

than the dipole by which they were produced (large-Q^2 limit), the kernel of Eq. (15) becomes

$$\int_\rho d^2\tilde{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \to x_{01}^2 \pi \int_{x_{01}^2}^{1/\Lambda_{\rm QCD}^2} \frac{dx_{02}^2}{(x_{02}^2)^2}, \tag{16}$$

where $x_{02} \approx x_{12} \gg x_{01}$, and the upper cutoff of the x_{02} integration is given by the inverse momentum scale characterizing the nucleus, $1/\Lambda_{\rm QCD}^2$. Since this integration is done in the region of large transverse sizes the ultraviolet cutoff ρ is no longer needed. One can easily see that including virtual corrections would bring in the exponential factor $e^{-(\alpha C_F/\pi)Y}$ in Eqs. (7) and (15) instead of $\exp[-(4\alpha C_F/\pi)\ln(x_{01}/\rho)Y]$. In the double logarithmic approximation $\alpha Y \ln(Q^2/\Lambda_{\rm QCD}^2) \gtrsim 1$ and $\ln(Q^2/\Lambda_{\rm QCD}^2) \gg 1$, therefore $\alpha Y \lesssim 1$. That way the factor of $e^{-(\alpha C_F/\pi)Y}$ can be neglected. The resulting limit of Eq. (15) is

$$\begin{aligned} N(\mathbf{x}_{01},\mathbf{b}_0,Y) = &- \gamma(\mathbf{x}_{01},\mathbf{b}_0) \\ &+ \frac{\alpha C_F}{\pi} x_{01}^2 \int_0^Y dy \int_{x_{01}^2}^{1/\Lambda_{\rm QCD}^2} \frac{dx_{02}^2}{(x_{02}^2)^2} \\ &\times [2\, N(\mathbf{x}_{02},\mathbf{b}_0,Y) \\ &- N(\mathbf{x}_{02},\mathbf{b}_0,Y)\, N(\mathbf{x}_{02},\mathbf{b}_0,Y)], \end{aligned}$$

which after differentiation with respect to Y yields

$$\begin{aligned} \frac{\partial N(\mathbf{x}_{01},\mathbf{b}_0,Y)}{\partial Y} = &\frac{\alpha C_F}{\pi} x_{01}^2 \int_{x_{01}^2}^{1/\Lambda_{\rm QCD}^2} \frac{dx_{02}^2}{(x_{02}^2)^2} [2\, N(\mathbf{x}_{02},\mathbf{b}_0,Y) \\ &- N(\mathbf{x}_{02},\mathbf{b}_0,Y)\, N(\mathbf{x}_{02},\mathbf{b}_0,Y)], \end{aligned} \tag{17}$$

where, for simplicity, we suppressed the difference in the impact parameter dependence of N on the left- and right-hand sides of Eq. (17). This is done in the spirit of the large cylindrical nucleus approximation. Also, one should keep in mind that for this double logarithmic limit in the definition of $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$ given by Eq. (11), the integration over the dipole's transverse sizes should be also done from x_{01}^2 to $1/\Lambda_{\rm QCD}^2$.

Now we have to make a connection between $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$ and the gluon distribution function $xG_A(x,Q^2)$ of a nucleus. $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$ is a forward scattering amplitude of a $q\bar{q}$ pair on a nucleus and is a well-defined physical quantity. However, there is some freedom in the definition of the gluon distribution. If one makes use of the general definition of the gluon distribution as a matrix element of leading twist operator, then an attempt to take into account higher twist operators would lead only to renormalization of their matrix elements (see [32], and references therein). The evolution equation for xG would be linear, with all the nonlinear saturation effects included in the initial conditions. The goal of the GLR type of approach is to put these nonlinear effects in the evolution equation. Therefore, in the double logarithmic approach one usually defines the gluon distribution function through a cutoff operator product expansion, i.e., as a matrix element of the $A_\mu A_\mu$ operator, with Q^2 an ultraviolet cutoff imposed on the operator (see the discussion on pp. 442–443 of [6]). In the spirit of this approach we define the gluon distribution by

$$N(\mathbf{x}_{01},\mathbf{b}_0,Y) = \frac{\alpha\pi^2}{2N_c S_\perp} \mathbf{x}_{01}^2 xG_A(x,1/x_{01}^2), \tag{18}$$

with the coefficient fixed by the two gluon exchange between the quark-antiquark pair and the nucleus (in the large-N_c limit). Substituting Eq. (18) into Eq. (17) one obtains

$$\begin{aligned} \frac{\partial xG_A(x,1/x_{01}^2)}{\partial Y} = &\frac{\alpha C_F}{\pi} \int_{x_{01}^2}^{1/\Lambda_{\rm QCD}^2} \frac{dx_{02}^2}{x_{02}^2} \Big[2\, xG_A(x,1/x_{02}^2) \\ &- x_{02}^2 \frac{\alpha\pi^2}{2N_c S_\perp} [xG_A(x,1/x_{02}^2)]^2 \Big]. \end{aligned}$$

Differentiating the resulting equation with respect to $\ln(1/x_{01}^2\Lambda_{\rm QCD}^2)$ and using $x_{01} \sim 2/Q$, which is valid in the double logarithmic limit, we end up with

$$\begin{aligned} \frac{\partial^2 xG_A(x,Q^2)}{\partial \ln(1/x)\, \partial \ln(Q^2/\Lambda_{\rm QCD}^2)} = &\frac{\alpha N_c}{\pi} xG_A(x,Q^2) \\ &- \frac{\alpha^2\pi}{S_\perp} \frac{1}{Q^2} [xG_A(x,Q^2)]^2, \end{aligned} \tag{19}$$

which exactly corresponds to the GLR equation [5,6], with the factors matching those corresponding to cylindrical nucleus case in Refs. [17,18].

One has to note that the problems with the definition of the gluon distribution function outlined above bear no consequence on Eq. (15). This equation describes the evolution of $N(\mathbf{x}_{01},\mathbf{b}_0,Y)$ in the leading $\ln(1/x)$ and does not assume

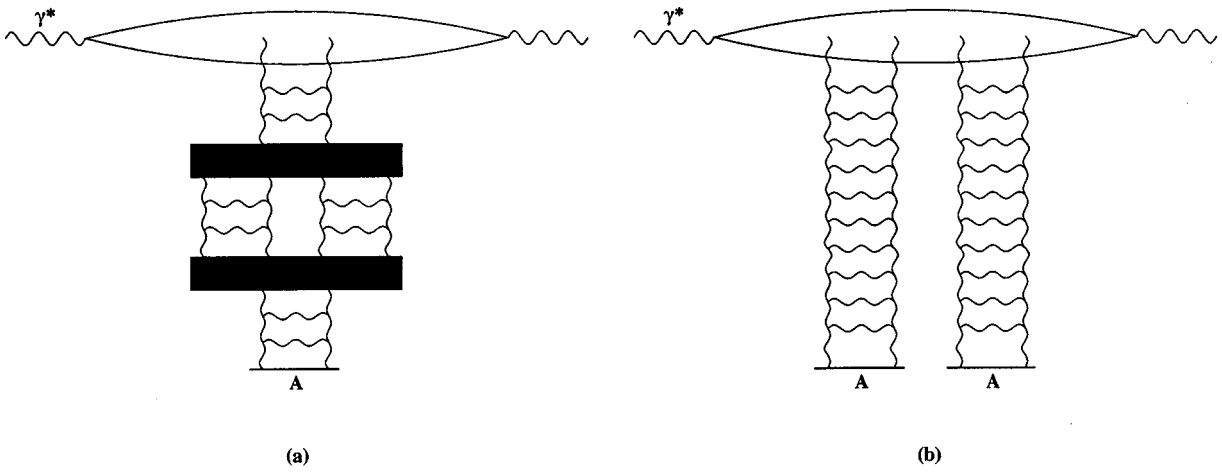

FIG. 4. (a) Diagrams which are not included in our analysis. (b) A diagram which is included in Eq. (15).

collinear factorization or impose transverse momentum cutoffs, therefore posing no problems like the mixing of operators of different twists [32].

One would also like to rederive the equation for xG_A derived earlier in [17,18,21], given by Eq. (2) here. In the double logarithmic limit we did not reproduce those results, given by formula (48) in [18], formula (3.19) in [21], and, most explicitly by formula (41) in [25]. Nevertheless, this equation could be obtained from Eq. (15) in the following peculiar limit. Consider the scattering of a virtual photon with moderately large virtuality Q on a nucleus composed of very heavy quarks, so that the typical momentum scale characterizing the nucleus given by the mass of the heavy quarks M is much larger than Q. In that case the double logarithmic limit would correspond to production of small dipoles in the dipole wave function. Now, using certain freedom we have in the definition of the gluon distribution function, we can treat Eq. (6b) as a definition of xG_A, which should be substituted in Eq. (6b) instead of AxG. Plugging the generalized Eq. (6b) into Eq. (15) taken in the small-dipole limit we obtain Eq. (2). That way we will rederive the results of [17,18,21], however not quite for the same process as the one for which they were derived originally.

Finally, we note that we failed to find a limit in which Eq. (15) reproduces formula (39) in [25],that was given by Eq. (3) above, which is another candidate for the double logarithmic evolution of xG including all multiple Pomeron exchanges.

IV. CONCLUSIONS

One of the diagrams that were not included in our analysis above is shown in Fig. 4(a). There a hard Pomeron ladder splits into two and then the two ladders again fuse into one which connects to the nucleus. In general this diagram is of the same order in coupling and rapidity as the usual two-pomeron exchange diagram of Fig. 4(b), and, therefore should be also considered. The effects of that type can play an important role in unitarization and saturation of the onium-onium scattering amplitude [33]. In the dipole wave function language the graph in Fig. 4(a) can be interpreted in two ways. In one case it could be a part of the dipole wave function, which should be redefined to include this graph in the form of color quadrupoles. On the other hand, the graph in Fig. 4(a) could correspond to the usual ''two dipole'' part of wave function (n_2) with both dipoles interacting with the same nucleon in the nucleus. However, for the given problem of scattering on a large nucleus, the graph in Fig. 4(a) is suppressed by powers of atomic number A compared to the graph in Fig. 4(b) for both cases.

The inclusion of the Pomeron fusion effects in the dipole wave function is a difficult task [33]. Equation (7) does not take them into account. Construction of dipole wave function which includes the diagrams shown in Fig. 4(a) is an interesting and important problem, which is still to be solved. One should note that inclusion of the graphs of the type shown in Fig. 4(a) in the dipole wave function would result in appearance of color quadrupoles [33]. In the case of onium-onium center-of-mass scattering this wave function can be either considered as suppressed by factors of N_C^2 compared to the one-dipole wave function, or, more correctly, one may note that its contribution to the scattering amplitude will be suppressed by the factor of $e^{(\alpha_P-1)Y/2}$ [3], with α_P the intercept of the BFKL Pomeron. However, this argument does not apply for the case when one of the onia is at rest [33], or to our case when the nucleus is at rest.

The graph in Fig. 4(a) in our case of nucleus at rest brings in suppression by powers of A. The reason for that is very simple: in the first diagram [Fig. 4(a)] there is only one Pomeron ladder (dipole) interacting with the nucleus below, whereas in the second diagram [Fig. 4(b)] there are two ladders. Since each dipole is convoluted with its propagator through the nucleus, each ladder brings in a factor of $\gamma(\mathbf{x},\mathbf{b})d^2b$. For large transverse size dipoles this factor is proportional to $A^{2/3}$ and for small dipoles it scales as A^1. In any case one can see that the graph in Fig. 4(a), having the same parametrical dependence on α and Y as the graph in Fig. 4(b), is suppressed by some power of A compared to this second graph. Therefore, one should note that by neglecting the effects of the graph in Fig. 4(a) we assume that we are

doing the leading calculation in the powers of the atomic number of the nucleus A. That allows us to avoid complications associated with the incorporation of the diagram in Fig. 4(a) in the evolution equation (15).

Equation (15) can also be derived directly from Eq. (7) by setting $u(\mathbf{x}_{01},\mathbf{b}_0)=\gamma(\mathbf{x}_{01},\mathbf{b}_0)+1$ in it and defining $N=1-Z$. That way we have a method of resumming all multiple Pomeron exchanges for any higher-order corrections to the dipole kernel. If one calculates the dipole kernel, say, at the next-to-lowest order, then we can write down an equation for generating functional Z, similar to Eq. (7). Though the next-to-lowest order equation will in addition have cubic terms in Z on the right-hand side. Then, setting $u=\gamma+1$ and $Z=1-N$ one would easily obtain an equation resumming multiple Pomeron exchanges in the subleading logarithmic approximation. Therefore, the dipole model provides us with a relatively straightforward way of taking into account the multiple Pomeron exchanges once the one-pomeron exchange contribution has been calculated. In other words, if the dipole kernel is known at any order in the coupling constant one can easily generalize the resulting equation for generating functional to include the multiple Pomeron exchanges on a nucleus.

Finally, we note that it would be interesting to try fitting the recent HERA data [16] using the evolution of the F_2 structure function given by Eq. (15).

ACKNOWLEDGMENTS

I would like to thank Alex Kovner, Genya Levin, Larry McLerran, and Alfred Mueller for a number of informative and very helpful discussions. This work is supported by DOE Grant No. DE-FG02-87ER40328.

[1] A.H. Mueller, Nucl. Phys. **B415**, 373 (1994).
[2] A.H. Mueller and B. Patel, Nucl. Phys. **B425**, 471 (1994).
[3] A.H. Mueller, Nucl. Phys. **B437**, 107 (1995).
[4] Z. Chen and A.H. Mueller, Nucl. Phys. **B451**, 579 (1995).
[5] L.V. Gribov, E.M. Levin, and M.G. Ryskin, Nucl. Phys. **B188**, 555 (1981); Phys. Rep. **100**, 1 (1983).
[6] A.H. Mueller and J. Qiu, Nucl. Phys. **B286**, 427 (1986).
[7] E.A. Kuraev, L.N. Lipatov, and V.S. Fadin, Zh. Éksp. Teor. Fiz. **72**, 377 (1977) [Sov. Phys. JETP **45**, 199 (1977)].
[8] Ya.Ya. Balitsky and L.N. Lipatov, Yad. Fiz. **28**, 43 (1978) [Sov. J. Nucl. Phys. **28**, 21 (1978)].
[9] A. H. Mueller, Proceedings of the 4th Workshop on Small-x and Diffractive Physics, Batavia, Illinois, 1998.
[10] V.S. Fadin and L.N. Lipatov, Phys. Lett. B **429**, 127 (1998), and references therein.
[11] M. Ciafaloni and G. Camici, Phys. Lett. B **430**, 349 (1998), and references therein.
[12] Yu.V. Kovchegov and A.H. Mueller, Phys. Lett. B **439**, 428 (1998).
[13] E. Levin, hep-ph/9806228.
[14] N. Armesto, J. Bartels, and M.A. Braun, Phys. Lett. B **442**, 459 (1998).
[15] A.H. Mueller, Phys. Lett. B **396**, 251 (1997).
[16] ZEUS Collaboration, J. Breitweg *et al.*, Eur. Phys. J. C **7**, 609 (1999).
[17] A.L. Ayala, M.B. Gay Ducati, and E.M. Levin, Nucl. Phys. **B493**, 305 (1997).
[18] A.L. Ayala, M.B. Gay Ducati, and E.M. Levin, Nucl. Phys. **B551**, 355 (1998).
[19] A.H. Mueller, Nucl. Phys. **B335**, 115 (1990).
[20] L. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994); **50**, 2225 (1994).
[21] J. Jalilian-Marian, A. Kovner, L. McLerran, and H. Weigert, Phys. Rev. D **55**, 5414 (1997).
[22] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Nucl. Phys. **B504**, 415 (1997).
[23] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Phys. Rev. D **59**, 014014 (1999).
[24] J. Jalilian-Marian, A. Kovner, and H. Weigert, Phys. Rev. D **59**, 014015 (1999).
[25] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Phys. Rev. D **59**, 034007 (1999); **59**, 099903(E) (1999).
[26] Yu.V. Kovchegov, Phys. Rev. D **54**, 5463 (1996); **55**, 5445 (1997).
[27] G.P. Salam, Nucl. Phys. **B461**, 512 (1996).
[28] Yu.V. Kovchegov and A.H. Mueller, Nucl. Phys. **B529**, 451 (1998).
[29] R. Baier, Yu.L. Dokshitzer, A.H. Mueller, S. Peigne, and D. Shiff, Nucl. Phys. **B484**, 265 (1997).
[30] D. Kharzeev, Yu.V. Kovchegov, L. McLerran, and R. Venugopalan (unpublished).
[31] N.N. Nikolaev and B.G. Zakharov, Z. Phys. C **49**, 607 (1991).
[32] E.M. Levin, M.G. Ryskin, and A.G. Shuvaev, Nucl. Phys. **B387**, 589 (1992).
[33] Yu.V. Kovchegov, A.H. Mueller, and S. Wallon, Nucl. Phys. **B507**, 367 (1997).