Possible effects of quantum mechanics violation induced by certain quantum-gravity effects on neutrino oscillations

Chao-Hsi Chang,[1,3] Wu-Sheng Dai,[4] Xue-Qian Li,[1,3,4] Yong Liu,[5] Feng-Cai Ma,[6] and Zhi-jian Tao[1,2,3]
[1]*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China*
[2]*Institute of High Energy Physics, Academia Sinica, Beijing 100039, China*
[3]*Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, China*
[4]*Department of Physics, Nankai University, Tianjin, 300071, China*
[5]*Nankai Institute of Mathematics, Nankai University, Tianjin, 300071, China*
[6]*Department of Physics, Liaoning University, Shenyang 110036, China*
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In this work we extensively apply the EHNS postulation to neutrino oscillation problems. Namely, we try to apply quantum mechanics violation (QMV) effects induced by the quantum gravity of black holes to understand neutrino oscillation phenomena. In addition to the original EHNS postulation, especially, the Lorentz transformation property of QMV is taken into account. The possibilities for observing such effects in neutrino experiments (in progress and/or accessible in the near future) are discussed. Of them, an interesting feature is outlined. [S0556-2821(99)05113-9]

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I. INTRODUCTION

It is known that neutrino oscillation is a very possible solution for the long-standing puzzle of the ''deficiency'' of the solar neutrino flux [1] and atmospheric neutrino problem [2]. Neutrino experiments are difficult but progress has been achieved steadily recently. For instance, the Super-Kamiokande Collaboration is collecting solar and atmospheric neutrino events with a very high rate since it began operation, and some evidence for neutrino oscillations has been reported recently [3]. In addition, as planned, several long-base-line neutrino experiments for the oscillation in matter are in progress.

Hawking, based on the principle of quantum mechanics and gravity, proposed a very interesting conjecture that the quantum gravity effects of black holes may cause it to emit particles in a thermal spectrum [4]. According to this conjecture, a black hole may create particles in pairs and one of them may fall back into the black hole while the other one will escape ''away'' thermodynamically; thus part of the information about the state of the system would be lost. Because of these effects, a quantum mechanical system in a pure quantum state may transform into a mixed one; i.e., it may manifest quantum mechanics violation (QMV).

To describe a mixed quantum system from a pure state to a mixed one, instead of the wave function, a density matrix description has to be adopted [5]. In such an evolution, where the QMV effects are involved, CP, and probably CPT, can be violated due to nonlocal quantum gravity effects. Thus Hawking's suggestion has received careful consideration.

At first, more than ten years ago Ellis, Hagelin, Nanopoulos, and Srednicki (EHNS) were motivated to track the ''sources'' for CP and CPT violation, and proposed to observe the QMV induced by quantum gravity effects in the K^0-$\bar{K}^0$ system with additional reasonable assumptions [6], and then the authors of [7,8] reexamined and formulated the effects with more care, and refreshed the bounds on the parameters of δH for QMV were obtained.

Based on the same conjecture, the authors of [9] discussed the possibility of observing the effects in the B^0-$\bar{B}^0$ system. The authors of [10,11] tried to study such effects on neutrino oscillation. In general relativity, the kinetic energies and the masses at rest stand on the same footing in the stress-energy tensor, no matter whether the neutrino masses are zero or finite (a tiny nonzero mass has not been ruled out yet). Thus the energy of neutrinos should play a role similar to mass in the K-$\bar{K}$ system almost at rest in the stress-energy tensor when considering gravity effects. Therefore we adopt this consideration in choosing the parameters α,β,γ appearing in the extra term of the modified Schrödinger equation in EHNS notation.

EHNS [6] suggested that α,β,γ are proportional to $O(E^2/M_{pl})$ where E is a typical energy scale of the system under discussion. Meanwhile, a serious problem about the Lorentz convariance of the formulation was raised, especially for the neutrino case where a very small mass is concerned and all subjects are extreme relativistic. This point has been discussed in detail by Sredwicki [12]. With the constraint of keeping the Lorentz convariance of the formulation, we assume that α,β,γ are proportional to E/M_{pl}; so we can write $\alpha,\beta,\gamma = E\times(\hat{\alpha},\hat{\beta},\hat{\gamma})$ where $\hat{\alpha},\hat{\beta},\hat{\gamma}$ are universal and dimensionless, while E is the typical energy scale of the system under discussion. Thus a simple extension can keep the modified Schrödinger equation Lorentz convariant (see below for more details).

Now the neutrino experiments with matter as the medium, the so-called ''long base-line'' experiments with very different distances on Earth, are under construction and will be carried out in the near future. We think that there might have some advantage to observe the QMV effects; at least, it is interesting to consider them and to see the effects that happen to the neutrino experiments quantitatively. Thus, in this paper, considering the present and planned neutrino experiments we will extensively apply the EHNS formulation to the neutrino cases, and discuss the effects of QMV affecting various neutrino oscillation observations.

To follow the notation of EHNS, let us repeat briefly their formulation for later convenience.

To describe such systems which may turn a pure state into a mixed one, commonly instead of the wave function, it is convenient to employ the density matrix. The density matrix of a pure state can always be written as

$$\rho_{pure}=|\psi\rangle\langle\psi|, \tag{1}$$

while a mixed state then should be in the form

$$\rho_{mix}=\sum_a P_a|\psi_a\rangle\langle\psi_a| \quad \text{with} \quad \sum_a P_a=1, \tag{2}$$

where $|\psi\rangle$ and $\psi_a\rangle$ are the regular wave functions respecting the superposition rule and normalization $\langle\psi|\psi\rangle=1$, $\langle\psi_a|\psi_a\rangle=1$ (not to sum over a). Note that

$$\mathrm{Tr}(\rho_{pure})=\mathrm{Tr}(\rho_{mix})=1, \tag{3}$$

but

$$\mathrm{Tr}(\rho_{pure}^2)=\mathrm{Tr}(\rho_{pure})=1, \qquad \mathrm{Tr}(\rho_{mix}^2)<1. \tag{4}$$

The Schrödinger equation for the density matrix is accordingly written as

$$i\frac{\partial}{\partial t}\rho=[H,\rho], \tag{5}$$

where ρ can be either ρ_{pure} or ρ_{mix}. Indeed so far it is exactly equivalent to the regular form of the Schrödinger equation for the wave functions. It is easy to prove that with Eq. (5) one has

$$\frac{d}{dt}\mathrm{Tr}(\rho^2)=0;$$

namely, pure and mixed states never interchange. However, as EHNS suggested [6], Hawking's quantum gravity effects may violate quantum mechanics, i.e., modify the Schrödinger equation significantly. For simplicity, here we derive all formulas for a two-energy-level system as an illustration. Generalizing the Schrödinger equation of a two-energy-level system, one can expand the 2×2 matrix form of ρ and H in terms of σ_0 and σ_i, where σ_0 is a 2×2 unit matrix and $\sigma_i(i=1,2,3)$ are the well-known Pauli matrices, i.e.,

$$\rho=\rho_0\sigma_0+\rho_i\sigma_i, \quad H=H_0\sigma_0+H_i\sigma_i. \tag{6}$$

Thus, in addition to the trivial ρ_0 component, Eq. (5) can be recast into a tensor form as [7]

$$i\frac{d}{dt}\rho=2\epsilon^{ijk}H^i\rho^j\sigma^k \quad (i,j,k=1,2,3). \tag{7}$$

Because of the QMV effects being induced by the concerned quantum gravity, EHNS introduced a non-Hermitian piece to Eq. (7):

$$i\,\delta H\rho=-h^{0j}\rho^j\sigma_0-h^{j0}\rho^0\sigma_j-h^{ij}\sigma_i\rho^j. \tag{8}$$

Since probability is conserved, and its entropy should not decrease, it is required that

$$h^{0j}=h^{j0}=0.$$

The modified Schrödinger equation can be in the form

$$\frac{d}{dt}\rho=2\epsilon^{ijk}H^i\rho^j\sigma^k-h^{ij}\rho^j\sigma^i.$$

Since h^{ij} is proportional to α,β,γ, the equation retains Lorentz convariance as the original equation (7) as long as α,β,γ are of the form $E\times(\hat{\alpha},\hat{\beta},\hat{\gamma})$.

EHNS [6] and the authors of Ref. [7] applied this modified Schrödinger equation to the K^0-$\overline{K}^0$ system. By ''putting on'' different conservation laws on the effects, h^{ij} would be constrained. If a physical quantity is conserved, its corresponding operator O must commute with the Hamiltonian and requires $d/dt(\mathrm{Tr}\,O\rho)=0$. Hence

$$\mathrm{Tr}(O\,\delta H\rho)=0.$$

EHNS and the authors of [7] assumed $O=\sigma_1$ which corresponds to strangeness being conserved ($\Delta S=0$) in the neutral kaon system:

$$\langle K^0|\sigma_1|K^0\rangle=-1 \quad \text{while} \quad \langle\overline{K^0}|\sigma_1|\overline{K^0}\rangle=+1.$$

Then $h_{\mu\nu}$ of Eq. (8) can be written as a 4×4 matrix:

$$h_{\mu\nu}=2\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&-\alpha&-\beta\\0&0&-\beta&-\gamma\end{pmatrix}, \tag{9}$$

in addition, EHNS also proposed an alternative parameter set by assuming that the conservation operator is $O=\sigma_3$, and it is the case that energy and some of the other quantum numbers such as leptonic number, etc., are requested to be conserved. Thus the matrix $h_{\mu\nu}$ reads

$$h_{\mu\nu}=2\begin{pmatrix}0&0&0&0\\0&-\alpha&-\beta&0\\0&-\beta&-\gamma&0\\0&0&0&0\end{pmatrix}. \tag{10}$$

Note that here we have added an extra factor of 2 in front of the matrix in Eq. (10) which is a different parametrization from the notation given in [6], the reason being to make the form similar to that in Eq. (9) where the authors of [6] had put a factor of 2 [see Eqs. (2.31) and (3.15) in [6]].

For the parametrization of Eq. (10), to avoid systems having complex entropy, $\mathrm{Tr}\,\rho^2$ can never exceed unity; so it requires

$$\rho_\alpha H_{\alpha\beta}\rho_\beta \leqslant 0,$$

and thus

$$\alpha>0, \quad \gamma>0, \quad \alpha\gamma>\beta^2. \tag{11}$$

By fitting data of ϵ and the semileptonic decays of the K system, EHNS obtained [6]

$$\alpha+\gamma \leqslant 2\times 10^{-21}\ \text{GeV},$$

while Huet and Peskin [7] updated the values as

$$\beta=(3.2\pm 2.9)\times 10^{-19}\ \text{GeV},$$

$$\gamma=(-0.2\pm 2.2)\times 10^{-21}\ \text{GeV},$$

and recently Ellis *et al.* renewed the estimate as [8]

$$\alpha \leqslant 4\times 10^{-17}\ \text{GeV}, \quad |\beta| \leqslant 3\times 10^{-19}\ \text{GeV},$$

$$\gamma \leqslant 7\times 10^{-21}\ \text{GeV}. \tag{12}$$

Indeed, the only important issue quoted here is the order of magnitudes of (α,β,γ) and the concrete coefficients are not of much significance.

Since the QMV effects considered here are caused by quantum gravity, EHNS suggested first that $\alpha,\ \beta,\ \gamma$ be proportional to E^2/M_{pl}, where M_{pl} is the Planck mass and E is an energy or mass (in the rest frame) scale of the concerned physical process in the concerned quantum system (neutral kaon or neutrino under discussion) [6]. As argued above, to retain Lorentz convariance, we formulate the parameters:

$$(\alpha,\beta,\gamma)=E\times(\hat{\alpha},\hat{\beta},\hat{\gamma}).$$

According to general relativity, the energy-momentum tensor plays the same role in all gravity effects, so does the QMV, no matter what the neutrio system or the K system; thus the neutrino system and the K system may have the same ''texture'' in $h_{\mu\nu}$, the parameters of the systems have the same connection to the energy-momentum tensor. Correspondingly, in this expression, having put the parameters on the same footing, $(\hat{\alpha},\hat{\beta},\hat{\gamma})$ should be dimensionless and universal; so if it is true, we can relate the neutrino parameters $(\alpha,\beta,\gamma)_\nu$ to those for the kaon system $(\alpha,\beta,\gamma)_K$:[1]

$$(\alpha,\beta,\gamma)=\frac{E_\nu}{M_K}(\alpha,\beta,\gamma)_K,$$

and the values of $(\alpha,\beta,\gamma)_K$ are given as in Eq. (12).

With the postulate, let us turn to the scenario for neutrino oscillations.

Quantum gravity effects play a certain role in all quantum systems, but the crucial problem is if they are observable or not, and in which system first. The neutrino oscillations among different species neutrinos should be really affected by the concerned QMV effects and neutrino experiments are planned to observe oscillations at very different distances in a ''vacuum'' and in matter as well; thus one may expect that there are some advantages for observing the QMV effects in neutrino oscillation systems than in the K^0-$\overline{K}^0$ system or elsewhere. In the two-generation neutrino oscillation case, the $h_{\mu\nu}$ has just the form as Eq. (6) in the basis of Pauli matrices, whereas in the three-generation neutrino oscillation case it becomes more complicated; namely, the Pauli matrices will be replaced by the SU(3) Gell-Mann matrices [10]. In this work, as stated above, for simplicity we will restrain ourselves only to formulate the two-generation neutrino case. We will only consider the form of δH given in Eqs. (9) and (10). It certainly is interesting to note here that besides the expected effects, the lepton number is allowed to be violated[2] even if the neutrinos are massless. Furthermore, one will see that the effects themselves may induce a certain pattern of oscillations; so they are observable in neutrino oscillation experiments accessible at present or the near future.

The physical picture may be imagined as follows. As neutrino ν_i interacts with a ''micro black hole'' and due to quantum effects, the black hole will create a neutrino-antineutrino pair of a certain species, the neutrino-antineutrino pair interacts with the incoming neutrino in a certain (coherent or incoherent) manner, and afterwards, a neutrino and an antineutrino will fall into the black hole but the other will escape away. The escaping neutrino does not need to be the same species as the incoming one, and which one will escape, besides by chance, depends on its coupling to the micro black hole via gravity. For two different species $(a\neq b)$ the neutrinos have different couplings to the black holes $\alpha \neq \gamma$ in QMV terms. h_{ij} of Eqs. (9) and (10) indeed reflect all the facts.

Thus we can apply this scenario to neutrino oscillations. A bald assumption was suggested before [11], but there the Lorentz convariance constraint was not kept, so that the formulation and numerical results somehow may be of problems. Now, taking into account the Lorentz convariance constraints seriously and making an assumption of the form of the parameters α,β,γ as discussed above, we are to study the possible phenomenological effects of QMV on neutrino flux detection.

Note that the parameters can be very different from those listed above, but we just assume them as a reference for later discussions. If assuming the solar neutrino deficit is due to neutrino oscillation [13,14], the parameter set, Eq. (12), will be restricted by data. Later on we will show that the solar

[1]So far the problem of the QMV is open totally; the relation between the systems of kaons and neutrinos in their parameters, we take here, should be considered as a guess essentially. In fact, here we should consider it as a reasonable working assumption.

[2]Here the lepton number is violated due to the interaction of the black holes.

neutrino and other neutrino experiments on Earth may set some substantial constraints on the parameters.

Now let us discuss the meaning of the solutions obtained from Eqs. (9) and (10).

II. ASYMPTOTIC BEHAVIOR OF QMV EVOLUTION OF THE NEUTRINO SYSTEM

It is easy to realize that the expressions (9) and (10) would lead to different behavior for the neutrino oscillations.

(a) With the form of δH given in Eq. (9), one has the solution that a probability for a ν_e transition to another ν_x in vacuum (x can be μ,τ or a sterile neutrino flavor):

$$P_{\nu_e\to\nu_x}=\frac{1}{2}-\frac{1}{2}e^{-\gamma L}\cos^2 2\theta_v -\frac{1}{2}e^{-\alpha L}\sin^2 2\theta_v\cos\left(\frac{\Delta}{2E_\nu}L\right), \tag{13}$$

where $\Delta\equiv|m^2_{\nu_1}-m^2_{\nu_2}|$, L is the distance from the production spot of ν_e to the detector, and θ_v is the mixing angle of ν_e and ν_x in vacuum. To obtain the above formula, $\beta \ll \alpha,\gamma$ is assumed. In fact, this approximation is not necessary, but here it is only for demonstration convenience; otherwise, the formula would become tedious. In the work of Ref. [11], more precise numerical results were given.

It is noted that in the basis of mass, because $|\nu_e\rangle =(\cos\theta_v|\nu_1\rangle+\sin\theta_v|\nu_2\rangle)$ and $|\nu_\mu\rangle=(-\sin\theta_v|\nu_1\rangle +\cos\theta_v|\nu_2\rangle)$, so we have

$$\langle\nu_e|\sigma_1|\nu_e\rangle=2\sin\theta_v\cos\theta_v,$$

$$\langle\nu_\mu|\sigma_1|\nu_\mu\rangle=-2\sin\theta_v\cos\theta_v.$$

As in the case of the K^0-$\bar{K}^0$ system, the conservation of σ_1 means that flavor conserves and there would be no transition among different flavors. In fact, since the original Hamiltonian does mix the flavors for massive neutrinos, although the $\delta H\rho$ term does not, the new term δH causes the transition attributed to the original Hamiltonian to be strengthened or weakened. One can see that in this case an exponential factor exists in front of the harmonic oscillation which is our familiar expression of neutrino oscillation in a vacuum. Thus this extra factor changes the oscillation behavior, but does not cause it.

When the neutrinos are massless, it is another story. Then the mixing disappears, i.e., $\theta_v=0$, and then $\langle\nu_e|\sigma_1|\nu_e\rangle =\langle\nu_\mu|\sigma_1|\nu_\mu\rangle=0$. It implies that the two states are degenerate[3] in the regular QM framework. But as long as there are extra terms such as the QMV, the degeneracy is broken and an oscillation can occur due to the new effects. Hence, in this case, σ_1 conservation does not forbid such a transition between different flavors (because the expectation value of σ_1 is zero for all flavors).

Namely, if neutrinos are massless and considering Eq. (7) only, we have $m_\nu=0$, and ν_1,ν_2 are exactly ν_e,ν_μ, but with the extra $\delta H\rho$ in the evolution equation, where different couplings stand in $h_{\mu\nu}$ in Eq. (9), and different ''flavor'' neutrinos manifest: the oscillation is still expected. Thus ν_1 and ν_2 have different behaviors as they propagate in an environment full with micro black holes and ''oscillations'' between them appear.

(b) With expression (10), we have the solution [15]

$$P_{\nu_e\to\nu_x}=\frac{1}{2}\sin^2 2\theta_v\left[1-e^{-(\alpha+\gamma)L}\cos\left(\frac{\Delta}{2E_\nu}L\right)\right]. \tag{14}$$

Note that to obtain the above result, we have assumed that $(\alpha,\beta,\gamma)_\nu \ll \Delta/2E_\nu$.

In fact, the exact result depends on whether the factor κ^2 is greater, equal, or smaller than zero with the definition

$$\kappa^2\equiv 4\left[(\alpha-\gamma)^2+4\beta^2-\frac{\Delta^2}{4E_\nu^2}\right]. \tag{15}$$

If κ^2 is less than zero, the oscillating form of Eq. (14) results; when κ^2 is greater or equal to zero, the expression turns into a purely damping solution. The precise version of Eq. (14) is

$$P_{\nu_e\to\nu_x}=\frac{1}{2}\sin^2 2\theta_v\left\{1-e^{-(\alpha+\gamma)L}\left[\frac{\alpha-\gamma}{\kappa}(e^{\kappa L/2}-e^{-\kappa L/2}) +\frac{1}{2}(e^{\kappa L/2}+e^{-\kappa L/2})\right]\right\}. \tag{16}$$

Indeed, when $\kappa^2<0$, κ is imaginary, and the solution contains an oscillatory factor; otherwise, it is attenuative.

We will discuss the phenomenological significance of Eqs. (13) and (14) later on.

III. EQUATIONS (13) AND (14) LEAD TO COMPLETELY DIFFERENT ASYMPTOTIC LIMITS AS $r\to\infty$ (OR $L\to\infty$)

The exponentially damping term in Eq. (13) would wash out any information of neutrino mixing as long as the detector is placed far enough from the source. In that case, $P_{\nu_e\to\nu_\mu}(t\to\infty)=\frac{1}{2}$ for two generations and, if generalizing the result to the n-generation structure [11],

$$P_{\nu_e\leftrightarrow\nu_\mu}(t\to\infty)=\frac{1}{n},\quad P_{\nu_e\leftrightarrow\nu_\tau}(t\to\infty)=\frac{1}{n},$$

$$P_{\nu_\tau\leftrightarrow\nu_\mu}(t\to\infty)=\frac{1}{n}.$$

On the contrary, Eq. (14) would lead to a different consequence as

[3]Even though they are degenerate for the QM scenario, they are different due to their different couplings to quantum gravity, so that oscillations still may be realized in this case.

$$P_{\nu_e\to\nu_\mu}(t\to\infty)=\frac{1}{2}\sin^2 2\theta_v;$$

i.e., the mixing angle between ν_e and ν_μ is still there. For the three-generation case we will have a similar result; only the simple ''Cabibbo-like'' angle θ_v should be replaced by the ''Kobayashi-Maskawa-like'' entries [10].

All the above expressions can apply to the $\nu_a\leftrightarrow\nu_b$ case with a,b being any pair of e,μ,τ as long as $a\neq b$.

IV. SOLAR NEUTRINO PROBLEM VS QMV EFFECTS

(a) For $\Delta\simeq 10^{-5}$ eV^2, i.e., Mikheyev-Smirnov-Wolfenstein (MSW) solution for the solar neutrino puzzle, one expects the averaged effect of oscillation term $\cos[(\Delta/2E_\nu)L]$ vanishes. Therefore the transition probability can be rewritten as follows.

(i) In the case of Eq. (9),

$$P(\nu_e\to\nu_e)=\frac{1}{2}[1+(1-2X)e^{-\gamma L}\cos 2\tilde{\theta}_0\cos 2\theta], \tag{17}$$

where $\tilde{\theta}_0$ is the neutrino mixing angle in the center of the Sun. X is the jumping probability from one neutrino mass eigenstate to another in the MSW resonant region. For the large angle solution $X\simeq 0$ and for the nonadiabatic solution it can be close to 1. From Eq. (17) we may see that $e^{-\gamma L}\gg 1$ is not favored to fit the solar neutrino data in addition to violating the condition Eq. (11). In this case we obtain a constant suppression 0.5, and it is disfavored [16]. As a result, the bound $\gamma L\leqslant 1$ is enforced. If $\gamma L\ll 1$, the violation effects are negligible. So only for $\gamma L\sim O(1)$ should the MSW solution for the solar neutrino problem be modified. Here L is the distance between the Sun and Earth. Generally we get $\gamma\leqslant 6\times 10^{-9}$ km^{-1}.

(ii) In the case of Eq. (10), the new effects are averaged to be zero over the distance L. The situation is exactly the same as the MSW solution without the QMV terms. In this case one cannot obtain any information about QMV from fitting the solar neutrino data.

(b) With the vacuum oscillation solution to the solar neutrino problem, $\Delta\sim 10^{-10}$ eV^2 is requested and in the case the oscillation term is not averaged to be zero. The transitional probability is given in Eqs. (13), (14), and (16). Again we obtain the bound $\alpha L,\gamma L\leqslant 1$ in order to solve the solar neutrino puzzle. And only for $\alpha L,\gamma L\sim O(1)$ should the parameter region of the vacuum oscillation solution be modified.

V. VERY INTERESTING FEATURE INDICATED BY EQ. (13)

Even if neutrinos are massless, the micro-black-hole effects can induce a neutrino transition from one flavor state to another. For an n-flavor neutrino case, the oscillation probability could be simplified as

$$P_{\nu_e\to\nu_x}=\frac{1}{n}-\frac{1}{n}e^{-\gamma L},$$

$$P_{\nu_e\to\nu_e}=\frac{1}{n}+\frac{n-1}{n}e^{-\gamma L}, \tag{18}$$

where SU(n) for n neutrino species should replace SU(2) for two neutrino species.

Indeed it is interesting to examine if such an oscillation probability alone is enough to solve the solar neutrino problem without requiring nonzero neutrino mass.

First of all, it is realized that γ cannot be a constant; otherwise, the ν_e suppression is energy independent which disagrees with the solar neutrino data [16]. By dimensional analysis, one may assume $\gamma=\hat{\gamma}\times E_\nu\sim E_\nu/M_{pl}$ for massless neutrinos. With this assumption we see that the larger neutrino energy corresponds to a larger suppression factor $e^{\gamma L}$; thus, in the solar neutrino experiment the ^{8}B neutrino is suppressed most, which is $1/n$. The ^{7}Be neutrino is suppressed less but very close to $1/n$; the pp neutrino is suppressed least which is between $1/n$ and 1. After careful study we find that the solar neutrino data can be fitted best with $n=3$. Hence we will discuss the three-species neutrino case in more detail [10]. We adopt the standard solar model [Bahcall and Basu (BP98) [17]] for our discussion. The predicted neutrino flux for H_2O experiments is, for $\nu_e\to\nu_\mu,\nu_\tau$ oscillation,

$$\Phi^{\rm th}_{\rm H_2O}=\left(2.21\begin{matrix}+0.19\\-0.14\end{matrix}\right)\times 10^6\ \ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}, \tag{19}$$

and for $\nu_e\to\nu_\mu,\nu_s$ oscillation (ν_s is a sterile neutrino),

$$\Phi^{\rm th}_{\rm H_2O}=\left(2.01\begin{matrix}+0.19\\-0.14\end{matrix}\right)\times 10^6\ \ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}. \tag{20}$$

The observed flux $\Phi^{\rm expt}_{\rm H_2O}$ is $2.42\pm 0.06^{+0.10}_{-0.07}\times 10^6$ cm^{-2} s^{-1} from Super-Kamiokande [3] (note that $2.80\pm 0.19\pm 0.33\times 10^6$ cm^{-2} s^{-1} from Kamiokande). The ratio $\Phi^{\rm th}_{\rm H_2O}/\Phi^{\rm expt}_{\rm H_2O}$ is estimated to be 0.92 ± 0.17 and 0.84 ± 0.17 for $\nu_e\to\nu_\mu,\nu_\tau$, and $\nu_e\to\nu_\mu,\nu_s$ oscillations, respectively. Theory agrees with the experiment within 1σ.

The neutrino capture rate in the chlorine experiments is obtained as

$$S^{\rm th}_{\rm Cl}=2.6\pm 0.4\quad \mathrm{SNU}. \tag{21}$$

To compare with the observed $S^{\rm expt}_{\rm Cl}=2.55\pm 0.25$ SNU [18], the ratio is estimated:

$$S^{\rm th}_{\rm Cl}/S^{\rm expt}_{\rm Cl}=1.0\pm 0.1. \tag{22}$$

So the theoretical expectation is in very good agreement with the experiment.

For the gallium experiments, if the parameter γ_ν falls in the region $(1.5\text{–}3.7)\times 10^{-8}$ km^{-1}, we will obtain the capture rate as

$$S^{\rm th}_{\rm Ga}=(68\text{–}79)\ \ \mathrm{SNU}. \tag{23}$$

This agrees with the experimental value (73.4 ± 5.7) SNU [18] at the 1σ level. At the 2σ level γ_ν can be taken as a value of $(0.66\text{–}5.4)\times10^{-8}\ \mathrm{km}^{-1}$.

VI. SCENARIO FOR THE "ATMOSPHERIC NEUTRINO" PROBLEM

With three-generation neutrinos and $\hat{\gamma}$ given in Sec. V, by fitting solar neutrino data at the level of 2σ errors, we may estimate some of the observables further for the atmospheric neutrino observations.

The up-down asymmetries of μ-like and e-like events for $\cos\Theta>0.2$ (down) and $\cos\Theta<-0.2$ (up), where Θ is the zenith angle, are denoted by Y_e, Y_μ. In the present scenario Y_e is always close to 1, independent of the energy and the traveling distance of the neutrinos. It is in agreement with the data $Y_e(\text{sub-GeV})=1.13\pm0.08$ and $Y_e(\text{multi-GeV})=0.83\pm0.13$ [3]. However Y_μ is estimated as 0.62–0.98 and 0.5 for multi-GeV and sub-GeV events, respectively; i.e., at the 1σ level, it agrees with the measured $Y_\mu(\text{multi-GeV})=0.54\pm0.07$, but at the 4σ level with the measured $Y_\mu(\text{sub-GeV})=0.78\pm0.06$. It is a little more involved to calculate the ratio of the total μ-like and e-like events. Here we only give a rough estimate on the double ratio by approximating that the downgoing neutrino flux is almost unsuppressed, while the upgoing and the horizontal neutrino flux is suppressed by a factor of 1/3. They are estimated to be 0.6 and 0.5–0.6 for sub-GeV and multi-GeV events, respectively, with the scenario. This is also not bad but in agreement with the measured ones $0.61\pm0.03\pm0.05$ and $0.66\pm0.06\pm0.08$ [3]. We conclude here that our scenario with zero neutrino mass can fit all the measurements of the solar and atmospheric neutrino experiments, except Y_μ (sub-GeV).

VII. CONSTRAINTS ON THE PARAMETER SET BY PRESENT DATA

As estimated in the previous sections, QMV effects can serve as an alternative mechanism for the solar and atmospheric neutrino flux shortage, if assuming

$$(\alpha,\beta,\gamma)_\nu=(\alpha,\beta,\gamma)_K\left(\frac{E_\nu}{M_K}\right). \tag{24}$$

The factor (E_ν/M_K) appearing here is due to the conjecture that there could be a simple relation between neutrino and K^0-$\overline{K}^0$ systems for the QMV effects induced by quantum gravity. Namely, as discussed above, to enforce the Lorentz-convariant form of the modified Schrödinger equation, we will have $(\alpha,\beta,\gamma)_K\propto M_K$ for the K^0-$\overline{K}^0$ system and a similar parametrization for neutrinos $(\alpha,\beta,\gamma)_\nu\propto E_\nu$. Thus the factor (E_ν/M_K) appears in Eq. (24). In the solar neutrino case, $E_\nu\sim0.3\text{–}10$ MeV; the factor suppresses $(\alpha,\beta,\gamma)_\nu$ by a factor of the order of $10^{-3}\text{–}10^{-2}$. This postulation should be tested by experiments on Earth.

The data on $\nu_\mu\to\nu_\tau$ oscillations by the CHARM II Collaboration [19] claimed that no evidence of the neutrino flux change had been observed. In the experiment, $E_\nu\sim27$ GeV and $L\sim0.6$ km. Considering the errors, $|\alpha L|$ must be smaller than 10^{-3}. This constraint requires

$$(\alpha,\beta,\gamma)_\nu\leqslant3.3\times10^{-22}\ \mathrm{GeV},$$

$$(\hat{\alpha},\hat{\beta},\hat{\gamma})\leqslant6.2\times10^{-8}\ \mathrm{MeV\,km^{-1}}\sim1.2\times10^{-23}. \tag{25}$$

Combining Eq. (25) with the enhancement factor $(E_\nu/M_K)\sim54$, it indicates that

$$(\alpha,\beta,\gamma)_K\leqslant6.2\times10^{-24}\ \mathrm{GeV}.$$

This number is much below the upper bounds given by the authors of [7,8]. If this is the case, the violation effects of quantum mechanics would hardly influence the ϵ value in the neutral kaon system. This constraint also excludes the ν_e, ν_μ, ν_τ oscillations discussed in Secs. V and VI. Hence a sterile neutrino must be introduced and the τ neutrino must be treated differently from the other species.

However, as pointed out above, the parameters for the neutrino and neutral kaon systems do not need to be as assumed here; so this comparison is interesting for further study of the problem.

VIII. SCENARIO IN LONG-BASE-LINE EXPERIMENTS

In the planned long-base-line experiments, KEK-Super-Kamiokande (250 km), CERN-GranSasso (730 km), and Fermilab-Sudan II (730 km), the average energies of the ν_μ beams are approximately 1 GeV, 6 GeV, and 10 GeV [20]. Accordingly, the suppression factor $e^{-\gamma L}$ for the experiments should cause obvious changes of the neutrino flux on the way from source to detector. If we substitute the values of (α,β,γ) given in Eq. (12) into the formulation for the three experimental sets, $e^{-\alpha L}$ turns out to be very small, so that the transition probability $P_{\nu_e\to\nu_x}\to\frac{1}{3}$ $(x\neq e)$ for the three-generation case. But if there exists a sterile neutrino, the probability would differ from 1/3.

By contrast, if we take the lower bound of γ given in Eq. (26) as $(\hat{\alpha},\hat{\beta},\hat{\gamma})\sim1.2\times10^{-23}$, then the suppression factors for the three experiments are

$$0.98,\quad 0.76,\quad 0.64, \tag{26}$$

respectively; the corresponding transition probabilities $P_{\nu_e\to\nu_x}$ are

$$0.0067,\quad 0.077,\quad 0.12.$$

QMV effects should be either observable in these long-base-line experiments or observation in the experiments will make more stringent constraints on the parameters of the effects.

IX. SUMMARY

The problem discussed here is an interesting subject in both aspects: first, the conclusion would indicate, even indirectly, if there are the mysterious micro-black-hole effects and, second, if this picture is valid. It is pointed out that existence of the effects may be non-negligible in certain physical processes, especially, the neutrino oscillations would be affected and the resultant neutrino-flux attenuation may become observable at the planned long-base-line experiments.

Moreover, as we indicated above, if $(\alpha,\beta,\gamma)_\nu > \Delta/4E_\nu$ in Eq. (15), the harmonic oscillation form would turn into a purely exponential damping form.

The *Ansätze* (9) and (10) lead to different asymptotic limits as $t\to\infty$, with $a\neq b$ for $P(\nu_a\to\nu_b)$. If the distance between detector and source is large enough, this difference of Eqs. (9) and (10) is distinguishable.

Especially, expression (19) makes very clear observational predictions with the parametrization given by EHNS and more stringent constraint (26), the neutrino flux can vary remarkably with respect to the distances of the long-base-line experiments. The predicted behavior of the neutrino flux is definitely distinguished from other mechanisms, and the effects would be observed at high-precision detectors which are under way as expected [21].

If a sophisticated neutrino detector is located in Beijing to receive neutrino flux sent from KEK, CERN, and Fermilab, as suggested by He and his collaborators [22], the distance is sufficiently large; then according to the above analysis, the observational prospect of such phenomena is optimistic.

In summary, neutrino oscillation experiments may put a stronger bound on the QMV effects than the K meson system if the E_ν dependence postulation which retains the Lorentz-convariant property of the evolution equation is valid. Even if neutrinos are massless, the QMV induced by micro black holes may ''cause'' neutrino oscillation. We find that this oscillation has an interesting prediction for solar neutrino and atmospheric behavior. Moreover, long-base-line experiments on neutrino oscillation may provide us with valuable information about the neutrinos and QMV effects. Anyhow, the physical picture of micro black holes has phenomenological significance, especially to the neutrino oscillation problem. It is worth further and better studies.

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