Pair production of charged vector bosons in supercritical magnetic fields at finite temperatures

V. R. Khalilov
Department of Physics, Moscow State University, Moscow 119899, Russia

Choon-Lin Ho
Department of Physics, Tamkang University, Tamsui 25137, Taiwan
(Received 29 January 1999; published 25 June 1999)

The thermodynamic properties of an ideal gas of charged vector bosons (with mass m and charge e) is studied in a strong external homogeneous magnetic field no greater than the critical value $B_{cr}=m^2/e$. The thermodynamic potential, after appropriate analytic continuation, is then used in the study of the spontaneous production of charged spin-one boson pairs from vacuum in the presence of a supercritical homogeneous magnetic field at finite temperature. [S0556-2821(99)02513-8]

PACS number(s): 13.40.−f, 05.30.Jp

I. INTRODUCTION

As is well known [1] the energy spectrum of the vector boson with mass m, charge e, spin $S=1$, and gyromagnetic ratio $g=2$ in a constant uniform magnetic field $\mathbf{B}=(0,0,B)$ is given by the formula (we set $c=\hbar=1$)

$$E_n(p)=\sqrt{m^2+(2n+1-2S)|eB|+p^2}, \quad S=-1,\ 0,\ 1. \tag{1}$$

The integer n ($n=0,1,2,\ldots$) labels the Landau level, and p is the momentum along the direction of the field. For $n=0$, $p=0$, $S=1$, E_0 vanishes at $B=B_{cr}\equiv m^2/e$. When $B>B_{cr}$, E_0 becomes purely imaginary. Such behavior of the energy E reflects a quantum instability of an electrically charged vector boson field in the presence of an external uniform magnetic field. The source of this instability is due to the interaction of the external field with the additional (anomalous) magnetic moment of the bosons, which, owing to the gyromagnetic ratio $g=2$, appears already in the tree approximation.

Charged spin-1 particles with the gyromagnetic ratio $g=2$ are not minimally coupled to an external electromagnetic field (if they were coupled in such a way the above ratio had to be $g=1$). However, the quantum theory of relativistic charged spin-1 bosons with $g=2$ in the presence of external electromagnetic fields is a linear approximation of gauge field theory in which the local $SU(2)$ symmetry is spontaneously broken into $U(1)$ symmetry [2]. So one anticipates that the perturbative vacuum of the Weinberg-Salam electroweak model in the linear approximation would exhibit instability in a homogeneous superstrong magnetic field.

When B becomes equal to B_{cr} the lowest energy levels of charged spin-1 particle and antiparticle ''collide'' with each other at $E_0=0$. One finds similar behavior in the case of scalar particles in a deep potential well which acts as the external field [3]. In this latter case, one usually interprets the behavior of energies as follows: when the binding energy of a state exceeds the threshold for particle creation, pairs of scalar particle-antiparticle may be spontaneously produced giving rise to the so-called condensate. The number of boson pairs produced by such a supercritical external field (here the depth of the well) may be limited if only the mutual interaction of the created particles is taken into account. In the framework of (second) quantized field theory the behavior of bosons in supercritical external fields was first considered in Ref. [4], which assumes a self-interaction of the ϕ^4-type, with the conclusion that the vacuum is, in fact, stabilized by the extremely strong (mutual) vacuum polarization. For a thorough discussion on the problem of electron-positron and scalar boson pair production in external electromagnetic fields see Ref. [5].

The case of vector bosons was considered in Ref. [6] by taking into account only the ground state of the charged spin-1 bosons in the superstrong external magnetic field and assuming a self-interaction for this state similar to the one of the W_μ vector boson field in the Weinberg-Salam electroweak gauge theory, namely, the $|W|^4$ interaction. In this work the condensate energy of charged spin-1 boson pairs was found, and a scheme for quantizing the W field in the neighborhood of the new classical vacuum with $W_{classical}\neq 0$ near the threshold for condensate production $B-B_{cr}\ll B_{cr}\equiv m_W^2/e$ (m_W is the mass of the W boson) was presented.

Using the complete electroweak Lagrangian the authors of Ref. [7] have managed to construct new ''classical'' static magnetic solutions for a W condensate in the tree approximation. They also show that the instability of the W field does not occur owing to the $|W|^4$ self-interaction term in the electroweak Lagrangian. Moreover, the electroweak gauge symmetry may be restored in the presence of a superstrong magnetic field with $B=m_H^2/e$ (m_H is the mass of the Higgs boson) if $m_H>m_W$.

In the one-loop approximation of the effective Lagrangian of the charged spin-1 boson field (without a self-interaction term), radiative corrections may induce, in the presence of a strong uniform magnetic field, the production of charged vector boson pairs in the lowest energy states, i.e., the condensate. It is of interest to see what happens with the vacuum when not only an external magnetic field is present but also when the temperature is finite.

In this paper we shall investigate the problem of pair production of charged vector bosons induced by the unstable mode in the presence of a supercritical magnetic field at fi-

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nite temperature. To study the vacuum effects we need to compute the effective potential density, which is closely related to the thermodynamic potential. To this end we shall first try to treat the problem in the framework of standard quantum statistical physics for the case $B<B_{\rm cr}$ when quantum statistical quantities such as the thermodynamic potential are unambiguous and may be well defined. After deriving the thermodynamic potential in the region $B<B_{\rm cr}$ we shall perform an analytic continuation of this quantity into the supercritical region $B>B_{\rm cr}$. This will give us the imaginary part of the effective potential, from which we can derive the expression for the rate of pair production. The contribution in the thermal one-loop effective action from gauge boson field in a constant homogeneous magnetic field was previously considered in Ref. [8] in connection with the question of symmetry restoration. But in that work contribution from the unstable modes was explicitly ignored.

II. THERMODYNAMIC POTENTIAL

The thermodynamic potential Ω for a gas of real (not virtual) charged vector bosons as a function of the chemical potential μ, the magnetic induction of external field B, and the gas temperature $T\equiv 1/\beta$ is defined by

$$\Omega=\frac{eBV}{4\pi^2\beta}\int dp\,\ln\{1-\exp\beta[\mu-(m^2-eB+p^2)^{1/2}]\}+\frac{eBV}{4\pi^2\beta}\sum_{n=0}^{\infty}g_n\int dp\times\ln\{1-\exp\beta[\mu-(m^2+(2n+1)eB+p^2)^{1/2}]\},\quad(2)$$

where V is the volume of the gas, and $g_n=3-\delta_{0n}$ counts the degeneracy of the excited states.

By expanding the logarithms and integrating over p, one can recast Ω into [9]

$$\Omega(\mu)=\Omega_1+\Omega_2+\Omega_3=-\frac{VeB}{2\pi^2\beta}\sum_{k=1}^{\infty}k^{-1}\exp(k\beta\mu)[M_-K_1(k\beta M_-)-M_+K_1(k\beta M_+)]-\frac{3VeB}{2\pi^2\beta}\sum_{n=0}^{\infty}\sqrt{M_+^2+2neB}\times\sum_{k=1}^{\infty}k^{-1}\exp(k\beta\mu)K_1(k\beta\sqrt{M_+^2+2neB}),\quad(3)$$

where $M_\mp\equiv\sqrt{m^2\mp eB}$ and $K_n(x)$ is the modified Bessel function of order n. If both particles and antiparticles are present, the factor $\exp(k\beta\mu)$ in Eq. (3) has to be replaced by $2\cosh(k\beta\mu)$. The thermodynamic potential as a function of the chemical potential is real-valued for real values of μ that are to satisfy for particles and antiparticles with mass M_- the inequality $|\mu|\leqslant M_-$. This condition comes from the physical requirement that the density (and the occupation numbers) of particles and antiparticles with the mass M_- is positive for any real values of momenta p.

In weak field $B\ll m^2/e$ and at temperatures $eB/m\ll T<m$ when the spacing between Landau levels is still considerably less than the thermal energy, one can approximate Ω as follows. For Ω_1 and Ω_2 in Eq. (3), we set $M_\mp\approx m(1\mp\chi/2)$ with $\chi\equiv eB/m^2$, and use the following formula

$$K_1[k\beta m(1\mp\chi/2)]=K_1(k\beta m)+\chi\frac{dz}{d\chi}\frac{dK_1(z)}{dz},\quad(4)$$

where $z=k\beta m(1\mp\chi/2)$. Evaluation of Ω_3 can be done by first replacing the summation over n by an integral using the Euler formula

$$\sum_{n=0}^{\infty}f(n+1/2)=\int_0^{\infty}f(x)dx+(1/24)f'(0),\quad(5)$$

and then by using the formula [10]

$$\int_1^{\infty}dzz^2\;K_1(kmz\beta)=\frac{1}{km\beta}K_2(km\beta).\quad(6)$$

The thermodynamic potential and density of spin-1 bosons at equilibrium can then be obtained as

$$\Omega\simeq-\frac{VT^{1/2}m^{3/2}}{(2\pi)^{3/2}}\left[3T^2{\rm Li}_{5/2}(e^{\beta(\mu-m)})+\frac{7(eB)^2}{8m^4}e^{\beta(\mu-m)}\right],\quad(7)$$

$$\rho\simeq3\left(\frac{Tm}{2\pi}\right)^{3/2}\zeta(3/2),\quad(8)$$

where ${\rm Li}_s(x)=\Sigma_{k=1}^{\infty}x^k/k^s$ is the polylogarithmic function of order s, and ${\rm Li}_s(1)=\zeta(s)$. The magnetization of the gas under the above conditions is a positive function[1] of the magnetic field induction and temperature because paramagnetic (spin) contribution dominate

$$M_z(B)=-\frac{1}{V}\frac{\partial\Omega}{\partial B}=\frac{7e^2BT^{1/2}}{4(2\pi)^{3/2}m^{1/2}}e^{\beta(\mu-m)}.\quad(9)$$

When $B\approx B_{\rm cr}$ transitions of bosons from level $n=0$ to any excited levels $n\geqslant1$ will not be allowed if $T<eB/m$ and all bosons in quantum state with $n=0$ that are available may be considered as condensate in a two-dimensional ''momentum'' space in the plane perpendicular to the magnetic field with values of ''effective momenta'' $k<(eB)^{1/2}$. True condensate, however, will not actually be formed in three-dimensional momentum space because longitudinal momenta of bosons may have values outside this region.

For low temperature T such that $\beta M_-\gg1$, contributions to the thermodynamic potential (3) from all the excited states of the vector bosons are exponentially small compared with that from the state with $n=0$ and $S=1$. Hence only the first term Ω_1 in Eq. (3) needs be considered in this limit

[1]We take this opportunity to correct the expression for the magnetization in Ref. [9].

$$\Omega(\mu)=\Omega_1(\mu)=-\frac{VeBM_-}{2\pi^2\beta}\sum_{k=1}^{\infty} k^{-1}\exp(k\beta\mu)K_1(k\beta M_-). \tag{10}$$

Subsequently, the boson density is

$$\rho_g=\frac{eBM_-}{2\pi^2}\sum_{k=1}^{\infty} K_1(k\beta M_-)\exp(k\beta\mu). \tag{11}$$

When $M_-\gg T$, $M_--\mu<T$, we can get for the total density at equilibrium

$$\rho\cong\frac{eB(TM_-)^{1/2}}{(2\pi)^{3/2}}\left[\left(\frac{\pi T}{M_--\mu}\right)^{1/2}-1.46\right]. \tag{12}$$

The first (leading) term of Eq. (12) reduces exactly to the one obtained in Ref. [11]. The total boson density (11) for relatively ''high'' temperature for which $T>M_-$ but $T\ll m$ is

$$\rho\cong\frac{eBT}{2\pi^2}e^{\mu/T}, \tag{13}$$

for $-\mu\gg T$ and

$$\rho\cong\frac{eBT}{2\pi^2}\ln(T/|M_-+\mu|) \tag{14}$$

for $-\mu\rightarrow M_-$.

It follows from formulas (12) and (14) that significant amount of vector bosons persists to fill the states with nonzero momentum projections on the magnetic field direction. These states now should be considered as excited ones. Hence, any density of bosons can be accommodated outside the ground state (with $p=0$) at any temperature. This means that there is no true Bose-Einstein condensation in the presence of finite magnetic fields. We mention here that it was Feynman [12] who first showed that true BEC is impossible in a classical one-dimensional gas.

An exact expression for the magnetization of the vector boson gas in the lowest energy state in a strong magnetic field may be derived from Eq. (10) in the form [9]

$$M_z(B)=\frac{e}{2\pi^2\beta}\sum_{k=1}^{\infty}\left[\frac{M_-}{k}K_1(k\beta M_-)+\frac{eB\beta}{2}K_0(k\beta M_-)\right]\exp(k\beta\mu). \tag{15}$$

The magnetization is also a positive function of the magnetic field and temperature.

III. PAIRS PRODUCTION OF VECTOR BOSONS

Let us now turn to discussing the problem of pair production of vector bosons in a supercritical magnetic field at finite temperature. There are two possible mechanisms for this process: (1) pairs may be produced as a result of thermal collisions of real charged bosons in the external field, (2) pairs may be spontaneously produced by a constant magnetic field when $B>B_{cr}$ from the vacuum, just as electron-positron pairs are produced by an external electric field [13].

Before we proceed to the second mechanism, let us first give some estimates of the density of spin-1 boson pairs (in the lowest energy state only) that may be produced as a result of thermal collisions of real bosons in the external field with $B\approx B_{cr}$. If the density of the created pairs is much greater than that of the bosons present initially, we may apply formula (11) with $\mu=0$ to find the density of the pairs produced by thermal collisions. For low ($\beta M_->1$ but $T\ll m$) and high ($\beta M_-<1$, $T<m$) temperature, we obtain, respectively,

$$\rho_T\cong\frac{eB(M_-T)^{1/2}}{(2\pi)^{3/2}}\exp(-M_-/T), \tag{16}$$

$$\rho_T\cong\frac{eBT}{2\pi^2}\ln(T/M_-). \tag{17}$$

Now we come to the second mechanism. As is known (see, for example, Ref. [14]) the quantum electrodynamics vacuum in the presence of an external electromagnetic field can be described by the total transition amplitude from the vacuum state $|0_{in}\rangle$ in time $t\rightarrow-\infty$ to the vacuum state $\langle 0_{out}|$ in time $t\rightarrow\infty$ as follows:

$$C_v=\langle 0_{out}|0_{in}\rangle=\exp[iW(\mathcal{E},B)], \tag{18}$$

where W is the effective action for a given quantum field. W defines the effective Lagrangian L_{eff} according to $W=\int d^4x L_{eff}$. The effective action is a classical functional depending on the external electric ($\mathcal{E}$) and magnetic (B) fields. When the external electromagnetic field is homogeneous the effective action is equal to $W(\mathcal{E},B)=-[E(\mathcal{E},B)-E(\mathcal{E}=0,B=0)]V\Delta t$, where $E(\mathcal{E},B)$ is nothing but the density of vacuum energy in the presence of the external field, V is the volume, and Δt is the transition time. It is worthwhile to note that the effective action contains all divergencies of the theory but they are in the real part of $W(\mathcal{E},B)$. C_v is the probability amplitude when the external electromagnetic field does not change and so this applies for the vacuum.

For external fields smoothly changing both in space and time one has

$$|C_v|^2=\exp[2\,\mathrm{Im}\,L_{eff}(\mathcal{E},B)V\Delta t]. \tag{19}$$

The imaginary part of the effective Lagrangian density $\mathrm{Im}\,L_{eff}$, or of the vacuum energy, is finite and describes production of particles by the external electromagnetic field. It also signals that an instability of the vacuum occurs. The imaginary part of the effective Lagrangian density reduces at $T=0$ to the imaginary part of the effective potential density. The latter (for the case under consideration) arises from the lowest energy of the charged massive vector boson being imaginary at $B>B_{cr}$. At $T=0$ the imaginary part of the effective potential is [1,15]

$$\operatorname{Im} V_0 = -\frac{eB(eB-m^2)}{16\pi}. \quad (20)$$

Here V_0 is the zero-point energy density of the vacuum at $T=0$. From Eq. (19) one concludes that the quantity $\Gamma(B) \equiv -2\operatorname{Im} V_0(B)$ is the production rate per unit volume of vector boson pairs (or, equivalently, the decay rate of the vacuum) in the external magnetic field at $T=0$, which, in this case, is given by

$$\Gamma(B) = \frac{eB(eB-m^2)}{8\pi}. \quad (21)$$

When $T\neq 0$ one must use the thermal one-loop effective potential density which is related to the thermodynamic potential by (see, for example, Ref. [8])

$$V(B,T) = V_0 + \Omega(\mu=0,B,T)/V. \quad (22)$$

The first term in Eq. (22) does not depend on temperature, while the second term coincides with the thermodynamic potential (per unit volume) of noninteracting gas of bosons at $\mu=0$. We see that the thermodynamic potential at $\mu=0$ play the role of the free energy of the vacuum of quantum field system in the presence of the external field at finite temperature.

Since the unstable mode contributes to Ω_1, it follows from Eq. (10) that the temperature-dependent part of the effective potential also becomes complex when $B>B_{\rm cr}$. The point $B=B_{\rm cr}$ is the branch point of the effective potential considered as a function of product eB. One could avoid the complex values of the effective potential by taking account of the ''physical'' part of energy spectrum (1) which may be well defined only when $B<B_{\rm cr}$.

To find the effective potential in the region $B>B_{\rm cr}$ we must perform an analytic continuation of $K_1(z)$ as a function of variable $z=k\beta M_-$ into the complex range. Let us denote the argument of function $K_1(z)$ in the region $B>B_{\rm cr}$ as $z=\pm ik\beta\sqrt{eB-m^2}\equiv \pm ikt$. Then, by Schwartz symmetry principle, we have

$$K_1(\pm ikt) = K_1^*(\mp ikt) = -\frac{\pi}{2}[J_1(kt)\mp iY_1(kt)], \quad (23)$$

where $J_1(kt)$ and $Y_1(kt)$ are the first- and second-order Bessel functions, respectively. From Eqs. (10) and (23) we get

$$\Omega_1(B,T) = \frac{Vm^3}{4\pi\beta}\chi\sqrt{\chi-1}\sum_{k=1}^{\infty}\frac{1}{k}[\pm iJ_1(k\beta m\sqrt{\chi-1}) + Y_1(k\beta m\sqrt{\chi-1})]. \quad (24)$$

Here $\chi=eB/m^2$ as defined before. For the imaginary part of $\Omega_1(B,T)$ one can obtain another form using the following formula which is valid for small temperatures such that $t>2\pi$:

$$\sum_{k=1}^{\infty}\frac{J_1(kt)}{k} = 1-\frac{t}{4}+\frac{2}{t}\sum_{n=1}^{l}\sqrt{t^2-(2\pi n)^2}, \quad (25)$$

where l is the integral part of $t/2\pi$.

The production rate at finite temperature and magnetic field is now defined to be $\Gamma(B,T)\equiv -2\operatorname{Im} V(B,T)$ according to Eq. (19). Since $\Gamma(B,T)$ must be positive at any finite temperature, $\operatorname{Im} V(B,T)$ must always be negative. This requirement allows one to perform the analytic continuation unambiguously. It follows from Eqs. (20), (22), (24), and (25) that we must take Eq. (24) with the lower sign in order to obtain a non-negative decay rate:

$$\Gamma(B,T) = \frac{m^4}{8\pi}\chi(\chi-1)\left[1+\frac{4}{x}\sum_{k=1}^{\infty}\frac{J_1(kx)}{k}\right], \quad (26)$$

where $x\equiv \beta m\sqrt{\chi-1}$. This is the total pair production rate of vector bosons induced by the unstable mode in a supercritical magnetic field at finite temperatures. We note here that the expression inside the square bracket of Eq. (26) depends on β, m, e, and B only through the combination x.

For low temperature ($\sqrt{eB-m^2}\gg T$) formula (26) may be simplified. Applying the asymptotic expansion for $J_\nu(z)$ at $z\to\infty$ in the form

$$J_\nu(z) = \sqrt{2/\pi z}\cos(z-\nu\pi/2-\pi/4),$$

replacing the summation over k by the integration and then taking into account the following formula for the Fourier integral [16]:

$$\int_a^{\infty} e^{ixt}f(t)dt = ie^{iax}f(a)/x + o(1/x) \quad (x\to\infty) \quad (27)$$

[by the Riemann-Lebesgue lemma the last formula is valid if the integral converges uniformly in (a,∞) at all large enough x], we obtain [up to the term $o(1/x)$]

$$\Gamma(B,T) \sim \frac{m^4\chi(\chi-1)}{4\pi}\left[1+\left(\frac{2}{x\pi^{1/5}}\right)^{5/2}\cos\left(x-\frac{\pi}{4}\right)\right]. \quad (28)$$

As $T\to 0$ $(x\to\infty)$, Eq. (28) reduces to the zero temperature expression (21).

IV. RESUME

Our calculations have been performed for the case of the constant (in time) magnetic field. When pairs are spontaneously produced by the constant magnetic field exclusively the background magnetic field will be changed in time. One may suppose that the background magnetic field is likely to remain constant in time only during a characteristic time $\delta t \sim 1/\sqrt{eB-m^2}$.

We see that the number of boson pairs produced by such a supercritical external field increases with increasing magnetic field. This number may be limited if the mutual interactions of the created particles are taken into account. As mentioned in the Introduction the vacuum may be stabilized with the appearance of vector boson condensate (at $T=0$) in the tree approximation. Another possibility to stabilize the vacuum is the one-loop radiative corrections to the mass of the charged spin-1 boson field in the critical field region $B \to B_{cr}$ [17].

ACKNOWLEDGMENTS

This work was supported in part by the Republic of China through Grant No. NSC 88-2112-M-032-002. V.R.K. would like to thank the Department of Physics of Tamkang University (R.O.C.) for kind hospitality and financial support.

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