String junctions and bound states of intersecting branes

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We study four-dimensional black hole configurations which result from wrapping M5-branes on a Calabi-Yau manifold, as well as *U*-dual realizations. Our aim is to understand the microscopic degrees of freedom responsible for the existence of bound states of multiple branes. The details depend on the chosen *U* frame; in some cases, they are massless string junctions. We also identify a perturbative description in which these states correspond to twisted strings of intersecting D3-branes at an orbifold singularity. In each case, these are the preponderant states of the spacetime infrared conformal field theory and account for the entropy of the black hole. [S0556-2821(99)04514-2]

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I. INTRODUCTION

It is by now well known that systems of intersecting branes correspond to black holes, and the entropy of such a system may be accounted for by enumerating string states [1]. At least when sufficient supersymmetry is preserved, the configuration of branes is a bound state at threshold. In many cases, these bound states signal the existence of degrees of freedom localized on the intersection manifold. It will be the aim of this note to understand in more detail the nature of these new states.

We are interested here in an intuitive problem: what is the detailed mechanism for binding together a collection of many (more than two) branes, and in particular, what are the relevant microscopic degrees of freedom? For a bound state of a pair of branes, we can certainly expect that ordinary strings stretching between them are responsible for the binding. However, in intersections of more than two branes, binding by ordinary strings cannot account for the entropy of the configuration, as we will discuss in some detail below.

The system that we will have in mind throughout this paper is the four-dimensional black hole obtained from an M5-brane wrapped on a divisor of a Calabi-Yau threefold. However, it will be useful to consider directly a collection of three types of M5-branes wrapped on orthogonal cycles of a T^6. In much of the paper, we will discuss directly the case of T^6, although we explore Calabi-Yau manifolds in the final section. In the case of T^6, we may take the M5-branes to be arranged as follows:

Brane	0	10	1	2	3	4	5	6	7	8	9
$M5_1$	●	●	●	●	●	–	–	●	–	–	–
$M5_2$	●	●	●	–	–	●	●	●	–	–	–
$M5_3$	●	–	–	●	●	●	●	●	–	–	–
P_L	–	–	–	–	–	–	–	●	–	–	–

The four-dimensional black hole has an $E_{7,7}$ U-duality group; a useful diagonal basis identifies four charges as the number of M5-branes of each of three types plus momentum along the eleventh direction. The entropy of this black hole is given, at least to leading order, by the product of these charges, and may be thought of as counting all of the excitations of the black hole.

Let us briefly review what is known about this system. There are several points of view. In the limit where the compact manifold is small, one attains an effective description in terms of a $(1+1)$-dimensional field theory on the intersection manifold of the M5-branes. This theory is a superconformal field theory in the infrared, with (0,4) supersymmetry. First, there is an important analysis of Ref. [2] (see also Ref. [3]) which computes the central charge of this theory in terms of the cohomology of the complex divisor upon which the M5-branes are wrapped. Thus the entropy is computed, in leading order, by the triple-self-intersection number of the divisor. String states stretching between two types of branes would only account for double intersections, and thus fall short. Whatever this spacetime conformal field theory (CFT) is, it is known that on T^6 it must have a moduli space of deformations given by $F_{4(4)}(\mathbb{Z})\backslash F_{4(4)}/Sp(2)\times Sp(6)$ [4].

The low energy physics of the bound states may be understood in terms of deformation theory. Locally, we can discuss the triple intersection in $\mathbb{C}^3$, coordinatized by z^1,z^2,z^3. An equation for the divisor is of the form

$$P_{N_1,N_2,N_3}(z^1,z^2,z^3)=0=P_{N_1}(z^1)P_{N_2}(z^2)P_{N_3}(z^3) \quad (1)$$

where N_i are the degrees of each polynomial. The zeroes of this polynomial correspond to the position of each M5-brane. The holomorphic deformations of the divisor are of the form

$$P_{N_1,N_2,N_3}(z^1,z^2,z^3)+Q_{N_1-1,N_2-1,N_3-1}(z^1,z^2,z^3)=0. \quad (2)$$

The degrees of the polynomial Q have been chosen such that this deformation does not alter the asymptotic form. We can choose to write the deformations in the following form:

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$$Q_{N_1,N_2,N_3}=\sum_{i,j,k} a_{ijk}\frac{P_{N_1}(z^1)P_{N_2}(z^2)P_{N_3}(z^3)}{(z^1-r_i^1)(z^2-r_j^2)(z^3-r_k^3)}. \tag{3}$$

The a_{ijk} are the localized deformations, and appear as fields in the low energy description. The number of degrees of freedom then is simply counted as the number of triple intersections; because of supersymmetry, these must come in supermultiplets, with $c=6$. When we compactify, care must be taken with boundary conditions, and so not all of these deformations are allowed. One expects, however, that these effects are subleading compared to the number of triple intersections.

Furthermore, the near-horizon limit of this black hole displays the geometry $\mathrm{AdS}_3\times \mathrm{S}^3/\mathbb{Z}_N\times M_4$; the supergravity spectrum on AdS_3 has been computed [5], and recently, the quantization of strings [4] in this background has been considered. In this paper, we are not directly interested in such SCFT descriptions. Instead, we would like to elucidate the microscopic stringy physics responsible for the existence of the bound state. We discuss several different U-frames here; perhaps the most intuitively appealing picture is within a type IIB frame, where the binding of three branes is related to the existence of massless string junctions localized at the triple intersection. The identification of these non-perturbative states is hampered by the absence of Bogomol'nyi-Prasad-Sommerfield (BPS) states in this background, although we give strong arguments for the existence of the bound states. Another type IIB frame involves intersecting D3-branes localized at an orbifold singularity; the bound states are understood in terms of twisted strings. The latter frame leads to a perturbative UV gauge theory description of this system.

II. STRING JUNCTIONS

We begin with a short review of the essential properties of string junctions. In type IIB string theory, 1-branes are classified by a pair of integers (p,q). In this notation, the fundamental string is a (1,0)-brane, and the $D1$-brane a (0,1)-brane. It is known that, subject to some conditions, there is a BPS state consisting of three such branes meeting at a junction. Since p and q are the charges with respect to the 2-forms B_{NS} and B_R, they must be conserved at the vertex:

$$\sum_i p_i=\sum_i q_i=0. \tag{4}$$

In addition, there is a zero force condition which depends on the coupling [6,7].

Now note that there is a U-duality frame in which the 3 M5-branes become an Neveu-Schwarz (NS) 5-brane, a D5-brane and a D3-brane in type IIB string theory. This is attained (referring to the table in Sec. I) by compactifying the 10-direction, then performing T-duality along, say, the 2-direction. These three branes intersect along a string as did the M5-branes. The low energy theory then is expected to be a $(1+1)$-dimensional CFT with (0,4) supersymmetry:

Brane	0	1	2	3	4	5	6	7	8	9
$D3$	●	●	–	●	–	–	●	–	–	–
$D5$	●	●	●	–	●	●	●	–	–	–
$NS5$	●	–	●	●	●	●	●	–	–	–
P_L	–	–	–	–	–	–	●	–	–	–

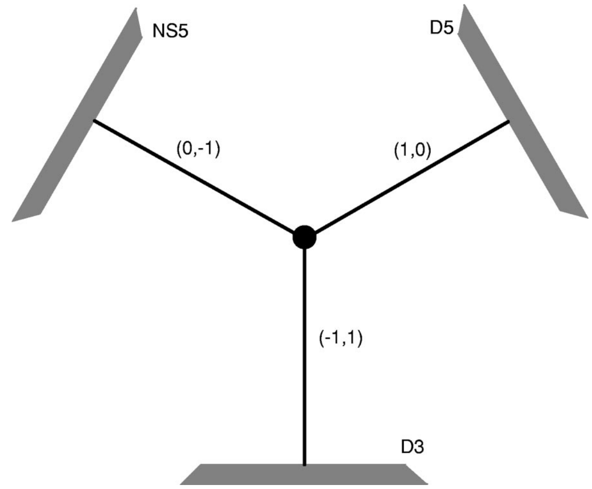

FIG. 1. Cartoon of junction between branes.

From the point of view of charge conservation, the state shown in Fig. 1 exists and is stable. Furthermore, the string junction is massless when the three branes intersect; the junction may be made massive by moving the branes away from each other in the 789-directions.

Now, each of the ends of the string junction may terminate on any of the N branes of the appropriate type. Thus, we see that there are of order $N_1N_2N_3$ states present here. Furthermore, since the junction must organize itself into a representation of the (0,4) supersymmetry, there are $4N_1N_2N_3$ bosonic states and their superpartners. String junctions then account for the entropy of this configuration. Note that in this frame, open string states stretching between branes are not this numerous. Thus, at least to leading order, the entropy is accounted for by non-perturbative states.

There are several potential problems with this picture however, and we now turn to a discussion of the relevant issues. We have claimed above that the string junctions are massless when the branes intersect. This is true geometrically at the classical level, however, it is not true that the mass of a massive state is protected. To understand the relevant issues, we should consider the details of (0,4) supersymmetry algebra in two dimensions [8]. The algebra takes the form

$$\{Q,Q\}=P_R. \tag{5}$$

In particular, there are no central charges as that requires both left and right moving supersymmetries. The BPS bound is thus simply $P_R\geqslant 0$; the only states saturating the bound are *massless* and may have $P_L\neq 0$. This implies that in any ultraviolet description, only the massless states with $P_R = 0$ will necessarily survive down to the infrared conformal theory and contribute to the entropy of the configuration we are studying. For this massless state to be present then, we must argue that the classical moduli space is unmodified quantum mechanically, at least at the origin. Indeed, we do not expect such modifications because of the (0,4) supersymmetry. This is actually more restrictive than (2,2); for example, the metric of the target space manifold must be hyper-Kähler.

If we identify the states localized at the intersection to be of a non-perturbative origin, then we must become comfortable with the idea that the conformal field theory of ordinary string states is somehow insufficient. Indeed, we can think of this situation as akin to a conifold singularity—at the origin, there is a new branch of the moduli space, parameterized by VEV's of the fields corresponding to string junctions. This is not obviously inconsistent, as near the NS5-five branes the string theory is strongly coupled which invalidates perturbation theory.

III. THE ORBIFOLD FRAME

In this section, we discuss another U frame which is perturbative, and the localized states at the intersection are twisted strings. To attain this, we may begin with the configuration used in the last section, and perform a T duality along $X^{1,2}$:

Brane	0	1	2	3	4	5	6	7	8	9
$D3_1$	●	–	●	●	–	–	●	–	–	–
$D3_2$	●	–	–	–	●	●	●	–	–	–
$KK5$	●	×	●	●	●	●	●	–	–	–

The interpretation of this configuration is that of a pair of D3-branes intersecting along a line (X^6), at a $\mathbb{Z}_{N_3}$ orbifold singularity.[1] Here, N_3 is the number of NS5-branes in the original picture, and there are N_1 (N_2) D3-branes of each type. Note that in this frame, there is no manifest triality between N_1,N_2 and N_3. This occurs simply because of taking a definite U-duality frame; triality will be recovered in U-invariant quantities, such as the entropy.

This is an interesting configuration in its own right. We find a gauge theory description of the $(1+1)$-dimensional intersection. This theory is an ultraviolet description where gravity has been decoupled, which will flow to the relevant conformal field theory in the infrared. In this theory, we will be able to identify the states that are localized at the intersection, and which contribute the predominant amount of entropy. Since the configuration is perturbative, the analysis is reliable.

The spectrum of this gauge theory may be obtained via a straightforward application of familiar techniques. Note first that if we concentrate on the states of a single D3-brane but dimensionally reduce along a two torus, we expect to see multiplets of (4,4) supersymmetry. The supersymmetry preserved by each of the two D3-branes is incompatible, and at the end we are only left with (0,4) supersymmetry; the string states connecting $D3_1$ to $D3_2$ do not form full (4,4)-multiplets.

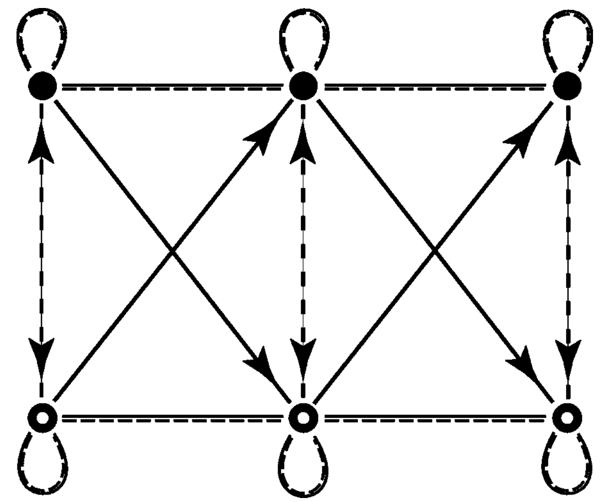

FIG. 2. A portion of the quiver diagram. The open (closed) circles represent images of the N_1 (N_2) collections of $D3$-branes. Bosonic string states and superpartners are represented by dashed lines, left-moving fermions by solid lines.

To construct the spectrum, account for the orbifolding by N_3 images of the collections of N_1 (N_2) D3-branes. String states that stretch between D3-branes of the same type, as mentioned, give multiplets of (4,4) supersymmetry—the fermions and bosons are in the same gauge multiplets. Those multiplets which correspond to string states between branes at the same image, turn out to be hypermultiplets, whereas those stretching horizontally (see Fig. 2) are vector multiplets, (this nomenclature comes from looking at the four dimensional theory on the intersection of two D5-branes, where the vector directions are along the intersection manifold, and the hypermultiplet directions are orthogonal). The resulting gauge group is then[2]

$$\prod_{k=1}^{N_3} [U(N_1)\times U(N_2)]. \tag{6}$$

The string states that stretch between D3-branes of different type however are acted upon non-trivially by the orbifold. It should be noted that the $\mathbb{Z}_{N_3}$ acts chirally on the $SU(2)\times SU(2)$ R symmetry on either of the D3-branes. First, this particular orbifold action is important for preserving (0,4) supersymmetry and the resulting hyper-Kähler structure.

[1]In the table, the symbol × refers to the Taub-NUT (Newman-Unti-Tamlewino) direction. We take $X^{1,7,8,9}$ to be noncompact. This ensures that the singularity is isolated.

[2]We consider the low energy ultraviolet theory, and so do not concern ourselves with the possible decoupling of $U(1)$'s.

Given this orbifold action, bosons and right moving fermions form supermultiplets, and the left moving fermions are singlets under supersymmetry. The field content is summarized in Fig. 2. The fields are supermultiplets for the vertical lines, and left-moving fermions for the diagonal lines. The nodes and edges have a supermultiplet and left-moving fermion singlets, as is required in order to complete representations of (4,4) supersymmetry. Note that this portion of the spectrum is an example of ''misaligned supersymmetry'' of Ref. [9], as bosons and fermions are degenerate but they are in different representations of the symmetry groups. Thus much of the structure of a (4,4)-supersymmetric theory is present; only the gauge representations are aware of the breaking to (0,4).

As the configuration is made only out of D3-branes, the value of the type IIB coupling constant is not fixed at any value, and we can actually take a weakly coupled limit, so that the field theory analysis is accurate.

Next, we would like to count (gauge invariant) modes, in order to probe the entropy of the corresponding black hole. To facilitate this, we move on the moduli space to a generic point, where the gauge group is broken as much as possible. To this end, we move all D3-branes apart in the 2345-directions (but not away from the orbifold singularity). The gauge group is Abelian, $U(1)^{N_1}\times U(1)^{N_2}$, and massless charged states are present. While most of the (4,4) vectors and hypermultiplets have been lifted, the twisted states survive. These states are localized at the orbifold singularity, and have multiplicity $N_1N_2N_3$ (since they are in $(N_1,\overline{N_2})$ representations, and there are N_3 images). For each of these, we have two complex bosonic modes and two complex fermionic modes (as in Fig. 2). In a sector with fixed P_L, these states dominate the entropy, giving by a standard argument $S=2\pi\sqrt{6N_1N_2N_3P_L}+\cdots$.

It was found in Ref. [4] that the central charge of the spacetime conformal theory contains no subleading corrections. However, in the present construction, it appears that there is a problem. There are massless fields which are the remnant of the adjoint hypermultiplets. There is one such supermultiplet remaining per vertex of Fig. 2, and thus one would expect that these fields contribute to the entropy at order $(N_1+N_2)N_3$. It is possible that the correct central charge is nevertheless obtained by canceling this contribution. We have assumed that all triple intersections contribute an independent supersymmetric degree of freedom to the entropy, but this is not really true, as not all of the local deformations can produce a smooth manifold. This means that some fraction of the (vertical) fields have a superpotential and therefore do not contribute to the entropy. It is quite possible that this correction to the leading term in the entropy above precisely cancels the effect of the adjoint fields. A similar mechanism is known to occur in the D1-D5 system [10,11]—the dimension of the moduli space is smaller than the number of fields because of D-term constraints (in our case we have F-terms).

Now note that we expect that this discussion of the spectrum is robust—the entropy is accounted for by twisted string states, as long as the singularity itself is not modified by quantum corrections. Furthermore, this description of the states localized at the intersection is T-dual to the description in the previous section in terms of string junctions, this is, from one description to the other we do a discrete Fourier transform. We regard this then as definitive evidence (if duality is to be believed) for the existence of massless string junctions in that frame, and hence for their contribution to the entropy.

IV. OTHER *U* FRAMES

It is of interest to consider other U frames in the same context. We confine ourselves to brief discussions of two such frames; in most cases, an understanding of the localized states is considerably more difficult.

A. M theory and 3 M5-branes

First, we consider the original M-theory configuration, and account for the entropy there [12]. This may be understood by beginning with the string junction; if we lift this to M theory, we find that the junction becomes a M2-brane ''pants section.'' Each (p,q)-leg has one direction wrapped along the vector (p,q) in the $X^{2,10}$ torus [13]. Thus the bound state degrees of freedom are these pants sections; at a triple intersection point of the M5-brane, they have zero area and so should go massless. A smooth point in moduli space then is attained by turning on vevs for these low energy fields.

B. Type IIA and the 4440 system

By compactifying the M-theory configuration along X^6, we obtain a system of three different types of D4-branes, plus D0-branes from momentum along X^6. This is a system that has been well-studied in the black hole context [14,15]. The pants section of the preceding paragraph descends to a similar D2-brane, while the momentum descends to a constant D0-flux through the D2-brane, $F_2=P_L dVol$; where the volume is normalized to unity.

The localized states of this system are counted as follows. At a given intersection, a pants section is massless, and with an arbitrary D0-flux it's energy is just the D0 brane charge. States with fixed D0-flux P_L are then obtained by partitioning that flux over n pants sections (where $1\leqslant n\leqslant P_L$, in the normalization where P_L is an integer.) Thus we find factorial growth of states exactly like the free field theory calculation, with[3] $S\simeq 2\pi\sqrt{6N_0N_1N_2N_3}$. Notice that the $D2$ branes are in a sense auxiliary to the construction as the total $D2$ brane charge is zero. This is a description of these bound states in terms of some remnant, along the lines of Refs. [16,17].

V. M-BRANES ON CALABI-YAU THREEFOLDS

It is also of interest to discuss the case of M5-branes on Calabi-Yau 3-folds more directly. To begin, consider the

[3]This may be obtained by taking the partition function $Z\simeq[\eta(q)\theta(q)]^{4N_1N_2N_3}$, for fixed $\langle E\rangle=N_0$.

case of $K3\times T^2$, with M5-branes wrapped on different complex 2-cycles. In particular, there are M5-branes wrapped on the whole $K3$ manifold; in a type IIB description, these branes give rise to an A_{N_3} singularity times a $K3$ surface. Other M5-branes that wrap the T^2 as well as a 2-cycle of the $K3$ correspond to D3-branes. Again, we can go to a weakly coupled type IIB picture and repeat the steps to get the open string quiver diagram corresponding to the configuration. The twisted open strings are again the relevant degrees of freedom.

This construction can be immediately generalized to an elliptically fibered Calabi-Yau manifold M with a section. Clearly, we should distinguish M5-branes which wrap a cycle on the base plus the elliptic fiber from those which wrap the base completely. The latter appear as KK monopoles while the former become D3-branes wrapped on 2-cycles of the base, once we turn to the dual IIB F-theory configuration.

The description in terms of twisted open strings is good only locally on the D3 branes, yet the choice of which string is light changes as we move around the D3 brane, and they certainly become massless at the intersection points of these. Their contribution to the central charge is nvertheless protected by the (0,4) supersymmetry.

VI. CONCLUDING REMARKS

In this note, we have considered configurations of branes which form bound states at threshold. The entropy of these objects may be understood from the counting of (not necessarily perturbative) states which becomes massless when the different constituents of the black hole are brought together. The identification of these modes as string junctions is particularly appealing, as all of the degrees of freedom can be seen geometrically, but are never perturbative in this U-frame.

We have also found a perturbative picture in which the microscopic states are twisted string states on the intersection of D3-branes at an orbifold singularity. The ultraviolet theory then is a gauge theory. We have been unable, by deforming the moduli space, to find a description of the spacetime infrared conformal field theory in terms of free fields however, either on the torus, or for those Calabi-Yau manifolds for which the construction makes sense.

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