

Three-point Green function of the stress-energy tensor in the AdS-CFT correspondence

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We compute the three-point function of the stress-energy tensor in d -dimensional conformal field theory CFT from AdS_{d+1} gravity. For $d=4$ the coefficients of the three linearly independent conformally covariant forms entering the three-point function are exactly the same as given by the free-field computations in the $\mathcal{N}=4$ SYM theory just as expected from the known renormalization theorems. For $d=3$ and $d=6$ our results give the value of the corresponding three-point function in the theories of strongly coupled $\mathcal{N}=8$ superconformal scalar and (2,0) tensor multiplets, respectively. [S0556-2821(99)01114-5]

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I. INTRODUCTION

The Maldacena conjecture [1–3] relating the large- N limit of certain conformal field theories (CFT's) in d dimensions with supergravity on the product of the $(d+1)$ -dimensional anti-de Sitter (AdS) space with a compact manifold has been recently tested by explicit computation of many two- and three-point gauge theory correlation functions from AdS supergravity [4–20]. Considerable progress was also achieved in studying four-point correlators [21–27], whose complete computation, however, requires the knowledge of the supergravity action on the AdS background beyond the quadratic [28] approximation.

An important question being yet unsolved with three-point functions is the computation of the three-point function of the stress-energy tensor $T_{ij}(\vec{x})$. Unlike the other three-point functions that are determined by the conformal symmetry almost completely (usually up to one constant), the three-point function of $T_{ij}(\vec{x})$ in general dimension d admits five independent conformally covariant forms, two of them being fixed by the gauge-theory conservation law $\partial_i T_{ij}(\vec{x}) = 0$ [29]. Three constants undetermined neither conformal symmetry nor the conservation law might be computed from the AdS supergravity and confronted with their free-field counterparts. This obviously provides a further nontrivial test for the AdS-CFT correspondence.

In this paper we therefore address the problem of computing the three-point function of the stress-energy tensor in the d -dimensional CFT from AdS_{d+1} gravity.

According to the AdS-CFT conjecture the CFT current of conformal weight zero coupled to the stress-energy tensor of conformal dimension d is extended to the interior of the AdS space as the on-shell graviton field. In comparison with three-point functions of other gauge-invariant composite op-

erators computation of the three-point function of $T_{ij}(\vec{x})$ is complicated by two things. First to clear up the infrared divergences of the gravity action, one can introduce the boundary of the AdS space. The Hamiltonian formulation [13] of the AdS-CFT correspondence then naturally requires additional boundary terms [8], so that the pure gravity action is not given only by the standard Einstein-Hilbert term. It is worth stressing that the account of the boundary terms is absolutely necessary since they provide the fulfillment of the Ward identities in the boundary CFT. The second thing is that after introducing the boundary terms the gravity action loses its manifest conformal as well as gauge invariance.

In our computation of the three-point function we account the boundary terms in the following way. We start with the standard Einstein-Hilbert term (with the cosmological constant) and decompose it up to the cubic order in metric perturbation $h_{\mu\nu}$. Removing all terms linear in second derivatives as well as all total derivative terms we are left with an action that according to [13] differs from the action one should use in the AdS-CFT correspondence only by terms that do not contribute to the Green functions. Then, by the on-shell perturbation theory, one may find that the contribution of the quadratic terms to the value of the three-point is exactly zero. Thus, the remaining action is just the sum of the cubic bulk and the boundary (noncovariant) terms, the latter arise due to the removal procedure. Fortunately, the cubic boundary terms do not contribute to the value of the three-point function and by this reason can be disregarded. The bulk term is manifestly covariant with respect to the AdS isometries as well as on-shell gauge symmetry and that apparently solves the second difficulty.

Choosing the covariant gauge we then explicitly compute the remaining bulk integral and get the three-point function. For the physically most interesting case $d=4$ we realize that the coefficients of the conformal tensors of the three-point function are exactly the same as the ones found by the free-field computations. This is obviously in agreement with the earlier results by [12,30,31], whose essence is that in four dimensions the superconformal symmetry is powerful

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enough to protect some three-point functions in $\mathcal{N}=4$ Yang-Mills against quantum corrections.

As to the other cases of particular interest $d=3$ and $d=6$, at the moment we are unaware of the gauge-theory calculations and may suggest that our results give the value of the corresponding three-point function in the theories of the strongly coupled superconformal scalar and (2,0) tensor multiplets, respectively.

The paper is organized as follows. In Sec. II we define the gravity action one should use in the AdS-CFT correspondence and obtain its decomposition up to cubic order in metric perturbation. In Sec. III we compute the on-shell value of the gravity action in the de Donder gauge and obtain the three-point function. We then write down explicitly the coefficients of the conformal tensors occurring in the three-point function in dimensions $d=2,3,4,5,6$ and comment on the most interesting cases. Some details of the calculation are collected in the Appendix.

II. GRAVITY ACTION AT THIRD ORDER OF PERTURBATION

We begin by fixing the basic notation. Let \mathcal{M} be a $(d+1)$ -dimensional manifold with a d -dimensional boundary. Throughout the paper the indices μ, ν, \dots run the set $0, \dots, d$, while i, j, \dots are reserved for the d -dimensional boundary and take values $1, \dots, d$. The coordinates x_μ are then split as $x_\mu = (x_0, \vec{x})$ with $\vec{x} = \{x_i\}$. Let \mathcal{M} be also supplied with a positive signature metric $G_{\mu\nu}$.

We will deal with the Euclidean version of the AdS_{d+1} space that is described as the upper half space $\mathcal{M} = \{x_i \in \mathbf{R}; x_0 > 0\}$ endowed with the metric

$$ds^2 = \frac{1}{x_0^2} dx^\mu dx^\mu.$$

The boundary of the AdS_{d+1} space is at $x_0=0$ and can be identified with the Euclidean space. Since the boundary is infinitely distant from any interior point, the gravity action on the AdS background suffers from infrared divergences. A natural regularization is then provided by setting the boundary of AdS_{d+1} space at $x_0=\varepsilon$ and considering the part with $x_0 \geq \varepsilon$. The physical fields are required to vanish when $x_0 \rightarrow \infty$.

The total gravity action \mathbf{S} one should use for computing gauge theory correlation functions is given by the sum

$$\mathbf{S} = S + S^{(1)} + S^{(2)} \quad (1)$$

of the standard Einstein-Hilbert term [with the cosmological constant $\lambda = \frac{1}{2}d(d-1)$]

$$S = \int \sqrt{G} (R - 2\lambda) \quad (2)$$

and two boundary terms $S^{(1)}$ and $S^{(2)}$ [8]. Explicitly, $S^{(1)}$ is the Gibbons-Hawking term [32]

$$S^{(1)} = 2 \int_{\partial M} d^d x \sqrt{\bar{G}} K, \quad (3)$$

where K is the trace of the fundamental form on the boundary and \bar{G} is the determinant of the induced metric. The second boundary term $S^{(2)}$ is a term proportional to the volume of the boundary:

$$S^{(2)} = 2(1-d) \int_{\partial M} d^d x \sqrt{\bar{G}}. \quad (4)$$

As was shown in [13], adding the boundary terms is equivalent to removing from the bulk action (2) all terms linear in second derivatives and all total derivative terms. The gravity action obtained in such a way differs from Eq. (1) only by terms which do not contribute to Green functions. Therefore, we can restrict our attention to considering the bulk term.

If we assume $g_{\mu\nu}$ to be the background AdS metric and perturb $G_{\mu\nu}$ near the background value: $G_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$, then equations of motion $R_{\mu\nu} = -d g_{\mu\nu}$ up to the second order in $h_{\mu\nu}$ can be written as follows:

$$L_{\mu\nu} = V_{\mu\nu}, \quad (5)$$

where two tensors

$$L_{\mu\nu} = \nabla^\rho \nabla_\rho h_{\mu\nu} + \nabla_\mu \nabla_\nu h - \nabla_\mu \nabla^\rho h_{\nu\rho} - \nabla_\nu \nabla^\rho h_{\mu\rho} + 2(h_{\mu\nu} - g_{\mu\nu} h), \quad (6)$$

and

$$V_{\mu\nu} = -\nabla_\rho [h^{\rho\sigma} (\nabla_\mu h_{\nu\sigma} + \nabla_\nu h_{\mu\sigma} - \nabla_\sigma h_{\mu\nu})] + \nabla_\nu (h^{\rho\sigma} \nabla_\mu h_{\rho\sigma}) + \frac{1}{2} (\nabla_\mu h_{\nu\rho} + \nabla_\nu h_{\mu\rho} - \nabla_\rho h_{\mu\nu}) \nabla^\rho h - \frac{1}{2} \nabla_\mu h_{\rho\sigma} \nabla_\nu h^{\rho\sigma} + \nabla_\rho h_{\mu\sigma} \nabla^\rho h_\nu^\sigma - \nabla_\sigma h_{\mu\rho} \nabla^\rho h_\nu^\sigma \quad (7)$$

were introduced. Here the covariant derivatives are taken with respect to the background metric.

Introduce the notation

$$R_{\mu\nu} = \dot{R}_{\mu\nu} + R_{\mu\nu}^{(1)} + R_{\mu\nu}^{(2)} + \dots = \dot{R}_{\mu\nu} + \delta R_{\mu\nu} + \frac{1}{2!} \delta^{(2)} R_{\mu\nu} + \dots$$

for decomposition of the Ricci tensor around the background $\dot{R}_{\mu\nu}$ and the analogous one for decomposition of the curvature.

Now we are ready to analyze gravity action (1) up to the third order in metric perturbation. We start with working out decomposition of Eq. (2):

$$S = \dot{S} + \delta S + \frac{1}{2!} \delta^{(2)} S + \frac{1}{3!} \delta^{(3)} S + \dots \quad (8)$$

Computing the first variation δS of Eq. (2) one then represents it in the form

$$\delta S[G, h] = - \int_M \sqrt{G} \left(R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \lambda G_{\mu\nu} \right) h^{\mu\nu} + T, \quad (9)$$

where T is the following boundary term:

$$T = \int_M \sqrt{G} \nabla_\mu (\nabla_\nu h^{\mu\nu} - \nabla^\mu h) = - \int_{\partial M} \sqrt{G} (\nabla_\mu h^{0\mu} - \partial^0 h).$$

In Eq. (9) we consider $\delta S[G, h]$ as the variation of $S[G]$ at a ‘point’ $G_{\mu\nu}$, i.e., we do not assume the metric $G_{\mu\nu}$ to be equal to its background value. Now the simple algorithm to find decomposition (8) is to consider the successive variations of $\delta S[G, h]$.¹ Since we are interested in decomposition of the total action (1) we can omit the total derivative term T .

Thus, varying the terms in parentheses in Eq. (9) and reducing the result to the background we get

$$R_{\mu\nu}^{(1)} - \frac{1}{2} g_{\mu\nu}^{(1)} \dot{R} - \frac{1}{2} g_{\mu\nu} R^{(1)} + \lambda g_{\mu\nu}^{(1)} = - \frac{1}{2} \left(L_{\mu\nu} - \frac{1}{2} g_{\mu\nu} L \right),$$

where the relation $R_{\mu\nu}^{(1)} + dh_{\mu\nu} = -\frac{1}{2} L_{\mu\nu}$ was used.

For the second variation we find

$$\begin{aligned} & \delta^{(2)} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \delta^{(2)} R - \frac{1}{2} \delta^{(1)} g_{\mu\nu} \delta^{(1)} R \\ &= 2 \left(R_{\mu\nu}^{(2)} - \frac{1}{2} g_{\mu\nu} (g^{\rho\sigma} R_{\rho\sigma}^{(2)}) \right) - \frac{1}{2} g_{\mu\nu} h^{\rho\sigma} L_{\rho\sigma} + \frac{1}{2} h_{\mu\nu} L. \end{aligned}$$

Here $R_{\mu\nu}^{(2)} - \frac{1}{2} g_{\mu\nu} (g^{\rho\sigma} R_{\rho\sigma}^{(2)}) = \frac{1}{2} (V_{\mu\nu} - \frac{1}{2} g_{\mu\nu} V)$, where $V = \nabla_\mu^\mu$.

With these formulas at hand it is now easy to find the action (2) up to the third order in $h_{\mu\nu}$:

$$\begin{aligned} S = \dot{S} + \int_M \sqrt{g} & \left[\frac{1}{4} \left(L_{\mu\nu} - \frac{1}{2} g_{\mu\nu} L \right) h^{\mu\nu} - \frac{1}{6} \left(V_{\mu\nu} - \frac{1}{2} g_{\mu\nu} V \right) \right. \\ & \left. \times h^{\mu\nu} - \frac{1}{6} \left(\frac{1}{8} h^2 L - \frac{3}{4} h h^{\mu\nu} L_{\mu\nu} + h^{\mu\rho} h_\rho^\nu L_{\mu\nu} \right) \right], \quad (10) \end{aligned}$$

where we again omitted nonessential total derivative terms. Note that the last term in the integrand coincides with the third-order term in the decomposition of \sqrt{G} , in which one tensor $h_{\mu\nu}$ is replaced by $L_{\mu\nu}$. The action (10) depends on second derivative terms. To remove these terms one should add to Eq. (10) total derivative terms which can be easily found by using explicit expressions (6) and (7) for L and V , respectively. A simple consideration then shows that the quadratic terms in the resulting action do not contribute to the three-point Green function.

¹The covariant derivatives in T are also with respect to the metric $G_{\mu\nu}$.

Thus, we see that to find the three-point function we need to compute the on-shell value of \mathbf{S} which is given by

$$\mathbf{S} = - \frac{1}{6} \int_M \sqrt{g} \left(V_{\mu\nu} - \frac{1}{2} g_{\mu\nu} V \right) h^{\mu\nu} + \text{cubic}. \quad (11)$$

Here $h_{\mu\nu}$ is a solution to the linearized equation of motion: $L_{\mu\nu}[h] = 0$ and cubic refers to the unwritten explicitly total derivative terms of the cubic order. Since these terms may deliver only a local contribution to the value of the three-point function, in what follows we disregard them.

III. THREE-POINT GREEN FUNCTION

The radiation gauge for the AdS gravity that is effectively used for computing the two-point Green function of the stress-energy tensor in the boundary CFT obviously breaks the invariance of the gravity action under isometries. For the three-point function this fact leads to severe difficulties in computing the bulk integrals. Thus, to handle the problem we choose the covariant gauge of the de Donder type:

$$\nabla_\mu \left(h_\nu^\mu - \frac{1}{2} \delta_\nu^\mu h \right) = 0. \quad (12)$$

In this gauge the solution of the linearized equations of motion reads as [8]

$$\begin{aligned} h_\mu^\nu(x_0, \vec{x}) = \kappa_G \int d^d y & \mathcal{K}(x, \vec{y}) J_\mu^j(x - \vec{y}) \\ & \times J_j^\nu(x - \vec{y}) \mathcal{E}_{ij,kl} h_{kl}(\vec{y}), \quad (13) \end{aligned}$$

where

$$J_\mu^\nu(x) = \delta_\mu^\nu - 2 \frac{x_\mu x^\nu}{|x|^2}, \quad \mathcal{K}(x, \vec{y}) = \frac{x_0^d}{[x_0^2 + (\vec{x} - \vec{y})^2]^d},$$

the coefficient $k_G = (d+1)/(d-1)\Gamma(d)/\pi^{d/2}\Gamma(d/2)$, $h_i^k(\vec{y})$ represents the boundary data of the graviton and $\mathcal{E}_{ij,kl}$ is the traceless symmetric projector:

$$\mathcal{E}_{ij,kl} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{kj}) - \frac{1}{d} \delta_{ij} \delta_{kl}. \quad (14)$$

Note that tensor h_μ^ν has the vanishing trace.

In the de Donder gauge the remaining bulk term of the on-shell action (11) can be represented in the following form most suitable for further computations:

$$\mathbf{S} = -\frac{1}{6} \int_M \sqrt{-g} \left(V_{\mu\nu} - \frac{1}{2} g_{\mu\nu} V \right) h^{\mu\nu} \quad \kappa_v = \frac{d+1}{d} \frac{2\Gamma(d)}{\pi^{d/2}\Gamma(d/2)}, \quad (17)$$

$$= \frac{1}{6} \int_M \sqrt{-g} \left(\nabla_\sigma (h^{\mu\nu} h_{\mu\rho} \nabla^\rho h_\nu^\sigma) - \frac{1}{2} \nabla_\mu (h^{\mu\nu} h^{\rho\sigma} \nabla_\nu h_{\rho\sigma}) \right.$$

$$\left. - \nabla_\sigma (h_\mu^\sigma h_\nu^\rho \nabla_\rho h^{\mu\nu}) \right) - \frac{1}{4} \int_M \sqrt{-g} \left(\nabla_\sigma \nabla_\rho h_{\mu\nu} h^{\mu\nu} h^{\rho\sigma} \right.$$

$$- 2 \nabla_\sigma \nabla_\rho h^{\mu\nu} h_\mu^\rho h_\nu^\sigma + \frac{2}{3} h_\mu^\nu \nabla_\rho h_\sigma^\mu \nabla^\rho h_\nu^\sigma$$

$$\left. - \frac{2}{3} (d+1) h_\mu^\nu h_\sigma^\mu h_\nu^\sigma \right),$$

where we used the explicit form of $V_{\mu\nu}$ and the vanishing of h_μ^μ .

Again omitting the total derivatives being the cubic order boundary terms we see that the on-shell action in the de Donder gauge is essentially given by the bulk integral

$$\mathbf{S} = -\frac{1}{4} \int_M \sqrt{-g} \left(\nabla_\sigma \nabla_\rho h_{\mu\nu} h^{\mu\nu} h^{\rho\sigma} - 2 \nabla_\sigma \nabla_\rho h^{\mu\nu} h_\mu^\rho h_\nu^\sigma \right.$$

$$\left. + \frac{2}{3} h_\mu^\nu \nabla_\rho h_\sigma^\mu \nabla^\rho h_\nu^\sigma - \frac{2}{3} (d+1) h_\mu^\nu h_\sigma^\mu h_\nu^\sigma \right). \quad (15)$$

By using the equation of motion that in the covariant gauge reads as $\nabla_\rho \nabla^\rho h_{\mu\nu} = -2h_{\mu\nu}$ we then rewrite Eq. (15) in the form

$$\mathbf{S} = \frac{1}{4} \int_M \sqrt{-g} \left(2 \nabla_\sigma \nabla_\rho h^{\mu\nu} h_\mu^\rho h_\nu^\sigma - \nabla_\sigma \nabla_\rho h_{\mu\nu} h^{\mu\nu} h^{\rho\sigma} \right.$$

$$\left. + \frac{2}{3} d h_\mu^\nu h_\sigma^\mu h_\nu^\sigma \right). \quad (16)$$

The computation of Eq. (16) is a rather cumbersome but purely technical task that can be performed by the inversion method of [7]. Before plugging into details we make some comments about the relation between the bulk and the boundary gauge transformations. The symmetry group of action (16) is now reduced to the gauge transformations that preserves the de Donder gauge. This group of residual gauge transformations is generated by vectors ξ^μ obeying an equation $\nabla_\rho \nabla^\rho \xi_\mu - d\xi_\mu = 0$. Explicitly, the solution satisfying the gauge $\nabla_\mu \xi^\mu = 0$ reads as

$$\xi^\mu(x_0, \vec{x}) = \kappa_v \int d^d y \frac{x_0^{d+2}}{[x_0^2 + (\vec{x} - \vec{y})^2]^{d+1}} J_i^\mu(x - \vec{y}) \xi^i(\vec{y}),$$

where the coefficient κ_v is fixed by requiring $\xi^i(x_0, \vec{x}) \rightarrow \xi^i(\vec{x})$ when $x_0 \rightarrow 0$. In particular, the component ξ^0 is

$$\xi^0(x_0, \vec{x}) = \frac{\kappa_v}{d+1} \int d^d y \frac{x_0^{d+3}}{[x_0^2 + (\vec{x} - \vec{y})^2]^{d+1}} \partial_i \xi^i(\vec{y}).$$

One may see that on the boundary ($\varepsilon \rightarrow 0$) the residual gauge transformations are reduced to

$$\delta h_i^j = \partial_i \xi^j + \partial_j \xi^i - \frac{2}{d} \delta_{ij} (\partial_k \xi^k), \quad (18)$$

i.e., to the usual gauge transformations of a traceless symmetric tensor. In other words, the transformations (18) of the boundary data can always be prolonged to the bulk gauge fields, which preserve the de Donder gauge. We, therefore, expect the three-point function $T_{ij,kl,mn}(\vec{x}, \vec{y}, \vec{z}) = \langle T_{ij}(\vec{x}) T_{kl}(\vec{y}) T_{mn}(\vec{z}) \rangle$ of the stress-energy tensor T_{ij} to obey the conservation law

$$\partial_i T_{ij,kl,mn}(\vec{x}, \vec{y}, \vec{z}) = 0, \quad \text{for noncoincident } \vec{x}, \vec{y} \text{ and } \vec{z}.$$

As was already mentioned in the Introduction in arbitrary dimension $d > 3$ there are five independent conformal tensors occurring in the expression for the three-point function of the stress-energy tensor. The conservation law then fixes the value of two from five coefficients. It is clear that the three-point function defined by S is conformally covariant and the only reason to find its explicit expression is to make comparison of the coefficients of conformal tensors to the ones found on the gauge theory side.

Thus, substituting Eq. (13) in Eq. (16) we see that according to the AdS-CFT prescription [2,3] the three-point function is defined as²

$$T_{ij,kl,mn}(\vec{x}, \vec{y}, \vec{z}) = 8 \sum I_{ij,kl,mn}(\vec{x}, \vec{y}, \vec{z}), \quad (19)$$

where the sum is taken over all possible permutations of sets of indices and points (ij, \vec{x}) , (kl, \vec{y}) , and (mn, \vec{z}) of the following tensor:

²We assume that the coupling of $T_{ij}(\vec{x})$ with $h_{ij}(\vec{x})$ on the boundary of AdS_{d+1} is given by $\int d^d x [\frac{1}{2} T_{ij}(\vec{x}) h_{ij}(\vec{x})]$ and this explains the number 8 in Eq. (19). Later on we show that this coupling also leads to the correct Ward identity.

$$\begin{aligned}
I_{ij,kl,mn}(\vec{x},\vec{y},\vec{z}) &= 2\kappa_G^3 \mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'} \mathcal{E}_{mn,m'n'} \int \frac{d^{d+1}\omega}{\omega_0^{d+1}} \mathcal{K}(\omega,\vec{y}) \mathcal{K}(\omega,\vec{z}) \times \left[2\nabla^\sigma \nabla_\rho (\mathcal{K} J_{i'}^\mu J_{j'}^\nu)(\omega-\vec{x}) (J_\mu^{k'} J_{l'}^\rho)(\omega-\vec{y}) (J_\sigma^{m'} J_{n'}^\nu)(\omega-\vec{z}) \right. \\
&\quad \times (\omega-\vec{z}) - \nabla_\sigma \nabla^\rho (\mathcal{K} J_{i'}^\mu J_{j'}^\nu)(\omega-\vec{x}) (J_\mu^{k'} J_{l'}^\nu)(\omega-\vec{y}) (J_\rho^{m'} J_{n'}^\sigma)(\omega-\vec{z}) \\
&\quad \left. + \frac{2d}{3} (\mathcal{K} J_{i'}^\mu J_{j'}^\nu)(\omega-\vec{x}) (J_\mu^{k'} J_{l'}^\rho)(\omega-\vec{y}) (J_\rho^{m'} J_{n'}^\nu)(\omega-\vec{z}) \right]. \tag{20}
\end{aligned}$$

Recall that here and in what follows the bulk (boundary) indices are contracted with respect to the AdS metric (Euclidean) and, therefore, only their positions matter. Note that the tensor $I_{ij,kl,mn}$ itself is not conformally covariant.

Following the method of [7] we now set in Eq. (20) $\vec{x}=0$ and perform the change of variables $\omega'_\mu = \omega_\mu/\omega^2$ and $x'_i = x_i/x^2$. This is just the inversion transformation under which the derivatives ∇_μ transform covariantly. In particular,

$$\nabla_\rho [\mathcal{K}(\omega,\vec{x}) J_\mu^i(\omega-\vec{x}) J_j^\nu(\omega-\vec{x})] = |\omega'|^2 J_\mu^\lambda(\omega') J_\beta^\nu(\omega') J_\rho^\sigma(\omega') \nabla'_\sigma [\mathcal{K}(\omega',\vec{x}') J_\lambda^\alpha(\omega'-\vec{x}') J_b^\beta(\omega'-\vec{x}')] \frac{J_a^i(\vec{x}) J_j^b(\vec{x})}{|x|^{2d}}$$

and

$$\nabla_\rho [\mathcal{K}(\omega,0) J_\mu^i(\omega) J_j^\nu(\omega)] = |\omega'|^2 J_\mu^\lambda(\omega') J_\beta^\nu(\omega') J_\rho^\sigma(\omega') \nabla'_\sigma ((\omega'_0)^d \delta_\lambda^\alpha \delta_j^\beta),$$

where the covariant derivatives ∇'_μ are with respect to the connection

$$\Gamma'^\mu_{\rho\sigma}(\omega') = -\frac{1}{\omega'_0} (\delta_\rho^0 \delta_{\mu\sigma} + \delta_\sigma^0 \delta_{\mu\rho} - \delta_\mu^0 \delta_{\rho\sigma}).$$

Thus, after substituting the change of variables all internal Jacobians depending on the variable ω alone cancel against each other and one is left with the following expression:

$$\begin{aligned}
I_{ij,kl,mn}(0,\vec{y},\vec{z}) &= 2\kappa_G^3 \mathcal{E}_{ij,i'j'} \frac{\mathcal{I}_{kl,k'l'}(\vec{y})}{|y|^{2d}} \frac{\mathcal{I}_{mn,m'n'}(\vec{z})}{|z|^{2d}} \int \frac{d^{d+1}\omega}{\omega_0} \mathcal{K}(\omega,\vec{y}') \mathcal{K}(\omega,\vec{z}') \times \left[-\frac{4d}{3} (J_{i'}^{k'} J_{l'}^s)(\omega-\vec{y}') (J_s^{m'} J_{n'}^{j'}) (\omega-\vec{z}') \right. \\
&\quad + 2 \left(d^2 + \frac{d}{3} + 2 \right) (J_{i'}^{k'} J_{l'}^0)(\omega-\vec{y}') (J_{j'}^{m'} J_{n'}^0)(\omega-\vec{z}') - (d^2 - d - 2) (J_{i'}^{k'} J_{l'}^{j'}) (\omega-\vec{y}') (J_0^{m'} J_{n'}^0)(\omega-\vec{z}') \\
&\quad \left. + 2d (J_0^{k'} J_{l'}^0)(\omega-\vec{y}') (J_{i'}^{m'} J_{n'}^{j'}) (\omega-\vec{z}') \right], \tag{21}
\end{aligned}$$

where the concise notation $\mathcal{I}_{ij,kl}(\vec{x}) = \mathcal{E}_{ij,i'j'} (J_{i'k} J_{j'l})(\vec{x})$ was introduced.

In view of Eq. (19) it is further more convenient to deal with the integral $I_{ij,kl,mn}^{\text{sym}}$ being the symmetrization of Eq. (21) with respect to (kl,\vec{y}) and (mn,\vec{z}) :

$$I_{ij,kl,mn}^{\text{sym}}(0,\vec{y},\vec{z}) = I_{ij,kl,mn}(0,\vec{y},\vec{z}) + I_{ij,mn,kl}(0,\vec{z},\vec{y}). \tag{22}$$

The computation of I^{sym} is sketched in the Appendix and below we present the result

$$\begin{aligned}
I_{ij,kl,mn}^{\text{sym}}(0, \vec{y}, \vec{z}) &= 2\pi^{d/2} \kappa_G^3 \frac{\Gamma(d/2)[\Gamma(d/2+1)]^2}{(d+1)^2 \Gamma(d-1) \Gamma(d+2)} \mathcal{E}_{ij,i'j'} \frac{\mathcal{I}_{kl,k'l'}(\vec{y})}{|y|^{2d}} \frac{\mathcal{I}_{mn,m'n'}(\vec{z})}{|z|^{2d}} \frac{1}{|t|^d} \\
&\times \left\{ a_1 \mathcal{E}_{i'j',ab} \mathcal{E}_{k'l',ac} \mathcal{E}_{m'n',bc} + a_2 \mathcal{E}_{i'j',ab} \mathcal{E}_{k'l',ac} \mathcal{E}_{m'n',bd} \frac{t_c t_d}{t^2} \right. \\
&+ a_3 [\mathcal{E}_{k'l',ab} \mathcal{E}_{m'n',bc} \mathcal{E}_{i'j',ad} + (k'l') \rightarrow (m'n')] \frac{t_c t_d}{t^2} + a_4 \left[\mathcal{E}_{k'l',i'j'} \left(\frac{t_{m'} t_{n'}}{t^2} - \frac{1}{d} \delta_{m'n'} \right) + (k'l') \rightarrow (m'n') \right] \\
&+ a_5 \mathcal{E}_{k'l',m'n'} \left(\frac{t_i t_{j'}}{t^2} - \frac{1}{d} \delta_{i'j'} \right) + a_6 \left[\mathcal{E}_{k'l',ab} \mathcal{E}_{i'j',ac} \left(\frac{t_{m'} t_{n'}}{t^2} - \frac{1}{d} \delta_{m'n'} \right) + (k'l') \rightarrow (m'n') \right] \frac{t_b t_c}{t^2} \\
&+ a_7 \mathcal{E}_{k'l',ab} \mathcal{E}_{m'n',ac} \left(\frac{t_i t_{j'}}{t^2} - \frac{1}{d} \delta_{i'j'} \right) \frac{t_b t_c}{t^2} + a_8 \left(\frac{t_i t_{j'}}{t^2} - \frac{1}{d} \delta_{i'j'} \right) \left(\frac{t_{k'} t_{l'}}{t^2} - \frac{1}{d} \delta_{k'l'} \right) \left(\frac{t_{m'} t_{n'}}{t^2} - \frac{1}{d} \delta_{m'n'} \right) \left. \right\}. \quad (23)
\end{aligned}$$

In the last formula the Latin indices (a, b, c, d) are used to distinguish the boundary summation indices and the variable $t_i = z'_i - y'_i$. The coefficients a_i , $i = 1, \dots, 8$ are expressed through the constants $a_i^{(k)}$, $k = 1, 2, 3$ found in the Appendix by the following formula:

$$a_i = -\frac{4d}{3} a_i^{(1)} + 2 \left(d^2 + \frac{d}{3} + 2 \right) a_i^{(2)} - (d^2 - 3d - 2) a_i^{(3)}. \quad (24)$$

If we now restore the \vec{x} dependence, the variable t_i ,

$$t_i = (z-x)'_i - (y-x)'_i = \frac{(x-y)_i}{(\vec{x}-\vec{y})^2} - \frac{(x-z)_i}{(\vec{x}-\vec{z})^2}$$

turns into the conformal vector X_i : $t_i = -X_i$ with a remarkable property to transform covariantly $X_i \rightarrow X'_i$ under the inversion $x'_i = x_i/x^2$ [29]:

$$X'_i = x^2 J_{ij}(\vec{x}) X_j.$$

Then by using the two identities

$$J_{ij}(\vec{x}-\vec{z}) Z_j = -\frac{(\vec{x}-\vec{y})^2}{(\vec{z}-\vec{y})^2} X_j, \quad J_{ik}(\vec{x}-\vec{z}) J_{kj}(\vec{z}-\vec{y}) = J_{ij}(\vec{x}-\vec{y}) + 2(\vec{x}-\vec{y})^2 X_i Y_j,$$

one may finally represent the three-point function (19) in the form

$$\begin{aligned}
T_{ij,kl,mn} &= \frac{1}{|\vec{x}-\vec{y}|^{2d} |\vec{y}-\vec{z}|^{2d} |\vec{x}-\vec{z}|^{2d}} \left\{ \mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'} \mathcal{E}_{mn,m'n'} [\mathcal{A} J_{i'k'}(\vec{x}-\vec{y}) J_{l'm'}(\vec{y}-\vec{z}) J_{j'n'}(\vec{z}-\vec{x}) + \mathcal{B} J_{i'k'}(\vec{x}-\vec{y}) J_{j'n'} \right. \\
&\times (\vec{x}-\vec{z}) Y_{l'} Y_{m'} (\vec{y}-\vec{z})^2 + \text{cycl. perm.}] + \mathcal{C} \left[\mathcal{I}_{ij,kl} \left(\frac{Z_n Z_m}{Z^2} - \frac{1}{d} \delta_{mn} \right) + \text{cycl. perm.} \right] + \mathcal{D} \left[\mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'} X_i Y_{k'} \right. \\
&\times (\vec{x}-\vec{y})^2 J_{j'l'}(\vec{x}-\vec{y}) \left(\frac{Z_m Z_n}{Z^2} - \frac{1}{d} \delta_{mn} \right) + \text{cycl. perm.} \left. \right] + \mathcal{E} \left(\frac{X_i X_j}{X^2} - \frac{1}{d} \delta_{ij} \right) \left(\frac{Y_k Y_l}{Y^2} - \frac{1}{d} \delta_{kl} \right) \left(\frac{Z_m Z_n}{Z^2} - \frac{1}{d} \delta_{mn} \right) \left. \right\}, \quad (25)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{A} &= 3\Delta_d a_1, \quad \mathcal{B} = \Delta_d (2a_1 + a_2 - 2a_3), \quad \mathcal{C} = \Delta_d (2a_4 + a_5), \\
\mathcal{D} &= \Delta_d (4a_5 + a_7 - 4a_3 - 2a_6), \quad \mathcal{E} = \Delta_d (12a_5 + 6a_7 + 3a_8), \quad (26)
\end{aligned}$$

and

$$\Delta_d = 2\pi^{d/2} \kappa_G^3 \frac{\Gamma(d/2)[\Gamma(d/2+1)]^2}{(d+1)^2 \Gamma(d-1) \Gamma(d+2)} = \frac{d\Gamma(d)}{2\pi^d (d-1)^2}.$$

As was expected formula (25) is just the conformally cova-

TABLE I. Coefficients $a_i^{(k)}$ for $d=4$.

$a_i^{(k)}$	$a_1^{(k)}$	$a_2^{(k)}$	$a_3^{(k)}$	$a_4^{(k)}$	$a_5^{(k)}$	$a_6^{(k)}$	$a_7^{(k)}$	$a_8^{(k)}$
$k=1$	547/21	-1163/42	-659/42	-25/84	-109/84	843/42	465/42	-92/7
$k=2$	39/14	-78/7	-22/7	-9/7	-2/7	66/7	24/7	-48/7
$k=3$	-29/21	116/21	74/21	113/21	8/21	-74/21	-32/21	64/7

ariant three-point function of the stress-energy tensor in the d -dimensional conformal field theory and it involves five independent conformal tensors.

The coefficients $\mathcal{A}, \dots, \mathcal{E}$ computed for any dimension $d \geq 2$ represent our basic result. We first discuss the most interesting case $d=4$. In this case by using Table I in the Appendix one finds for $\mathcal{A}, \dots, \mathcal{E}$ the following values:

$$\mathcal{A} = -\frac{8 \times 128}{9 \pi^4}, \quad \mathcal{B} = -\frac{8 \times 392}{9 \pi^4}, \quad \mathcal{C} = -\frac{8 \times 184}{9 \pi^4},$$

$$\mathcal{D} = -\frac{8 \times 472}{9 \pi^4}, \quad \mathcal{E} = -\frac{8 \times 304}{9 \pi^4}. \quad (27)$$

Now we are ready to confront the coefficients (27) with the ones found by the free-field computations in $\mathcal{N}=4$ SYM. To this end one needs to restore the gravity coupling g_G^{-2} that enters as an overall constant in front of the total action \mathbf{S} . The value of the coupling constant is fixed by the type-IIB supergravity on the $\text{AdS}_5 \times \text{S}^5$ background and is equal to

$$g_G^2 = \frac{8 \pi^2}{N^2}.$$

Clearly, to restore the g_G^2 dependence of the three-point function we should multiply Eq. (27) on g_G^{-2} and obtain

$$\mathcal{A} = -\frac{128}{9 \pi^6} N^2, \quad \mathcal{B} = -\frac{392}{9 \pi^6} N^2, \quad \mathcal{C} = -\frac{184}{9 \pi^6} N^2. \quad (28)$$

Recall that the two-point function of $T_{ij}(\vec{x})$ found from the AdS_{d+1} gravity is given by

$$\langle T_{ij}(\vec{x}), T_{kl}(\vec{y}) \rangle = \frac{C_d}{|\vec{x}-\vec{y}|^d} \mathcal{T}_{ij,kl}(\vec{x}-\vec{y}) \quad (29)$$

with the central charge $C_d = 2k_G d / g_G^2$. In particular, for $d=4$, one gets $C_4 = 80 / \pi^2 g_G^2 = (10 / \pi^4) N^2$.

In Ref. [33] it was shown that for any four-dimensional free-field theory given by n_s scalars, by n_f Dirac fermions, and by n_v vector fields the coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are as follows:

$$\mathcal{A} = \frac{1}{\pi^6} \left(\frac{8}{27} n_s - 16 n_v \right),$$

$$\mathcal{B} = -\frac{1}{\pi^6} \left(\frac{16}{27} n_s + 4 n_f + 32 n_v \right),$$

$$\mathcal{C} = -\frac{1}{\pi^6} \left(\frac{2}{27} n_s + 2 n_f + 16 n_v \right).$$

Substituting here the field-theoretical content of the $\mathcal{N}=4$ $\text{SU}(N)$ SYM:

$$n_s = 6(N^2 - 1), \quad n_f = 2(N^2 - 1), \quad n_v = N^2 - 1,$$

we, thus, arrive at

$$\mathcal{A} = -\frac{128}{9 \pi^6} (N^2 - 1), \quad \mathcal{B} = -\frac{392}{9 \pi^6} (N^2 - 1),$$

$$\mathcal{C} = -\frac{184}{9 \pi^6} (N^2 - 1). \quad (30)$$

The central charge C_4 can be found by taking into account the Ward identity that relates C_4 with coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}$ of the three-point function [29]:

$$C_4 = \frac{\pi^2}{12} (9\mathcal{A} - \mathcal{B} - 10\mathcal{C}) = \frac{10}{\pi^4} (N^2 - 1).$$

It is now obvious that in the large- N limit the coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and C_4 of the $\mathcal{N}=4$ $\text{SU}(N)$ SYM coincide with the ones found from the AdS_5 gravity.

Thus, the coefficients \mathcal{A}, \mathcal{B} , and \mathcal{C} of the conformal tensors obtained from the AdS_5 gravity and reflecting thereby the strong-coupling behavior of the corresponding gauge theory do not receive corrections to their free-field (one-loop) values. This fact finds a good agreement with the results by [12,30,31]. Indeed, the traceless conserved stress-energy tensor occurs in the multiplet of the supercurrent $T = \text{tr}(W^2)$, where W is an analytic superfield describing the $\mathcal{N}=4$ Yang-Mills strength multiplet. In [12] it was checked for the leading components of T being the scalar fields that their three-point functions computed from $\text{AdS}_5 \times \text{S}^5$ supergravity coincide with the one-loop results in the large- N limit. The same conclusion about vanishing of the radiative corrections at order g^2 was achieved in [30] even for finite N . By considering the anomaly in the superconformal symme-

try, it has been argued in [31] (see also [7]) that the two- and three-point functions of T should actually have a one-loop nature.

Having discussed the four-dimensional case, we now list explicitly the coefficients $\mathcal{A}, \dots, \mathcal{E}$ for dimensions $d = 2, 3, 5, 6$ that can be found from Eq. (24) together with (A1)–(A3).

$d = 2$:

$$\begin{aligned} \mathcal{A} &= -\frac{32}{\pi^2}, & \mathcal{B} &= -\frac{40}{\pi^2}, & \mathcal{C} &= -\frac{12}{\pi^2}, \\ \mathcal{D} &= -\frac{32}{\pi^2}, & \mathcal{E} &= -\frac{16}{\pi^2}. \end{aligned} \quad (31)$$

$d = 3$:

$$\begin{aligned} \mathcal{A} &= -\frac{3^4}{2\pi^3}, & \mathcal{B} &= -\frac{19 \times 3^2}{2\pi^3}, & \mathcal{C} &= -\frac{11 \times 3^3}{2^3 \pi^3}, \\ \mathcal{D} &= -\frac{41 \times 3^2}{2^2 \pi^3}, & \mathcal{E} &= -\frac{11 \times 3^4}{2^4 \pi^3}. \end{aligned} \quad (32)$$

$d = 5$:

$$\begin{aligned} \mathcal{A} &= -\frac{3 \times 5^4}{2^2 \pi^5}, & \mathcal{B} &= -\frac{303 \times 5^2}{2^2 \pi^5}, & \mathcal{C} &= -\frac{117 \times 5^3}{2^4 \pi^5}, \\ \mathcal{D} &= -\frac{257 \times 3 \times 5^2}{2^3 \pi^5}, & \mathcal{E} &= -\frac{137 \times 3 \times 5^3}{2^5 \pi^5}. \end{aligned} \quad (33)$$

$d = 6$:

$$\mathcal{A} = -\frac{2^8 \times 3^5}{5^2 \pi^6}, \quad \mathcal{B} = -\frac{181 \times 2^6 \times 3^3}{5^2 \pi^6},$$

$$\mathcal{C} = -\frac{59 \times 2^5 \times 3^4}{5^2 \pi^6}, \quad \mathcal{D} = -\frac{119 \times 2^7 \times 3^3}{5^2 \pi^6}, \quad \mathcal{E} = -\frac{2^7 \times 3^7}{5^2 \pi^6}. \quad (34)$$

In [29] it was shown that the conservation law implies the fulfilment of the following two identities:

$$(d^2 - 4)\mathcal{A} + (d + 2)\mathcal{B} - 4d\mathcal{C} - 2\mathcal{D} = 0,$$

$$(d - 2)(d + 4)\mathcal{B} - 2d(d + 2)\mathcal{C} + 8\mathcal{D} - 4\mathcal{E} = 0.$$

It is needless to say that coefficients (27) and Eqs. (31)–(34) satisfy both of them. It is, of course, only the check that we have done the computation of Eq. (16) correctly. One should be also aware of the fact that for $d = 2$ and $d = 3$ the number of linearly independent conformal tensors is reduced to 1 and 2, respectively [33].

The cases $d = 3$ and $d = 6$ are of particular interest since according to the AdS-CFT conjecture they correspond to compactifications of the 11d supergravity on $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$, respectively. We, therefore, expect that coefficients (32) describe the three-point function of $T_{ij}(\vec{x})$ in the strongly coupled three-dimensional gauge theory of the $\mathcal{N} = 8$ conformal scalar multiplet, while Eq. (34) are expected to be found in the six-dimensional gauge theory of the (0,2) tensor multiplet [34–37].

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APPENDIX

We sketch here for the reader's convenience some details of computation of integral (22).

Clearly, Eq. (22) can be written as the following sum of the integrals I^k , $k = 1, 2, 3$:

$$I_{ij,kl,mn}^{\text{sym}} = \frac{\kappa_G^3}{2} \mathcal{E}_{ij,i'j'} \frac{\mathcal{I}_{kl,k'l'}(\vec{y})}{|y|^{2d}} \frac{\mathcal{I}_{mn,m'n'}(\vec{z})}{|z|^{2d}} \left[-\frac{4d}{3} I_{i'j',k'l',m'n'}^1 + 2 \left(d^2 + \frac{d}{3} + 2 \right) I_{i'j',k'l',m'n'}^2 - (d^2 - d - 2) I_{i'j',k'l',m'n'}^3 \right],$$

where I^k are given by

$$\begin{aligned} I_{ij,kl,mn}^1(0, \vec{y}, \vec{z}) &= \mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'} \mathcal{E}_{mn,m'n'} \int \frac{d^{d+1} \omega}{\omega_0} \mathcal{K}(\omega, \vec{y}') \mathcal{K}(\omega, \vec{z}') (J_{i'}^{k'} J_{l'}^s)(\omega - \vec{y}') (J_s^{m'} J_{n'}^{j'}) (\omega - \vec{z}'), \\ I_{ij,kl,mn}^2(0, \vec{y}, \vec{z}) &= \mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'} \mathcal{E}_{mn,m'n'} \int \frac{d^{d+1} \omega}{\omega_0} \mathcal{K}(\omega, \vec{y}') \mathcal{K}(\omega, \vec{z}') (J_{i'}^{k'} J_{l'}^0)(\omega - \vec{y}') (J_j^{m'} J_n^0)(\omega - \vec{z}'), \\ I_{ij,kl,mn}^3(0, \vec{y}, \vec{z}) &= \mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'} \mathcal{E}_{mn,m'n'} \int \frac{d^{d+1} \omega}{\omega_0} \mathcal{K}(\omega, \vec{y}') \mathcal{K}(\omega, \vec{z}') \\ &\quad \times \frac{1}{2} [(J_j^{k'} J_{l'}^{j'}) (\omega - \vec{y}') (J_0^{m'} J_n^0)(\omega - \vec{z}') + (J_0^{k'} J_{l'}^0)(\omega - \vec{y}') (J_j^{m'} J_{n'}^{j'}) (\omega - \vec{z}')]. \end{aligned}$$

Then by using the identities

$$\begin{aligned} \mathcal{E}_{kl,k'l'}(J_i^{k'} J_{l'}^j)(\omega) \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^d} &= \mathcal{E}_{ij,k'l'} \left(\frac{d-1}{d+1} \delta_i^{k'} \delta_{l'}^j \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^d} - \frac{1}{2d(d+1)} \partial_{k'} \partial_i \frac{\delta_{l'}^j \omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^{d-1}} \right. \\ &\quad - \frac{1}{2d(d+1)} \partial_{l'} \partial_j \frac{\delta_i^{k'} \omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^{d-1}} + \frac{1}{2(d-1)d(d+1)} \partial_{k'} \partial_{l'} \frac{\delta_i^j \omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^{d-1}} \\ &\quad \left. + \frac{1}{4(d-2)(d-1)d(d+1)} \partial_i \partial_j \partial_{k'} \partial_{l'} \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^{d-2}} \right), \\ \mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'}(J_i^{k'} J_{l'}^{j'}) (\omega) \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^d} &= \mathcal{E}_{ij,i'j'} \mathcal{E}_{kl,k'l'} \\ &\quad \times \left(\frac{d-1}{d+1} \delta_i^{k'} \delta_{l'}^{j'} \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^d} - \frac{1}{d(d+1)} \partial_{k'} \partial_{j'} \frac{\delta_{l'}^{j'} \omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^{d-1}} \right. \\ &\quad \left. + \frac{1}{4(d-2)(d-1)d(d+1)} \partial_i \partial_{j'} \partial_{k'} \partial_{l'} \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^{d-2}} \right), \\ \mathcal{E}_{kl,k'l'}(J_i^{k'} J_{l'}^0)(\omega) \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^d} &= \frac{d-1}{d(d+1)} \mathcal{E}_{kl,im} \partial_m \frac{\omega_0^{d+1}}{(\omega_0^2 + \vec{\omega}^2)^d} - \frac{1}{2(d-1)d(d+1)} \mathcal{E}_{kl,k'l'} \partial_{k'} \partial_{l'} \partial_i \frac{\omega_0^{d+1}}{(\omega_0^2 + \vec{\omega}^2)^{d-1}}, \end{aligned}$$

and

$$\mathcal{E}_{ij,i'j'}(J_i^0 J_{l'}^{j'}) (\omega) \frac{\omega_0^d}{(\omega_0^2 + \vec{\omega}^2)^d} = \frac{1}{d(d+1)} \mathcal{E}_{ij,i'j'} \partial_i \partial_{j'} \frac{\omega_0^{d+2}}{(\omega_0^2 + \vec{\omega}^2)^d}.$$

One can rewrite every I^k as derivatives with respect to the external variable $t_i = z'_i - y'_i$ of the standard integrals [7]:

$$\begin{aligned} I_{b,c}^a &= \int d^{d+1} \omega \frac{\omega_0^a}{[\omega_0^2 + \vec{\omega}^2]^b [\omega_0^2 + (\vec{\omega} - \vec{t})^2]^c} = \frac{\pi^{d/2}}{2} \frac{\Gamma(a/2 + 1/2) \Gamma(b+c-d/2-a/2-1/2)}{\Gamma(b) \Gamma(c)} \\ &\quad \times \frac{\Gamma(1/2+a/2+d/2-b) \Gamma(1/2+a/2+d/2-c)}{\Gamma(1+a+d-b-c)} |\vec{x} - \vec{y}|^{1+a+d-2b-2c}. \end{aligned}$$

After straightforward calculations one obtains that all integrals I^k result in the same form

$$\begin{aligned} I^k &= \frac{\pi^{d/2}}{2} \frac{\Gamma(d/2) [\Gamma(d/2+1)]^2}{(d+1)^2 \Gamma(d-1) \Gamma(d+2)} \frac{1}{|t|^d} \left\{ + a_1^{(k)} \mathcal{E}_{ij,ab} \mathcal{E}_{kl,ac} \mathcal{E}_{mn,bc} + a_2^{(k)} \mathcal{E}_{ij,ab} \mathcal{E}_{kl,ac} \mathcal{E}_{mn,bd} \frac{t_c t_d}{t^2} \right. \\ &\quad + a_3^{(k)} [\mathcal{E}_{kl,ab} \mathcal{E}_{mn,bc} \mathcal{E}_{mn,ad} + (kl) \rightarrow (mn)] \frac{t_c t_d}{t^2} + a_4^{(k)} \left[\mathcal{E}_{kl,ij} \left(\frac{t_m t_n}{t^2} - \frac{1}{d} \delta_{mn} \right) + (kl) \rightarrow (mn) \right] + a_5^{(k)} \mathcal{E}_{kl,mn} \left(\frac{t_i t_j}{t^2} - \frac{1}{d} \delta_{ij} \right) \\ &\quad + a_6^{(k)} \left[\mathcal{E}_{kl,ab} \mathcal{E}_{ij,ac} \left(\frac{t_m t_n}{t^2} - \frac{1}{d} \delta_{mn} \right) + (kl) \rightarrow (mn) \right] \frac{t_b t_c}{t^2} + a_7^{(k)} \mathcal{E}_{kl,ab} \mathcal{E}_{mn,ac} \left(\frac{t_i t_j}{t^2} - \frac{1}{d} \delta_{ij} \right) \frac{t_b t_c}{t^2} \\ &\quad \left. + a_8^{(k)} \left(\frac{t_i t_j}{t^2} - \frac{1}{d} \delta_{ij} \right) \left(\frac{t_k t_l}{t^2} - \frac{1}{d} \delta_{kl} \right) \left(\frac{t_m t_n}{t^2} - \frac{1}{d} \delta_{mn} \right) \right\} \end{aligned}$$

but with different coefficients $a_i^{(k)}$, where $i=1, \dots, 8$ and $k=1, 2, 3$.

For the first integral I^1 the coefficients $a_i^{(k)}$ are found to be

$$\begin{aligned}
a_1^{(1)} &= \frac{4(d-1)(d+1)}{d} + \frac{(d-1)(8d+12)}{d^2} + \frac{8d+4}{d^2} \\
&\quad + \frac{12(d+2)+8(d-2)(d+3)}{d^2(d-1)(d+3)}, \\
a_2^{(1)} &= -\frac{2(d-1)(2d+3)}{d} - \frac{8d+4}{d} \\
&\quad - \frac{12(d+2)+8(d-2)(d+3)}{d(d-1)(d+3)}, \\
a_3^{(1)} &= -6 - \frac{2(d-1)(d+1)}{d} \\
&\quad - \frac{12(d+2)+8(d-2)(d+3)}{d(d-1)(d+3)}, \\
a_4^{(1)} &= -\frac{(d+3)(d-3)+3(d+2)}{d(d-1)(d+3)}, \\
a_5^{(1)} &= -\frac{d-1}{d} - \frac{3(d+2)+2(d-2)(d+3)}{d(d-1)(d+3)}, \\
a_6^{(1)} &= \frac{(d+2)(2d+1)}{d} \\
&\quad + \frac{(d+2)[6(d+2)+4(d-2)(d+3)]}{d(d-1)(d+3)}, \\
a_7^{(1)} &= \frac{(d-1)(d+2)}{d} \\
&\quad + \frac{(d+2)[6(d+2)+4(d-2)(d+3)]}{d(d-1)(d+3)},
\end{aligned}$$

$$a_8^{(1)} = -\frac{(d+2)(d+4)[3(d+2)+2(d-2)(d+3)]}{2d(d-1)(d+3)}. \quad (\text{A1})$$

For the second integral I^2 the coefficients $a_1^{(2)}, \dots, a_8^{(2)}$ are as follows:

$$\begin{aligned}
a_1^{(2)} &= \frac{2(d-1)}{d} + \frac{4}{d} + \frac{4(d+2)}{d(d-1)(d+3)}, \\
a_2^{(2)} &= -2(d-1) - 4 - \frac{4(d+2)}{(d-1)(d+3)}, \\
a_3^{(2)} &= -2 - \frac{4(d+2)}{(d-1)(d+3)}, \quad a_4^{(2)} = -1 - \frac{(d+2)}{(d-1)(d+3)}, \\
a_5^{(2)} &= -\frac{(d+2)}{(d-1)(d+3)}, \quad a_6^{(2)} = d+2 + \frac{2(d+2)^2}{(d-1)(d+3)}, \\
a_7^{(2)} &= \frac{2(d+2)^2}{(d-1)(d+3)}, \quad a_8^{(2)} = -\frac{(d+2)^2(d+4)}{2(d+3)(d-1)},
\end{aligned} \quad (\text{A2})$$

while for the third one they are

$$\begin{aligned}
a_1^{(3)} &= -\frac{4}{d} - \frac{4(d+4)}{d(d-1)(d+3)}, \quad a_2^{(3)} = 4 + \frac{4(d+4)}{(d-1)(d+3)}, \\
a_3^{(3)} &= 2 + \frac{4(d+4)}{(d-1)(d+3)}, \quad a_4^{(3)} = d+1 + \frac{(d+4)}{(d-1)(d+3)}, \\
a_5^{(3)} &= \frac{(d+4)}{(d-1)(d+3)}, \quad a_6^{(3)} = -d-2 - \frac{2(d+2)(d+4)}{(d-1)(d+3)}, \\
a_7^{(3)} &= -\frac{2(d+2)(d+4)}{(d-1)(d+3)}, \quad a_8^{(3)} = \frac{(d+2)(d+4)^2}{2(d-1)(d+3)}.
\end{aligned} \quad (\text{A3})$$

The coefficients $a_i^{(k)}$ are obtained for the general dimension d . We then specify their value for $d=4$ in Table I.

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