# Conformal symmetry of supergravities in AdS spaces

Renata Kallosh
*Physics Department, Stanford University, Stanford, California 94305-4060*

Antoine Van Proeyen
*Instituut voor theoretische fysica, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium*

(Received 30 April 1998; published 27 May 1999)

We show that the background field method applied to supergravity in AdS space-time provides the path integral for the theory in the bulk with conformal symmetry associated with the isometry of the AdS space. This in turn allows us to establish the rigid conformal invariance of the generating functional for the supergravity correlators on the boundary. [S0556-2821(99)07612-2]

PACS number(s): 11.25.Hf, 04.65.+e

## I. INTRODUCTION

In this paper we present an observation that the effective gauge-fixed supergravity actions have conformal (and $R$ symmetry *in the bulk and at the boundary* in cases when the $AdS_{p+2}\times S^{d-p-2}$ background (with form fields) defines a vacuum of the theory. This sheds light on the status of the recent conjecture [1] about the conformal field theory (CFT) and AdS correspondence between the large $N$ limit of the SU($N$) Yang-Mills (YM) theory and supergravity. In particular, we will find out *what part of Maldacena's conjecture* [1] *can be proved and what part remains to be studied*.

Our observation is based on the existence of the background functional method, first introduced for gravitational field theories by De Witt [2] and extended later for the case of supergravities in [3]. We use here the existence of the so-called background covariant gauges in which the effective actions of the theory as well as the generating functional of the Green functions are background invariant.

In fact we are using here precisely the same idea which has allowed us to construct a conformal theory of branes in [4]. There we had a class of actions with world volume local symmetries and rigid symmetries due to isometries of AdS space. After gauge fixing local symmetries, we found that the gauge-fixed theory has conformal symmetry. The basic idea is the same in supergravity: the gauge-fixed action can be constructed in background-covariant gauges which respect the symmetry of the background. In the case of interest it is a conformal symmetry associated with the AdS background.

Our general analysis applies to the following class of theories. (i)The so-called Poincaré supergravities (ungauged) for which the $AdS_{p+2}\times S^{d-p-2-k}\times E_k$ background with some form fields are a solution of classical field equations. (ii) Any of the gauged supergravities for which AdS space is known to be a consistent solution of field equations. In some cases these gauged supergravities correspond to the massless modes of the higher dimensional supergravity compactified on a sphere or on some other compact space. This is the simplest case and for $AdS_5$ it will be studied in detail.

We first recall the essential features of the background field method for supergravity in general and specify it for the case of a background with isometries in Sec. II. In Sec. III we derive the conformal symmetry of the generating functional for the Green functions of the supergravity in AdS space in the bulk. In Sec. IV we deduce the simplified rigid conformal symmetry for correlators at the boundary of AdS space. In Sec. V we suggest how to compare our results with available calculations. Finally in the Discussion we explain that the part of the Maldacena's conjecture about conformal symmetry in supergravity is proved. We explain why this is not yet a proof of the exact correspondence with conformal Yang-Mills theory.

## II. BACKGROUND FIELD METHOD IN SUPERGRAVITY AND ISOMETRIES

The background method is best described in condensed DeWitt's notation [2] where the gauge symmetry of the classical action means that

$$S_{cl}[\Phi]=S_{cl}[\Phi+\delta\Phi], \tag{2.1}$$

where $\Phi^i$ includes all fields of supergravity and

$$\delta_{loc}\Phi^i=R^i{}_\alpha[\Phi]\xi^\alpha_{loc}, \tag{2.2}$$

and $\xi^\alpha_{loc}$ is the infinitesimal parameter of the local symmetry. In the presence of a background field $\phi^i$, the fields of supergravity are shifted and the classical action is given by $S_{cl}[\phi+\Phi]$. This action is invariant under *two types of symmetries*.

(1) Transformations which affect only the quantum fields $\Phi$ which will become the integration variables in the path integral:

$$\delta_{quant}\phi^i=0,\quad \delta_{quant}\Phi^i=R^i{}_\alpha[\phi+\Phi]\xi^\alpha_{loc}. \tag{2.3}$$

This results in

$$\delta_{loc}(\phi^i+\Phi^i)=R^i{}_\alpha[\phi+\Phi]\xi^\alpha_{loc}, \tag{2.4}$$

which is a symmetry of the action $S_{cl}[\phi+\Phi]$.

(2) Background symmetry. Transformations of the background fields $\phi$ are accompanied by transformations of the quantum fields $\Phi$:

$$\delta_G\phi^i=R^i{}_\alpha[\phi]\xi^\alpha_{loc},\quad \delta_G\Phi^i=R^i{}_{\alpha,j}[\phi]\Phi^j\xi^\alpha_{loc}. \tag{2.5}$$

 ©1999 The American Physical Society

This results in

$$\delta_G(\phi^i+\Phi^i)=R^i{}_\alpha[\phi+\Phi]\xi^\alpha_{loc}, \tag{2.6}$$

so that the action $S_{cl}[\phi+\Phi]$ is invariant.

In view of the local symmetries, to define the path integral we have to add to the classical action of supergravity some gauge-fixing conditions and the ghost actions. The generating functional of the Green functions in supergravity is given by

$$e^{iW[\phi,J]}=\int d\Phi d\bar{c}dcdc_3\, e^{iS[\phi,\Phi,\bar{c},c,c_3,J]}. \tag{2.7}$$

Here $S$ consists of four terms, a classical action in the background, the gauge-fixing action, the ghost action, and the source term:

$$S[\phi,\Phi,\bar{c},c,J]=S_{cl}[\phi+\Phi]+S_{gf}[\phi,\Phi]+S_{ghost}[\phi,\Phi,\bar{c},c,c_3]+J_i\Phi^i. \tag{2.8}$$

The integration in the path integral is performed over all quantum supergravity fields $\Phi^i$ and over the Faddeev-Popov antighosts $\bar{c}$ and ghosts $c$ and over the third ghost $c_3$. The term $S_{gf}[\phi,\Phi]$ is designed to break all local symmetries of the classical action. In this way the local gauge symmetry of the first type $\delta_{quant}$ is broken and replaced by the Becchi-Rouet-Stora-Tyutin (BRST) symmetry due to the presence of ghosts. However, the second, the background symmetry of the classical action, is preserved by both the gauge-fixing condition in background covariant gauges, by the ghost action, and by the source term:

$$S[\phi,\Phi,\bar{c},c,J]=S[\phi+\delta_G\phi,\Phi+\delta_G\Phi,\bar{c}+\delta_G\bar{c},c+\delta_G c,c_3+\delta_G c_3,J+\delta_G J]. \tag{2.9}$$

Since we are mostly interested here in the tree approximation of supergravity, we will not provide here the background symmetry transformations of the ghosts; it is sufficient to notice that it exists. However, the transformation of source fields which preserves the background symmetry of the integrand in the path integral is important for our purpose and it is given below:

$$\delta_G J_i=J_j R^j_{\alpha,i}[\phi]\xi^\alpha. \tag{2.10}$$

For example, the gravitational part of our integrand (2.8) in the path integral (2.7) is

$$S=S_{cl}[g_{\mu\nu}+h_{\mu\nu}]+S_{gf}[g_{\mu\nu},h_{\mu\nu}]+S_{ghost}[g_{\mu\nu},h_{\mu\nu},\bar{c},c]+\int d^dx\sqrt{g}\;\; h_{\mu\nu}J^{\mu\nu}, \tag{2.11}$$

where $\sqrt{g}\equiv\sqrt{|\det g_{\mu\nu}|}$ and

$$S_{gf}[g_{\mu\nu},h_{\mu\nu}]=\int d^dx\sqrt{g}\;\;\frac{1}{2}D^\rho h_{\rho\mu}g^{\mu\nu}D^\sigma h_{\sigma\nu}, \tag{2.12}$$

and $D^\mu$ is a covariant derivative in the background metric $g_{\mu\nu}$. For the vector fields we have

$$S_{gf}[g_{\mu\nu},A_\mu]+S_{source}=\int d^dx\sqrt{g}\;\;\frac{1}{2}(D^\mu A_\mu)^2+\int d^dx\sqrt{g}\;\; A_\mu J^\mu. \tag{2.13}$$

It is usual to consider the gauge-fixing term quadratic in quantum fields, as in the examples above; in DeWitt's notation this is $\frac{1}{2}\Phi^i F_{ij}[\phi]\Phi^j$. Together with the part of the classical action quadratic in fields, $S^2_{cl}$, it gives

$$(S^2_{cl}+S_{gf})[\phi,\Phi]=\frac{1}{2}\Phi^i(S_{,ij}[\phi]+F_{ij}[\phi])\Phi^j. \tag{2.14}$$

This defines the free-background-dependent Green functions $G^{ij}[\phi]$:

$$-(S_{ij}+F_{ij})[\phi]G^{jk}[\phi]=\delta_i^k. \tag{2.15}$$

The basic property of the differential operators $(S_{ij}+F_{ij})[\phi]$ and higher order vertices $S_{,ijk\ldots}[\phi]$ in the background field method is that they transform covariantly under background field transformations. This means that for each upper index the transformation is as in Eq. (2.5) for $\Phi^i$ and for each index down as for the source $J_i$.

Thus we have recalled here the well-known property of the background field method for supergravity which allows us to construct the generating functional for the Green functions of the theory in a background covariant way, i.e., by keeping the following symmetry:

$$W[\phi,J]=W[\phi+\delta_G\phi,J+\delta_G J]. \tag{2.16}$$

This background symmetry is also present in the effective action of the theory:

$$\Gamma[\phi,\tilde{\Phi}]=W[\phi,J]-J_i\tilde{\Phi}^i, \tag{2.17}$$

where $J$ is replaced by the function of the background and classical fields according to the solution of

$$\frac{\delta W}{\delta J_i}[\phi,J(\phi,\tilde{\Phi})]=\tilde{\Phi}^i. \tag{2.18}$$

Here one has to take into account that the background transformation of the field $\tilde{\Phi}^i$ is given by

$$\delta_G\tilde{\Phi}^i=R^i{}_{\alpha,j}[\phi]\tilde{\Phi}^j\xi^\alpha. \tag{2.19}$$

In the case that the background has isometries, which means that for some part of $\xi^\alpha$, which will be denoted by $\xi^\alpha_K$,

$$\delta_G^{Killing}\phi^i=R^i{}_\alpha[\phi]\xi^\alpha_K=0, \tag{2.20}$$

The generating functional for the Green functions of the theory has the following symmetry:[1]

$$W[\phi,J]=W[\phi,J+\mathcal{L}_{\xi_K}J]. \tag{2.21}$$

Here the transformation of the sources $J$ for the particular case we consider, when the background has isometries, is reduced to an action of the Lie derivative for a given field with respect to the Killing vectors. For the effective action we have the following symmetry:

$$\Gamma[\phi,\tilde{\Phi}]=\Gamma[\phi,\tilde{\Phi}+\mathcal{L}_{\xi_K}\tilde{\Phi}]. \tag{2.22}$$

When the path integral of supergravity is expanded near the saddle point $\Phi=\tilde{\Phi}$, one gets in the first approximation the generating functional for the correlators and the effective action describing all tree diagrams:

$$W_{tree}[\phi,J]=S_{cl}[\phi+\tilde{\Phi}]+S_{gf}[\phi,\tilde{\Phi}]+J_i\tilde{\Phi}^i, \tag{2.23}$$

$$\Gamma_{tree}[\phi,\tilde{\Phi}]=S_{cl}[\phi+\tilde{\Phi}]+S_{gf}[\phi,\tilde{\Phi}], \tag{2.24}$$

$$-\frac{\delta\Gamma_{tree}[\phi,\tilde{\Phi}]}{\delta\tilde{\Phi}^i}=J_i\ . \tag{2.25}$$

The iterative solution of Eq. (2.25) gives $\tilde{\Phi}^i$ as a functional of the sources. It involves background covariant free Green functions $G^{ij}[\phi]$ and vertices $S_{,ijk}[\phi],S_{,ijkl}[\phi],\ldots$ of the theory. Symbolically the tree solution for the field $\tilde{\Phi}^i$ can be written as

$$\tilde{\Phi}^i=\left(J_j+\frac{1}{2}J_pG^{pm}S_{,mjn}[\phi]J_qG^{qn}+\cdots\right)G^{ji}[\phi]. \tag{2.26}$$

By inserting this tree field back into the right-hand side of Eq. (2.23) for $W_{tree}$ one gets the generating functional of all connected Green functions of the theory in the tree approximation.

## III. CONFORMAL SYMMETRY OF SUPERGRAVITIES IN ADS SPACES IN THE BULK

The new interesting feature of the background field method comes from the fact that when we consider supergravity in the fixed AdS background we are allowed to consider only those symmetries of the generating functional for the Green functions or effective action which do not change the background. The background has isometries generated by the Killing vector fields $\xi_K$. Therefore the symmetry of the generating functional is reduced to the action of the Lie derivatives with respect to the Killing vectors of AdS space.

[1] We assume here that the Jacobian of the background transformations on the quantum fields is trivial, i.e., that the background symmetry has no anomalies.

The metric of our background $AdS_{p+2}\times S^{d-p-2}$ geometry is

$$ds^2=\left(\frac{r}{R}\right)^{2/w}dx^m\eta_{mn}dx^n+\left(\frac{R}{r}\right)^2[dr^2+r^2d\Omega^2]. \tag{3.1}$$

One can also rewrite this metric as $(m=0,\ldots,p;\ m'=p+1,\ldots,d-1;\ r^2=y_{m'}y_{m'})$

$$ds^2=\left(\frac{r}{R}\right)^{2/w}dx^m\eta_{mn}dx^n+\left(\frac{R}{r}\right)^2dy^{m'}\delta_{m'n'}dy^{n'}. \tag{3.2}$$

In these coordinates the infinitesimal action of the SO($p+1$,2) isometry group is [4]

$$\delta_{AdS}(\xi)x^m=-\hat{\xi}^m(x,r)=-\xi^m(x)-(wR)^2\left(\frac{R}{r}\right)^{2/w}\Lambda_K^m,$$

$$\delta_{AdS}(\xi)y^{m'}=-\hat{\xi}^{m'}(x,r)=w\Lambda_D(x)y^{m'},$$

$$\delta_{AdS}(\xi)r=-\hat{\xi}^r(x,r)=w\Lambda_D(x)r, \tag{3.3}$$

where

$$\xi^m(x)=a^m+\lambda_M^{mn}x_n+\lambda_Dx^m+(x^2\Lambda_K^m-2x^mx\cdot\Lambda_K),$$

$$\Lambda_D(x)=\frac{1}{d}\partial_m\xi^m=\lambda_D-2x\cdot\Lambda_K \tag{3.4}$$

and $a^m,\lambda_M^{mn},\lambda_D,\Lambda_K^m$ are the constant parameters associated with translations $P_m$, Lorentz transformations $M_{mn}$, dilatations $D$, and special conformal transformations $K_m$. In case that $d>p+2$, there is also a sphere $S^{d-p-2}$ and we have SO($d-p-1$) $R$ symmetry:

$$\delta_{SO(d-p-1)}y^{m'}=\Lambda^{m'}{}_{n'}y^{n'}. \tag{3.5}$$

Now we can apply the AdS and $R$ transformations to study the symmetry of the generating functional of the supergravity in the background-covariant gauges.

We will explain the main result for the simpler case of only an AdS background in the case of gauged supergravity. The generalization to the more general situation when the sphere is present is technically more involved. Also for definiteness we will focus on the case of $AdS_5$ which describes the massless part of ten-dimensional supergravity compactified on $AdS_5\times S^5$. This part of Maldacena's conjecture was developed in [5–9] where actual calculations supporting the conjecture were presented.

The path integral of the five-dimensional $N=8$ gauged supergravity in the $AdS_5$ background is symmetric under the isometries generated by the Killing vector fields, $\delta_{AdS}(\xi)g_{\mu\nu}=0$, and we will use the notation $\mu=0,1,2,3,r$. Here $m=0,1,2,3$ are coordinates of the four-dimensional boundary which is at $r\to\infty$.

First we focus on supergravity in the bulk. The action of AdS symmetry on all sources to the supergravity fields is generated by the Lie derivative with respect to the Killing vectors $\xi$. These in turn coincide with what is known in the supergravity literature as a ''general covariance'' transformation of various fields in the action. By combining these two notions we are getting the following symmetry transformations for the sources of the gravitational, vector, and scalar fields, $J^{\mu\nu}$, $J^{\mu}$, and $J$, which are coupled to the supergravity fields. The generating functional for the Green functions (2.21) is symmetric under the following transformations of sources:

$$-\mathcal{L}_{\xi}J^{\mu\nu}=\hat{\xi}^{\lambda}\partial_{\lambda}J^{\mu\nu}-\partial_{\lambda}\hat{\xi}^{\mu}J^{\lambda\nu}-\partial_{\lambda}\hat{\xi}^{\nu}J^{\mu\lambda}, \tag{3.6}$$

$$-\mathcal{L}_{\xi}J^{\mu}=\hat{\xi}^{\lambda}\partial_{\lambda}J^{\mu}-\partial_{\lambda}\hat{\xi}^{\mu}J^{\lambda}, \tag{3.7}$$

$$-\mathcal{L}_{\xi}J=\hat{\xi}^{\lambda}\partial_{\lambda}J, \tag{3.8}$$

and the corresponding transformations on fermionic sources. Here $\hat{\xi}^{\lambda}=\hat{\xi}^{m}$ for $\lambda=0,1,2,3$ is defined in Eq. (3.3) and the last one, the $r$ component $\hat{\xi}^{r}$, is equal to $-w\Lambda_D(x)r$, as one can see from Eq. (3.3).

The effective action (2.22) is symmetric under the following transformations of the effective fields:

$$-\mathcal{L}_{\xi}\tilde{h}_{\mu\nu}=\hat{\xi}^{\lambda}\partial_{\lambda}\tilde{h}_{\mu\nu}+\partial_{\mu}\hat{\xi}^{\lambda}\tilde{h}_{\lambda\nu}+\partial_{\nu}\hat{\xi}^{\lambda}\tilde{h}_{\mu\lambda}, \tag{3.9}$$

$$-\mathcal{L}_{\xi}\tilde{A}_{\mu}=\hat{\xi}^{\lambda}\partial_{\lambda}\tilde{A}_{\mu}+\partial_{\mu}\hat{\xi}^{\lambda}\tilde{A}_{\lambda}, \tag{3.10}$$

$$-\mathcal{L}_{\xi}\tilde{\Phi}=\hat{\xi}^{\lambda}\partial_{\lambda}\tilde{\Phi}, \tag{3.11}$$

and the corresponding transformations on fermionic fields.

The transformations shown in Eqs. (3.6), (3.7), and (3.8) represent the conformal symmetry of the generating functional for the correlators of the supergravity fields in the bulk at finite values of $r$. Note that so far our sources are defined in the bulk and differentiating the generating functional over the sources one can get all correlators of supergravity fields in the bulk.

If the actual calculation of the generating functional is performed, e.g., if the term quadratic in $J$ is found, it takes the form

$$W(g_{\mu\nu}(r),J^{\mu\nu}(x,r),J^{\mu}(x,r),J(x,r),\dots)$$
$$=\frac{1}{2}\int d^4x\,dr\,d^4x'\,dr'\,J(x,r)G(x,r;x',r')J(x',r')+\dots. \tag{3.12}$$

This expression has to be invariant under the transformations (3.8) which puts constraints on the Green function $G$ in the bulk. The same takes place for any other correlator.

## IV. CONFORMAL SYMMETRY OF SUPERGRAVITIES ON THE BOUNDARY OF THE AdS SPACES

The basic idea developing Maldacena's conjecture [1] was suggested by Gubser, Klebanov, and Polyakov [6] and Witten [7]. It was to place the sources to the supergravity fields at the boundary and calculate the correlators of the fields on the boundary and compare them with those of the YM theory.

In our context the advantage of considering the sources to the supergravity fields located only at the boundary of AdS space is the dramatic simplification of the conformal symmetry comparative to the one in the bulk.

To find these symmetries we have to consider the limit of our conformal symmetries in the bulk to the boundary at $r\to\infty$. We specify the case with $\mathrm{AdS}_5$, $w=1$. We will first change the variables to $z=R^2/r$ in which the AdS part of the metric is conformally flat, $ds^2=(R^2/z^2)(dx_m^2+dz^2)$. In these variables the boundary is at $z\to 0$. Clearly,

$$\hat{\xi}^{m}(x,z)_{z\to 0}=(\xi^{m}(x)+z^2\Lambda_K^{m})_{z\to 0}\Rightarrow\xi^{m}(x) \tag{4.1}$$

and

$$(\hat{\xi}^{z})_{z\to 0}\Rightarrow\Lambda_D(x)z, \tag{4.2}$$

where the parameters of the rigid conformal symmetry $\xi^{m}(x)$ and $\Lambda_D(x)$ are given in Eq. (3.4). We will denote the sources placed at the boundary, which are $z$ independent by $\mathcal{J}$. To find the boundary limit of the conformal transformations from the bulk we note that on $z$-independent functions,

$$[\hat{\xi}^{m}(x,z)\partial_m+\hat{\xi}^{m}(x,z)\partial_z]_{z\to 0}\Rightarrow\xi^{m}(x)\partial_m. \tag{4.3}$$

To study the limit to the boundary on tensors we need to use

$$[\partial_m\hat{\xi}^{n}(x,z)]_{z\to 0}\Rightarrow\partial_m\xi^{n}(x), \tag{4.4}$$

$$[\partial_z\hat{\xi}^{z}(x,z)]_{z\to 0}\Rightarrow\Lambda_D(x), \tag{4.5}$$

$$[\partial_z\hat{\xi}^{n}(x,z)]_{z\to 0}\Rightarrow 2\Lambda_K^{n}(x)z\Rightarrow 0, \tag{4.6}$$

$$[\partial_m\hat{\xi}^{z}(x,z)]_{z\to 0}\Rightarrow\partial_m\Lambda_D(x)z\Rightarrow 0. \tag{4.7}$$

It follows that the components of the contravariant tensors in the bulk direction $z$ are not mixed anymore with those in directions $x$:

$$-\mathcal{L}_{\xi}^{bound}\mathcal{J}^{mn}=\xi^{l}(x)\partial_l\mathcal{J}^{mn}-\partial_l\xi^{m}\mathcal{J}^{ln}-\partial_l\xi^{n}\mathcal{J}^{ml}, \tag{4.8}$$

$$-\mathcal{L}_{\xi}^{bound}\mathcal{J}^{m}=\xi^{l}\partial_l\mathcal{J}^{m}-\partial_l\xi^{m}\mathcal{J}^{l}, \tag{4.9}$$

$$-\mathcal{L}_{\xi}^{bound}\mathcal{J}=\xi^{l}\partial_l\mathcal{J}, \tag{4.10}$$

with $\xi^{m}(x)$ defined in Eq. (3.4).

The transformation of the remaining components of the sources is

$$-\mathcal{L}_{\xi}^{bound}\mathcal{J}^{mz}=\xi^{l}(x)\partial_l\mathcal{J}^{mz}-\partial_l\xi^{m}\mathcal{J}^{lz}-\Lambda_D(x)\mathcal{J}^{mz}, \tag{4.11}$$

$$-\mathcal{L}_{\xi}^{bound}\mathcal{J}^{zz}=\xi^{l}(x)\partial_l\mathcal{J}^{zz}-2\Lambda_D(x)\mathcal{J}^{mz}, \tag{4.12}$$

$$-\mathcal{L}_{\xi}^{bound}\mathcal{J}^{z}=\xi^{l}\partial_l\mathcal{J}^{z}-\Lambda_D(x)\mathcal{J}^{z}. \tag{4.13}$$

The generating functional depending on boundary sources has the following symmetry:

$$W^{bound}[\mathcal{J}]=W[\mathcal{J}+\mathcal{L}_{\xi}^{bound}\mathcal{J}], \tag{4.14}$$

where the transformation of all sources at the boundary is given in Eqs. (4.8)–(4.13).

Note that we have suppressed here the dependence on internal indices of the vectors and scalars since for our simple example conformal symmetries do not act on them; the only relevant ones are those which show the behavior under general coordinate transformations.

## V. COMPARISON WITH AVAILABLE CALCULATIONS

Consider here the calculations of the two-point and three-point correlators available in the literature [6,7,9]. For massless scalars in a Euclidean signature we have

$$W^{(2)}[\mathcal{J}]=c\int d^4x d^4y \frac{\mathcal{J}(x)\mathcal{J}(y)}{|x-y|^8}. \tag{5.1}$$

This answer is in agreement with our form of conformal symmetry which states that under the transformations (4.10) the functional $W$ has to be symmetric. These transformations include translation, dilatation, and Lorentz transformations, which are all obvious, but also the special conformal transformations, which are not so obvious. The transformation of $W^{(2)}[\mathcal{J}]$ is

$$\begin{aligned}\delta_\xi W^{(2)}[\mathcal{J}]&=-c\int d^4x\ \ d^4y\frac{1}{|x-y|^8}\{[\mathcal{L}_\xi\mathcal{J}(x)]\mathcal{J}(y)+\mathcal{J}(x)\\&\quad\times[\mathcal{L}_\xi\mathcal{J}(y)]\}\\&=c\int d^4x\ \ d^4y\frac{1}{|x-y|^8}\{[\xi(x)^m\partial_m\mathcal{J}(x)]\mathcal{J}(y)\\&\quad+\mathcal{J}(x)[\xi(y)^m\partial_m\mathcal{J}(y)]\}.\end{aligned} \tag{5.2}$$

After integration by parts, and using Eq. (3.4), we obtain

$$\begin{aligned}\delta_\xi W^{(2)}[\mathcal{J}]&=c\int d^4x\ \ d^4y\frac{1}{|x-y|^8}\mathcal{J}(x)\mathcal{J}(y)\bigg[-4\Lambda_D(x)\\&\quad-4\Lambda_D(y)+8\frac{[\xi(x)^m-\xi(y)^m](x_m-y_m)}{|x-y|^2}\bigg].\end{aligned} \tag{5.3}$$

Inserting the explicit expression of $\xi^m$ in Eq. (3.4) leads to the cancellation of the terms in the square brackets.

For vectors in [7,9] the relevant correlators are given for two-point functions and in [9] for three-point functions. Instead of doing a direct verification of our symmetries when $\mathcal{J}^m$ sources are nonvanishing, as shown for scalars above, we may use some important properties of the correlators established in [9] which will allow us to confirm that there is an agreement with our form of symmetries. In [9] the boundary correlators are found to be covariant under inversion which means that the generating functional $W[\mathcal{J}^m]$ is invariant under inversion. The relevant nonlocal functional is also invariant under translational symmetry. Therefore by applying inversion, translation, and another inversion one can find out that the generating functional is indeed invariant under conformal symmetry as predicted by our analysis. The functional for the two-point correlators of vector fields found in [7,9] has the form

$$\begin{aligned}W^2[\mathcal{J}^m]&=c\int d^4xd^4y\,\mathcal{J}_a^m(x)\mathcal{J}_b^n(y)\,\delta^{ab}(\partial_l\partial^l\delta_{mn}\\&\quad-\partial_m\partial_n)\frac{1}{|x-y|^4}.\end{aligned} \tag{5.4}$$

The $f^{abc}$ part of the three-point correlator was found in [9]:

$$W^3[\mathcal{J}^m]=c\int d^4xd^4yd^4z\,\mathcal{J}_a^m(x)\mathcal{J}_b^n(y)\mathcal{J}_c^l f^{abc} \tag{5.5}$$

$$\times[k_1D_{mnl}^{\rm sym}(x,y,z)+k_2C_{mnl}^{\rm sym}(x,y,z)], \tag{5.6}$$

where the tensors $D$ and $C$ have particular form presented in [9]. It may be difficult but not impossible to establish that the generating functional for the correlators of supergravity fields on the boundary calculated in [9] indeed has the required symmetry properties required by Eqs. (4.14), (4.9). This would confirm that the choice of the inversion-covariant bulk-to-boundary Green functions in [9] corresponds to the correct choice of the background-field-covariant gauge in the AdS background.

## VI. DISCUSSION

In this paper we have addressed the problem which naturally comes to the mind of anybody familiar with the difference between ungauged supergravity, gauged supergravity, and conformal supergravity. As phrased by Stelle [10], why should Poincaré supergravity know anything about conformal symmetry at all? The answer is that in general, indeed, only conformal supergravity has local conformal symmetry and both ungauged and gauged supergravities do not have anything close to conformal symmetry. However, when placed in the consistent AdS background with symmetries isomorphic to the conformal group these two versions of supergravity upon gauge fixing do have conformal symmetry as conjectured in [1]. In particular the generating functional for all correlators of supergravity fields everywhere in AdS space-time has conformal symmetries of the type which were found previously on the world volumes [4] of brane actions. They are characterized by an unusual form of the special conformal symmetry. Now we have found the analogous symmetries in the space-time supergravity.

We also studied a particular case when the sources of the supergravity fields are placed only on the boundary of AdS space-time, as proposed in [6,7]. The conformal symmetry of supergravity in the bulk is simplified and reduced to the simple rigid conformal symmetry, consisting as usual of translations, Lorentz transformations, dilatations, and special conformal transformations. For example we found that the generating functional of the correlators of scalar fields of supergravity on the boundary is symmetric under the transformations of the $z$-independent scalar source $\mathcal{J}$ placed at the boundary $z \to 0$:

$$\delta \mathcal{J}(x) = [a^m + \lambda_M^{mn} x_n + \lambda_D x^m + (x^2 \Lambda_K^m - 2x^m x \cdot \Lambda_K)] \frac{\partial}{\partial x^m} \mathcal{J}(x). \tag{6.1}$$

It should also be stressed that in Witten diagrams [7,9], as our analysis shows, the Green functions which are not touching the boundary are in fact bulk-to-bulk Green functions which are different from bulk-to-boundary correlators used in the calculations performed so far. Such bulk-to-bulk correlators will be always present in the tree level boundary supergravity starting with four-point correlators as well as in all loop diagrams. These Green functions are conformal covariant in the bulk and not only at the boundary, as well as all vertices of the theory.

Thus we have found a clear explanation via a background field method why in ungauged or gauged supergravity one encounters rigid conformal symmetry. Note that our proof never used the tree approximation to the supergravity correlators; it is correct for any loop approximation under the condition that the AdS background remain a consistent solution of equations of motion with quantum corrections. It will be interesting to establish whether this is true.

The observation made here does not indicate yet that the results of calculations of correlators in tree level supergravity on the boundary have to be the same as the one in large $N$ limit of $\mathcal{N}=4$ supersymmetric Yang-Mills theory.

In conclusion, we have shown a surprisingly simple picture as to how a conformal (and superconformal) symmetry is present in supergravities in AdS spaces. This part of the famous CFT-AdS correspondence [1] is no conjecture anymore: if one uses the correct Feynman rules for the calculations of tree diagrams in supergravity in the particular setup, explained in this paper, conformal symmetry is guaranteed to be there, in the form of Eq. (2.21) in the bulk and Eq. (4.14) at the boundary.

*Note added.* After this paper was written we saw a recent paper of Liu and Tseytlin [11] where new calculations of supergravity correlators on the boundary of AdS space are performed. In particular the aim was to find the graviton-dilaton-dilaton correlator ‘‘without making *a priori* assumptions about the conformal invariance of the result.’’ A detailed study of this set of calculations (and any new one which may appear soon) would be interesting to carry out from the perspective of our proof of the conformal symmetry of the full generating functional for the boundary correlators of supergravity on AdS.

## ACKNOWLEDGMENTS

We are grateful to the organizers and the participants of the ‘‘Super Five Branes and Physics in $5+1$ dimensions’’ conference in Trieste for stimulating discussions of the preliminary form of the result presented in this paper. R.K. also would like to acknowledge particularly useful conversations related to this work with A. Matusis, A. Rajaraman, and R. Siebelink. The work of R.K. is supported by NSF grant PHY-9219345, and the work of A.V.P. is supported by the European Commission TMR program ERBFMRX-CT96-0045 and by FWO, Belgium.

[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); J. Maldacena and A. Strominger, J. High Energy Phys. **12**, 005 (1998).
[2] B. DeWitt, Phys. Rev. **162**, 1195 (1967); **162**, 1239 (1967).
[3] R. Kallosh, Nucl. Phys. **B141**, 141 (1978).
[4] P. Claus, R. Kallosh, and A. Van Proeyen, Nucl. Phys. **B518**, 117 (1998); R. Kallosh, J. Kumar, and A. Rajaraman, Phys. Rev. D **57**, 6452 (1998); P. Claus, R. Kallosh, J. Kumar, P. Townsend, and A. Van Proeyen, J. High Energy Phys. **06**, 004 (1998).
[5] S. Ferrara and C. Fronsdal, Class. Quantum Grav. **15**, 2153 (1998).
[6] S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[7] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[8] Sergio Ferrara, Christian Fronsdal, and Alberto Zaffaroni, Nucl. Phys. **B532**, 153 (1998).
[9] D.Z. Freedman, S.D. Mathur, A. Matusis, and L. Rastelli, Nucl. Phys. **B546**, 96 (1998).
[10] K. Stelle (private communications).
[11] H. Liu and A.A. Tseytlin, Nucl. Phys. **B533**, 88 (1998).