Axionic boson stars in magnetized conducting media

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Axions are possible candidates for the dark matter in the present Universe. They have been argued to form axionic boson stars with small masses $10^{-14}M_{\odot}$–$10^{-11}M_{\odot}$. Since they possess oscillating electric fields in a magnetic field, they dissipate their energies in magnetized conducting media such as white dwarfs or neutron stars. At the same time the oscillating electric fields generate a monochromatic radiation with an energy equal to the mass of the axion. We argue that the effect of the energy dissipation can be seen in old white dwarfs. In particular, we show that colliding with sufficiently cooled white dwarfs, plausible candidates of MACHOs, the axionic boson stars dissipate their energies in the dwarfs and heat up the dwarfs. Consequently the white dwarfs in the halo can emit a detectable amount of thermal radiation with the collision. On the other hand, monochromatic radiation can be seen only during the collision, a period of the dwarf passing the axionic boson star. Assuming that MACHOs are dark white dwarfs, we show that there is a threshold in the luminosity function of the white dwarfs below which the number of white dwarfs in the halo increases discontinuously. The threshold in the luminosity function is expected to be located around $10^{-5.5}L_{\odot}$–$10^{-7}L_{\odot}$. Its precise value is determined by the mass of the axionic boson stars dominant in the halo. [S0556-2821(99)03512-2]

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I. INTRODUCTION

The axion is the Goldstone boson associated with Peccei-Quinn symmetry [1], which was introduced to solve naturally the strong CP problem. In the early Universe some of the axions condense and form topological objects [2,3], i.e., strings and domain walls, although they decay below the temperature of the QCD phase transition. After their decay, however, they have been shown to leave a magnetic field [4] as well as a cold axion gas as relics in the present Universe; the field is a candidate for a primordial magnetic field supposed to lead to galactic magnetic fields observed in the present Universe.

In addition to this topological objects, the existence of axionic boson stars has been argued [5,6]. It has been shown numerically [6] that in the early Universe axion clumps are formed around the period of 1 GeV owing to both the nonlinearity of an axion potential, leading to an attractive force among the axions, and the inhomogeneity of coherent axion oscillations. Namely, when the temperature of the Universe decreases and the axion potential is generated by QCD instantons, the inhomogeneity of the coherent axion oscillation on the scale beyond the horizon gives rise to localized clumps due to the attractive force of the potential. These clumps are called axitons since they are similar to solitons in the sense that their energy is localized. Then, the axitons contract gravitationally to axionic boson stars [7,8] after separating out from the cosmological expansion. They are solitons of coherent axions bounded gravitationally. (The axions are represented with a real scalar field, which has been shown to possess solutions of oscillating boson stars [8].) The masses of the axionic boson stars have been estimated roughly to be of the order of $\sim 10^{-12}M_{\odot}$. Eventually we expect that in the present Universe, axionic boson stars as well as the axion gas as dark matter candidates. It has been estimated [9] that a fair amount of the fraction of axion dark matter is composed of the axionic boson stars.

In this paper we wish to point out an intriguing observable effect associated with coherent axionic boson stars; we call them axion stars. Namely, they dissipate their energies in magnetized conducting media such as magnetic white dwarfs or neutron stars so that the temperature of the media increases and thermal radiation is emitted. The phenomena are caused by oscillating electric fields generated by the axion field under external magnetic fields; their frequency is given by the mass of the axion. The electric fields induce electric currents in the conducting media and lose their energies owing to the existence of electric resistances. Consequently the axion stars dissipate their energies in the magnetized conducting media. Although the electric fields themselves are small, the total amount of the energy dissipation is very large because dissipation arises all over the volume of the axion stars or the volume of the magnetized conducting media: Radii of the white dwarfs (neutron stars) are typically 10^9 cm(10^6 cm), while radii of the axion stars of our concern are such as 10^6 cm–10^{10} cm. Consequently a detectable amount of radiation is expected from media heated this way. In particular we are concerned with old white dwarfs which are plausible candidates of massive compact halo objects (MACHOs) [10] and have been cooled sufficiently; they are dark enough to be invisible. Thus the effect of the energy dissipation of the axion star in such white dwarfs possessing small specific heats is so large that the luminosity of the dwarfs after collisions with axion stars increases such that the dwarfs become observable.

In particular, we point out that there is a threshold in the luminosity function of the halo white dwarfs (the luminosity function describes, roughly speaking, the number density of the white dwarfs as a function of luminosity). Namely, the luminosity function increases discontinuously below the threshold of a certain luminosity. To explain it, we first point out that the white dwarfs in the halo are sufficiently old so that their luminosity is quite low to be invisible. Especially, according to a recent cooling model, the white dwarfs with a

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helium-rich atmosphere are expected to have been cooled so that their luminosity is less than $10^{-7}L_{\odot}$ at their age of $\sim 1.2\times 10^{10}$ yr. Here we assume that MACHOs are such white dwarfs with a helium-rich atmosphere. Thus they are invisible with present observational apparatus unless they are located quite near the Sun. Then, since they have few internal thermal energies, their temperatures increase dramatically with the dissipation of the energy of the axion star in the white dwarfs. We show that the luminosity gained by the white dwarfs with the collision is about $10^{-5.5}L_{\odot}$–$10^{-7}L_{\odot}$ depending on the mass of the axion star; the mass of our concern is $10^{-11}M_{\odot}$–$10^{-14}M_{\odot}$ ($L_{\odot}$ denotes the luminosity of the Sun). Therefore, this luminosity is expected to be the threshold luminosity of the white dwarf luminosity function in the halo.

In addition to this thermal radiation from the magnetized medium, oscillating electric fields associated with the axion star in the magnetized medium generate monochromatic radiation with energy equal to the mass of the axion. Since the electric fields arise only during the axion star being exposed to an external magnetic field of the magnetized media, the radiation is emitted only in a period of the media passing the axion star. Therefore, we expect that we will first observe monochromatic radiation in a short period and then observe the thermal radiation which is emitted until the dissipation energy deposited by the axion star is exhausted. This thermal radiation is observed possibly as a nova which is an old white dwarf heated by the collision. Here we should mention that since the rate of the collision between the white dwarfs and the axion stars is very small, it seems difficult to observe monochromatic radiation.

Because the strength of the electric fields is proportional to the strength of the magnetic field, the phenomena are distinctive of strongly magnetized media. We show that the amount of energy dissipated in white dwarfs, for instance, with mass $\sim 0.5M_{\odot}$ and with a magnetic field larger than 10^5 G, is approximately given by 10^{42} erg $M_a/10^{-12}M_{\odot}$ where $M_a(M_{\odot})$ is the mass of the axion star (the Sun). Namely, in almost all cases, the whole energy of the axion star is dissipated in such white dwarfs. In our discussions we neglect, for simplicity, gravitational effects of the magnetized conducting media on the axion stars when they collide with the media. Later we discuss this point.

In Sec. II we present our solutions of the axionic boson stars with small masses. The axion field of the solutions oscillates with the frequency of the axion mass. We show explicitly relations between the mass and the radius of the axion star, which are exploited for an the estimation of the amount of energy dissipation in magnetized conducting media. In Sec. III we describe an intriguing phenomenon: that the axionic boson star induces an oscillating electric field when it is exposed to an external magnetic field. This electric field induces electric currents carried by ordinary charged particles in the magnetized conducting media. Thus axion stars dissipate their energies in such media. The axion star in magnetic field possesses another oscillating electric current even in nonconducting media such as vacuum owing to an interaction of the axion field with the electromagnetic fields. The current [11] is composed of the axion field although the axions are neutral. Owing to these two types of oscillating currents, the axion stars dissipate their energies in the conducting media. Thus the media are heated so that thermal radiations are emitted. The application of these phenomena is discussed in Secs. IV and V where white dwarfs and neutron stars are discussed as magnetized conducting media, respectively, separately. We summarize our results in Sec. VI.

II. AXIONIC BOSON STAR

Let us first explain our solutions of the axionic boson stars. Originally Seidel and Suen [8] found solutions of a real scalar axion field a coupled with gravity. Their solutions represent spherical oscillating axion stars with masses of the order of $10^{-5}M_{\odot}$; the solution possesses oscillation modes with various frequencies. (Static regular solutions have been shown not to exist in the massless real scalar field coupled with gravity. Even in the massive real scalar field such solutions have not yet been obtained.) Although these axion stars are oscillating, they are stable solitons composed of the axions coupled with gravity; they are similar to the ''breather'' solution of the $(1+1)$-dimensional sine-Gordon model. The axion stars of our concern, on the other hand, are ones with much smaller masses, $\sim 10^{-14}M_{\odot}$. This is because according to the arguments of Kolb and Tkachev [6,7,9] the axion stars produced in the early Universe have masses typically such as $10^{-12}M_{\odot}\Omega_a h^2$ where Ω_a is the ratio of the axion energy density to the critical density in the Universe and h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}. They have been produced after the period of the QCD phase transition with axions contracting due to effects of both the gravitational attraction and the attraction of the axion potential.

So in order to find such solutions and to obtain explicit relations among the parameters, e.g., radius, R_a, mass, M_a, etc., of these axion stars, we have numerically obtained solutions of the spherical axionic boson stars [12,8,13] in the limit of a weak gravitational field. Relevant equations are a free field equation of the axion and Einstein equations:

$$\ddot{a}=\frac{(\dot{h}_t-\dot{h}_r)\dot{a}}{2}+a''+\left(\frac{2}{r}+\frac{h_t'-h_r'}{2}\right)a'-m_a^2a, \tag{1}$$

$$h_t'=\frac{h_r}{r}+4\pi Gr(a'^2-m_a^2a^2+\dot{a}^2), \tag{2}$$

$$h_r'=-\frac{h_r}{r}+4\pi Gr(a'^2+m_a^2a^2+\dot{a}^2), \tag{3}$$

where we have assumed gravity as being small, i.e., $h_{t,r}\ll 1$, so that the metric is such that $ds^2=(1+h_t)dt^2-(1+h_r)dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2)$; r, θ, and ϕ denote the polar coordinates. The first equation is the equation of the axion field a. The second and the third equations are Einstein equations. An overdot (prime) indicates a derivative in time $t(r)$. $G(m_a)$ is the graviational constant (the mass of the axion). The potential term $\sim\sin(a)$ of the axion has been neglected because the amplitude of the field a is sufficiently small for nonlinearity not to arise since the mass of the axion star is small enough. Actually the masses we are concerned with are such as $\sim 10^{-14} M_\odot$, while nonlinearity has been found numerically to arise only for axion stars with masses larger than $\sim 10^{-9} M_\odot$. We need to impose a boundary condition such as $h_r(r=0)=0$ for the regularity of the spacetime.

Changing the scales such that $\tau = m_a t$, $x = m_a r$, and $b = a/m_a$, we rewrite the equations as follows,

$$\ddot{b} = \dot{V}\dot{b} + b'' + \left(\frac{2}{x} + V'\right) b' - b, \tag{4}$$

$$V' \equiv \frac{h_t' - h_r'}{2} = \epsilon \left(\frac{\int_0^x dx x^2 (b'^2 + \dot{b}^2 + b^2)}{x^2} - x b^2 \right), \tag{5}$$

with $\epsilon = 4\pi G m_a^2$, where we have expressed V' in terms of the field b, solving Eqs. (2) and (3); here an overdot (prime) denotes a derivative in $\tau(x)$. We understand that if the gravitational effect is neglected ($\epsilon = 0$), the equation of b is reduced to a Klein-Gordon equation. Thus the frequency ω of the field b receives a small gravitational effect of the order of $\epsilon, \omega = 1 - o(\epsilon)$.

We look for a solution [8] such that

$$b = A_0 B(x) \sin \omega\tau + o(\epsilon) \sin 3\omega\tau, \tag{6}$$

where $B(x)$ represents coherent axions bounded gravitationally with their spatial extension representing the radius of the axion star; $B(x)$ is normalized such as $B(x=0)=1$. Later we find that A_0 is a free parameter determining the mass or radius of the axion star. The second term is a small correction of the order of ϵ. Here we comment that previous solutions [8] representing axion stars with larger masses possess more oscillating terms such as $\sin(2n-1)\omega$ with $n = 1,2,\ldots$. Inserting Eq. (6) into Eqs. (4) and (5) and taking account of gravitational effects only with the order of ϵ, we find that

$$k^2 B = B'' + \left[\frac{2}{x} + \epsilon A_0^2 \left(T + \frac{3U'}{4} \right)\right] B' + \frac{\epsilon A_0^2 (U+v) B}{2}, \tag{7}$$

with

$$T \equiv \frac{\int_0^x z^2 B^2 dz}{x^2} \quad \text{and} \quad U \equiv \int_0^x dy \left(\frac{\int_0^y z^2 B'^2 dz}{y^2} - y B^2 \right), \tag{8}$$

where $k^2 (=1-\omega^2)$ is the binding energy of axions. We have imposed a boundary condition for consistency such that $V(x=0) = h_t(x=0)/2 = \epsilon A_0^2 v\, \omega \sin 2\omega\tau$; this is the definition of constant v in the above formula.

We can see that the parameter ϵA_0^2 can take an arbitrary value and that it represents the gravitational effect of this system. Namely, the mass of the axionic boson star is determined by choosing a value of the parameter. Note that the normalization of B has been fixed in Eq. (7) although the equation is linear in B.

We may take the value of v without loss of generality such that $v = -U(x=\infty)$. Then it turns out that the inverse k^{-1} of the binding energy represents the radius of the axion star; B decays exponentially such as $\exp(-kx)$ for $x \to \infty$. It turns out from Eq. (7) that the choice of small values of ϵA_0^2 leads to solutions representing axion stars with small masses and large radii k^{-1}.

Before solving Eq. (7) numerically, it is interesting to rewrite the equation as follows. That is, we rewrite the equation by taking only dominant terms of a "potential" V_b in Eq. (7),

$$V_b = \epsilon A_0^2 \left[\left(T + \frac{3U'}{4} \right) B' + \frac{(U+v)B}{2} \right] \tag{9}$$

in the limit of the large length scale; setting $x = \lambda y$, we take a dominant term as $\lambda \to \infty$. This corresponds to taking the axion stars with their spatial extension being large.

Then, since the dominant term in the limit is the last term in Eq. (9), $U + v \sim \int_x^\infty dx x B^2$, we obtain the following equation:

$$\bar{B} = \bar{B}'' + \frac{2\bar{B}'}{z} + \frac{\bar{B} \int_z^\infty dy y \bar{B}^2}{2}, \tag{10}$$

where we have scaled the variables such that $B^2 = k^4 \bar{B}^2 / \epsilon A_0^2$ and $x = k^{-1} z$; an overbar denotes a derivative in z. This equation is much simpler than Eq. (7), where we need to find each eigenvalue of k for each value of ϵA_0^2 given, in order to obtain solutions of the axion stars with various masses. On the other hand, we need only to find an appropriate value of $\bar{B}(z=0) = \epsilon A_0^2 / k^4$ in order to obtain such solutions in Eq. (10). A relevant solution we need to find is the solution without any nodes. Obviously, the solution is characterized by one free parameter k^4 or ϵA_0^2, which is related to the mass of the axion star. Namely, the choice of a value of the mass determines uniquely the properties of the axion star, e.g., the radius of the star, distribution of the axion field a, etc.

Although Eq. (10) governs only the axion star with a large radius, we have confirmed by solving numerically the original equation (7) that the stars of our concern can be controlled by Eq. (10).

We have confirmed that our numerical solutions may be approximated by the explicit formula

$$a = f_{PQ} a_0 \sin(m_a t) \exp(-r/R_a), \tag{11}$$

where $t(r)$ is the time (radial) coordinate and f_{PQ} is the decay constant of the axion. The value of f_{PQ} is constrained from cosmological and astrophysical considerations [3,2] such as 10^{10} GeV $< f_{PQ} < 10^{12}$ GeV. Corresponding to this constraint, m_a is constrained roughly such as 10^{-5} eV $< m_a < 10^{-3}$ eV.

In the limit of the small mass of the axion star we have found a simple relation [12] between the mass M_a and the radius R_a of the axion star,

$$M_a = 6.4 \frac{m_{pl}^2}{m_a^2 R_a}, \tag{12}$$

with Planck mass m_{pl}. Numerically, for example, $R_a = 1.6 \times 10^{10} m_5^{-2} M_{14}^{-1}$ cm; hereafter we use the notation, $M_n \equiv M_a / 10^{-n} M_\odot$ and $m_n \equiv m_a / 10^{-n}$ eV. A similar formula has been obtained in the case of boson stars of complex scalar fields [13]. We have also found an explicit relation [12] between the radius and the dimensionless amplitude a_0 in Eq. (1),

$$a_0 = 1.73 \times 10^{-8} \frac{(10^8 \ \mathrm{cm})^2}{R_a^2} \frac{10^{-5} \ \mathrm{eV}}{m_a}. \tag{13}$$

These explicit formulas are used for the evaluation of the dissipation energy of axion stars in magnetized conducting media.

III. AXION STARS IN A MAGNETIC FIELD

We now proceed to explain that the axionic boson stars generate an electric field in an external magnetic field. It will turn out, below, that the field gives rise to intriguing astrophysical phenomena. Thus it is important to understand the mechanism of producing the field. The point is that the axion couples [2] with electromagnetic fields in the following way:

$$L_{a\gamma\gamma} = c \alpha a \vec{E} \cdot \vec{B} / f_{PQ} \pi, \tag{14}$$

with $\alpha = 1/137$, where $\vec{E}$ and $\vec{B}$ are electric and magnetic fields, respectively. The value of c depends on the axion models [14,15]; typically it is of order 1.

It follows from this interaction that Gauss' law is given by

$$\vec{\partial} \vec{E} = -c \alpha \vec{\partial} \cdot (a \vec{B}) / f_{PQ} \pi + \text{``matter,''} \tag{15}$$

where the last term ''matter'' denotes contributions from ordinary matter. The first term on the right hand side represents a contribution from the axion. Thus it turns out that the axion field has an electric charge density $\rho_a = -c \alpha \vec{\partial} \cdot (a \vec{B}) / f_{PQ} \pi$ under the magnetic field $\vec{B}$ [11]. This charge density does not vanish only when the field configuration a is not spatial uniform since $\vec{\partial} \vec{B} = 0$. Thus the electric field E_a associated with this axion charge is produced such that $\vec{E_a} = -c \alpha a \vec{B} / f_{PQ} \pi$. Note that both ρ_a and E_a oscillate with the frequency given by the mass of the axion in the case of the axion star since the field a itself oscillates.

Obviously, this field induces an oscillating electric current $J_m = \sigma E_a$ in magnetized conducting media with electric conductivity σ. In addition to the current J_m carried by ordinary matter, e.g., electrons, there appears an electric current J_a associated with the oscillating charge ρ_a owing to current conservation [11] $(\partial_0 \rho_a - \vec{\partial} \vec{J_a} = 0)$. This is given such that $\vec{J_a} = -c \alpha \partial_t a \vec{B} / f_{PQ} \pi$. It is important to note that this electric current is present even in nonconducting media such as a vacuum as far as the axion star is exposed to the magnetic field. On the other hand, the current J_m is present only in magnetized conducting media.

Since $\partial_t a \sim m_a a$ in the case of the axion star, the ratio of J_m / J_a is given by σ / m_a. Hence, J_a is dominant in media with $\sigma < 10^{12}/s$, while J_m is dominant in media with $\sigma > 10^{12}/s$; note that $10^{10}/s < m_a < 10^{12}/s$, corresponding to the above constraint on f_{PQ}. Astrophysically, the interiors of neutron stars or white dwarfs [16], which are our concern in this paper, possess electric conductivities large enough for J_m to be dominant. On the other hand, envelopes or surfaces of the white dwarfs may have the small conductivities so as for J_a to be dominant, although the envelopes of the neutron stars have still much large conductivities so that J_m is dominant.

Now we discuss some implications of these electric currents. First we show that these electric currents, especially, J_m yield thermal energies to the conducting media owing to Joul heat. It implies that the axion stars dissipate their energy in the media. It also implies that the media increase their temperatures and as a result they radiate thermal photons more than before. In particular, it will be shown in the next section that the dark white dwarfs become bright enough to be detectable, when they collide with axion stars. Such white dwarfs have been supposed to be plausible candidates for MACHOs and to be cooled sufficiently so that the optical detection of them is difficult without the heat produced a collision.

Second we show that since the currents are oscillating, radiation is emitted. However, most of the radiation is absorbed inside of the conducting media. Thus observable radiation is the only one emitted around the surfaces of the media. In particular, the emission from the surfaces of the neutron stars and the white dwarfs is important, to be discussed. But there exists the problem that we cannot estimate precisely its luminosity since we do not have enough information about the physical properties of the surfaces, e.g., electric conductivity, opacity, etc., of magnetized stars. Hence our estimation of the luminosity is necessarily ambiguous. However, it will turn out that a sufficient amount of emission for observation is expected in the case of neutron stars since electric conductivities of the surfaces (so called envelopes) are much larger than those of normal metals.

In order to see these expectations, first of all, we would like to calculate the dissipation energy of an axion star exposed to a magnetic field. The field is associated with the neutron star or the white dwarf [16]. We suppose in such a situation that these magnetic stars collide with an axion star. As the radius of the axion star depends on M_a and m_a, we need to treat the two cases separately: the case of the radius of the axion star being larger than the radius R of the medium, $R_a > R$, and the inverse case $R_a < R$. Namely, dissipation arises when the media are inside the axion star and it arises when the axion star is inside of the media.

Denoting the average electric conductivity of the media by σ and assuming Ohm's law, we find that the axion star dissipates an energy W per unit time,

$$W_{>}=4\sigma\alpha^2c^2B^2a_0^2R^3/3\pi \quad \text{for} \quad R_a>R \tag{16}$$

$$=5.5c^2\times 10^{31}\ \text{erg/s}\ \frac{\sigma}{10^{22}/\text{s}}\frac{m_a^6}{(10^{-5}\ \text{eV})^6}\times\frac{M^4}{(10^{-14}M_\odot)^4}\frac{R^3}{(10^9\ \text{cm})^3}\frac{B^2}{(10^6\ \text{G})^2} \tag{17}$$

$$=5.5c^2\times 10^{38}\ \text{erg/s}\frac{\sigma}{10^{26}/\text{s}}\frac{m_a^6}{(10^{-5}\ \text{eV})^6}\times\frac{M^4}{(10^{-14}M_\odot)^4}\frac{R^3}{(10^6\ \text{cm})^3}\frac{B^2}{(10^{12}\ \text{G})^2}, \tag{18}$$

$$W_{<}=\sigma\alpha^2c^2B^2R_a^3a_0^2/8\pi \quad \text{for} \quad R_a<R \tag{19}$$

$$=4c^2\times 10^{38}\quad \text{erg/s}\ \frac{\sigma}{10^{22}/\text{s}}\frac{M}{10^{-12}M_\odot}\frac{B^2}{(10^6\ \text{G})^2} \tag{20}$$

$$=4c^2\times 10^{54}\ \text{erg/s}\ \frac{\sigma}{10^{26}/\text{s}}\frac{M}{10^{-12}M_\odot}\frac{B^2}{(10^{12}\ \text{G})^2}, \tag{21}$$

with $c\sim 1$. We have used the formulas (11), (12), and (13). Since the field a oscillates [12,8] with a frequency given by the mass of the axion, m_a, we have taken an average in time over the period, m_a^{-1}. When R is much smaller than R_a, we set $\exp(-r/R_a)=1$ in Eq. (1) in the derivation of Eq. (16), $r<R$. W in Eqs. (17) and (19) is for the white dwarf [16] with $R\sim 10^9$ cm and $B\sim 10^6$ G, and W in Eqs. (18) and (21) is for the neutron star [16] with $R\sim 10^6$ cm and $B\sim 10^{12}$ G, respectively. The values of σ have been taken tentatively.

Note that $R_a=1.6\times 10^{10}m_5^{-2}M_{14}^{-1}$ cm. Hence formula $W_{<}$ is applied to the white dwarf with $R=10^9$ cm only when $m_5^2M_{14}>10^2$, e.g., $m_5>10$ and $M_{14}>1$. On the other hand, $W_{<}$ is applied to the neutron star with $R=10^6$ cm only when $m_5^2M_{14}>10^4$, e.g., $m_5>10$ and $M_{14}>1$.

We comment that the formula may be applied to the conducting media where Ohm's law holds even for oscillating electric fields with frequencies $m_a=10^{10}$–10^{12} Hz. In general the law holds in media where electrons interact sufficiently many times in a period of m_a^{-1} with each other or other charged particles and diffuse their energies acquired from the electric field. Actually the law holds with the white dwarfs and neutron stars of our concern.

We would like to point out that although the electric field $\vec{E}_a=-c\alpha a\vec{B}/f_{PQ}\pi$ is very small owing to the large factor of f_{PQ}, the amount of dissipation energy W becomes large because W is proportional to the volume (R^3) of the media or the volume (R_a^3) of the axion stars.

Next we calculate the luminosity of the monochromatic radiation emitted around the surface of the magnetized media. It arises associated with the oscillation of the current J_a or J_m. Here we are only concerned with the case $R_a>R$. This is because the radiation is emitted only around the surface of the media; it is possible when the media is inside the axion star.

We denote the depth of the region from the surface by d, in which radiation is emitted and can escape from the magnetized conducting media. We also denote the average electric conductivity in the region by σ. These values are not well known so that we take them as free parameters. Noting that only radiation from a semisphere facing observers can arrive at them; we calculate the electromagnetic gauge potentials A_i of the radiation with an appropriate gauge condition,

$$A_i=\frac{1}{R_0}\int_{\text{surface}}J_m(t-R_0+\vec{x}\cdot\vec{n})d^3x \tag{22}$$

$$=\frac{c\alpha\sigma a_0B_i}{\pi R_0}\int_{\text{surface}}\sin m_a(t-R_0+\vec{x}\cdot\vec{n})d^3x \tag{23}$$

$$=\frac{2c\alpha\sigma a_0B_iR}{R_0m_a^2}[m_ad\cos m_a(t-R_0) -2\cos m_a(t-R_0+R-d/2)\sin(m_ad/2)], \tag{24}$$

where we have integrated it over the region around the surface with depth $d\ll R$. Here R_0 is the distance between the observer and the media ($R_0\gg R$). Here we have used the current $J_m=\sigma E_a$ with the field a in the approximate formula (11) by setting $\exp(-r/R_a)=1$; the media are involved fully in the axion star so that $r/R_a\ll 1$. On the other hand, the current J_a should be used for $\sigma\leqslant m_a$, in which case σ should be replaced by m_a in the above formula. Using the gauge potentials, we evaluate the luminosity of the monochromatic radiation with the frequency of m_a,

$$L=\frac{8}{3}\left(\frac{\sigma}{m_a}\right)^2c^2a_0^2B^2R^2K^2, \tag{25}$$

where we have taken an average both in time and the direction of the magnetic field. K^2 is given such that

$$K^2=[m_a^2d^2+4\sin^2(m_ad/2) -4m_ad\cos(m_aR)\sin(m_ad/2)]/2 \tag{26}$$

$$\cong m_a^2d^2/2 \quad \text{for} \quad m_ad\gg 1 \tag{27}$$

$$\cong m_a^2d^2[1-\cos(m_aR)] \quad \text{for} \quad m_ad\ll 1. \tag{28}$$

In both limits K^2 is proportional to $m_a^2d^2$. Thus it turns out that L is proportional to σ^2d^2 for $m_a\leqslant\sigma$ or to $m_a^2d^2$ for $\sigma\leqslant m_a$. We should note that the luminosity is proportional to the surface area R^2 of the magnetized stars; it is of the order of $(10^6)^2$ cm^2 for neutron stars or $(10^9)^2$ cm^2 for white dwarfs. Thus the quantity is enhanced even if the luminosity per unit area in the surface is quite weak. This is the point we wish to stress. As we have stated before, phenomena caused by the axion are too faint to be detected owing to the small factor of m_a/f_{PQ}. But in our case we have a large factor $R^2m_a^2$ of the order of, for example, 10^{18} in the case of the white dwarfs.

To evaluate numerically the value of L we need to know the depth d and the electric conductivity σ around the surface. It seems that the quantities depend on each physical condition of the surface of the magnetized stars, e.g., temperature, constituents, etc. Thus it is difficult to estimate generally the luminosity. In later sections we discuss it by assuming that the depth is given by the penetration depth of the radiation. Then the depth is written in terms of the conductivity σ. With this simplification, only the conductivity around the surface remain as an ambiguous quantity.

IV. WHITE DWARF

White dwarfs [16] are stars in the final stage of their lives with intermediate masses $1\,M_\odot \sim 8\,M_\odot$. As is well known, they are composed of C or O with atmosphere of H or He, and their mass (radius) is typically given by $0.5M_\odot(10^9\ \text{cm})$. The pressure in the white dwarfs is dominated by the pressure of degenerate electrons whose density is much larger than normal metals. On the other hand, the internal energy is dominantly given by the kinetic and potential energies of nuclei such as C and O. Then, radiation from them reduces their internal energies stored inside. As a result their temperatures decrease and their luminosities become small with time because they never generate nuclear energies. Hence old white dwarfs are expected to be so dark that it is difficult to observe them. This fact leads to the natural expectation that the white dwarfs in the halo are candidates of MACHOs detected with a gravitational microlense effect. Actually some of cooling models [17] of white dwarfs support this expectation, although this point is still controversial. There are several arguments against this possibility of the MACHO being a white dwarf [18].

Here we assume that the MACHO is just a dark white dwarf with sufficiently low temperature whose population is $2\times10^{11}M_\odot/0.5M_\odot \sim 4\times10^{11}$; note that the total mass of the halo is about $4\times10^{11}M_\odot$, half of which is expected to be the mass of the white dwarfs. We show that the dark white dwarfs become bright again with the collision of the axion stars. We also calculate the rate of the collision in a halo. As a result we find that the number of white dwarfs in the halo increases discontinuously with luminosities below a certain luminosity ($10^{-5.5}L_\odot$–$10^{-7}L_\odot$) gained by the white dwarfs with the collision.

These white dwarfs may have strong magnetic fields typically such as 10^6 G. Then when they collide with axion stars, the axion field generates currents J_a and J_m in the white dwarfs. Consequently, thermal energies are produced with the dissipation of the axion field energy. The energies ($\propto \sigma J^2$) are expected to be large owing to the large electric conductivity in the white dwarfs. In particular, the inside of sufficiently cooled white dwarfs is crystallized just like solid metals and reaches a stage of a fast Debye cooling [16]. It means that the old white dwarfs have been sufficiently cooled [17] so that their core temperature is much lower than 10^4 K, for example. In such a case, the conductivity of electrons has been found [19] theoretically to be large, $\sigma \sim 10^{26}(T/10^3\ \text{K})/\text{s}$ at density $\sim 10^7\ \text{g/cm}^3$ where T represents the core temperature inside a white dwarf.

Then, colliding with the axion star, the white dwarf gains dissipation energy per unit time as follows:

$$W_> = 5.5c^2\times10^{35}\ \text{erg/s}\frac{\sigma}{10^{26}/\text{s}}\frac{m_a^6}{(10^{-5}\ \text{eV})^6} \times\frac{M_a^4}{(10^{-14}M_\odot)^4}\frac{R^3}{(10^9\ \text{cm})^3}\frac{B^2}{(10^6\ \text{G})^2} \quad \text{for } R_a>R, \tag{29}$$

$$W_< = 4c^2\times10^{42}\ \text{erg/s}\ \frac{\sigma}{10^{26}/\text{s}} \times\frac{M}{10^{-12}M_\odot}\frac{B^2}{(10^6\ \text{G})^2} \quad \text{for } R_a<R, \tag{30}$$

where we have taken the above value of the conductivity. Noting that the constant c is of order of 1, we found that the white dwarf gains so much thermal energy when it goes through the axion star.

However, the maximal energy the axion star can deposit per unit time is the energy possessed by a part of the axion star which the white dwarf sweeps per unit time, when the dwarf is smaller than the axion star ($R_a>R$). This is owing to energy conservation. As the relative velocity v between the white dwarf and the axion star in the halo is approximately given by $10^{-3}\times$ light velocity, the energy stored in that part can be estimated such as $3R^2vM/4R_a^3 \sim 10^{35}\ \text{erg/s}\ (M_a/10^{-14}M_\odot)^4(m_a/10^{-5}\ \text{eV})^6$. This is smaller than $W_>$ estimated naively. Therefore, the real amount of energy the white dwarf can gain is at most given by $W_{real}\sim10^{35}\ \text{erg/s}\ (M_a/10^{-14}M_\odot)^4(m_a/10^{-5}\ \text{eV})^6$. The gain of energy continues until the white dwarf passes through the axion star. Thus the total energy gained by the white dwarf (or dissipated by the axion star) is $2R_a/v \times W_{real}\sim10^{38}\ \text{erg}\ (M_a/10^{-14}M_\odot)^3(m_a/10^{-5}\ \text{eV})^4$ in the case of the radius of the axion star being larger than that of the white dwarf. This is the energy gained by the white dwarf when it passes the axion star without trapping the star. If the white dwarf traps the axion star, the white dwarf may gain a larger amount of energy. Maximally all of the mass $M_a \simeq 1.8\times10^{40}\ \text{erg}\ (M_a/10^{-14}M_\odot)$ can be transformed into thermal energy.

On the other hand, when the dwarf is larger than the axion star, it is obvious from the formula (30) that the whole energy of the axion star, $M_a\simeq1.8\times10^{42}\ \text{erg}\ (M_a/10^{-12}M_\odot)$, is dissipated within 1 s. Namely, such an axion star evaporates soon after it enters the white dwarf.

Suppose that a white dwarf with mass $0.5\times M_\odot$ and radius 10^9 cm is in the stage of Debye cooling. Then, its specific heat c_v per ion is given approximately by [16] $c_v \simeq 16\pi^4 (T/\theta_D)^3/5$, where θ_D is the Debye temperature, typically being 10^7 K. Hence the injection of energy, 10^{38} erg$(M/10^{-14}M_\odot)^3(m_a/10^{-5}\ \text{eV})^4$ [or $M_a \simeq 10^{42}$ erg$(M_a/10^{-12}M_\odot)$], increases the core temperature of the white dwarf to $\simeq 1.5\times 10^4$ K$(M/10^{-14}M_\odot)^{3/4}(m_a/10^{-5}\ \text{eV})$ (or $\simeq 1.5\times 10^5$ K$(M_a/10^{-12}M_\odot)^{1/4}$) when the initial temperature is much less than these. In order to evaluate the luminosity of the dwarf, we need to know the surface temperature. Generally, the surface temperature is much lower than the core temperature. It depends on the opacity of the atmosphere of the white dwarf. Recently it has been shown [20] that a white dwarf with an atmosphere of helium has much lower opacity at surface temperature ($<6\times 10^3$ K) than the ones estimated previously. Such a white dwarf cools more rapidly. According to the model [20] of the cooling, such white dwarfs of ages $\simeq 10^{10}$ yr have been cooled with core temperature $\simeq 2.3\times 10^5$ K and with corresponding surface temperature 2000 K; its luminosity is $10^{-5.6}L_\odot$ which is less than the minimum luminosity $\sim 10^{-4.8}L_\odot$ of a white dwarf observed at present. Thus white dwarfs older than 10^{10} yr must be cooler than this one. It is expected that the ages of the halo white dwarf population is around 1.2×10^{10} yr. Hence if an axion star with mass larger than $10^{-12}M_\odot$ $(10^{-9}M_\odot)$ collides with a white dwarf whose radius is larger than that of the axion star, then the core temperature reaches more than $\simeq 1.5\times 10^5$ K$(\simeq 8.4\times 10^5$ K) and its luminosity reaches more than $L \simeq 10^{-6}L_\odot(\simeq 10^{-4.5}L_\odot)$ according to the cooling model; here we have simply extrapolated its result to the case of lower luminosity than that $(\sim 10^{-5.6}L_\odot)$ addressed in the model. It takes about 1.4×10^7 yr $(5\times 10^8$ yr) for the dwarf to lose the energy injected.

Now we proceed to estimate the rate of collisions between the white dwarf and the axion star in the halo. Especially, we are concerned with the event rate observed in a solid angle, $5°\times 5°$, for example. We assume that as indicated by recent observations of gravitational microlensing, half of the halo is composed of white dwarfs with mass $M=0.5\times M_\odot$ and radius $R=10^9$ cm. The other half is assumed to be composed of axion stars. The total mass of the halo is supposed to be $\sim 4\times 10^{11}M_\odot$. Furthermore, the distribution [21] of the halo is taken such that its density $\propto (r^2+3R_c^2)/(r^2+R_c^2)^2$ with $R_c=4$ kpc where r denotes a radial coordinate with the origin being the center of the galaxy (the final result does not depend practically on the value of $R_c=2$–8 kpc). Then it is easy to evaluate the event rate of the collisions;

$$0.5\ \text{per year}\times\frac{(10^{-14}M_\odot)^3}{M_a^3}\,\frac{(10^{-5}\ \text{eV})^4}{m_a^4}\times\frac{\Omega}{5°\times 5°}\quad\text{for}\quad R<R_a, \tag{31}$$

$$5\times 10^{-5}\ \text{per year}\times\frac{10^{-12}M_\odot}{M_a}\,\frac{\Omega}{5°\times 5°}\quad\text{for}\quad R>R_a, \tag{32}$$

where Ω is a solid angle. We have taken into account the fact that Earth is located at about 8 kpc from the center of our galaxy, simply by counting the number of the collisions arising in the region from $r=8$ kpc to 50 kpc. Here we have assumed that the collisions take place with their cross section simply given by $\pi(10^{10}\ \text{cm})^2(10^{-14}M_\odot/M_a)^2 m_5^{-4}$ for $R<R_a$ and $\pi(10^9\ \text{cm})^2$ for $R>R_a$, respectively. Namely the cross section is a geometrical one of the axion star or the white dwarf. But we expect that the actual cross section is much larger than this; a tidal force of the white dwarf may decrease the kinetic energy of the axion star by tearing the star.

We also calculate the rate of the collision in the neighborhood of Earth. In particular, we wish to see how many the collisions occur within a volume $(1\ \text{kpc})^3$ in a year around Earth. This is because, since the luminosity of the white dwarfs expected in the collisions is very small, the dwarfs need to be located near Earth in order to be detected. We assume that local density of the halo [3] is given by 0.5×10^{-24} g cm^{-3}, half of which is composed of white dwarfs and the other half is composed of axion stars. Then it is easy to find that the rate of collisions is given by $\sim 0.06M_{14}^{-3}m_5^{-4}$ per year for $R<R_a$ and $\sim 6\times 10^{-6}M_{12}^{-1}$ per year for $R>R_a$, respectively, where we used the relative velocity being equal to 3×10^7 cm/s. It seems that it is difficult to detect the collisions. The real population of the white dwarfs and the axion stars in the halo may be smaller than the one we have assumed in the analysis. Then the collision rate is smaller than the above value and it is more difficult to detect collisions with the observation of monochromatic radiation discussed below. However, if we take into account the fact that the actual collision cross section must be much larger than the geometrical one of the axion star due to gravitational attraction, the rate will increase. To see this we need to analyze numerically the collision in detail.

Here we wish to discuss how many white dwarfs heated with collisions are present in a region around Earth whose volume is assumed to be $(1\ \text{kpc})^3$. For this purpose we note that white dwarfs heated with collisions lose the energies injected, taking many years. Hence, even if the collision rate is small, the number of such white dwarfs increases with time and saturates at a balance point between the decay and the production. For example it takes 1.4×10^7 yr $(5\times 10^8$ yr) for a white dwarf with luminosity $10^{-6}L_\odot(10^{-4.5}L_\odot)$ to lose the energy which is injected by an axion star with $M=10^{-12}M_\odot(M=10^{-9}M_\odot)$. On the other hand, the rate of collisions producing the dwarf with the luminosity is given by $\simeq 6\times 10^{-6}$ per year $(\simeq 6\times 10^{-9}$ per year) in the region with volume $(1\ \text{kpc})^3$. Thus it turns out that there are present 84(3) such white dwarfs in the region. The number of such white dwarfs becomes larger as their luminosities gained with the collisions are smaller. For example, with a collision of the axion star $M=10^{-11}M_\odot$, a white dwarf gains a luminosity $10^{-5.5}L_\odot$ and loses its energy in 3.7×10^7 yr. On the other hand, the production rate of the dwarfs is 6×10^{-7} per year. Thus there are present approximately 22 such dwarfs. Accordingly, we understand that as the mass of the axion stars is smaller, the energy gained by the white dwarfs is smaller and their resultant luminosity is smaller, but the number of such dwarfs is larger.

Although we do not know theoretically the masses of the axion stars, observations of dark white dwarfs enable us to determine the mass of the axion stars. Namely, if there is a threshold luminosity, e.g., $10^{-5.5}L_{\odot}$ below which the number of white dwarfs increases discontinuously, it implies that such white dwarfs with threshold luminosity are produced with collisions of axion stars with a corresponding mass, e.g., $10^{-11}M_{\odot}$. In this discussion, we have assumed that old white dwarfs dominant in the halo have been cooled sufficiently and their core temperature is quite low (e.g., less than 10^4 K).

Until now, we have discussed the thermal radiation caused by an axion star which dissipates its energy and makes the white dwarf warmer than before.

We proceed to discuss the monochromatic radiation generated by oscillating electric currents. The radiation is emitted only around the surface of the white dwarf. Otherwise, the radiation is absorbed inside the white dwarf. There are difficult problems in estimating the amount of such radiation because the physical parameters (conductivity, opacity, etc.) around the surface of the cooled white dwarfs are not yet known so well [17,20]. Thus our estimation is inevitably ambiguous. However, with the assumption that the depth d of the region in which the radiation is generated and can go out of the white dwarf is given by the penetration depth of the radiation, we can obtain the amount of radiation definitely with one ambiguous parameter, the penetration depth, which is written in terms of the electric conductivity and the frequency of the radiation.

With use of Eqs. (25) and (26), we find the luminosity of the radiation,

$$L \sim 10^{21}\ \text{erg/s}\ B_6^2 R_9^2 \sigma_{20} m_5^5 M_{14}^4 c^2 \quad \text{for } m_a \ll \sigma, \tag{33}$$

$$\sim 10^{21}\ \text{erg/s}\ B_6^2 R_9^2 m_5^8 M_{14}^4 c^2/\sigma_5^2 \quad \text{for } m_a \gg \sigma \tag{34}$$

where

$$B_6 = \frac{B}{10^6\ \text{G}}, \quad R_9 = \frac{R}{10^9\ \text{cm}}, \quad \sigma_{20} = \frac{\sigma}{10^{20}/\text{s}}, \quad \sigma_5 = \frac{\sigma}{10^5/\text{s}}. \tag{35}$$

Here we have only addressed the case that the radius of the white dwarf is smaller than R_a. We have used the penetration depth $d = 1/2\pi\sigma$ for $m_a \gg \sigma$ and $d = \sqrt{1/2\pi\sigma m_a}$ for $m_a \ll \sigma$, respectively. In both cases we have simply assumed that values of both the dielectric constant and magnetic permeability are the same as those of the vacuum. Numerically, $d \sim 10^5$ cm$/\sigma_5$ for $m_a \gg \sigma$ and $d \sim 10^{-5}$ cm$(\sigma_{20} m_5)^{-0.5}$ for $m_a \ll \sigma$. We should note that as a source of the radiation, the matter current J_m carried by electrons is dominant for $m_a \ll \sigma$, while the axion current J_a is dominant for $m_a \gg \sigma$. In each case we have represented the luminosity in Eqs. (33) and (34), respectively.

This monochromatic radiation is emitted only during the collisions with the axion star. This is because after the white dwarf passes the axion star, there are no oscillating currents around its surface; the currents arise only when the dwarf is passing inside the axion star. Therefore, the period of the emission continuing is given by $2R_a/v \sim 3.2 \times 10^{10}$ cm $m_5^{-2} M_{14}^{-1}/(3\times 10^7$ cm/s$) \sim 10^3$ s for $m_5^2 M_{14} = 1$.

It seems that the luminosity is small for the observation of this radiation. But it depends heavily on the mass of the axion especially in the case of $m_a \gg \sigma$. For example, if $m_a = 10^{-4.5}$ eV where $R \sim R_a$, then $L \sim 10^{25}$ erg/s in the case of $m_a \gg \sigma$, Eq. (34). This may be large enough for observations. Furthermore, we point out that there are huge ambiguities in the evaluation of the luminosity as we have mentioned before. Thus we cannot determine definitely whether the luminosity of the radiation is large enough for observation or not. Although there exist some parameter ranges for the radiation being observed, it seems difficult to observe the radiation caused by the collision unless the luminosity is so large for the radiation from the distance 10 kpc to be detectable. Note that the rate of collision is given approximately by $60 M_{14}^{-3} m_5^{-4}$ per year per $(10\ \text{kpc})^3$.

In the above discussion we have assumed that the axion star does not receive any gravitational effects from the white dwarf in the collision. However, actually it receives strong gravitational effects from the white dwarf. This is because the mass of the white dwarf is much bigger than that of the axion star. So we need to take into account the effects in order to see whether or not some of the assumptions are changed, in particular, the number of axion stars which was estimated naively under the assumption of the stars not decaying within the age of the Universe.

First of all, we examine whether or not the axion star decays by a tidal force when the white dwarf passes near it. We suppose that the axion star decays if the energy difference caused by the tidal force between different parts of the axion star is larger than the binding energy of the axion star. In particular, we wish to estimate how many axion stars survive without decaying in the present Universe whose age is approximately 10^{10} yr.

Suppose that an axion star with mass $10^{-14}M_{\odot}$ is composed of two parts which are bounded gravitationally with each other; the distance between the two parts is assumed to be R_a. When a white dwarf passes the axion star, each part receives a different gravitational force owing to the difference of their distances from the white dwarf. In order to examine whether or not this tidal force tears the axion star, we compare the energy difference between the gravitational energy received by a part of the axion star from the white dwarf and the energy received by the other part of the axion star, with binding energy GM_a^2/R_a of the axion star. It is reasonable to think that if the energy difference is larger than the binding energy, the tidal force tears the axion star. The energy difference is dependent on the impact parameter of the collision between the axion star and the white dwarf. Assuming a relative velocity of $10^{-3}\times$ light velocity, we find with rough estimation that the axion star decays with the tidal force when they approach within 10^{15} cm of each other. Since the number of axion stars in the halo is given by

$\sim 2\times 10^{11} M_\odot / 10^{-14} M_\odot = 2\times 10^{25}$, and the number of white dwarfs is $\sim 2\times 10^{11} M_\odot / 0.5\times M_\odot = 4\times 10^{11}$, the number of collisions leading to the decay of axion stars is at most of the order of 10^{23} within the age of the Universe. Therefore, it turns out that almost of all axion stars (more than 99%) have survived against such collisions. Note that axion stars with masses larger than $10^{-14} M_\odot$ can survive even more against the tidal force of the white dwarfs. This is because the collision cross section becomes smaller for such axion stars and so the decay rate is smaller.

Although we found that there are still present quite large numbers of axion stars in our halo, we do not know how actually direct collisions between the axion star and the white dwarf take place; a direct collision implies a collision with their closest distances being less than $R_a \sim 10^{10}$ cm. Namely, its cross section is given by $\pi[1.6\times 10^{10}\ \mathrm{cm}(10^{-14} M_\odot / M_a)]^2$, i.e., the geometrical cross section of the axion star itself. It seems that the gravitational force deforms strongly the configulation of the axion star, but the coherence of the axion field may hold and generation of electric currents still arises. Thus we expect the heating up of cooled white dwarfs and the resultant emission of thermal radiation as well as monochromatic radiation.

V. NEUTRON STARS

Now we proceed to discuss the influence on neutron stars of their collision with the axion stars. The neutron star [16] is highly dense nuclear matter composed of neutrons and protons, although the number of protons is much less than that of neutrons. Since their radius R_n is typically given by 10^6 cm, we are only concerned with the case $R_n < R_a \sim 10^8 m_5^{-2} M_{12}^{-1}$ cm. Since there are also highly dense electrons, the conductivity is quite higher than that of the white dwarf. As is well known, a strong magnetic field is present. Its strength is typically given by 10^{12} G. Hence, the amount of energy dissipation of the axion star per unit volume in the neutron star is much larger than that in the white dwarf. But as we mentioned in the previous section, the actual amount of energy dissipation is restricted owing to energy conservation. It is given by $3R_n^2 v M_a / 4R_a^3 \sim 10^{29}$ erg/s$(M_a/10^{-14} M_\odot)^4 (m_a/10^{-5}\ \mathrm{eV})^6$. The total amount of the energy deposited after the collision is 10^{32} erg $(M_a/10^{-14} M_\odot)^3 m_5^4$. This is a small fraction of the thermal energy possessed by the neutron star. Therefore, a collision with the axion star does not affect significantly the thermal content of the neutron star, contrary to the case of the white dwarf.

Although the thermal energy does not change so much with the collision, a detectable amount of radiation from the surface of the neutron star arises during the collision with the axion star. As we mentioned, the axion field of the axion star generates an oscillating electric field E_a when it is under a magnetic field. Thus the oscillating current $J_m = \sigma E_a$ is induced around the surface, which generates obviously the radiation. In the case of neutron stars the electric conductivity is so large [19] even at the surface that the luminosity of the radiation is large enough for it to be detectable.

Using the formulas (25) and (26), we obtain the luminosity of the radiation,

$$L \sim 10^{27}\ \mathrm{erg/s}\ B_{12}^2 R_6^2 \sigma_{20} m_5^5 M_{14}^4, \tag{36}$$

with

$$B_{12} = \frac{B}{10^{12}\ \mathrm{G}}, \quad R_6 = \frac{R_n}{10^6\ \mathrm{cm}}, \quad \sigma_{20} = \frac{\sigma}{10^{20}/\mathrm{s}}, \tag{37}$$

where we have used the above formulas (12) and (13) for expressing a_0 in terms of the mass M_a of the axion star. We have assumed for convenience that the conductivity takes a value such as 10^{20}/s. It is reasonable to take such a value of σ because the number density of electrons at the surface must be much larger than that of normal metals. Then, since it is much larger than m_a, the depth d is taken such as $d = \sqrt{1/2\pi\sigma m_a}$ which is the penetration depth of the radiation. This luminosity is that of monochromatic radiation with frequency $m_a/2\pi = 2.4\times 10^9 m_5$ Hz.

Emission continues until the axion star passes through the neutron stars. It takes $2R_a/v \sim 3.2\times 10^{10}$ cm $m_5^{-2} M_{14}^{-1}/(3\times 10^7\ \mathrm{cm/s}) \sim 10^3$ s $m_5^{-2} M_{14}^{-1}$, assuming that the velocity of the axion star is 3×10^7 cm/s, which is the typical velocity of matter composing the halo in our galaxy.

The monochromatic radiation emitted at a distance D from Earth is detected at Earth with the following strength:

$$\sim 1\ \mathrm{Jy}\ B_{12}^2 R_6^2 \sigma_{20} m_5^4 M_{14}^4 (D_{\mathrm{kpc}})^{-2} \tag{38}$$

with $D_{\mathrm{kpc}} = D/1$ kpc, where we have assumed that the frequency ν of the radiation is Doppler broadened with a characteristic width $\Delta\nu \sim 10^{-3}\nu$ owing to the velocity of the axion stars. Units of Jy denotes Jansky: 1 Jy $= 10^{-23}$ erg cm^{-2} s^{-1} Hz^{-1}. It turns out that the strength of the radiations is large enough to be detectable although the duration $\sim 10^3$ s of radiation emission is very short.

In order to estimate how frequent the events of the collisions between the neutron star and the axion star occur in the neighborhood of Earth, for example, $D \leqslant 1$ kpc, we need to know the number density of the neutron stars including old invisible ones in our galaxy. Assuming the number of the axion stars in the halo being given by $2\times 10^{11} M_\odot / 10^{-14} M_\odot \simeq 10^{25}$ and their uniform distribution within a volume $\sim (50\ \mathrm{kpc})^3$, we find that the events in a volume $\sim (1\ \mathrm{kpc})^3$ occur with a rate of the order of $\sim 10^{-9}$ per year when the number of neutron stars in the volume is just 1. Here the collision cross section is assumed to be given by the geometrical one $\pi(10^{10})^2$ cm^2 of the axion star. Therefore it is impossible in practice to detect such phenomena unless the number density of the invisible old neutron stars in the neighborhood of Earth is much larger than the one we expect or the real collision cross section including the gravitational effect of the attraction is much larger than the geometrical one.

VI. DISCUSSION

We have shown that axionic boson stars dissipate their energies in magnetized conducting media such as white dwarfs or neutron stars. Among them, the old halo white dwarfs with a sufficiently low core temperature ($<10^3$ K–10^4 K), possible candidates of MACHOs, are heated in this mechanism and emit thermal radiation. Their luminosities have been estimated to achieve more than the luminosity $10^{-6}L_\odot(10^{-7.1}L_\odot)$ of a white dwarf when M_a is larger than $10^{-12}M_\odot(10^{-14}M_\odot)$; in the case of mass $10^{-14}M_\odot$, we have assumed the axion stars being trapped to the white dwarf and dissipating their whole energy. These dwarfs lose their energies with the radiation and become dark with time, while new such dwarfs are produced with collisions of axion stars. The masses of almost of all axion stars could be fixed to a certain value in a range of $10^{-14}M_\odot$–$10^{-11}M_\odot$ when they are produced at a QCD phase transition. Thus the core temperature or the luminosity of the dwarf heated with a collision must be almost the same as each other. Therefore, we expect that there exists a threshold luminosity below which the number of halo white dwarfs increases discontinuously. The threshold is determined with the mass of axion stars which collide and heat the dark dwarfs with sufficiently low temperature. In our estimation the threshold luminosity is given by $10^{-5.5}L_\odot$–$10^{-7}L_\odot$ corresponding to the masses of axion stars quoted above.

The threshold luminosity is quite small so that the collision needs to occur near Earth for the detection of such white dwarfs. Thus we have also estimated the rate of collisions within a volume of $(1\ \text{kpc})^3$ around Earth. With assumptions of both populations of the white dwarfs and the axion stars being given by half the halo, the rate has turned out to be $\sim 0.06M_{14}^{-3}m_5^{-4}$ per year for $R<R_a$ ($6\times10^{-6}M_{12}^{-1}$ per year for $R>R_a$), when the relative velocity between the dwarf and the axion star is equal to 3×10^7 cm/s, $R_a=1.6\times10^{10}m_5^{-2}M_{14}^{-1}$ cm. This indicates that the detection of phenomena such as a nova caused by a collision would be difficult. But in the estimation we have not included gravitational attraction. When we take into account this effect, the collision cross section will be quite larger than the naive geometrical one we have used in the estimation. So we may expect that the cross section will become large so that the actual rate of collisions is large for us to be able to observe the phenomena. On this point we need to simulate numerically the collision and to know how the collision occurs. In particular we wish to know the collision cross section and also to know whether or not the axion star is torn by the tidal force of the white dwarf.

We have also shown that monochromatic radiation is emitted during collisions between axion stars and magnetized conducting media. The radiation is produced around the surface of the media by oscillating current, $J_m=\sigma E_a$. Especially, strong radiation is expected from neutron stars since their electric conductivity σ is still large even at their envelope. However, the number density of neutron stars in our galaxy may be much smaller for the rate of collisions to be fairly rare, $\sim10^{-9}/\text{kpc}^3$ per year when their number density is given by $1/\text{kpc}^3$. Thus it is difficult to observe monochromatic radiation from neutron stars.

On the other hand, although the collision with white dwarfs does not lead to strong radiation such as that from neutron stars, the number density of dwarfs is supposed to be much larger than that of neutron stars so that the rate of collisions is much larger than one in the case of neutron stars. It is approximately given by $0.06/\text{kpc}^3$ per year with use of $M_a=10^{-14}M_\odot$ and with use of the geometrical cross section $(=\pi R_a^2)$. Hence, unless the luminosity of the radiation is large enough for it from the distance 10 kpc, for example, to be detectable, the value is a little bit too small for observation. Furthermore, our assumption of the population of white dwarfs and axion stars is dubious. In general the population is smaller than the one we have assumed. Then the collision rate is smaller than one in the above estimation. Hence the real rate of collisions may be small enough so as for the collision to be undetectable. But taking into account the gravitational attraction between the axion star and the dwarf, the real cross section may be larger than the naive geometrical one. Thus the rate of collisions may be larger than one we have estimated. On this point we need to analyze numerically the collision in detail.

Finally, we mention that our estimation is ambiguous in the sense that we do not know many physical parameters associated with the phenomena, for instance, the precise values of both the mass and the population of axion stars, the mass of the axion itself, the physical properties of invisible old white dwarfs, the actual cross section of the collision, and how the axion star collides with magnetic stars. Although there are many ambiguous points in the disscusion, it is important to note that the observation of monochromatic radiation makes us determine precisely the mass of the axions and the observation of the threshold in the luminosity function of the halo white dwarfs makes us determine the mass of the axion stars.

Theoretically, if we detect monochromatic radiation which is emitted in a short period, it is a good signal indicating that a collision between the axion star and the magnetized media actually occurs. After the detection we expect to see a nova in the direction of the radiation; the nova is a white dwarf heated with the energy dissipation of the axion star in the old white dwarf.

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