Possible constraints on the time variation of the fine structure constant from cosmic microwave background data

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The formation of the cosmic microwave background radiation (CMBR) provides a very powerful probe of the early universe at the epoch of recombination. Specifically, it is possible to constrain the variation of fundamental physical constants in the early universe. We calculate the effect of a varying electromagnetic coupling constant (α) on the CMBR and find that new satellite experiments should provide a tight constraint on the value of α at recombination which is complementary to existing constraints. An estimate of the obtainable precision is $|\dot{\alpha}/\alpha| \lesssim 7 \times 10^{-13}$ yr^{-1} in a realistic experiment. [S0556-2821(99)01814-7]

I. INTRODUCTION

A fundamental question in physics is whether or not the physical constants are actually constant. Some unifying physical theories such as superstring theories do in fact suggest that the physical ''fine structure'' constants change in time [1]. It is therefore of considerable importance to find methods of detecting a possible time evolution of these quantities. In the present paper, we wish to discuss specifically the electromagnetic fine structure constant α. Its present value is known quite precisely to be [2]

$$\alpha_0^{-1} = (e^2/4\pi)^{-1} = 1/137.0359895(61). \qquad (1)$$

One option for detecting time variation is, of course, to measure its value in the laboratory and constrain its time derivative in this way. However, this has the major drawback that, even though quite minute changes are detectable, the time differences are so small that only a moderate sized time derivative is detectable.

Therefore one often turns to other methods. For instance, it is possible to use astrophysical arguments to constrain the evolution of α, the most commonly used method being the use of differential changes in quasar absorption lines. This method offers both a long look back in time [for $z \simeq 3$, one has $t/t_0 \simeq 1/8$, assuming a standard flat cold dark matter (CDM) cosmology] and the ability to detect rather small changes in α. Another possibility is to use big bang nucleosynthesis, but this method suffers from the problem that constraints on α are based on a specific assumption on how the neutron to proton mass difference depends on α.

As a possible probe that is complementary to all the others discussed, we investigate the sensitivity of the cosmic microwave background radiation (CMBR) to changes in α. It is well known that the fluctuation spectrum of the CMBR is extremely sensitive to the physical conditions at recombination [3] and, using inversion technique, it should therefore be possible to determine the physical parameters at recombination given sufficiently good observations.

The fluctuations are usually described in terms of spherical harmonics

$$T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi), \qquad (2)$$

where the coefficients are related to C_l coefficients by

$$C_l \equiv \langle |a_{lm}|^2 \rangle. \qquad (3)$$

These fluctuations were first detected in 1992 by the Cosmic Background Explorer (COBE) satellite [4], but only for $l \lesssim 20$. At such low l, the power spectrum is almost degenerate in the cosmological parameters and no real constraints are obtainable. In the next few years, however, the power spectrum will be measured out to $l \simeq 2500$ by two new probes, the Microwave Anisotropy Probe (MAP) and the PLANCK surveyor [5], and using this data should yield precision measurements of the physical parameters at recombination. It should also be possible to constrain new exotic physics such as nonstandard neutrinos [6,7] or, indeed, a change in α.

In the next sections, we discuss the physical consequences of changing α and calculate an estimate of how precisely we can hope to measure such a change with the CMBR data. To calculate actual CMBR power spectra, we have used the CMBFAST package developed by Seljak and Zaldarriagga [8]. Finally, as will be discussed later, there is also a terrestrial method which offers long look back times, namely, to use the Oklo natural fission reactor in Gabon. This method offers the currently most stringent limit on time variation in α.

II. CONSEQUENCES OF CHANGING α

Since the formation of the CMBR is based entirely on electromagnetic processes, changing the strength of these interactions is bound to change the CMBR fluctuation spectrum. First of all, it changes the Thomson scattering cross section for all interacting particles. Second, it also changes the recombination of hydrogen. The second effect is far more subtle than the first, since it also involves changing all the energy levels of the hydrogen (and helium) atom. In the following, we shall neglect the impact on helium and only concentrate on hydrogen. Notice that there is also a small secondary effect from the change in helium abundance from nucleosynthesis which we shall also neglect in the present paper.

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Thomson scattering. By far the most efficient equilibration mechanism for thermalizing the photon gas in the early universe is Thomson scattering on free electrons (not protons, since the rate for this process is suppressed by a factor $m_e^2/m_p^2 \simeq 3 \times 10^{-7}$). The fundamental cross section for this process is given by [9]

$$\sigma_T = \frac{1}{6\pi} \frac{e^4}{m_e^2}, \tag{4}$$

meaning that it has a α^2 dependence.

Recombination. The phenomenon of recombination is of paramount importance for the formation of the CMBR, since photon equilibration is mediated by Thomson scattering on free electrons. Prior to recombination, the photons are tightly coupled to the electron-baryon fluid, whereas subsequent to recombination, the photons are therefore essentially free particles. Thus, the epoch of CMBR formation is directly linked to the recombination epoch. The recombination of hydrogen has been extensively studied by many authors and we shall follow the treatment by Ma and Bertschinger [10] which is based on the earlier treatment by Peebles [11].

Recombination directly to the ground state is strongly prohibited in the early universe since it leads to immediate reionization. Instead, it proceeds via two-photon emission from the $2s$ level or via the redshift of Ly-α photons out of the line center [11]. Putting together these two effects with the appropriate recombination coefficients to all exited levels, one obtains an equation for the time-evolution of the ionization fraction, $x_e \equiv n_e/n_H$, with respect to conformal time:

$$\frac{dx_e}{d\tau} = a C_r [\beta(T_b)(1-x_e) - n_H \alpha^{(2)}(T_b) x_e^2], \tag{5}$$

where the first term on the right-hand side describes collisional ionization from the ground state and the second describes the recombination rate. a is the cosmological scale factor, normalized so as to be equal to one at present, n_H is the total number density of hydrogen nuclei, and T_b is the baryon temperature. The other factors are given in the following way:

$$\alpha^{(2)}(T_b) = \frac{64\pi}{(27\pi)^{1/2}} \frac{e^4}{m_e^2} \left(\frac{T_b}{B_1}\right) \phi_2(T_b), \tag{6}$$

$$\phi_2(T_b) \simeq 0.448 \log\left(\frac{B_1}{T_b}\right), \tag{7}$$

$$\beta(T) = \left(\frac{m_e T_b}{2\pi}\right)^{3/2} e^{-B_1/T_b} \alpha^{(2)}(T_b), \tag{8}$$

$$B_1 = m_e e^2/2 = 13.6 \text{ eV}. \tag{9}$$

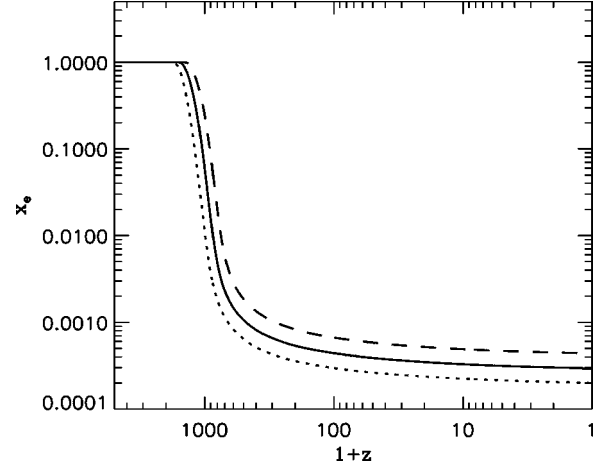The reduction factor C_r has been calculated by Peebles [11] and is given by

FIG. 1. The ionization fraction as a function of redshift for three different values of $x \equiv \alpha/\alpha_0$. The solid curve is for $x=1$, the dashed for $x=0.95$, and the dotted for $x=1.05$.

$$C_r = \frac{\Lambda_\alpha + \Lambda_{2s \to 1s}}{\Lambda_\alpha + \Lambda_{2s \to 1s} + \beta^{(2)}(T_b)}, \tag{10}$$

where

$$\beta^{(2)}(T_b) = \beta(T_b) e^{\omega_\alpha/T_b}, \tag{11}$$

$$\Lambda_\alpha = \frac{8\pi \dot{a}}{a^2 \lambda_\alpha^3 (1-x_e) n_H}, \tag{12}$$

$$\lambda_\alpha = \frac{8\pi}{3B_1}. \tag{13}$$

All these equations scale quite straightforwardly with α. The only thing left is to treat the two-photon process where, in the standard case, $\Lambda_{2s \to 1s} = 8.22458 \text{s}^{-1}$ [12]. Following Shapiro and Breit [13], one finds that this fundamental process has the very steep dependence

$$\Lambda_{2s \to 1s} \propto \alpha^8. \tag{14}$$

To see how the process of recombination changes with α, we have plotted the evolution of the ionization fraction x_e as a function of redshift for different values of $x \equiv \alpha/\alpha_0$ in Fig. 1. If α increases interactions become stronger and equilibrium is maintained longer, meaning that the final ionization fraction becomes smaller. This is exactly the trend seen in Fig. 1.

The fact that a lot of the parameters entering the recombination equations are extremely sensitive to changes in α brings hope that the CMBR spectrum is equally sensitive to changes in α. This is exactly the case, as will be discussed in the next section. In Fig. 2 we have shown the CMBR fluctuation spectrum for a standard CDM model with two different values of x. There are seen to be very substantial changes, even for a quite small change in x.
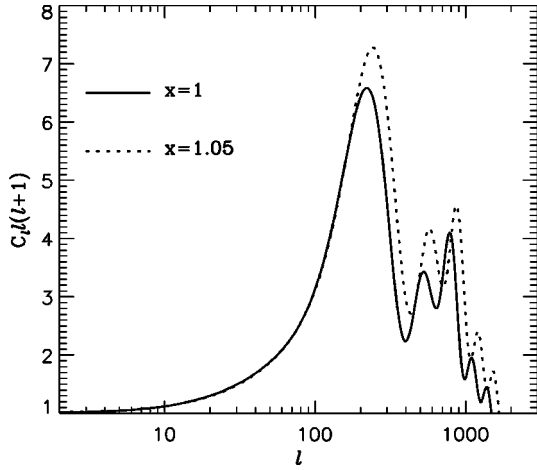
FIG. 2. CMBR fluctuation spectra for two different values of x. The spectrum has been normalized to the quadrupole fluctuation $6C_2$.

III. CMBR SENSITIVITY TO CHANGES IN α

The key question is now whether or not it will be possible to detect deviations in α relative to the standard value. In order to estimate the sensitivity of the CMBR data, we use a standard technique for this purpose. Since we have no usable data at present, we can only provide what is called *error forecasting* [3]. To do this, we choose an underlying cosmological model (in our case standard CDM) and determine how precisely the cosmological parameters can be determined. This method has been described in great detail elsewhere [3,14] and we shall not go into details. The cosmological model can be described by a vector of parameters and in our calculations, we work with the following set

$$\Theta = (\Omega, \Omega_b, \Lambda, h, n, N_\nu, \tau, \alpha). \tag{15}$$

Here τ is the optical depth due to possible reionization and n is the spectral index. The standard CDM model which we choose as our reference is then given by the vector

$$\Theta_{CDM} = (1, 0.08, 0, 0.5, 1, 3, 0, \alpha_0). \tag{16}$$

The main point is then to calculate the so-called Fisher matrix, which is given by

$$I_{ij} = \sum_{l=2}^{l_{max}} (2l+1)[C_l + C_{l,\text{error}}]^{-2} \frac{\partial C_l}{\partial \theta_i} \frac{\partial C_l}{\partial \theta_j}, \tag{17}$$

where $C_{l,\text{error}}$ represents the experimental error. Following Lopez *et al.* [15], we shall neglect the experimental error and only take into account the ''error'' induced by cosmic variance. It can then be shown that the standard error in estimating any parameter is of the order $\sigma_i^2 \simeq (I^{-1})_{ii}$. Specifically, if all parameters are allowed to vary simultaneously, one obtains

$$\sigma_i^2 \simeq (I^{-1})_{ii}, \tag{18}$$

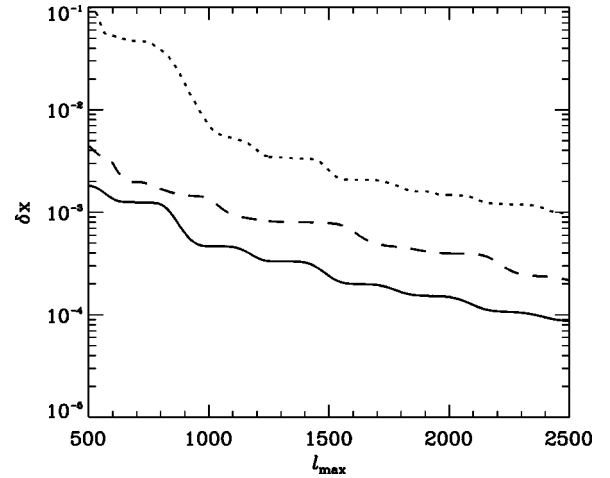whereas if all parameters except θ_i have been determined, it is

FIG. 3. The expected standard error δx as a function of the maximum measured l in a CMBR measurement of x. The solid curve assumes that all other parameters are known whereas the dotted line assumes no prior knowledge of any parameter. The dashed curve is with Ω_b and h held fixed, but all other parameters allowed to vary.

$$\sigma_i^2 \simeq (I_{ii})^{-1}. \tag{19}$$

Using our cosmological model as given above, we have calculated the expected precision to which $x \equiv \alpha/\alpha_0$ can be determined. The results of this calculation have been shown in Fig. 3 as a function of the maximum measured l-value. We note here that $l_{max}(MAP) \simeq 1000$ and $l_{max}(PLANCK) \simeq 2500$. Fortunately, since the CMBR spectrum is very sensitive to changes in x, it seems possible to detect changes as small as $10^{-3} - 10^{-2}$, even if all cosmological parameters must be determined simultaneously. To be on the conservative side, we estimate that $\delta x \lesssim 10^{-2}$ is a realistic obtainable precision.

Of course, in the event that all other parameters can be determined by other means, it should be possible to detect $\delta x \lesssim 10^{-4}$. This is surely not within reach in the foreseeable future, but it is still interesting to look at what other parameters are most important to determine in order to obtain a better constraint on δx. It turns out that x is most degenerate with Ω_b and h, and Fig. 3 we have also shown the standard error on x assuming that these two parameters are held fixed at their fiducial values $h = 0.5$ and $\Omega_b = 0.08$. If these two parameters can be determined by other means, a factor of 3–5 improvement in the precision should be possible. A possible determination of Ω_b should come from Big Bang nucleosynthesis (BBN) arguments. Especially the new measurements of deuterium in quasar absorption systems seem very promising in this regard [16]. As for a measurement of the Hubble parameter, perhaps the most promising method is to use what is called *cosmic complementarity*, namely, the fact that a joint use of CMBR measurements and large scale galaxy surveys like the Sloan Digital Sky Survey (SDSS) break some of the degeneracy in the CMBR measurements and allows for a more precise determination of h [17].

IV. DISCUSSION

To compare with other constraints, we convert the above constraint on δx to a constraint on redshift and time evolution of α and obtain

$$|\alpha^{-1}d\alpha/dz| \lesssim 9 \times 10^{-5}, \tag{20}$$

or

$$|\alpha^{-1}d\alpha/dt| \lesssim 7 \times 10^{-13} \text{ y}^{-1}. \tag{21}$$

This number should be compared with the constraints coming from other sources. Especially constraints coming from the line shift of quasar absorption systems have been extremely useful in providing constraints on the time evolution of α [18–20]. The most recent such measurement is that of Varshalovich *et al.* [20], who obtained

$$(\Delta\alpha/\alpha)_{z \simeq 3} \lesssim 1.6 \times 10^{-4}. \tag{22}$$

This would correspond to

$$|\alpha^{-1}d\alpha/dz| \lesssim 6 \times 10^{-5} \tag{23}$$

or

$$|\alpha^{-1}d\alpha/dt| \lesssim 1.6 \times 10^{-14} \text{ y}^{-1}. \tag{24}$$

It should be noted that there is actually a claim that time variation in α has been detected using data from quasar absorption systems (QSO) data [21]. Here, a change of

$$\Delta\alpha/\alpha = -1.5 \pm 0.3 \times 10^{-5} \tag{25}$$

has been reported.

It, thus, seems that the possible constraints on changes in α coming from the CMBR data will be almost as good as those from QSO absorption systems if α evolves linearly in time, and potentially better if the evolution is nonlinear. Moreover, it is important to have reliable constraints from different epochs in the evolution of our universe.

We also note that it is possible to constrain the evolution of α using arguments from big bang nucleosynthesis. However, these are much more model dependent than those obtainable from CMBR data. Kolb, Perry, and Walker [22] found an upper limit of

$$|\alpha^{-1}d\alpha/dt| \lesssim 1.5 \times 10^{-14} \text{ y}^{-1}, \tag{26}$$

but, as previously mentioned, this is based on a specific assumption of how changes in α affect the neutron to proton mass ratio, an assumption which is at best uncertain.

Finally, there are also quite severe constraints coming from laboratory experiments and other terrestrial sources. Presently, the best laboratory limit is that of Prestage, Tjoelker, and Maleki [23] who obtained

$$|\alpha^{-1}d\alpha/dt| \lesssim 3.7 \times 10^{-14} \text{ y}^{-1}. \tag{27}$$

The other very interesting terrestrial constraint comes from the Oklo natural fission reactor in Gabon [24,25]. The most recent discussion is that of Damour and Dyson [24] who derived the limit

$$|\alpha^{-1}d\alpha/dt| \lesssim 5 \times 10^{-17} \text{ y}^{-1}. \tag{28}$$

In conclusion, we have calculated the expected sensitivity of CMBR measurements to changes in the electromagnetic fine structure constant α. It was found that CMBR should provide a constraint which is on the same order of magnitude as other known constraints from cosmology and terrestrial sources. Also, this type of constraint is completely independent of all other existing limits, a fact which makes it very interesting. It should perhaps also be noted here that CMBR data can potentially also be used to constrain the time variation of other fundamental constants such as m_e, G_F, or G_N.

Note added. After this paper had been submitted, another paper by Kaplinghat, Scherrer, and Turner [26] on the same subject has appeared. Using the same methods, they reach conclusions very similar to those presented in the present paper.

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