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Relaxing the bounds on primordial magnetic seed fields

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We point out that the lower bound on the primordial magnetic field required to seed the galactic dynamo is significantly relaxed in an open universe or in a universe with a positive cosmological constant. In such universes, the increased age of galaxies gives a dynamo mechanism more time to amplify a small initial field. It is shown that, for reasonable cosmological parameters, primordial seed fields of strength 10^{-30} G or less at the time of galaxy formation could explain observed galactic magnetic fields. As a consequence, mechanisms of primordial magnetic seed-field generation that have previously been ruled out could well be viable. We also comment on the implications of the observation of micro-Gauss magnetic fields in galaxies at high redshift. [S0556-2821(99)50114-8]

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Magnetic fields pervade most astrophysical systems [1], but their origin is unknown. Spiral galaxies are observed to possess large-scale magnetic fields with strength of the order of 10^{-6} G and direction aligned with the rotational motion. A plausible explanation is that galactic magnetic fields result from the exponential amplification of an initially weak seed field by a mean-field dynamo [2,3]. Many proposals have been put forward regarding the origin of such a seed field. One suggestion is that it might arise spontaneously from non-parallel gradients of pressure and charge-density during galaxy formation [4]. A wider range of possibilities is offered if the seed field is of primordial origin. This category includes cosmological magnetic fields [5] as well as magnetic fields created by any of a number of early-universe particle-physics mechanisms [6] such as collisions of bubbles in a first-order phase transition [7] or false-vacuum inflation [8,9].

The seed-field strength required at the time of completed galaxy formation (t_{gf}) for a galactic dynamo to produce the present magnetic field strength $B_0 \sim 10^{-6}$ G is usually quoted in the range $\sim 10^{-23}$–10^{-19} G. Such lower bounds are obtained by considering the dynamo amplification in a flat universe with zero cosmological constant for ''typical'' values of the parameters of the $\alpha\omega$-dynamo. The seed field must also be coherent on a scale at least as large as the size of the largest turbulent eddy, ~ 100 pc [2]. Most proposed models of primordial seed-field generation fail to meet these requirements as formulated above.

In this paper, we address the issue in light of recent developments in cosmology. Observations of distant type-IA supernovas [10] and of anisotropies in the cosmic microwave background (CMB) [11] in combination have made it increasingly likely that the universe is less dense than the critical density and has a positive cosmological constant Λ. Most previous studies of magnetic fields have assumed a $\Lambda = 0$ universe with critical matter density.

We shall recalculate the constraints on primordial seed fields for general Friedmann universes with matter density parameter Ω_0 and vacuum energy density parameter $\lambda_0 \equiv \Lambda/(3H_0^2)$ such that $\Omega_0 + \lambda_0 \leqslant 1$ (the subscript 0 here indicates quantities at present time and H_0 is the Hubble parameter). In addition to finding revised bounds on the seed field at time t_{gf}, we shall trace the evolution of the magnetic field back to the time of radiation decoupling, t_{dec}. Prior to decoupling, the evolution of the magnetic field proceeds via complicated plasma processes and depends on the field's initial strength and correlation length [12]. After decoupling, there is sufficient residual ionization for the magnetic field to be frozen into the plasma; the evolution is simple and independent of the mechanism of generation. Thus t_{dec} is a natural epoch for imposing bounds on primordial magnetic fields.

We begin by considering the $\alpha\omega$-dynamo [2,3] which is a well-studied model of amplification of magnetic fields. It is powered by the differential rotation of the galaxy in combination with the small-scale turbulent motion of ionized gas. By separating the magnetic field into a large-scale mean field $\boldsymbol{B}$ and a random, turbulent field $\boldsymbol{b}$, one obtains a system of equations with exponentially growing solutions $B_\varphi \propto e^{\Gamma t}$ for the azimuthal component B_φ of the mean field in the plane of the disc. The dynamo amplification rate Γ appears as an eigenvalue that must be determined numerically.

Unfortunately, the value of Γ is rather sensitive to the parameters of the dynamo model [13], which include the root-mean-square (RMS) velocity and magnetic diffusivity of the turbulent plasma as well as the angular-velocity gradient $r\, d\omega(r)/dr$. For reasonable estimates of these quantities, one obtains Γ^{-1} in the range $0.2 < \Gamma^{-1} < 0.8$ [Gyr]. Because of the exponential growth, such an uncertainty quickly translates into an uncertainty of many orders of magnitude in the total amplification, which may or may not rule out various seed-field mechanisms. The point of this paper is not to linger on these uncertainties, but rather to emphasize the tremendous increase in amplification that will occur in an open universe or in one with a positive cosmological constant *for any value of* Γ. We shall present results for the two values that appear most frequently in the literature, $\Gamma^{-1} = 0.3$ Gyr [13] and $\Gamma^{-1} = 0.5$ Gyr [2].

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Any magnetic seed field is exponentially amplified until it reaches the equipartition energy ($B\sim$ few μG) when further growth is suppressed by dynamical back reaction of the Maxwell stresses on the turbulence. Assuming that the dynamo mechanism begins to operate around the time of completed galaxy formation $t_{\rm gf}$, the lower bound on the strength of the seed field at this epoch is given by

$$B_{\rm gf} \geqslant B_0 e^{-\Gamma(t_0 - t_{\rm gf})}, \tag{1}$$

where t_0 is the age of the universe obtained by integrating the Friedmann equation [14]. In particular, for a given value of H_0, open universes and universes with a positive cosmological constant are significantly older than the $\Omega=1$ Einstein–de Sitter universe.

The time of galaxy formation $t_{\rm gf}$ can be estimated from a spherical collapse model. As we shall show presently, galaxies of a given average density $\overline{\rho}_{\rm gal}$ have collapsed at approximately the same time after the Big Bang for all realistic cosmological models. Galaxies in an open or $\Lambda>0$ universe are therefore older, giving the dynamo mechanism more time $t_0 - t_{\rm gf}$ to operate. Consequently, a much smaller magnetic field $B_{\rm gf}$ can seed the dynamo and still give the observed micro-Gauss field B_0.

The spherical collapse model [15,16] describes the nonlinear collapse of a bounded spherical region with average local density $\overline{\rho}_{\rm i}$ larger than the critical density at some initial time $t_{\rm i}$ in the matter-dominated era. This overdensity causes the sphere to break away from the Hubble expansion, reach a maximal (turn-around) radius $r_{\rm m}$, and eventually collapse to form a gravitationally bound system. The general equation of motion for a shell of radius r enclosing mass M is [14]

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{GM}{r} - \frac{1}{6}\Lambda r^2 = E, \tag{2}$$

where E is a constant. The exact solution can be expressed in terms of incomplete elliptic integrals [17], but we choose instead to expand in the parameter $\beta=\Lambda r_{\rm m}^3/(6GM)$, obtaining the more convenient parametric solution

$$r = \frac{r_{\rm m}}{2}(1-\cos\theta), \tag{3}$$

$$t+T = \frac{1}{\sqrt{GM}}\left(\frac{r_{\rm m}}{2}\right)^{3/2} \times\Bigg((\theta-\sin\theta) + \frac{\beta}{96}(66\theta - 93\sin\theta + 15\sin 2\theta - \sin 3\theta) + \mathcal{O}(\beta^2)\Bigg), \tag{4}$$

where T is a constant which can be neglected [16]. We see that turn-around occurs at a time $t_{\rm m}$ corresponding to $\theta=\pi$. As the spherical region recollapses for $t>t_{\rm m}$, random nonradial particle velocities become important; the simple collapse model breaks down and the collapse is halted at a final radius $r_{\rm vir}$ given by the virial theorem. For universes with zero cosmological constant, $r_{\rm vir}=r_{\rm m}/2$. For $0\leqslant\beta<1/2$ (which is required for collapse to occur) Lahav *et al.* [14] showed that $1/2 \geqslant r_{\rm vir}/r_{\rm m} > 0.366$ and also obtained the approximate relation

$$\frac{r_{\rm vir}}{r_{\rm m}} = \frac{1-\beta}{2-\beta}. \tag{5}$$

We can estimate β for a galaxy: Taking $0\leqslant\lambda_0<1$, $M=10^{11}M_\odot \approx 2\times10^{45}$g and $r_{\rm vir}\lesssim 15$ kpc, we get

$$0\leqslant\beta<\frac{H_0^2}{2GM}\frac{r_{\rm vir}^3}{(0.366)^3}\sim 3\times10^{-5}. \tag{6}$$

The small value of β signifies that the vacuum energy density plays a negligible role compared to the matter density in the collapse of objects as small and dense as galaxies. From Eq. (5), it follows that we can set $r_{\rm vir}=r_{\rm m}/2$ for all realistic values of the cosmological constant. Moreover, we can neglect the β-dependent terms in Eq. (4).

It is generally assumed [16] that gravitational collapse is complete at the time $t_{\rm vir}>t_{\rm m}$ when r approaches zero in Eq. (3), corresponding to $\theta\approx 2\pi$.[1] This assumption is supported by N-body simulations and, because of the small value of β, remains valid in any realistic Friedmann cosmology. From Eq. (4) we then have $t_{\rm vir}\approx 2t_{\rm m}$ as well as $t_{\rm m}^2 = 3\pi/(32G\overline{\rho}_{\rm m})$, where $\overline{\rho}_{\rm m}\equiv 3M/(4\pi r_{\rm m}^3)$ is the average density of the spherical region at turn-around. It follows from $r_{\rm vir}=r_{\rm m}/2$ that $\overline{\rho}_{\rm gal}=8\overline{\rho}_{\rm m}$. Finally, with $t_{\rm gf}=t_{\rm vir}$, all these relations combine to give

$$\overline{\rho}_{\rm gal} = \frac{3\pi}{Gt_{\rm gf}^2}. \tag{7}$$

This is the result that we have sought. It shows that the relationship between the average density of a galaxy $\overline{\rho}_{\rm gal}$ and the time of completed galaxy formation $t_{\rm gf}$ is independent of cosmology. The current galactic density is a quantity which can be measured using methods that also do not depend on cosmology. It may at first seem mysterious that Ω_0 does not enter in Eq. (7) or any of the derivations leading to it. The reason is that (for $\Lambda=0$) the same average local density (larger than the critical density) is required for a spherical region to collapse, regardless of the density of the surrounding universe. By Birkhoff's theorem, the evolution therefore proceeds in an identical manner.

We are now in a position to calculate bounds on magnetic seed fields $B_{\rm gf}$ at the time of completed galaxy formation in different cosmologies. We take $B_0=10^{-6}$ G and $\overline{\rho}_{\rm gal}=10^{-24}\,{\rm g\,cm^{-3}}$. The latter value corresponds to the average density of the galactic halo rather than the central disc, whose density is $\sim10^{-23}\,{\rm g\,cm^{-3}}$. The reason for this choice is that the spherical collapse model uses the simplified as-

[1]The naive estimate, that collapse is complete when the radius r in Eq. (3) reaches the virial radius $r_{\rm vir}$ (corresponding to $\theta=3\pi/2$), is unrealistic as the radius decreases more slowly during virialization than the spherical collapse model would imply.

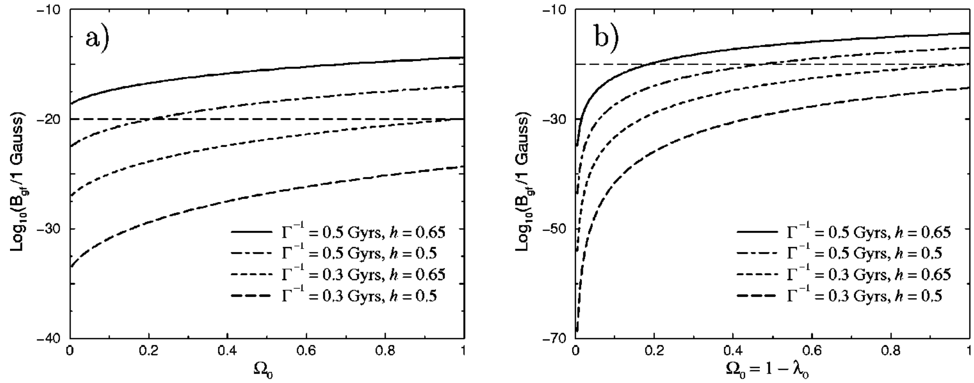

FIG. 1. Lower bound on the seed field at galaxy formation B_{gf} vs Ω_0: (a) universe with $\Lambda=0$, (b) flat Λ universe.

sumption of a uniform-density ''top-hat'' profile of the galactic density distribution, and since the halo comprises most of the volume of the galaxy, this value seems more appropriate. The precise value of $\overline{\rho}_{\mathrm{gal}}$ is of little importance as our results are quite insensitive to it.

The results are displayed in Fig. 1(a) for a $\Lambda=0$ universe and in Fig. 1(b) for a flat Λ universe $(\Omega_0+\lambda_0=1)$. The quantity h is the Hubble parameter H_0 in units of $100\,\mathrm{km\,sec^{-1}\,Mpc^{-1}}$. For comparison, the straight horizontal line in each plot shows the constraint of $B_{\mathrm{gf}}\geqslant 10^{-20}\,\mathrm{G}$ given by Rees [18]. It can be seen on these graphs that, in an open universe and, particularly, in a universe with a significant cosmological constant Λ, this requirement is too strong. For reasonable cosmological parameters and the same value of Γ, the dynamo mechanism could generate currently observed galactic magnetic fields from a seed field of the order of $10^{-30}\,\mathrm{G}$ or less at the completion of galaxy formation provided that the seed field is coherent on a scale $\xi_{\mathrm{gf}}\gtrsim 100\,\mathrm{pc}$.

An analytic approximation for the lower bound on B_{gf} can be derived using an approximate expression [19] for t_0. From Eqs. (1) and (7), one obtains

$$\ln\frac{B_{\mathrm{gf}}}{B_0}\geqslant -\Gamma\left(\frac{2\sinh^{-1}\sqrt{(1-\Omega_{\mathrm{a}})\Omega_{\mathrm{a}}}}{3H_0\sqrt{(1-\Omega_{\mathrm{a}})\Omega_{\mathrm{a}}}}-\sqrt{\frac{3\pi}{G\overline{\rho}_{\mathrm{gal}}}}\right), \qquad (8)$$

where $\Omega_{\mathrm{a}}=0.7\Omega_0+0.3(1-\lambda_0)$. The expression (8) is the exact result when $\Omega_0+\lambda_0=1$; for realistic parameters Ω_0 and Γ, it differs from the exact result by less than one order of magnitude in B_{gf}.

We shall now evolve the bounds of Fig. 1 back to the time of radiation decoupling, taking the conservative view that there is no magnetohydrodynamic turbulence or dynamo mechanism operating during gravitational collapse (see, however, Ref. [4] for more optimistic proposals). The magnetic field is assumed to be frozen into the plasma and its evolution is determined by flux conservation $Br^2=\mathrm{const}$, where r is a length scale evolving with the matter, i.e., $r\sim(\overline{\rho})^{-1/3}$. Care must be taken not to associate this length scale with the scale factor $a(t)$, as a collapsing galaxy is decoupled from the Hubble expansion. One obtains [8]

$$\frac{B_{\mathrm{gf}}}{B_{\mathrm{dec}}}=\left(\frac{\overline{\rho}_{\mathrm{gal}}}{\rho_{\mathrm{dec}}}\right)^{2/3}=\left(\frac{\overline{\rho}_{\mathrm{gal}}}{\rho_0}\right)^{2/3}\frac{1}{(1+z_{\mathrm{dec}})^2}, \qquad (9)$$

where we have used the energy conservation relation $\rho=\rho_0(1+z)^3$ for the matter component and the fact that the matter density ρ_{dec} at the epoch t_{dec} is very nearly uniform. The redshift of radiation decoupling, z_{dec}, is constrained to lie in the interval $1100\lesssim z_{\mathrm{dec}}\lesssim 1200$ [20].

Note that the magnetic field will *decrease* between t_{dec} and t_{gf}, since by virtue of the Hubble expansion, the physical volume of the galaxy is larger than the volume containing the same mass at t_{dec}. The depletion depends on the cosmological parameters via the present matter density $\rho_0\approx 1.88\times 10^{-29}\Omega_0 h^2\ [\mathrm{g\,cm^{-3}}]$. It can be seen that the depletion is somewhat smaller in universes with $\Omega_0<1$. This further increases the effect of cosmological parameters in relaxing the bounds on primordial seed fields. The resulting bounds for B_{dec} are shown in Fig. 2(a) and Fig. 2(b) for a $\Lambda=0$ universe and for a flat Λ universe, respectively.

We shall now address the issue of the correlation length of the magnetic field. In order for the galactic dynamo to begin to operate, the correlation length of the seed field at the time of completed galaxy formation must satisfy $\xi_{\mathrm{gf}}\gtrsim 100\,\mathrm{pc}$ [2].[2] Using the spherical collapse model, one can calculate the physical scale r_{dec} at the time of radiation decoupling that will evolve into the size of a galaxy. At any time before the onset of gravitational collapse the matter density follows the Hubble expansion and it makes sense to express r_{dec} in the constant comoving quantity x defined by $r=a(t)x$. The comoving scale x corresponding to a galaxy is given by [16]

$$x_{\mathrm{gal}}=0.95(\Omega_0 h^2)^{-1/3}M_{12}^{1/3}\ [\mathrm{Mpc}], \qquad (10)$$

where $M_{12}=M/(10^{12}M_{\odot})$.

The correlation length ξ can be written as a fraction of the radius of the galaxy, $\xi=\eta r_{\mathrm{vir}}$. With the simplified assumption of the spherical collapse model that the collapsing re-

[2] A more conservative bound, used by many authors, is $\xi_{\mathrm{gf}}\gtrsim 1\,\mathrm{kpc}$.

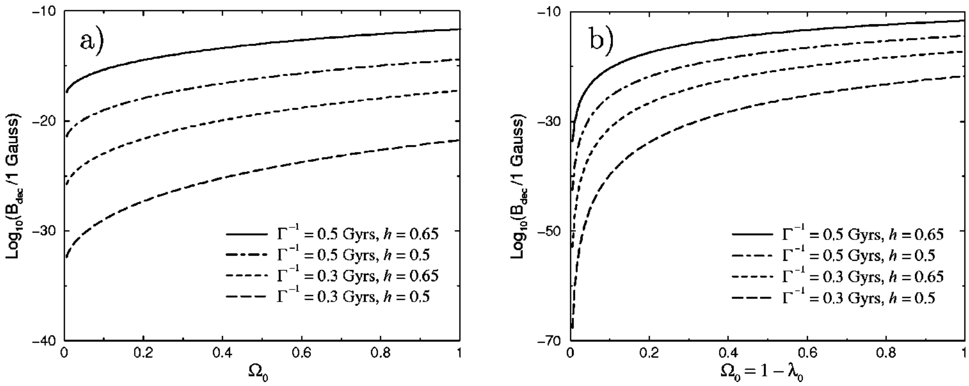

FIG. 2. Lower bound on the seed field at radiation decoupling B_{dec} vs Ω_0: (a) universe with $\Lambda=0$, (b) flat Λ universe.

gion has uniform density, the collapse is homogeneous and isotropic and different scales collapse proportionately. Assuming that the magnetic field is frozen into the plasma between t_{dec} and t_{gf}, we have $x_{\mathrm{corr}} = \eta x_{\mathrm{gal}}$. For a galaxy, $M_{12} \approx 0.1$, and the typical length scale of the turbulent motion, $\xi_{\mathrm{turb}} = 100$ pc, corresponds to $\eta \approx 1/150$, giving the following bound on the comoving correlation length

$$x_{\mathrm{corr}} \gtrsim x_{\mathrm{turb}} = 5-10 \ \ \mathrm{kpc} \tag{11}$$

for observationally realistic values $0.25 > \Omega_0 h^2 > 0.025$. This bound is somewhat higher than that stated in Ref. [12]. The bound should not be applied before t_{dec}, since the correlation length then evolves according to complicated magnetohydrodynamic processes and is not proportional to the scale factor $a(t)$ [12].

In general, primordial seed fields produced by particle-physics or field-theory mechanisms are too incoherent to meet the requirement posed by Eq. (11). However, there is a possibility even for a less correlated magnetic field to pass the requirement provided that it has sufficient strength to satisfy the bound on B_{dec} after RMS coarse-graining over the scale given by x_{turb}. The said procedure results in an RMS field

$$B_{\mathrm{RMS}} = \left(\frac{x_{\mathrm{dec}}}{x_{\mathrm{turb}}} \right)^{d/2} B_{\mathrm{dec}}, \tag{12}$$

where B_{RMS} is the quantity that must satisfy the bound given in Fig. 2, with B_{dec} and x_{dec} being the strength and correlation length in comoving units, respectively, of the primordial seed field evolved from formation to t_{dec}. The exponent d can equal 1, 2, or 3 depending on the averaging procedure used. This complicated issue [21] shall not be addressed in this paper.

There have been observations of micro-Gauss fields at redshifts of $z=0.395$ [22] and $z=2$ [23], although the latter has been criticized [24]. If correct, these observations are difficult to explain in a flat universe with $\Lambda=0$. They may, however, be easier to understand in an open or Λ universe. Applying our model to the $z=0.395$ case, with $B_{0.395} = 10^{-6}$ G, we obtain for a flat Λ universe the bounds at $t = t_{\mathrm{dec}}$ shown in Fig. 3. Hence, a seed field of 10^{-20} G at t_{dec}, or equivalently 10^{-23} G at t_{gf}, could account for this observation.

If we attempt a similar analysis in the $z=2$ case, the required seed field is sufficiently high that it would have other cosmological implications, e.g., on the CMB [25] or structure formation [26]. Consequently, we conclude that, unless the dynamo parameters are radically different for high column density Ly-α clouds (e.g., if they have fast-spinning cores and thereby have a higher angular-velocity gradient $|r\, d\omega/dr|$ [27]), these observations cannot be explained by amplification of a primordial seed field by a galactic dynamo.

In this paper, we have reconsidered the constraints on the primordial magnetic field required to seed the galactic dynamo in light of recent cosmological advances. We have shown that, in an open universe or a universe with $\Lambda>0$, a much smaller seed field is required to explain the observed micro-Gauss fields in galaxies. As a consequence, mechanisms of primordial magnetic seed-field generation that had previously been ruled out, on the grounds of giving too small strength or correlation length, could well be viable. We have evolved the bounds back to the epoch of radiation decoupling t_{dec} assuming that, from t_{dec} to the present, the magnetic field is frozen into the plasma and evolves first via flux conservation and thereafter by amplification via a dynamo

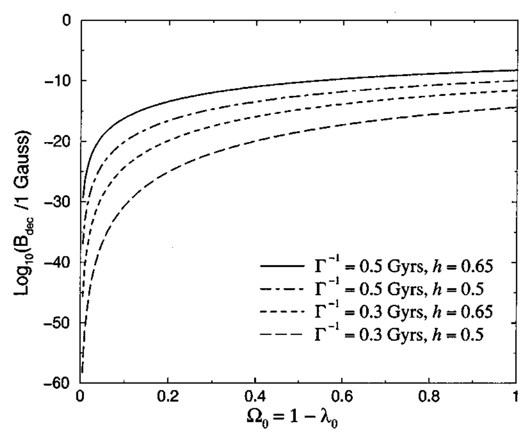

FIG. 3. Lower bound on B_{dec} vs Ω_0 for generating a field strength of 10^{-6} G at redshift $z=0.395$ via the dynamo mechanism in a universe with $\lambda_0 + \Omega_0 = 1$.

mechanism. The remaining problem is to evolve primordial magnetic fields from the time of their generation to $t_{\rm dec}$ taking into account various plasma effects. A step in this direction has been taken in Ref. [12]. This work needs to be generalized to different cosmologies, although the main cosmological effects are expected to occur at late times.

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