Comment on ''Hara's theorem in the constituent quark model''

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It is pointed out that current conservation *alone* does not suffice to prove Hara's theorem, as was claimed recently. By explicit calculation, we show that the additional implicit assumption made in such ''proofs'' is that of a sufficiently localized current. [S0556-2821(99)05201-7]

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Weak radiative hyperon decays proved to be a challenge to our theoretical understanding. Despite many years of theoretical studies, a satisfactory description of these processes is still lacking. In a recent review (see Ref. [1]) presenting the current theoretical and experimental situation in that field, attention was focused on the question of the validity of Hara's theorem [2]. This question was originally posed by the paper of Kamal and Riazuddin [3] who observed that in the quark model Hara's theorem is violated. There have been several attempts to understand the origin and meaning of this quark model result [4–7]. Here we want to comment on Ref. [6] wherein it is claimed that the argument made by Serot in Ref. [8] and discussed later in Ref. [9] is sufficient to prove Hara's theorem.

In Ref. [6] it is stated that the argument of Serot (upon which the claim of Ref. [6] is based) relies *only* on the multipole decomposition of the electromagnetic current matrix element and on the conservation of electromagnetic current. This statement should be treated with suspicion as in the standard proof of Hara's theorem, it is the absence of massless hadrons that—in addition to gauge invariance—is necessary for the proof to go through (see, e.g., Ref. [1]). Thus, one may suspect that the argument of Serot uses a somewhat similar additional *hidden* assumption. Rather than trying to identify such an implicit assumption, a large part of Ref. [6] (see also Ref. [9]) is then concerned with the demonstration of how to satisfy electromagnetic current conservation in actual calculations with composite states. Below we will demonstrate through an explicit calculation what implicit assumption is being made in Serot-like arguments.

There are two conserved electromagnetic currents entering into the discussion of Hara's theorem:

$$J_5^{\mu}=F_1(q^2)\overline{\psi_1}\left(\gamma^{\mu}-\frac{q^{\nu}\gamma_{\nu}}{q^2}q^{\mu}\right)\gamma_5\psi_2 \tag{1}$$

and

$$J_5'^{\mu}=F_3(q^2)\overline{\psi_1}i\sigma^{\mu\nu}q_{\nu}\gamma_5\psi_2. \tag{2}$$

In the limit of exact SU(3) ($m_1=m_2$), the coupling of photon to current J_5' vanishes due to its symmetry properties (see Sec. 3.1. in Ref. [1]). The only allowed current is then that of Eq. (1). According to Zeldovich and Perelomov [10], expansion of $F_1(q^2)$ around $q^2=0$ has to start with a term proportional to q^2. For a real photon, this entails a vanishing current matrix element. However, as discussed in Ref. [9] for a nonvanishing $F_1(0)$, one obtains current matrix element which is finite at $q^2=0$ and a vanishing parity-violating charge density. Thus, the form of Eq. (1) seems fully admissible also for $F_1(0)\neq 0$. In Ref. [9] it is then claimed that Serot managed to prove the vanishing of the relevant matrix element at $q^2=0$ using conservation of the electromagnetic current *only*. As remarked above, the proof of Serot most likely uses a hidden assumption. Let us therefore look at this proof in some detail.

In the nonrelativistic approximation, the current J_5^{μ} takes the form (see also Ref. [9]):

$$\mathbf{J}_5(q)=\frac{\mathbf{q}\times(\boldsymbol{\sigma}\times\mathbf{q})}{\mathbf{q}^2}=\boldsymbol{\sigma}-(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}, \tag{3}$$

where $\hat{\mathbf{q}}=\mathbf{q}/|\mathbf{q}|$ and we have put $F_1(0)=1$. For $\mathbf{J}_5(q)$ of Eq. (3), the transverse electric dipole is clearly nonzero.

On the other hand, the argument of Serot, which starts with a general formula for the transverse electric dipole, seems to show that for $\mathbf{q}^2\to 0$, this multipole vanishes as $\mathbf{q}^2$ anyway. Since the argument of Serot is made in position space, in order to analyze it, we have to find the shape of current $\mathbf{J}_5$ from Eq. (3) in position space. Let us therefore consider

$$\mathbf{J}_5^{\varepsilon}(\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3q\,\mathbf{J}(\mathbf{q})e^{-i\mathbf{qr}-\varepsilon\mathbf{q}^2}. \tag{4}$$

In Eq. (4) we have introduced a small parameter (ε) to regularize the emerging integrals.

The integral in Eq. (4) is composed of two pieces. The first [Fourier transform of $\mathbf{J}^{(1)\varepsilon}(\mathbf{q})\equiv\boldsymbol{\sigma}\exp(-\varepsilon\mathbf{q}^2)$] gives

$$\mathbf{J}^{(1)\varepsilon}(\mathbf{r})=\boldsymbol{\sigma}\cdot\delta_{\varepsilon}^3(\mathbf{r}) \tag{5}$$

with $\delta_{\varepsilon}^3(\mathbf{r})=\Pi_{i=1}^3\delta_{\varepsilon}(r_i)$, where $\delta_{\varepsilon}(r_i)$ is a one-dimensional regularized delta function:

$$\delta_{\varepsilon}(r_i)=\frac{1}{\sqrt{4\pi\varepsilon}}\exp[-r_i^2/(4\varepsilon)]. \tag{6}$$

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Calculation of the second piece [Fourier transform of the term $\mathbf{J}^{(2)\varepsilon}(\mathbf{q}) \equiv -(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}\exp(-\varepsilon\mathbf{q}^2)$] is more complicated and is sketched in the Appendix. Together one obtains

$$\mathbf{J}_5^{\varepsilon}(\mathbf{r}) = \mathbf{J}_5^{(1)\varepsilon}(\mathbf{r}) + \mathbf{J}_5^{(2)\varepsilon}(\mathbf{r})$$

$$= [\boldsymbol{\sigma} - (\boldsymbol{\sigma}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\delta_\varepsilon^3(\mathbf{r}) + \frac{1}{2\pi r^2}[\boldsymbol{\sigma} - 3(\boldsymbol{\sigma}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\delta_\varepsilon(r)$$

$$-\frac{1}{4\pi r^3}[\boldsymbol{\sigma} - 3(\boldsymbol{\sigma}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}] \times \mathrm{erf}\left(\frac{r}{2\sqrt{\varepsilon}}\right), \tag{7}$$

where $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x e^{-t^2}dt$ is the error function, $\hat{\mathbf{r}} = \mathbf{r}/r$, and $r = |\mathbf{r}|$.

The transverse electric dipole is defined as [6,9,11]

$$T_{1M}^{el} = \frac{1}{iq\sqrt{2}}\int d^3r\{-\mathbf{q}^2(\mathbf{J}_5\cdot\mathbf{r}) + (\nabla\cdot\mathbf{J}_5)[1 + (\mathbf{r}\cdot\nabla)]\} \times j_1(qr)Y_{1M}(\hat{\mathbf{r}}), \tag{8}$$

where $q = |\mathbf{q}|$.

One may check by direct calculation that current $\mathbf{J}_5^{\varepsilon}(\mathbf{r})$ of Eq. (7) is conserved [as it should be because for its Fourier transform $\mathbf{J}_5^{\varepsilon}(\mathbf{q})$, we obviously have from Eq. (3): $\mathbf{q}\cdot\mathbf{J}_5^{\varepsilon}(\mathbf{q}) = 0$]:

$$\nabla\cdot\mathbf{J}_5^{\varepsilon} = \left(-\frac{2\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}}{r}\delta_\varepsilon^3(\mathbf{r})\right) + \left(\frac{2\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}}{r}\delta_\varepsilon^3(\mathbf{r}) - \frac{\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}}{\pi r^3}\delta_\varepsilon(r)\right) + \left(\frac{\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}}{\pi r^3}\delta_\varepsilon(r)\right) = 0. \tag{9}$$

In Eq. (9) the three terms in parentheses come from the three terms on the right-hand side of Eq. (7). Clearly, all three terms in Eq. (7) are required for the cancellation of Eq. (9) to work. Since $\nabla\cdot\mathbf{J}_5^{\varepsilon} = 0$, in the calculation of the electric dipole in Eq. (8), only the first term of the integrand may give a nonzero result. However, according to the argument of Ref. [6,9] for small q, this term is proportional to q^2 after replacing the spherical Bessel function $j_1(qr)$ by its approximation for small arguments: $\frac{1}{3}qr$. Consequently, the argument seems to show that T_{1M}^{el} vanishes in the long wavelength limit $q^2 \to 0$.

Unfortunately, the above argument is not correct when one uses current $\mathbf{J}_5^{\varepsilon}$ of Eq. (7). Let us calculate

$$\mathbf{J}_5^{\varepsilon}(\mathbf{r})\cdot\mathbf{r} = -\frac{\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}}{\pi r}\delta_\varepsilon(r) + \frac{1}{2\pi r^2}\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\,\mathrm{erf}\left(\frac{r}{2\sqrt{\varepsilon}}\right). \tag{10}$$

The first term on the right-hand side of Eq. (10), being proportional to delta function, is sufficiently localized in space to permit the replacement of the spherical Bessel function j_1 by its approximation for small arguments, since only small values of qr are allowed ($q \to 0$ and r is small). With the second term, the situation is, however, different.

Introducing new variable $z \equiv qr$, the contribution of the second term in Eq. (10) to the transverse electric dipole is (for $\varepsilon \to 0$)

$$T_{1M}^{el} = \frac{i}{2\pi\sqrt{2}}\int_0^\infty dz\, j_1(z)\int d\Omega_{\hat{z}}\boldsymbol{\sigma}\cdot\hat{\mathbf{z}}Y_{1M}(\hat{\mathbf{z}}). \tag{11}$$

The right-hand side apparently does not depend on q. Using $(d/dz)j_0(z) = -j_1(z)$ and $Y_{1M}(\hat{\mathbf{z}}) = \sqrt{(3/4\pi)}\boldsymbol{\epsilon}_M\cdot\hat{\mathbf{z}}$, we calculate

$$T_{1M}^{el} = \frac{i}{\sqrt{6\pi}}\boldsymbol{\epsilon}_M\cdot\boldsymbol{\sigma} \tag{12}$$

a definitely nonvanishing result in agreement with Eq. (3). The origin of this nonzero result is clear from the above calculation: in Eq. (7) the third term of axial current $\mathbf{J}_5^{\varepsilon}(\mathbf{r})$ is *not* localized in space sufficiently well. Thus, the theorem of Serot is based not only on current conservation, but also on the assumption that the position-space current vanishes at infinity faster than $1/r^3$. To forbid such a behavior corresponds in standard proofs of Hara's theorem to assuming the absence of massless (infinite range) hadrons.

It is a different question whether the above-identified implicit assumption used by Serot should really be made. Conventional wisdom certainly requires the electromagnetic axial current of a baryon to be well localized in position space, which assumption—together with that of current conservation—leads to a vanishing parity-violating matrix element of the electromagnetic current at the real photon point.

Results of strict quark model calculations of Kamal and Riazuddin (KR) [3], which indicated the nonvanishing of this matrix element [for $SU(3)$-related strangeness-changing current $\Sigma^+ \to p$], were therefore treated with disbelief. However, since Serot theorem is not based on current conservation *only*, one cannot conclude from the violation of Hara's theorem obtained in Ref. [3] that gauge invariance must be broken in these calculations. In fact, by repeating KR calculations, one can convince oneself that gauge invariance *is preserved* in Ref. [3]. Thus, it seems that it is rather the other assumption used by Serot: that of a sufficiently well localized current, which is violated in the KR paper.

This tentative identification seems understandable if one thinks of quark model prescription in position space. Indeed, in the strict quark model, the initial and final states are described by sums of tensor products of plane-wave quark states spreading all over position space. In the calculation of KR, the intermediate quark (between the action of weak Hamiltonian and the emission of a photon) may also propagate to spatial infinity, reflecting total quark freedom. It should not come then as a surprise that the total electromagnetic current of the three-quark state contains a piece which is not sufficiently well localized.

To summarize let us repeat that the assumption of current conservation *alone* does not suffice to prove Hara's theorem.

The current of Eq. (7) is definitely conserved and yet the transverse electric dipole moment is nonzero. Thus, the considerations of Refs. [6,9] which concern the detailed manner in which current conservation is realized for composite systems cannot by themselves provide us with a proof of Hara's theorem.

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APPENDIX

Calculation of $\mathbf{J}^{(2)\varepsilon}(\mathbf{r})$ requires determination of the integral

$$I_{ml}(\mathbf{r}) = -\frac{1}{(2\pi)^3}\int d^3q \frac{q_m q_l}{q^2} e^{-i\mathbf{q}\cdot\mathbf{r}-\varepsilon\mathbf{q}^2} \tag{A1}$$

which may be evaluated as

$$I_{ml}(\mathbf{r}) = \frac{1}{(2\pi)^3}\frac{\partial^2}{\partial r^m \partial r^l}\int_\varepsilon^\infty d\xi \int d^3q e^{-\xi\mathbf{q}^2 - i\mathbf{q}\cdot\mathbf{r}}$$
$$= \frac{1}{4\pi^{3/2}}\frac{\partial^2}{\partial r^m \partial r^l}\frac{\sqrt{\pi}}{r}\operatorname{erf}\left(\frac{r}{2\sqrt{\varepsilon}}\right). \tag{A2}$$

Performing indicated differentiations, we obtain

$$I_{ml}(\mathbf{r}) = -\frac{1}{4\pi r^3}(\delta_{ml} - 3\hat{r}_m\hat{r}_l)\operatorname{erf}\left(\frac{r}{2\sqrt{\varepsilon}}\right)$$
$$-\frac{1}{\pi r^2}\hat{r}_m\hat{r}_l\frac{1}{\sqrt{4\pi\varepsilon}}\exp\left(-\frac{r^2}{4\epsilon}\right)$$
$$+\frac{1}{2\pi r^2}(\delta_{ml} - \hat{r}_m\hat{r}_l)\frac{1}{\sqrt{4\pi\varepsilon}}\exp\left(-\frac{r^2}{4\epsilon}\right)$$
$$-\hat{r}_m\hat{r}_l\frac{1}{(4\pi\varepsilon)^{3/2}}\exp\left(-\frac{r^2}{4\epsilon}\right). \tag{A3}$$

This leads to Eq. (7).

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