

# Light quarks in the instanton vacuum at finite baryon density

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We consider the finite density, zero-temperature behavior of quark matter in the instanton picture. Since the instanton-induced interactions are attractive in both  $\bar{q}q$  and  $qq$  channels, a competition ensues between phases of matter with condensation in either or both. It results in chiral symmetry restoration due to the onset of diquark condensation, a “color superconductor,” at finite density. Also possible is a state with both manners of condensation; however, such a phase is at best metastable for any chemical potential. The properties of quark matter in each phase are discussed, with emphasis on the microscopic effects of the effective mass and superconducting energy gap. [S0556-2821(99)05711-2]

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## I. INTRODUCTION

The idea that the QCD partition function is dominated by instanton fluctuations of the gluon field, with quantum oscillations about them, has successfully described much of what we know about the zero-temperature, zero-density hadronic world (for a review, see Refs. [1,2]). Instantons have been reliably identified in lattice simulations (for a review, see Ref. [3]), which demonstrate their relevance to hadronic observables [4]. From the theory side, the instanton vacuum constructed from the Feynman variational principle [5,6] gives an example of how the necessary “transmutation of dimensions” occurs in QCD, meaning that all dimensional quantities can be expressed through the QCD scale parameter,  $\Lambda_{\text{QCD}}$ . It can be added that in the solvable  $N=2$  supersymmetric version of QCD, instantons are sufficient to reproduce the expansion of the exact Seiberg-Witten prepotential [7].

More relevant to this paper, the instanton vacuum provides a reasonable and phenomenologically satisfying mechanism of spontaneous chiral symmetry breaking in QCD (for a review, see [1]). In addition, it has recently gained support from direct lattice studies [8]. Therefore, an expansion of instanton ideas to a new frontier, in this case to nonzero matter density, seems to be well justified.

The key point is that instantons induce interactions that are attractive not only in the  $\bar{q}q$  channel (leading to the spontaneous chiral symmetry breaking) but also in the  $qq$  channel (potentially leading to diquark condensation). Quark pairing was originally investigated as resulting from single gluon exchange in Ref. [9], and was later considered nonperturbatively in Ref. [10]. Both situations have been recently discussed in the literature by various authors. In the special case of two colors ( $N_c=2$ ) and two light quark flavors ( $N_f=2$ ), the phenomenon becomes particularly clear. Here the instanton-induced interactions possess not a global  $SU(2)\times SU(2)$  symmetry, as would be for any  $N_c>2$ , but a larger  $SU(4)$  symmetry [11] (often referred to as the Pauli-Gürsey symmetry). At  $N_c=2$  the attraction in the  $\bar{q}q$  and  $qq$

channels are exactly equal. For this reason  $\bar{q}q$  and  $qq$  condensates belong in fact to one phase: one condensate can be rotated to another along the Goldstone valley [11].<sup>1</sup>

Switching in a nonzero chemical potential  $\mu$  violates explicitly the global  $SU(4)$  symmetry of the  $N_c=2$  world, the degeneracy of the five-dimensional Goldstone valley is lifted, and one can ask which of the condensates becomes preferred. In Ref. [13] arguments have been given that it is the diquark condensate. In a previous publication [14] we have confirmed this expectation by direct calculations; see also below. It should be noted that at  $N_c=2$  singlet diquarks are nothing but the colorless “baryons” which happen to be bosons, and their condensation does not break the color symmetry.

In the opposite limit,  $N_c\rightarrow\infty$ , the diquark interaction is suppressed by a factor of  $\sim 1/N_c$  as compared to the  $\bar{q}q$  one, and so is the diquark condensate. The question is then, is our world with  $N_c=3$  closer to  $N_c=2$  or  $N_c=\infty$ ?

The possibility of diquark condensation at any  $N_c$ , as induced by instantons, has been studied in Ref. [15]. Contrary to the case of  $N_c=2$ , a diquark condensation at  $N_c\geq 3$  would inevitably spontaneously break the color symmetry, similarly to the Higgs breaking of the  $SU(2)$  gauge symmetry of electroweak interactions. For that reason the (possible) symmetry breaking of the color  $SU(N_c)$  has been named “dynamical Higgs mechanism” in Ref. [15], and a parallel with the superconductivity has been drawn. The presently used term is “color superconductivity,” to which we shall adhere.

<sup>1</sup>In the original paper [11] a general case of the  $SU(4)$  breaking has been considered, leading to nine Goldstone particles. A closer inspection shows, however, that the instanton-induced interactions possess an additional degeneracy, leading to only five Goldstone particles, instead of nine which would be the general case. Instantons apparently “know” about the Vafa-Witten theorem which guarantees that the symmetry-breaking pattern at  $N_c=2$  corresponds to five Goldstone particles [12].

For the zero chemical potential only a metastable diquark-condensed state has been found at  $N_c \geq 3$  [15]. Furthermore, at  $N_c = 3$  the scalar diquark already appears to be unbound in the vacuum, indicating that our world is in a sense closer to the idealized  $N_c \rightarrow \infty$  than to the  $N_c = 2$  world. Parametrically, the diquark mass is  $\sim 1/\bar{\rho} \sim 1$  GeV where  $\bar{\rho}$  is the average instanton size as explained below. This means that a scalar diquark correlation function should decay with the exponent corresponding to the ‘‘constituent’’ quark threshold  $2M(0) \approx 700$  MeV, which seems to be supported by recent lattice measurements [16]. Nevertheless, it has been suggested in Ref. [15] that  $qq$  condensates could be found as metastable states in heavy-ion collisions and in astrophysics.

More recently it has been argued by the Princeton-MIT and Stony Brook groups [13,17], both using instantons as a framework, that taking nonzero fermion density shifts the balance in favor of the diquark condensation, and that at a certain critical chemical potential  $\mu_c$  there should be a phase transition from the usual broken chiral phase to the color superconducting state.<sup>2</sup> Avoiding some of the unnecessary approximations made in the works of those before us [13,17,19], we arrive essentially at the same conclusions [14]. Our results share with these previous studies their primary feature, which is a competition between phases which spontaneously break different symmetries of QCD.

In this paper we study the competition between various quark channels in a more systematic way than in our previous publication [14]. Since ‘‘color superconductivity’’ implies color symmetry is broken, one can imagine several phases with chiral symmetry broken or restored for quarks of different colors. We explore these possibilities using what amounts to a virial expansion in the instanton density, and carrying out detailed calculations to first order. This truncation effectively constrains us to working in a density-independent instanton background, an approximation which loses validity for a quark chemical potential around 600 MeV [20]. Beyond this point one expects the instantons to be suppressed and perturbative QCD to become applicable at asymptotically high density. Thus our intent is to consider phenomena which occur at a chemical potential of a few hundred MeV.

The chiral broken phase is characterized by a nonzero order parameter  $\langle \bar{q}q \rangle \neq 0$  or, equivalently, by a nonzero dynamical or ‘‘constituent’’ quark mass which in fact is momentum dependent. We find that, as far as one remains inside this phase, the quark occupation number  $n(p)$  is a perfect Fermi step function, despite strong energy and momentum-dependent interactions between quarks. It should be contrasted to the color superconducting phase with  $\langle qq \rangle \neq 0$  but  $\langle \bar{q}q \rangle = 0$ . In this case the quark occupation number  $n(p)$  is distorted near the Fermi surface and is of a typical BCS type.

In our numerics we use the standard characteristics of the

instanton ensemble. They lead to very reasonable values of the quark condensate and of the constituent quark mass at zero chemical potential. However, the same instanton characteristics lead inevitably to a very early transition to the superconducting state: it occurs at quark densities less than the normal nuclear density; after the system jumps into the superconducting phase the density appears to be about twice the nuclear density. Taken literally, it suggests that the ordinary nuclear matter is a ‘‘boiling’’ mixture of the two phases [19], however, it most probably contradicts the lore of conventional nuclear theory.<sup>3</sup> To ‘‘save’’ the nuclear matter, one would probably need to go beyond the mean-field approximation actually used in this paper.

## II. QCD INSTANTON VACUUM

The construction of the QCD instanton vacuum has been reviewed in previous publications [1,14], therefore here we simply review the main steps.

The use of the Feynman variational principle applied to instantons leads to the stabilization of the grand canonical instanton ensemble, with the main characteristics of the instanton medium, namely, the average four-dimensional instanton density,  $N/V = 1/\bar{R}^4$ , and the average instanton size  $\bar{\rho}$ , expressed through the only dimensional quantity  $\Lambda_{\text{QCD}}$ . The two-loop calculations performed in [5,6] give

$$\bar{\rho} = \sqrt{\bar{\rho}^2} \approx \frac{0.48}{\Lambda_{\overline{MS}}}, \quad \bar{R} \approx \frac{1.35}{\Lambda_{\overline{MS}}}. \quad (1)$$

Taking  $\Lambda_{\overline{MS}} = 280$  MeV one finds  $\bar{\rho} \approx 0.35$  fm,  $\bar{R} \approx 0.95$  fm,  $\bar{\rho}/\bar{R} \approx 1/3$ . This small ratio has been previously suggested on phenomenological grounds by Shuryak [21]. The smallness of the  $\bar{\rho}/\bar{R}$  ratio implies that the packing fraction of instantons in the vacuum, i.e., the fraction of the  $4d$  volume occupied by the balls of radius  $\bar{\rho}$ , is quite small:

$$f = \frac{\pi^2}{2} \bar{\rho}^4 \frac{N}{V} = \frac{\pi^2}{2} \frac{\bar{\rho}^4}{\bar{R}^4} \sim \frac{1}{10}. \quad (2)$$

Though the packing fraction (2) is but numerically small, one can treat it as a formal algebraic parameter, and develop a perturbation theory in it. Taking nonzero matter density and/or nonzero temperature can only decrease the ratio  $\bar{\rho}/\bar{R}$ .

When one switches in light quarks on top of the instanton ensemble, the existence of the small packing fraction parameter enables one to separate contributions of high and low fermion eigenmodes. The high-frequency part is controllably factorizable into contributions from individual instantons. This part can be seen to renormalize somewhat the one-instanton weight or instanton ‘‘fugacity.’’ The low-frequency part is dominated by the would-be zero modes of individual instantons. The normalized (that is localized) zero

<sup>2</sup>See the Proceedings of the International Workshop of QCD at Finite Baryon Density (Bielefeld, Germany), where these matters have been extensively discussed [18].

<sup>3</sup>We take an opportunity to thank G. Brown and K. Langanke for an instructive discussion of this point.

modes in the background of individual instantons exist both in the vacuum ( $\mu=0$ ) [22] and at  $\mu \neq 0$  [23,20], see the Appendix. In the ensemble of instantons and anti-instantons they cease to be exactly zero modes (that is why we call them the ‘‘would-be zero modes’’). The spectral density of the Dirac operator at small eigenvalues is obtained through the diagonalization of the matrix made of the overlaps of the would-be zero modes belonging to individual instantons and anti-instantons [24]. A nonzero spectral density at zero Dirac eigenvalue signals chiral symmetry breaking: it is actually due to the delocalization of the would-be zero modes owing to the ‘‘hopping’’ of quarks from one instanton to another [25].

In fact, three seemingly different but actually equivalent methods have been developed to describe chiral symmetry breaking by instantons at zero  $\mu$ : (i) diagonalization of the random matrix made of the would-be zero modes’ overlaps [24,26]; (ii) finding the quark propagator in the random instanton ensemble [25,27], which exhibits the appearance of a momentum-dependent quark mass; (iii) derivation of the effective low-momentum theory for quarks with instanton-induced interactions [1,28].

The three approaches underline different sides of the physics involved, though mathematically they prove to be equivalent. It is straightforward to generalize them to the case of the nonzero chemical potential. In this paper we use only the third method, it being the most economical.

### III. EFFECTIVE FERMION ACTION FOR FINITE DENSITY

With the instanton solution representing a dominant, stable fluctuation of the gauge field, the original strategy for computing quark observables in the QCD vacuum was to calculate the quantity in the presence of a random instanton background and then average over an ensemble of such field configurations. However, the analysis here calls for a more transparent manifestation of the effects from instantons in order to resolve the possible mechanisms for symmetry breaking. This is obtained by first averaging over the ensemble of instanton and anti-instanton configurations to formulate an effective theory in terms of interacting quarks, where explicit instanton effects are absorbed into the form of the interaction.

This evidence of averaged instantons is retained in the interaction through the would-be fermion zero modes, one for each flavor. Thus the consequent interaction is a vertex involving  $2N_f$  quarks, commonly cited as a ’t Hooft interaction after the first author to specify their proper quantum numbers [22]. For two flavors, it may be cast to resemble the Vaks-Larkin or Nambu–Jona-Lasinio (NJL) model which has long been a popular model for quark and hadron phenomenology.

The field of a single instanton–anti-instanton determines a particular solution for the gauge field  $A_I^\mu$ , and in the absence of quantum fluctuations the zero mode  $\Phi(x, \mu)$ , is simply the solution of the Dirac equation with zero eigenvalue:

$$(i\partial - i\mu\gamma_4 - A_I)\Phi(x, \mu) = 0 \cdot \Phi(x, \mu) = 0. \quad (3)$$

The exact solution for nonzero  $\mu$  was found by Abrikosov [23] and is written in the Appendix. It can be decomposed into chiral components as

$$\Phi(x, \mu) = \begin{bmatrix} \Phi_{\bar{l}}(x, \mu) \\ \Phi_l(x, \mu) \end{bmatrix}, \quad (4)$$

meaning that a (right) left-handed zero mode is generated by an (anti-) instanton. This is a result of the instanton solution’s structure and the self-dual equations from which it comes. With  $\tilde{\Phi}_{l(\bar{l})}$  we will denote the conjugate zero mode, which takes the chemical potential argument of the opposite sign:  $\tilde{\Phi}_l(x, \mu) = \Phi_{\bar{l}}^\dagger(x, -\mu)$ , where the dagger means the Hermitian conjugate. This is a direct consequence of the non-Hermiticity of the  $\mu$  dependence which arises in the Dirac operator.

This solution is not physically realizable, since the QCD background is not modeled by one but many (a ‘‘liquid’’ of) instantons. Localized around each there is a would-be zero mode, from which the effective action is constructed. This action is determined by building a low-momentum partition function, the foundation of which is the Green function for a quark in the field of one instanton. It can be expressed as a sum over the complete set of eigenfunctions, and approximated as

$$\begin{aligned} S^l(x, y) &\equiv \langle \psi(x) \psi^\dagger(y) \rangle = - \sum_n \frac{\psi_n(x) \psi_n^\dagger(y)}{\lambda_n + im} \\ &\approx - \frac{\Phi(x) \tilde{\Phi}(y)}{im} + S_0(x, y; \mu). \end{aligned} \quad (5)$$

At low momenta this propagator is dominated by the zero modes while at large momenta it is essentially reduced to the free one,  $S_0(x, y; \mu)$ . Therefore, Eq. (5) can be regarded as an interpolation of the true propagator in the field of one instanton. It becomes exact both at large,  $p \gg 1/\bar{\rho}$ , and small,  $p \ll 1/\bar{\rho}$ , quark momenta.

The partition function which produces the necessary propagator (5) is of the form

$$\begin{aligned} \mathcal{Z} &= \int D\psi D\psi^\dagger \exp \left[ \sum_f \int \psi_f^\dagger (i\partial - i\mu\gamma_4) \psi_f \right] \\ &\times \left( \frac{Y_{N_f}^+}{V} \right)^{N_+} \left( \frac{Y_{N_f}^-}{V} \right)^{N_-}. \end{aligned} \quad (6)$$

The pre-exponential factors contain the instanton-induced interactions between fermions, and for  $N_f$  flavors are the specific nonlocal  $2N_f$ -fermion vertices

$$\begin{aligned} Y_{N_f}^\pm[\psi, \psi^\dagger] &= (-)^{N_f} \int d^4z dU \prod \int d^4x d^4y \psi_{L,R}^\dagger(x) \\ &\times (i\partial - i\mu)^\mp \tilde{\Phi}_{\bar{l},l}(x-z) \tilde{\Phi}_{\bar{l},l}(y-z) \\ &\times (i\partial - i\mu)^\pm \psi_{L,R}(y). \end{aligned} \quad (7)$$

Here we use the notation  $x^\pm = x^\mu \sigma_\mu^\pm$ , where the  $2 \times 2$  matrices  $\sigma_\mu^\pm = (\pm i \vec{\sigma}, 1)$  decompose the Dirac matrices into chiral components, and it is understood that  $\mu$  written as a four vector is  $\mu_\alpha = (\vec{0}, \mu)$ . Note that the zero mode in the field of an (anti) instanton couples to that of a (right-) left-handed quark, and the effective range of the interaction is the average instanton size  $\bar{\rho}$ .

It should be stressed that correlations between instantons induced by fermions are inherent in this approach; as to correlations induced by gluons, they are effectively taken care of by the use of the variational principle [5,6] resulting in the effective size distribution. For simplicity we freeze all the sizes at the average value  $\bar{\rho}$ , but average explicitly over random position ( $z$ ) and orientation ( $U$ ) variables.

Fermion operators in the pre-exponent are not convenient; these operators can be raised into the exponent with the help of a supplementary integration over a pair of Lagrange multipliers, denoted  $\lambda_\pm$ :

$$\begin{aligned} \mathcal{Z} = & \int d\lambda_+ d\lambda_- \int D\psi D\psi^\dagger \exp \left\{ \int d^4x \psi^\dagger (i\partial - i\mu\gamma_4) \psi \right. \\ & + \lambda_+ Y_{N_f}^+ + \lambda_- Y_{N_f}^- + N_+ \left( \ln \frac{N_+}{\lambda_+ V} - 1 \right) \\ & \left. + N_- \left( \ln \frac{N_-}{\lambda_- V} - 1 \right) \right\}. \end{aligned} \quad (8)$$

Indeed, integrating over  $\lambda_\pm$  by the saddle-point method one recovers Eq. (6). Notice that the saddle-point integration becomes exact in the thermodynamic limit:  $N_\pm, V \rightarrow \infty$  with  $N/V$  fixed. Through this procedure we obtain a purely exponential integrand which is the required effective action.

Two important consequences follow and should be emphasized. First, the coupling constant of the  $2N_f$ -fermion interaction, whose role is played by  $\lambda_\pm$ , is not fixed once and forever; its value is found from minimizing the free energy *after* integration over fermions is performed. Therefore, the strength of the interaction depends itself on the phase the fermion system assumes. Second, the saddle-point value of  $\lambda_\pm$  is *not* proportional to the instanton density, as one might naively think. For example, in the case of chiral symmetry breaking it behaves as  $\lambda \sim (N/V)^{1-N_f/2}$  [28]. Both circumstances are due to the peculiarity of the instanton-induced interactions dominated by the existence of the zero modes. Would these modes remain zero, the fermion determinant would be zero, thus suppressing the presence of instantons themselves. It is this intricate fermion self-supporting mechanism which makes the instanton-induced interactions different from a more naive NJL model where the strength of the effective four-fermion interactions are chosen once and forever. As will be shown below, this is the mechanism by which chiral symmetry is restored at the large chemical potential in the case of two flavors.

For practical applications it is favorable to use the Fourier-transformed expressions of the quark zero modes, which are written explicitly in the Appendix. These complex

functions of the four-momenta and chemical potential determine the (matrix) form factors attached to each fermion leg of the vertex:

$$\begin{aligned} \mathcal{F}(p, \mu) &= (p + i\mu)^- \varphi(p, \mu)^+, \\ \mathcal{F}^\dagger(p, -\mu) &= \varphi^*(p, -\mu)^- (p + i\mu)^+. \end{aligned} \quad (9)$$

With such definitions, the interaction terms may be written in momentum space,

$$\begin{aligned} Y^+[\psi, \psi^\dagger] &= \int dU \int \prod_f^{N_f} \left[ \frac{(d^4 p_f d^4 k_f)}{(2\pi)^8} \right] (2\pi)^4 \delta^4 \\ & \times \left( \sum (p_f - k_f) \right) \prod_f^{N_f} [\psi_{L f \alpha_j}^\dagger(p_f) \\ & \times \mathcal{F}(p_f, \mu)_{k_f}^{i_f} \epsilon_{k_f l}^{j_f} U_{l_f}^{\alpha_f} U_{\beta_f}^{\dagger o_f} \epsilon_{n_f o_f} \\ & \times \mathcal{F}^\dagger(k_f, -\mu)_{p_f}^{n_f} \psi_{L f}^{\beta_f} \epsilon_{p_f}^{j_f}(k_f)], \end{aligned} \quad (10)$$

with a similar form for  $Y^-$  which carries right-handed quarks. The first indices on the fermion operators refer to flavor, the Greek to color ( $1, \dots, N_c$ ), and the last denote spin (1,2). This formulation of the effective interaction retains the full  $p$  and  $\mu$  dependence of the zero modes, as opposed to the approximate treatments in other recent works [13,17]. At the same time this formalism enables one to reduce the problem of determining the phase structure to algebraic equations where the coefficients are given by certain integrals over the form-factor functions (9).

#### IV. ONE LIGHT FLAVOR (A PEDAGOGICAL ASIDE)

In the case of one flavor, the effective action assumes a relatively simple and hence instructive form. The form factors combine and appear as an effective mass term. The interaction remains nonlocal, thus one finds an induced mass  $\mathcal{M}$  with a specific momentum dependence:

$$\begin{aligned} S_{\text{INT}} &= -\lambda_+ \int \frac{d^4 p}{(2\pi)^4} \int dU \psi_{L \alpha i}^\dagger(p) \mathcal{F}(p, \mu)_{i_j}^j \\ & \times \epsilon^{jk} U_k^\alpha U_\beta^{\dagger l} \epsilon_{ml} \mathcal{F}^\dagger(p, -\mu)_n^m \psi_L^{\beta n}(p) - \lambda_- (L \leftrightarrow R) \\ & = \mathcal{M}(p, \mu) \psi^\dagger(p) \psi(p), \end{aligned} \quad (11)$$

where

$$\mathcal{M}(p, \mu) = \lambda (p + i\mu)_\alpha (p + i\mu)_\alpha \varphi_\beta(p, \mu) \varphi_\beta(p, \mu). \quad (12)$$

At finite  $\mu$  the form factors are complex, since the zero modes are solutions of Eq. (3). This interaction is sufficient to spontaneously break chiral symmetry. However, the coupling constant  $\lambda$  has here been introduced as a Lagrange multiplier in order to obtain a manageable form for the effective action, and is, in principle, an integration over all possible coupling strengths. This simplifies to a saddle-point approximation which becomes exact in the thermodynamic

limit of  $N, V \rightarrow \infty$ . Such an evaluation of the integral also naturally connects the instanton density to this coupling, in that the resulting ‘‘gap’’ equation is

$$\frac{N}{V} = 4N_c \int \frac{d^4 p}{(2\pi)^4} \frac{\mathcal{M}(p, \mu)^2}{(p + i\mu)^2 + \mathcal{M}(p, \mu)^2}. \quad (13)$$

This equation is a direct generalization of the gap equation found for  $\mu = 0$  in Ref. [25]. Although  $\mathcal{M}$  and the  $\mu$  term are complex, one finds that the imaginary part of the integrand is odd in  $p_4$  and hence vanishes under energy integration. This is equivalent to the statement that while individual eigenvalues of the Dirac operator are imaginary, their product remains real. By preserving the exact form of the zero-mode solution, this property becomes manifest in our treatment. Through solving this equation self-consistently, one obtains a solution for the  $\lambda$  at any given quark chemical potential  $\mu$ . This in turn is proportional to an effective mass evaluated in the zero momentum limit,  $|\mathcal{M}(0, \mu)| = (2\pi\bar{\rho})^2 \lambda / N_c$ .

To first order in  $\lambda$ , this procedure amounts to averaging the effects of every instanton and anti-instanton which modify the quark propagator. In principle, the instanton density  $N/V$  is modified by a finite quark density. It scales with the fugacity,  $\chi \equiv Z(\mu)Z(0)^{-1}$ , as  $\chi^{4/b}$ , where  $b$  is the Gell-Mann-Low coefficient of QCD. Fugacity not equal to unity simply arises from the change with  $\mu$  of the fermion determinant in the partition function. Upon a systematic expansion of this determinant in  $\lambda$  one finds the first-order term vanishes, for this is exactly the saddle-point condition of Eq. (13). Thus modifications of the instanton background appear only when the determinant is expanded to order  $\lambda^2$ , which is beyond the scope of this work.

Without a first-order effect on  $N/V$ , there exists no mechanism in the theory to suppress the instanton background and modify the form of the gap equation above. While a finite quark chemical potential can and does modify the coupling strength and hence the effective mass, the coupling is never forced to zero. Therefore, chiral symmetry remains broken at any finite baryon density. This result can of course change when effects of higher order in the instanton density are taken into account. The picture is also radically altered with more than one flavor, as possible quark pairing offers an alternative mechanism for chiral symmetry restoration.

## V. TWO LIGHT FLAVORS

The remainder of this paper investigates the case of massless  $N_f = 2$ , corresponding to a system of chiral up and down quarks. Furthermore, we will assume the  $CP$  invariant case of  $\theta = 0$ , which requires  $N_+ = N_- = N/2$  and hence  $\lambda_+ = \lambda_- = \lambda$ . With the definitions and notation of Eqs. (9) and (10), the interaction terms may be written in momentum space,

$$\begin{aligned} \lambda Y^+ = & \lambda \int \frac{d^4 p_1 d^4 p_2 d^4 k_1 d^4 k_2}{(2\pi)^{16}} (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) \\ & \times \int dU \psi_{L1\alpha_1 i_1}^\dagger(p_1) \mathcal{F}(p_1, \mu)_{k_1}^{i_1} \epsilon^{k_1 l_1} \\ & \times U_{l_1}^{\alpha_1} U_{\beta_1}^{\dagger o_1} \epsilon_{n_1 o_1} \mathcal{F}^\dagger(k_1, -\mu)_{p_1}^{n_1} \psi_{L1}^{\beta_1 p_1}(k_1) \psi_{L2\alpha_2 i_2}^\dagger(p_2) \\ & \times \mathcal{F}(p_2, \mu)_{k_2}^{i_2} \epsilon^{k_2 l_2} U_{l_2}^{\alpha_2} U_{\beta_2}^{\dagger o_2} \epsilon_{n_2 o_2} \\ & \times \mathcal{F}^\dagger(k_2, -\mu)_{p_2}^{n_2} \psi_{L2}^{\beta_2 p_2}(k_2), \end{aligned} \quad (14)$$

with a similar form for  $\lambda Y^-$ .

### A. Gap equations

Since the instanton-induced interactions (14) support both  $\bar{q}q$  and  $qq$  condensation, it is necessary to consider the two competing channels simultaneously. This means that one must calculate both the normal ( $S$ ) and anomalous ( $F$ ) quark Green functions. A color-flavor-spin ansatz compatible with the possibility of chiral and color symmetry breaking is

$$\begin{aligned} \langle \psi^{f\alpha i}(p) \psi_{g\beta j}^\dagger(p) \rangle &= \delta_g^f \delta_\beta^\alpha S_1(p)_j^i \quad \text{for } \alpha, \beta = 1, 2, \\ \langle \psi^{f\alpha i}(p) \psi_{g\beta j}^\dagger(p) \rangle &= \delta_g^f \delta_\beta^\alpha S_2(p)_j^i \quad \text{for } \alpha, \beta > 2, \\ \langle \psi_L^{f\alpha i}(p) \psi_L^{g\beta j}(-p) \rangle &= \langle \psi_R^{f\alpha i}(p) \psi_R^{g\beta j}(-p) \rangle \\ &= \epsilon^{fg} \epsilon^{\alpha\beta\gamma} \epsilon^{ij} F(p), \end{aligned} \quad (15)$$

where  $[\gamma]$  refers to some generalized direction(s) in color space, and it is this set of  $N_c - 2$  indices which signals the breaking of color symmetry. In the particular case of  $N_c = 3$ , where the color symmetry is broken as  $SU(3) \rightarrow SU(2) \times U(1)$  and our ansatz considers the  $\bar{3}$  channel [9], we will by convention take  $[\gamma] = 3$ ; for  $N_c = 4$  one can take  $[\gamma] = 34$  and so forth. In the event of color symmetry breaking, the standard propagators (and ensuing condensates) will lose their color degeneracy and the separation of  $S(p)$  into  $S_1(p)$  and  $S_2(p)$  becomes necessary; otherwise the Schwinger-Dyson-Gorkov equations do not close.

Written in the chiral  $L, R$  basis, the  $4 \times 4$  propagators  $S_{1,2}(p)$  are of the form

$$S(p) = \begin{bmatrix} G(p)\mathbf{1} & Z(p)\mathbf{S}_0(p)^+ \\ Z(p)\mathbf{S}_0(p)^- & G(p)\mathbf{1} \end{bmatrix}. \quad (16)$$

Here the off-diagonal, bare propagator  $\mathbf{S}_0(p)^\pm = [(p + i\mu)^\mp]^{-1}$  is modified by the scalar functions  $Z_{1,2}(p)$ , and is augmented on the diagonal by the scalar  $G_{1,2}(p)$  which, if nonzero, break chiral symmetry.

Using the instanton-induced interaction (14) one can build a systematic expansion for the  $F, G$  Green functions in the  $1/N_c$  and  $\bar{\rho}/\bar{R}$  parameters. In the leading order in both parameters we restrict ourselves to the one-loop approximation shown in Fig. 1. It corresponds to a set of self-consistent Schwinger-Dyson-Gorkov equations. An important  $\mu$  depen-

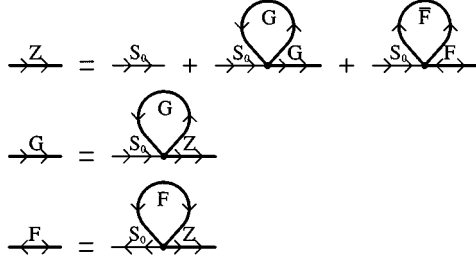


FIG. 1. Schwinger-Dyson-Gorkov diagrams to first order in  $\lambda$ , corresponding to five scalar equations in the text. The first is for the off-diagonal and the second for the diagonal components of  $S_1$  and  $S_2$ , the third is the anomalous diquark propagator.

dence enters through the form factors in Eq. (14). With  $\bar{F}(p) = F^*(p)$  these diagrams lead to the set of five algebraic equations for the scalar Green functions  $Z_{1,2}$ ,  $G_{1,2}$ , and  $F$ :

$$\begin{aligned} Z_1(p) &= 1 - G_1(p)A(p, \mu)M_1 - 2F(p)B(p, \mu)\Delta, \\ Z_2(p) &= 1 - G_2(p)A(p, \mu)M_2, \\ G_1(p) &= Z_1(p)\varphi_\alpha(p, \mu)\varphi_\alpha(p, \mu)M_1, \\ G_2(p) &= Z_2(p)\varphi_\alpha(p, \mu)\varphi_\alpha(p, \mu)M_2, \\ F(p) &= 2Z_1(-p)\varphi_\alpha(p, \mu)\varphi_\alpha(-p, \mu)\Delta. \end{aligned} \quad (17)$$

The constants  $M_1$ ,  $M_2$ , and  $\Delta$  will be defined below and the functions

$$\begin{aligned} A(p, \mu) &= (p + i\mu)_\alpha (p + i\mu)_\alpha \varphi_\beta(p, \mu)\varphi_\beta(p, \mu), \\ B(p, \mu) &= (p^2 + \mu^2)\varphi_\beta(p, \mu)\varphi_\beta(-p, \mu) \\ &\quad + (p + i\mu)_\alpha \varphi_\alpha(p, \mu)(p - i\mu)_\beta \varphi_\beta(-p, \mu) \\ &\quad - (p + i\mu)_\alpha \varphi_\alpha(-p, \mu)(p - i\mu)_\beta \varphi_\beta(p, \mu), \end{aligned} \quad (18)$$

are the form factors which arise from the zero modes (see the Appendix). At  $\mu=0$  we have  $A(p, 0) = B(p, 0)$ , but for any finite  $\mu$  the direction of the momentum flow through each vertex leg is critical.

The condensates  $g_1$ ,  $g_2$ , and  $f$  are the closed loops contributing to the quark self-energy. They are found by integrating the appropriate Green function, modified by the vertex form factors, over an independent momentum:

$$\begin{aligned} g_{1,2} &= \frac{\lambda}{N_c^2 - 1} \int \frac{d^4k}{(2\pi)^4} A(k, \mu) G_{1,2}(k), \\ f &= \frac{\lambda}{N_c^2 - 1} \int \frac{d^4k}{(2\pi)^4} B(k, \mu) F(k). \end{aligned} \quad (19)$$

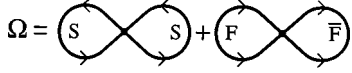
The quantities  $M_{1,2}$  and  $\Delta$  are linear combinations of the condensates  $g_{1,2}$  and  $f$ :

$$\begin{aligned} M_1 &= \left(5 - \frac{4}{N_c}\right)g_1 + \left(2N_c - 5 + \frac{2}{N_c}\right)g_2, \\ M_2 &= 2\left(2 - \frac{1}{N_c}\right)g_1 + 2(N_c - 2)g_2, \\ \Delta &= \left(1 + \frac{1}{N_c}\right)f. \end{aligned} \quad (20)$$

The  $M_{1,2}$  are measures of chiral symmetry breaking, which act as an effective mass modifying the standard quark propagation. On the other hand, the diquark loop  $2\Delta$  plays the role of twice the single-quark energy gap formed around the Fermi surface. The Fermi momentum, in the absence of chiral symmetry breaking, will remain at  $p_f = \mu$  regardless of the magnitude of  $\Delta$ . It should be noted that the effective masses  $M_{1,2}$  exhibit different behavior in  $N_c$  than the superconducting gap  $\Delta$ : the former are  $N_c$  times larger than the latter. Although, one can of course express a solution in terms of  $(g_1, g_2, f)$  as one in  $(M_1, M_2, \Delta)$ , we will retain both notations in the following text. The reason is the distinct physical interpretations for the two variable sets: the first quantifies the symmetry breaking by channel while the second measures its effects on the quark states with particular color.

After determining the five scalar functions through solving Eqs. (17) and inserting these solutions into Eqs. (19), we find the coupled equations for the condensates themselves:

$$\begin{aligned} g_1 &= \frac{\lambda M_1}{N_c^2 - 1} \int \frac{d^4k}{(2\pi)^4} \frac{\alpha(p, \mu)[1 - 4\beta(p, \mu)\Delta^2 + \alpha^*(p, \mu)M_1^2]}{[1 + \alpha(p, \mu)M_1^2][1 + \alpha^*(p, \mu)M_1^2] - 16\beta(p, \mu)^2\Delta^4}, \\ g_2 &= \frac{\lambda M_2}{N_c^2 - 1} \int \frac{d^4k}{(2\pi)^4} \frac{\alpha(p, \mu)}{1 + \alpha(p, \mu)M_2^2}, \\ f &= \frac{2\lambda\Delta}{N_c^2 - 1} \int \frac{d^4k}{(2\pi)^4} \frac{\beta(p, \mu)[1 - 4\beta(p, \mu)\Delta^2 + \alpha(p, \mu)M_1^2]}{[1 + \alpha(p, \mu)M_1^2][1 + \alpha^*(p, \mu)M_1^2] - 16\beta(p, \mu)^2\Delta^4}, \end{aligned} \quad (21)$$


 FIG. 2. Contributing diagrams to  $\Omega$  at order  $\lambda^1$ .

where yet another pair of functions has been introduced:

$$\begin{aligned}\alpha(p, \mu) &= A(p, \mu) \varphi_\alpha(p, \mu) \varphi_\alpha(p, \mu), \\ \beta(p, \mu) &= B(p, \mu) \varphi_\alpha(p, \mu) \varphi_\alpha(-p, \mu).\end{aligned}\quad (22)$$

Note that while  $\beta$  is real, the function  $\alpha$  is complex. Although the integrands above are complex, all imaginary parts are odd in  $p_4$  and thus vanish under integration. As usual for gap equations, there is the possibility of a solution where some (or even all) of the condensates vanish.

The magnitudes of these condensates are not yet determined, as this requires fixing the coupling constant  $\lambda$ . This is done through minimizing the partition function (8) and leads to the saddle-point condition on  $\lambda$ , replacing the integration with

$$\frac{N}{V} = \lambda \langle Y^+ + Y^- \rangle. \quad (23)$$

On the left-hand side is the instanton density, which we will here take to be fixed at its vacuum value, although in principle it will have some correction due to the finite quark density. Evaluating the right-hand side requires calculating the one-vertex contributions to the free energy, which in this case includes two-loop, figure-eight-type diagrams formed by joining the four fermion legs into two pairs as shown in Fig. 2. This can be written concisely in terms of the condensates:

$$\begin{aligned}\frac{N}{V} = \lambda \langle Y^+ + Y^- \rangle &= \frac{4(N_c^2 - 1)}{\lambda} \\ &\times [2g_1 M_1 + (N_c - 2)g_2 M_2 + 4f\Delta].\end{aligned}\quad (24)$$

As with the single flavor case, we are quenching by truncating our perturbative treatment in  $\lambda$ , and thus the instanton density on the left is held fixed. This equation for  $\lambda$  and the definitions of the condensates (21) comprise a system of equations which can be solved for all quantities. It follows from Eqs. (20), (21), and (24) that the mass and energy gaps  $M_{1,2}$  and  $\Delta$  are proportional to the *square root* of the instanton density  $N/V$ , while the saddle-point value of  $\lambda$  is, to the first approximation, independent of  $N/V$ .

Once these quantities are found the chiral condensate may be calculated, being a trace over the quark propagator. This is distinct from the effective masses  $M_{1,2}$ , as it does not contain contributions from the form factors present in the

loops from which the effective masses are computed. Present instead, is the solution for the propagator, in particular its diagonal elements:

$$\begin{aligned}-\langle \bar{\psi} \psi \rangle_{\text{Mink}} &= i \langle \psi^\dagger \psi \rangle_{\text{Eucl}} = i \int \frac{d^4 p}{(2\pi)^4} \text{Tr}[S(p)] \\ &= 4 \int \frac{d^4 p}{(2\pi)^4} [2G_1(p) + (N_c - 2)G_2(p)].\end{aligned}\quad (25)$$

As with the previous integrals, the integrand reflects in  $p_4$  such that the result is guaranteed real.

## B. Thermodynamic competition

As a consequence of having two different modes of quark condensation, one obtains multiple solutions for the characteristics of the quark medium at any fixed chemical potential. Specifically, there will be a competition between the following phases.

(0) *Free massless quarks*:  $g_1 = g_2 = f = 0$ .

(1) *Pure chiral symmetry breaking*:  $g_1 = g_2 \neq 0$ ,  $f = 0$ .

This is the standard vacuum scenario.

(2) *Pure diquark condensation*:  $g_1 = g_2 = 0$ ,  $f \neq 0$ . This is the ‘‘color superconducting’’ phase, and leaves quarks of color  $[\gamma]$  free, since they neither participate in the diquark formation nor exhibit broken chiral symmetry.

(3) *Mixed symmetry breaking*:  $g_1 \neq g_2 \neq 0$ ,  $f \neq 0$ . In fact, one need not have finite  $g_1$ , since it corresponds to chiral symmetry breaking by the quarks of transverse color to the diquark condensate; however, such a phase requires that there at least be chiral breaking in the parallel-colored quarks.

Phase (0), with all symmetries restored, would mean that the average four-fermion vertex,  $\langle Y^+ + Y^- \rangle$ , is zero. It would imply, via Eq. (8), that the saddle-point value of  $\lambda$  is infinite. Being substituted into Eq. (23), infinite  $\lambda$  means that the fermion determinant vanishes. Consequently, the whole instanton vacuum is severely suppressed, which means a large loss in the free energy. One can put it in another way: if the density of instantons  $N/V$  is considered fixed, there is no solution to the combined Eqs. (20),(21),(23) with all condensates being zero, at least in the one-loop approximation to the Schwinger-Dyson-Gorkov equation we are considering.

To resolve between the remaining, symmetry-breaking solutions the free energy is minimized. Consistent with the evaluation of the Green functions, it is calculated to first order in  $\lambda$ . Repeating the calculation of figure-eight diagrams and recalling the explicit dependence on  $\lambda$  in Eq. (8) we obtain the free energy

$$\begin{aligned} \frac{\Omega}{V_3} &= -\frac{1}{\beta V_3} \ln Z \\ &= \frac{\Omega_0}{V_3} - \frac{N}{V} \ln \left( \frac{N}{\lambda V} \right) + \frac{N}{V} - \frac{4(N_c^2 - 1)}{\lambda} \\ &\quad \times [2g_1 M_1 + (N_c - 2)g_2 M_2 + 4f\Delta]. \end{aligned} \quad (26)$$

Here  $V_3$  is the three volume, while  $V$  is the Euclidean four volume, and  $\Omega_0$  is the free energy for a gas of free quarks. The last two terms are precisely the quantity which must vanish under the saddle-point determination of  $\lambda$ , and thus we have

$$\frac{\Omega}{V_3} = \frac{\Omega_0}{V_3} + \frac{N}{V} \ln \left( \frac{\lambda}{N/V} \right). \quad (27)$$

Thus, the phase which features the *lowest* coupling  $\lambda$  is the thermodynamically favored one.

If we first concentrate on a competition between the two phases of simple chiral or color symmetry breaking, phase (1) and (2), the former is marked by nonzero  $g \equiv g_1 = g_2$  and the latter by nonzero  $f$ . Each case leads to its own value of  $\lambda$  through solving Eq. (24) for any given  $\mu$ . For the first, the gap equation reduces to

$$\lambda = \frac{8(N_c^2 - 1)g^2}{N/V}, \quad (28)$$

while for the second one finds

$$\lambda = \frac{16(N_c^2 - 1)(N_c + 1)f^2}{N_c(N/V)}. \quad (29)$$

At certain value of  $\mu = \mu_c$  these solutions for  $\lambda$  cross; this is the point where the phase transition occurs. This point is defined by the condition

$$\frac{f(\mu_c)}{g(\mu_c)} = \sqrt{\frac{N_c(N_c - 1)}{2}}. \quad (30)$$

When the ratio between the condensates is less than the constant on the right, phase (2) is favored; otherwise it is chiral symmetry that is spontaneously broken in phase (1). However, the phase structure is not so simple in general, considering the possible presence of phase (3). Calculations were carried out for  $N_c = 2$  and  $N_c = 3$ , taking the values for  $N/V$  and  $\bar{\rho}$  specified in Sec. II. From Eq. (30) one observes that at large  $N_c$  it becomes increasingly difficult to get to the color superconducting phase.

## VI. TWO COLORS

In this case it is obvious that color symmetry is not broken by diquark formation, which here corresponds to color-

singlet ‘‘baryons,’’ and hence there is only one possible chiral condensate  $g_1 = g$ . Not so obvious is that at  $\mu = 0$  the color and flavor SU(2) groups are arranged into the higher SU(4) symmetry. So no mixed phases exist; the theory has only one symmetry. The instanton vacuum accounts for this [11], and in the context of the analysis here this corresponds to  $f^2 + g^2$  being the only discernible quantity in the gap equations (21). Numerically, we find  $\sqrt{f^2 + g^2} = 147$  MeV.

At finite  $\mu$ , however, the SU(4) symmetry is explicitly broken and there is a thermodynamic competition between phases of pure chiral or diquark condensation. For all finite chemical potentials, numerical calculations reveal that  $f$  decreases faster than  $g$ . Since in this case the critical ratio of Eq. (30) is unity, we conclude that for any finite density the  $N_c = 2$  world prefers diquark condensation to chiral symmetry breaking. This finding is in agreement with the reasoning of Ref. [13] and the lattice results of Ref. [29].

## VII. THREE COLORS

With three colors, the more relevant case for QCD, the phase structure is richer due to the possible appearance of phase (3). Furthermore, the physics is naturally completely different, since here any diquark formation will spontaneously prefer a particular color. This dynamical consequence mimics the Higgs mechanism as a spontaneous breaking of the SU( $N_c$ ) gauge group.

### A. Numerical results

The system of gap equations is comprised of all three in Eq. (21), which are accompanied by the condition (24). For a given  $\mu$  a set of solutions can be obtained numerically, each of which corresponds to a phase of the quark matter characterized by the mass or energy gaps formed.

In the vacuum, where  $\mu$  strictly vanishes, there exist solutions for all three symmetry-breaking phases. The thermodynamically favored is one of spontaneous chiral symmetry breaking, and hence the equilibrium description of the vacuum in the instanton picture is recovered. Numerical calculations show that the constituent quark mass is  $M \equiv M_1 = M_2 = 346$  MeV, corresponding to  $\lambda^{(1)} = 0.311$ . Two metastable states are also present, these being manifestations of phases (2) and (3). They feature  $\lambda^{(2)} = 0.674$  and  $\lambda^{(3)} = 0.673$ , and are thus clearly only local minima of the free energy. It is noteworthy that the diquark gap in the metastable state is quite large:  $\Delta = 220$  MeV. This is in agreement with that found for  $\mu = 0$  in Ref. [15].<sup>4</sup>

Although it is an unstable state in vacuum and remains so as  $\mu$  is increased, the phase of mixed symmetry breaking is worthy of attention. It is marked by nonzero condensates  $g_2$  and  $f$ , which leads to not only a superconductivity energy gap but also finite and distinct effective quark masses,  $M_1$  and  $M_2$ . The magnitudes of these gaps are plotted as a function

<sup>4</sup>In that work the notation  $m_c = 2\Delta$  was used.



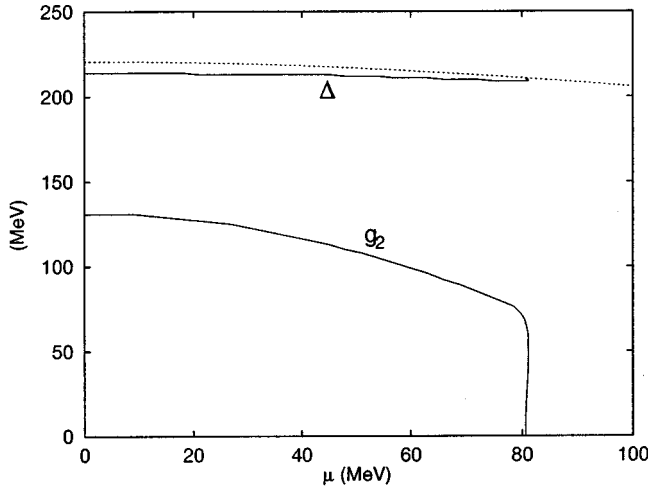


FIG. 3. Chiral and diquark gaps as a function of chemical potential in the phase of mixed symmetry breaking, (3). The dashed line is the phase (2) solution of the diquark energy gap.

of chemical potential in Fig. 3. In effect here is a separation between the phenomenology of quarks of different colors. Quarks orthogonal in color to the diquark condensate (colors 1 and 2) do not conspire to break chiral symmetry (so  $g_1 = 0$ ), and rather condense in color-asymmetric pairs. This leaves the quarks color parallel (color 3) to the diquark  $\bar{3}$  to spontaneously break chiral symmetry in the usual manner ( $g_2 \neq 0$ ). Thus, they acquire an effective mass due to self-interaction and bestow a mass upon the other two quarks through a similar interaction. Since the SU(3) color symmetry has been broken, these masses need not be equivalent, and in fact one finds that

$$M_1 = \frac{5}{3} g_2, \quad M_2 = 2 g_2. \quad (31)$$

In the vacuum,  $M_1 = 220$  MeV and  $M_2 = 260$  MeV. As the chemical potential increases, the coexisting diquark and chiral condensates compete for the coupling strength; this is the condition of Eq. (24). We find that at relatively low  $\mu$ , around 80 MeV, this limited resource becomes entirely consumed by the diquark. This is demonstrated in Fig. 3, where one notes the chiral condensate in the color-3 channel abruptly vanishes and the diquark condensate assumes its value of phase (2). Thus this solution is not only strictly a local minimum, but is also short-lived in density.

The remaining thermodynamic competition is thus between the two phases of pure chiral or color symmetry breaking. For three colors, the critical ratio of the condensates (30) is  $\sqrt{3}$ . At  $\mu=0$  we find  $f/g = 165 \text{ MeV}/65 \text{ MeV} > \sqrt{3}$ . It means that at low values of  $\mu$  the coupling constant  $\lambda$  is smaller in the chiral broken phase, hence this phase is energetically preferred. Furthermore, this is the  $\lambda$  which leads to a reasonable effective quark mass of  $M = 346$  MeV, as well as a chiral condensate of  $\langle \bar{\psi}\psi \rangle = -(255 \text{ MeV})^3$ . We thus recover the standard chiral symmetry breaking at low baryon density.

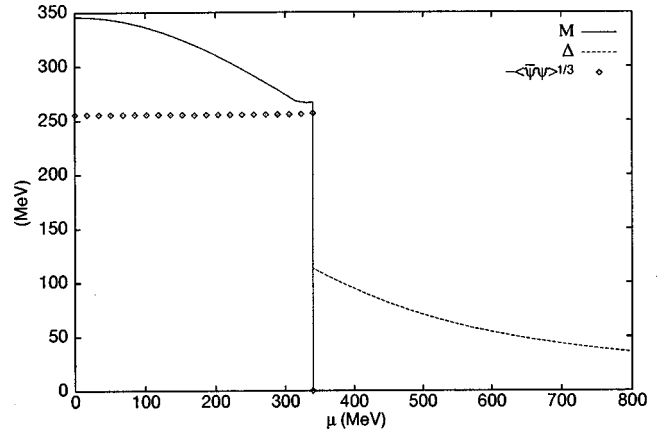


FIG. 4. Condensates for  $N_c=3$  as a function of  $\mu$ . Shown are the effective quark mass  $M$ , the quark condensate  $-\langle \bar{\psi}\psi \rangle^{1/3}$ , and the diquark energy gap per quark  $\Delta$ .

With increasing chemical potential both  $f$  and  $g$  condensates in their respective phases are reduced, however, as  $\mu$  surpasses the effective quark mass  $M$  around 300 MeV, the chiral-breaking  $g$  stabilizes while the color-breaking  $f$  continues to decrease in phase (2). At  $\mu \approx 340$  MeV, condition (30) is reached, and therefore for this and greater chemical potential chiral symmetry is restored and the ‘‘color superconducting’’ phase is realized. At the transition point, the quark gap is  $\Delta = 115$  MeV and slowly decreasing with rising density. Numerical results for the characteristic quantities of each phase are plotted in Fig. 4. It is noteworthy that the chiral condensate  $\langle \bar{\psi}\psi \rangle$  is distinct from the effective quark mass  $M$ , in that while the latter decreases with  $\mu$  the former remains practically at its vacuum value. For  $\mu > \mu_c$  they both vanish.

We note that other authors using different methods have found first-order chiral symmetry restoration at approximately the same chemical potential. One previous work obtained this via the same mechanism of diquark condensation in the context of the Nambu-Jona-Lasinio model [19], while others have found similar results when only analyzing the chiral sector [34,32].

## B. Properties of quark matter

The chemical potential is a useful concept only in that it constrains the particle number or density, the physical quantity we actually need to compute. The four-fermion interactions we are dealing with are somewhat unusual in that they are dependent on the energy, a consequence of using the zero-mode Green functions as the seed of the interaction.

For this reason we carefully define the density as

$$n = \int d^4x j_4(x), \quad (32)$$

and note that in this case one does not have the simple form  $j_\mu = \psi^\dagger \gamma_\mu \psi$  for the current which carries the conserved particle number. In our notation this corresponds to the current

of the high-momentum quarks, which without the zero modes would imply that the correct normalization is not preserved. We find the applicable current is

$$\begin{aligned}
j_\mu(q) = & -i\psi^\dagger(q)\gamma_\mu\psi(q) + i\lambda \int \frac{d^4p_1 d^4p_2 d^4k_1 d^4k_2}{(2\pi)^{16}} \\
& \times (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) \\
& \times \int dU \psi_{L1\alpha_1 i_1}^\dagger(p_1) \epsilon^{k_1 l_1} U_{l_1}^{\alpha_1} U_{\beta_1}^{\dagger o_1} \\
& \times \epsilon_{n_1 o_1} \psi_{L1\beta_1}^{\beta_1 p_1}(k_1) \psi_{L2\alpha_2 i_2}^\dagger(p_2) \epsilon^{k_2 l_2} \\
& \times U_{l_2}^{\alpha_2} U_{\beta_2}^{\dagger o_2} \epsilon_{n_2 o_2} \psi_{L2\beta_2}^{\beta_2 p_2}(k_2) \frac{\partial}{\partial q_\mu} \\
& \times [\mathcal{F}(p_1, \mu)_{k_1}^{i_1} \mathcal{F}^\dagger(k_1, -\mu)_{p_1}^{n_1} \mathcal{F}(p_2, \mu)_{k_2}^{i_2} \\
& \times \mathcal{F}^\dagger(k_2, -\mu)_{p_2}^{n_2}] + (L \leftrightarrow R), \tag{33}
\end{aligned}$$

where the bracketed momentum derivative is understood to be a sum of four terms, each with one form factor differentiated with respect to its momentum argument.

Although there has been some rearrangement, this is essentially the vertex (14) in which the form factors have been differentiated with respect to the momentum  $q_\mu$ . One can verify this current is conserved by considering the Dirac equation, written here for the left-handed, flavor-1 component:

$$\begin{aligned}
[(p+i\mu)^-]_j^i \psi_L^{1\alpha j}(q) - \lambda \int \frac{d^4p_2 d^4k_1 d^4k_2}{(2\pi)^{12}} (2\pi)^4 \\
\times \delta(q+p_2-k_1-k_2) \int dU \mathcal{F}(q, \mu)_{k_1}^i U_{l_1}^\alpha U_{\beta_1}^{\dagger o_1} \epsilon_{n_1 o_1} \\
\times \mathcal{F}^\dagger(k_1, -\mu)_{p_1}^{n_1} \psi_{L1\beta_1}^{\beta_1 p_1}(k_1) \psi_{L2\alpha_2 i_2}^\dagger(p_2) \mathcal{F}(p_2, \mu)_{k_2}^{i_2} \epsilon^{k_2 l_2} \\
\times U_{l_2}^{\alpha_2} U_{\beta_2}^{\dagger o_2} \epsilon_{n_2 o_2} \mathcal{F}^\dagger(k_2, -\mu)_{p_2}^{n_2} \psi_{L2\beta_2}^{\beta_2 p_2}(k_2) = 0. \tag{34}
\end{aligned}$$

Acting on this with  $q_\mu \psi_{L1}^\dagger(q)(\partial/\partial q_\mu)$ , and with similar operations on similar equations for the other components, one recovers the required  $q_\mu j_\mu(q) = 0$ .

For three colors we have found two phases which specify the stable equilibria, each having one broken symmetry. Phase (1) denotes spontaneously broken chiral symmetry, and it competes with phase (2) which breaks color symmetry. As an intermediate step in calculating the density, we first examine the quark occupation numbers for the two states,  $n(p)$  where in this context  $p \equiv |\vec{p}|$ , which is obtained via a  $p_4$  integration and precisely defined as

$$n = -i \int \frac{d^3p}{(2\pi)^3} n(p). \tag{35}$$

With this definition we have not separated the quark and antiquark distribution functions, since the phases we analyze do not necessarily retain these as basic degrees of freedom.

The conserved current written above is a sum of two parts. The first is computed with the Green functions obtained through a self-consistent solution of the Schwinger-Dyson-Gorkov equations, and to the density it contributes

$$\begin{aligned}
& \int \frac{d^4p}{(2\pi)^4} [-\psi^\dagger(p)\gamma_4\psi(p)] \\
& = -i \int \frac{d^4p}{(2\pi)^4} \text{Tr}[S(p)\gamma_4] \\
& = -i \int \frac{d^4p}{(2\pi)^4} \left[ \frac{p_4 + i\mu}{(p + i\mu)^2} \right] [2Z_1(p) + (N_c - 2)Z_2(p)]. \tag{36}
\end{aligned}$$

The scalar functions  $Z_{1,2}(p)$  are complex, and for a given solution set  $(M_1, M_2, \Delta)$  become

$$\begin{aligned}
Z_1(p) & = \frac{1 + \alpha^*(p, \mu)M_1^2 - 4\beta(p, \mu)\Delta^2}{[1 + \alpha(p, \mu)M_1^2][1 + \alpha^*(p, \mu)M_2^2] - 16\beta(p, \mu)^2\Delta^4}, \\
Z_2(p) & = \frac{1}{1 + \alpha(p, \mu)M_2^2}. \tag{37}
\end{aligned}$$

The functions  $\alpha(p, \mu)$  and  $\beta(p, \mu)$ , as defined previously, arise as combinations of the form factors evaluated with the appropriate momentum arguments. In general, the first is complex while the second is purely real and always positive, a consequence of this being the expression which accompanies the diquark propagation. Through pairing quarks, the complications due to the non-Hermiticity of the single-quark Dirac operator are avoided.

In the current's second term the form factors have been differentiated with respect to  $p_4$ . The expectation values were computed to the same  $\mathcal{O}(\lambda^1)$  as followed for the Green functions, and this generates another sibling  $\gamma(p, \mu)$  in our family of functions.

$$\begin{aligned}
\gamma(p, \mu) = & (p + i\mu)_\alpha (p + i\mu)_\alpha \varphi_\beta(p, \mu) \varphi_\beta(p, \mu) \varphi_\lambda \\
& \times \frac{\partial \varphi_\lambda(p, \mu)}{\partial p_4}. \tag{38}
\end{aligned}$$

Combining the two pieces, the density reads

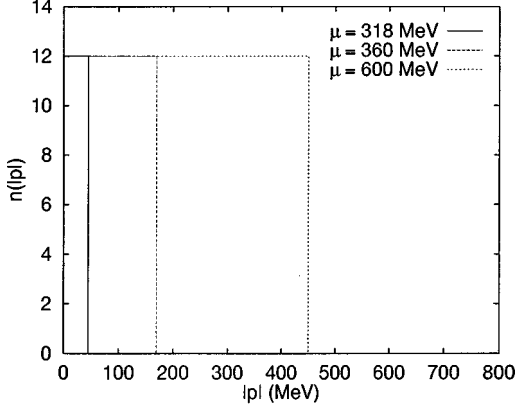


FIG. 5. Occupation number  $n(p)$  vs  $p$  in phase (1), where  $\mu = 318, 360,$  and  $600$  MeV. The corresponding  $p_F = 45, 170,$  and  $450$  MeV, respectively.

$$\begin{aligned}
 n = & -iN_f \int \frac{d^4 p}{(2\pi)^4} \left\{ \left[ \frac{p_4 + i\mu}{(p + i\mu)^2} \right] [2Z_1(p) + (N_c - 2)Z_2(p)] \right. \\
 & - \frac{8}{\varphi_\alpha(p, \mu)\varphi_\alpha(p, \mu)} \left[ \frac{p_4 + i\mu}{(p + i\mu)^2} \alpha(p) + \gamma(p) \right] \\
 & \times \left[ \left( 5 - \frac{4}{N_c} \right) G_1(p)g_1 + (N_c - 2)^2 G_2(p)g_2 \right. \\
 & \left. \left. + \left( 2N_c - 5 + \frac{2}{N_c} \right) (G_1(p)g_2 + G_2(p)g_1) \right] \right\}. \quad (39)
 \end{aligned}$$

The functions  $G_{1,2}(p)$  can be deduced from Eqs. (17) and (37). The integrand here is complex, however, under  $p_4$  reflection we note that  $\alpha(-p_4) = \alpha^*(p_4)$  and  $\gamma(-p_4) = -\gamma^*(p_4)$ . This follows from our preserving the non-Hermitian contributions exactly, again we note that upon  $p_4$  integrate we recover purely real occupation numbers and hence quark density.

We now consider the two phases individually. When phase (1) is assumed, and  $\Delta = 0$ , the density expression (39) simplifies to a form where the integrand contains a denominator of the form  $(p + i\mu)^2 + \mathcal{M}(p, \mu)^2$ . A chiral effective

mass is clearly present, which upon evaluation of Eqs. (21),(24) satisfies the same gap equation as in the  $N_f = 1$  case, Eq. (13).<sup>5</sup> The effective mass shifts the Fermi surface, which for free quarks is at  $p_F = \mu$ , to the solution of  $p_F = \sqrt{\mu^2 - \mathcal{M}(p_F, \mu)^2}$ . This, as shown in Fig. 5, is the only modification of the standard fermionic step function (of magnitude  $2N_f N_c = 12$ ) due to chiral symmetry breaking.

The occupation numbers are more interesting when the quarks fall into the color superconducting state. As is known in BCS theory, Cooper pairing of quarks spreads the Fermi surface as fermions pair with momenta above and below the chemical potential. As demonstrated in Fig. 6, this is precisely the result for the two quark colors which participate in the diquark. Quarks of the parallel color are unaffected by the energy gap, and furthermore, since chiral symmetry has been completely restored, appear as massless fermions with a degeneracy of  $2N_f$ . This is the origin of the discontinuity at  $p = \mu$ , since the Fermi momentum remains at this point for these free quarks. Thus the plot should be considered a combination of a smooth distribution of Bose-condensed diquarks added to a simple step function of magnitude 4. To be explicit, Eq. (39) can be rewritten as

$$\begin{aligned}
 n^{(2)} = & \frac{2N_f(N_c - 2)}{6\pi^2} \mu^3 + 2N_f \int \frac{d^4 p}{(2\pi)^4} \\
 & \times \frac{\mu(p_4^2 + \mu^2 - p^2)}{[p_4^2 + (p - \mu)^2][p_4^2 + (p + \mu)^2]} \\
 & \times \left[ \frac{1}{1 + 4\beta(p)\Delta^2} \right]. \quad (40)
 \end{aligned}$$

The first term is the density for  $N_c - 2$  free quarks, the second's a contribution from the remaining degrees of freedom which are no longer properly described as single quarks.

Performing the integration over all momenta, we arrive at the density as a function of chemical potential. In Fig. 7 this is shown for the equilibria states, demonstrating the large

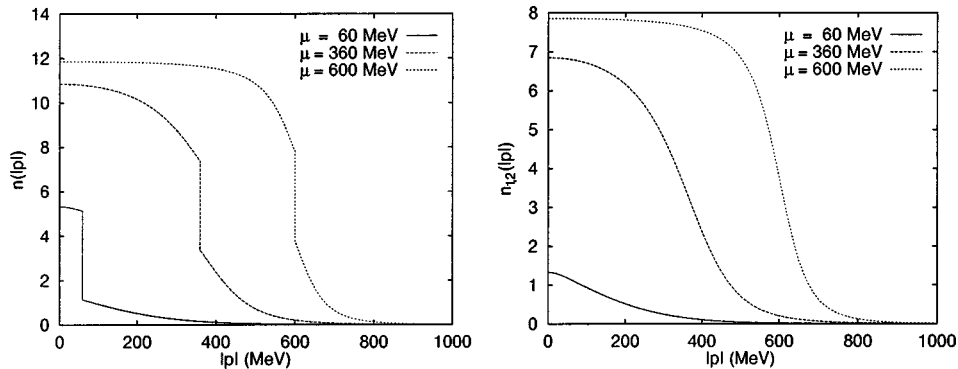


FIG. 6. Occupation number  $n(p)$  vs  $p$  in phase (2), where  $\mu = 60, 360,$  and  $600$  MeV. The plot on the left is for all three colors together, the right for the two which participate in the bosonic diquark condensate.

<sup>5</sup>In fact the same gap equation applies to order  $\lambda^1$  for any number of flavors [28].

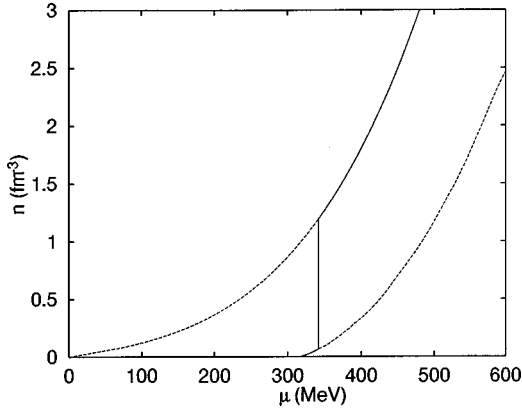


FIG. 7. Quark density as a function of chemical potential. The solid line denotes the equilibrium state of quark matter, the dashed lines the metastable continuations. Naive quark matter saturation density lies at  $\approx 0.45 \text{ fm}^{-3}$ .

discontinuity at the phase transition, where the dashed lines are the metastable continuations in each phase. Since the constituent mass of a quark in the chiral breaking phase remains near 300 MeV, no density is amassed below this chemical potential. This is consistent with our finding a virtually constant chiral condensate below the critical chemical potential, in that the absence of physical quark density should leave the physical condensate unchanged from its vacuum value. Soon after the point at which the Fermi surface moves from zero and physical states begin to be filled, the onset of the phase transition has been reached. Thus, for pure quark matter in this simple mean-field treatment, the maximum density of a purely chiral-broken phase is a rather small  $n = 0.062 \text{ fm}^{-3}$ . Taking a naive saturation density for quark matter as three times that for nuclear matter, we have  $n_0 = 0.445 \text{ fm}^{-3}$ , which lies within the coexistence line between the two phases. It is only above a density of  $n = 1.05 \text{ fm}^{-3} = 2.53n_0$  where one finds the pure superconducting phase. For reference, the density of free quarks at this  $\mu$  would be  $2.25n_0$ , somewhat lower than that which is found in the pairing scenario where the states are in some sense compressed through bosonic condensation.

This extremely low density for the onset of chiral restoration suggests that in the core of heavy nuclei quarks exist in some transitional, ‘‘boiling’’ state [17]. However, we have disregarded certain effects which could modify these results. Considering an extended mean-field ansatz of more than the three channels we consider, would likely result in additional interquark forces, which could lead to enhanced repulsion and extend the stability of the chiral-breaking phase to higher density [30]. We have also not taken into account the binding energy of the quarks within a nucleon, which would not modify our results at the quark level but could lead to some differences between quark and nuclear matter.

Another possibility is to vary the parameters of our theory. In particular, one can change the instanton packing fraction which has been fixed at  $\bar{\rho}/\bar{R} = 1/3$  in all calculations described here. This in turn scales the strength of the induced interaction. However, taking values from half to twice our choice does not lead to qualitative differences in the density

results; in every case chiral symmetry is restored at densities which are a small fraction of that for saturation.

The near-zero transition density also arises when diquark condensation is neglected and chiral symmetry is restored by other means [31,32]. These authors have undertaken analysis of extended mean-field possibilities [30] and interpolative models of quark and hadronic matter [33].

Finally, we note that the strong prediction one can make from this and similar treatments is the presence of a first-order transition which restores chiral symmetry. It is a conclusion which is shared by other recent works which consider chiral symmetry breaking at the quark level. In particular we note that the density discontinuity which we find at this transition,  $\Delta n \approx 2.4n_0$ , is close to that found in the NJL-type treatment of Berges and Rajagopal ( $\Delta n \approx 1.5n_0 - 2n_0$ ) [19] as well as the chiral random matrix model ( $\Delta n \approx 2.5n_0$ ) [34].

## VIII. CONCLUSIONS

We have formulated the effective low-energy action for two light fermions induced by instantons at nonzero chemical potential. In the resulting fermion vertex we have retained the full dependence on both momentum and chemical potential, which arises from the would-be zero modes. The overall interaction strength is given by a dynamically determined coupling, which is a consequence of the fact that the instanton weight itself is proportional to the fermion determinant. In these respects we differ from other studies such as the random-matrix model, Nambu-Jona-Lasinio models, and alternative instanton approaches.

In general, introduction of the fermion chemical potential leads to complications due to the resulting nonhermiticity of the Dirac operator. This is particularly pronounced through the complex nature of the quark interaction, induced by the zero-mode solution of the complex eigenvalue problem. By retaining the exact functional forms without simplifications we are able to avoid any complications from individual imaginary eigenvalues. This is an advantage over various numerical techniques, and should persist for all orders in the instanton density. The formalism outlined in this paper allows one to make such an expansion in a systematic way.

The effective action leads to a competition between two phases, one of chiral symmetry breaking and another characterized by diquark condensation. It was studied by solving a coupled system of gap equations to first order in the instanton density. For two massless flavors and three colors, a simple ansatz allowed us a detailed study of the various types of symmetry breaking. Although we did recover an interesting case of mixed condensation which broke both chiral and color symmetries, it was found to be thermodynamically disfavored. Considering the remaining possibilities, we find that spontaneously broken chiral symmetry is restored through a first-order phase transition, replaced by color breaking due to the formation of a diquark condensate. With our ‘‘standard’’ choice of the instanton ensemble,  $N/V = 1 \text{ fm}^{-4}$  and  $\bar{\rho}/\bar{R} = 1/3$ , we find the critical chemical potential is  $\approx 340 \text{ MeV}$ , where the superconducting energy

gap is 115 MeV and decreasing with rising  $\mu$ . This translates into the onset of chiral restoration at very low density of quark matter. Indeed, the critical  $\mu_c = 340$  MeV corresponds to the quark density of  $0.14 n_0$  in the chiral broken phase and to the density of  $2.53 n_0$  in the superconducting phase, where  $n_0 = 0.445 \text{ fm}^{-3}$  is the quark density corresponding to the standard nuclear matter. Taken literally, it suggests that the deep interior of heavy nuclei are in a boiling mixture of the two phases, as suggested earlier in Ref. [17]. It should be kept in mind however, that our calculations were to only first order in the instanton density and that we neglected the binding of constituent quarks into nucleons. Both circumstances might shift the critical density somewhat.

Restricting our discussion to the quark level, we computed the fermion occupation numbers in the two distinct phases which explicitly demonstrate the physics in each system. In the chiral broken phase, the reduced Fermi radius illustrates the effective mass, whereas in the diquark phase BCS-type behavior is demonstrated by the lack of a sharp Fermi surface.

It is expected that for large chemical potential higher-order corrections will self-consistently reduce the instanton background [20] and thus suppress the nonperturbative effects. At very high quark density, pairing has been predicted due to the perturbative attraction from single gluon exchange [9,35].

Finally, we note that our primary result, a first-order restoration of chiral symmetry at a chemical potential near 340 MeV, basically agrees with calculations performed previously using simpler models both with [17,13,19] and without [31,32,34] color superconductivity. These more schematic models of spontaneous symmetry breaking did not include some of the microscopic degrees of freedom retained in the instanton model used here. The general uniformity in predictions from these different approaches suggests our more detailed instanton calculations validate the formulation of these models which are designed to describe the dominant phenomena of nonperturbative QCD.

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#### APPENDIX: FOURIER TRANSFORMS OF FERMION ZERO MODES

The use of the exact fermion zero modes in the momentum space tremendously simplifies all calculations. The starting point is the exact fermion zero mode in the field of one (anti-)instanton in  $x$  space [23,20], which we cite for arbitrary instanton position  $z$ , size  $\rho$ , and orientation given by rectangular  $N_c \times 2$  matrix  $U$ . In the spinor representation for the Dirac matrices the zero modes are two-component Weyl spinors which can be written as

$$[\Phi_{R,L}(x-z)]^{\alpha i} = \frac{\rho}{\sqrt{2}\pi} e^{\mu(x_4-z_4)} \sqrt{\Pi(x-z)} \partial_\mu \times \left( \frac{e^{-\mu(x_4-z_4)} \Delta(x-z, \mu)}{\Pi(x-z)} \right) (\sigma_\mu^\pm)_j^i \epsilon^{jk} U_k^\alpha. \quad (\text{A1})$$

The single instanton solution is apparent in the functions

$$\Pi(x) = 1 + \frac{\rho^2}{x_4^2 + r^2 + \rho^2},$$

$$\Delta(x, \mu) = \frac{1}{x_4^2 + r^2} \left[ \cos(\mu r) + \frac{x_4}{r} \sin(\mu r) \right]. \quad (\text{A2})$$

The Fourier transform is defined as

$$\Phi(p, \mu) = i \int d^4x e^{-ip \cdot x} \Phi(x, \mu) \quad (\text{A3})$$

and has the structure

$$\Phi_{R,L}(p, \mu)^{\alpha i} = \varphi_\mu(p, \mu) (\sigma_\mu^\pm)_j^i \epsilon^{jk} U_k^\alpha. \quad (\text{A4})$$

Lorentz symmetry is broken at finite  $\mu$ , and the components become

$$\begin{aligned} \varphi_4(p_4, p; \mu) &= \frac{\pi \rho^2}{4p} \{ (p - \mu - ip_4) [(2p_4 + i\mu) f_{1-} \\ &\quad + i(p - \mu - ip_4) f_{2-}] + (p + \mu + ip_4) \\ &\quad \times [(2p_4 + i\mu) f_{1+} - i(p + \mu + ip_4) f_{2+}] \}, \\ \varphi_i(p_4, p; \mu) &= \frac{\pi \rho^2 p_i}{4p^2} \left\{ (2p - \mu)(p - \mu - ip_4) f_{1-} + (2p + \mu) \right. \\ &\quad \times (p + \mu + ip_4) f_{1+} + \left. \left[ 2(p - \mu)(p - \mu - ip_4) \right. \right. \\ &\quad \left. \left. - \frac{1}{p} (\mu + ip_4) [p_4^2 + (p - \mu)^2] \right] f_{2-} \right. \\ &\quad \left. + \left[ 2(p + \mu)(p + \mu + ip_4) + \frac{1}{p} (\mu + ip_4) \right. \right. \\ &\quad \left. \left. \times [p_4^2 + (p + \mu)^2] \right] f_{2+} \right\}, \quad (\text{A5}) \end{aligned}$$

where the scalar  $p = |\vec{p}|$ , the spatial  $i = 1-3$ , and the functions

$$f_{1\pm} = \frac{I_1(z_{\pm})K_0(z_{\pm}) - I_0(z_{\pm})K_1(z_{\pm})}{z_{\pm}},$$

$$f_{2\pm} = \frac{I_1(z_{\pm})K_1(z_{\pm})}{z_{\pm}^2} \quad (\text{A6})$$

are evaluated at  $z_{\pm} = \frac{1}{2}\rho\sqrt{p_4^2 + (p \pm \mu)^2}$ . With these expres-

sions it is explicitly verified that the normalization condition holds for any  $\mu$ :

$$1 = \int \frac{d^4p}{(2\pi)^4} \vec{\Phi}_l(p, \mu) \Phi_l(p, \mu)$$

$$= \int \frac{d^4p}{(2\pi)^4} [\varphi_4^*(-\mu)\varphi_4(\mu) + \vec{\varphi}^*(-\mu) \cdot \vec{\varphi}(\mu)]. \quad (\text{A7})$$

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