Gauge independent effective potential and the Higgs boson mass bound

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We introduce the Vilkovisky-DeWitt formalism for deriving the lower bound of the Higgs boson mass. We illustrate the formalism with a simplified version of the standard electroweak model, where all charged boson fields as well as the bottom-quark field are disregarded. The effective potential obtained in this approach is gauge independent. We derive from the effective potential the mass bound of the Higgs boson. The result is compared to its counterpart obtained from the ordinary effective potential. [S0556-2821(99)06211-6]

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I. INTRODUCTION

The effective potentials in quantum field theories are off-shell quantities. Therefore, in gauge field theories, effective potentials are gauge-dependent as pointed out by Jackiw in the early 1970s [1]. This property caused concerns on the physical significance of effective potentials. In a work by Dolan and Jackiw [2], the effective potential of scalar QED was calculated in a set of R_ξ gauges. It was concluded that only the limiting unitary gauge gives a sensible result on the symmetry-breaking behavior of the theory. This difficulty was partially resolved by the work of Nielsen [3]. In his paper, Nielsen derived the following identity governing the behavior of effective potential in a gauge field theory:

$$\left(\xi\frac{\partial}{\partial\xi}+C(\phi,\xi)\frac{\partial}{\partial\phi}\right)V(\phi,\xi)=0, \tag{1}$$

where ξ is the gauge-fixing parameter, ϕ is the order parameter of the effective potential, and $C(\phi,\xi)$ is the Green's function for certain composite operators containing a ghost field. The above identity implies that, for different ξ, the local extrema of V are located along the same characteristic curve on the (ϕ,ξ) plane, which satisfies $d\xi=d\phi/[C(\phi,\xi)/\xi]$. Hence covariant gauges with different ξ are equally good for computing V. On the other hand, a choice of the multi-parameter gauge $L_{gf}=-(1/2\xi)(\partial_\mu A^\mu+\sigma\phi_1+\rho\phi_2)^2$ [2], with $\phi_{1,2}$ the components of the scalar field, would break the homogeneity of Eq. (1) [3]. Therefore an effective potential calculated in such a gauge does not have a physical significance.

Recently, it was pointed out [4] that the Higgs boson mass bound, which one derives from the effective potential, is gauge dependent. The gauge dependence resides in the expression for the one-loop effective potential. Boyanovsky, Loinaz and Willey proposed a resolution [5] to the problem, which is based upon the *physical effective potential* constructed as the expectation value of the Hamiltonian in physical states [6]. They computed the *physical effective potential* of an Abelian Higgs model with an axial vector coupling of the gauge fields to the fermions. A gauge-independent lower bound for the Higgs boson mass is then determined from the effective potential. We note that their approach requires the identification of first-class constraints of the model and a projection to the physical states. Such a procedure is not manifestly Lorentz covariant. Consequently we expect that it is highly non-trivial to apply their approach to the standard model (SM). In our work, we shall introduce the Vilkovisky-DeWitt formalism [7,8] for constructing a gauge-independent effective potential, and therefore obtain a gauge-independent lower bound for the Higgs boson mass.

In the Vilkovisky-DeWitt formalism, fields are treated as vectors in the configuration space, and the *affine connection* of the configuration space is identified to facilitate the construction of an invariant effective action. Since this procedure is completely Lorentz covariant, the computations for the effective potential and the effective action are straightforward. We shall perform a calculation with respect to a toy model [9] which disregards all charged boson fields in the SM. It is easy to generalize our calculations to the full SM case. In fact, the applicability of Vilkovisky-DeWitt formalism to non-Abelian gauge theories has been extensively demonstrated in the literature [10].

The outline of this paper is as follows. In Sec. II, we briefly review the Vilkovisky-DeWitt formalism using the scalar QED as an example. We shall illustrate that the effective action of Vilkovisky and DeWitt is equivalent to the ordinary effective action constructed in the Landau-DeWitt gauge [11]. In Sec. III, we calculate the effective potential of the simplified standard model, and the relevant renormalization constants of the theory using the Landau-DeWitt gauge. The effective potential is then improved by the renormalization group analysis. In Sec. IV, the Higgs boson mass bound is derived and compared to that given by the ordinary effective potential in the Landau gauge. We conclude in Sec. V, with some technical details discussed in the Appendix.

II. VILKOVISKY-DEWITT EFFECTIVE ACTION OF SCALAR QED

The formulation of Vilkovisky and DeWitt is motivated by the parametrization dependence of the ordinary effective action, which can be written generically as [12]

$$\exp\frac{i}{\hbar}\Gamma[\Phi]=\exp\frac{i}{\hbar}\left(W[j]+\Phi^i\frac{\delta\Gamma}{\delta\Phi^i}\right)$$
$$=\int[D\phi]\exp\frac{i}{\hbar}\left(S[\phi]-(\phi^i-\Phi^i)\frac{\delta\Gamma}{\delta\Phi^i}\right), \tag{2}$$

where $S[\phi]$ is the classical action, and Φ^i denote the background fields. The dependence on the parametrization arises because the quantum fluctuation $\eta^i \equiv (\phi^i - \Phi^i)$ is not a vector in the field configuration space, hence the product $\eta^i \cdot \delta\Gamma/\delta\Phi^i$ is not a scalar under a reparametrization of fields. The remedy to this problem is to replace η^i with a two-point function $\sigma^i(\Phi,\phi)$ [7,8,13] which, at the point Φ, is tangent to the geodesic connecting Φ and ϕ. The precise form of $\sigma^i(\Phi,\phi)$ depends on the connection of the configuration space, Γ^i_{jk}. It is easy to show that [12]

$$\sigma^i(\Phi,\phi) = \eta^i - \frac{1}{2}\Gamma^i_{jk}\eta^j\eta^k + O(\eta^3). \tag{3}$$

For scalar QED described by the Lagrangian:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \lambda(\phi^\dagger\phi - \mu^2)^2, \tag{4}$$

with $D_\mu = \partial_\mu + ieA_\mu$ and $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$, the connection of the configuration space is given by [7,12]

$$\Gamma^i_{jk} = \left\{ \begin{matrix} i \\ jk \end{matrix} \right\} + T^i_{jk}, \tag{5}$$

where $\{{}^{\,i}_{jk}\}$ is the Christoffel symbol of the field configuration space and T^i_{jk} is a quantity induced by generators of the gauge transformation. The Christoffel symbol $\{{}^{\,i}_{jk}\}$ can be computed from the following metric tensor of scalar QED:

$$G_{\phi_a(x)\phi_b(y)} = \delta_{ab}\delta^4(x-y),$$

$$G_{A_\mu(x)A_\nu(y)} = -g^{\mu\nu}\delta^4(x-y),$$

$$G_{A_\mu(x)\phi_a(y)} = 0. \tag{6}$$

According to Vilkovisky's prescription [7], the metric tensor of the field configuration space is obtained by differentiating twice with respect to the fields in the kinetic Lagrangian. For the above metric tensor, we have $\{{}^{\,i}_{jk}\} = 0$ since each component of the tensor is field independent. However, the Christoffel symbol would be non-vanishing if one parametrizes Eq. (4) with polar variables ρ and χ such that $\phi_1 = \rho\cos\chi$ and $\phi_2 = \rho\sin\chi$. Finally, to determine T^i_{jk}, let us specify generators g^i_α of the scalar-QED gauge transformations:

$$g_y^{\phi_a(x)} = -\epsilon^{ab}e\phi_b(x)\delta^4(x-y),$$

$$g_y^{A_\mu(x)} = -\partial_\mu\delta^4(x-y), \tag{7}$$

where ϵ^{ab} is a skew-symmetric tensor with $\epsilon^{12} = 1$. The quantity T^i_{jk} is related to the generators g^i_α via [7]

$$T^i_{jk} = -B^\alpha_j D_k g^i_\alpha + \frac{1}{2}g^l_\alpha D_l g^i_\beta B^\alpha_j B^\beta_k + j \leftrightarrow k, \tag{8}$$

where $B^\alpha_k = N^{\alpha\beta}g_{k\beta}$ with $N^{\alpha\beta}$ being the inverse of $N_{\alpha\beta} \equiv g^k_\alpha g^l_\beta G_{kl}$. The expression for T^i_{jk} can be easily understood by realizing that $i,j,\ldots,l$ are function-space indices, while α and β are space-time indices. Hence, for example,

$$D_{\phi_1(z)}g_y^{A_\mu(x)} = \frac{\partial g_y^{A_\mu(x)}}{\partial\phi_1(z)} + \left\{ \begin{matrix} A_\mu(x) \\ j\,\phi_1(z) \end{matrix} \right\} g^j_y, \tag{9}$$

where the summation over j also implies an integration over the space-time variable in the function j.

The one-loop effective action of scalar QED can be calculated from Eq. (2) with each quantum fluctuation η^i replaced by $\sigma^i(\Phi,\phi)$. The result is written as [12]:

$$\Gamma[\Phi] = S[\Phi] - \frac{i\hbar}{2}\ln\det G + \frac{i\hbar}{2}\ln\det\tilde{D}^{-1}_{ij}, \tag{10}$$

where $S[\Phi]$ is the classical action with Φ denoting generically the background fields; $\ln\det G$ arises from the function-space measure $[D\phi] \equiv \Pi_x d\phi(x)\sqrt{\det G}$; and $\tilde{D}^{-1}_{ij}$ is the modified inverse propagator:

$$\tilde{D}^{-1}_{ij} = \frac{\delta^2 S}{\delta\Phi^i\delta\Phi^j} - \Gamma^k_{ij}[\Phi]\frac{\delta S}{\delta\Phi^k}. \tag{11}$$

To study the symmetry-breaking behavior of the theory, we focus on the effective potential which is obtained from $\Gamma[\Phi]$ by setting each background field Φ^i to a constant.

The Vilkovisky-DeWitt effective potential of scalar QED has been calculated in various gauges and different scalar-field parametrizations [11,12,14]. The results all agree with one another. In this work, we calculate the effective potential and other relevant quantities in the Landau-DeWitt gauge [15], which is characterized by the gauge-fixing term: $L_{gf} = -(1/2\xi)(\partial_\mu B^\mu - ie\eta^\dagger\Phi + ie\Phi^\dagger\eta)^2$, with $\xi \to 0$. In L_{gf}, $B^\mu \equiv A^\mu - A^\mu_{cl}$, and $\eta \equiv \phi - \Phi$ are quantum fluctuations while A^μ_{cl} and Φ are background fields. For the scalar fields, we further write $\Phi = (\rho_{cl} + i\chi_{cl})/\sqrt{2}$ and $\eta = (\rho + i\chi)/\sqrt{2}$. The advantage of performing calculations in the Landau-DeWitt gauge is that T^i_{jk} vanishes [11] in this case. In other words, the Vilkovisky-DeWitt formalism coincides with the conventional formalism in the Landau-DeWitt gauge.

For computing the effective potential, we choose $A^\mu_{cl} = \chi_{cl} = 0$, i.e., $\Phi = \rho_{cl}/\sqrt{2}$. In this set of background fields, L_{gf} becomes

$$L_{gf} = -\frac{1}{2\xi}(\partial_\mu B^\mu\partial_\nu B^\nu - 2e\rho_{cl}\chi\partial_\mu B^\mu + e^2\rho^2_{cl}\chi^2). \tag{12}$$

We note that $B_\mu - \chi$ mixing in L_{gf} is ξ dependent, and therefore would not cancel out the corresponding mixing term in the classical Lagrangian of Eq. (4). This induces mixed-propagators such as $\langle 0|T(A_\mu(x)\chi(y))|0\rangle$ or $\langle 0|T(\chi(x)A_\mu(y))|0\rangle$. The Faddeev-Popov ghost Lagrangian in this gauge reads

$$L_{FP} = \omega^*(-\partial^2 - e^2\rho^2_{cl})\omega. \tag{13}$$

With each part of the Lagrangian determined, we are ready to compute the effective potential. Since we choose a field-independent flat-metric, the one-loop effective potential is completely determined by the modified inverse-propagators $\tilde{D}^{-1}_{ij}$ [16]. From Eqs. (4), (11), (12) and (13), we arrive at

$$\tilde{D}^{-1}_{B_\mu B_\nu}=(-k^2+e^2\rho_0^2)g^{\mu\nu}+\left(1-\frac{1}{\xi}\right)k^\mu k^\nu,$$

$$\tilde{D}^{-1}_{B_\mu \chi}=ik^\mu e\rho_0\left(1-\frac{1}{\xi}\right),$$

$$\tilde{D}^{-1}_{\chi\chi}=\left(k^2-m_G^2-\frac{1}{\xi}e^2\rho_0^2\right),$$

$$\tilde{D}^{-1}_{\rho\rho}=(k^2-m_H^2),$$

$$\tilde{D}_{\omega*\omega}=(k^2-e^2\rho_0^2)^{-2}, \tag{14}$$

where we have set $\rho_{cl}=\rho_0$, which is a space-time independent constant, and defined $m_G^2=\lambda(\rho_0^2-2\mu^2)$, $m_H^2=\lambda(3\rho_0^2-2\mu^2)$. Using the definition $\Gamma[\rho_0]=-(2\pi)^4\delta^4(0)V_{eff}(\rho_0)$ along with Eqs. (10) and (14), and taking the limit $\xi\to 0$, we obtain $V_{eff}(\rho_0)=V_{tree}(\rho_0)+V_{1\text{-}loop}(\rho_0)$ with

$$V_{1\text{-}loop}(\rho_0)=\frac{-i\hbar}{2}\int\frac{d^nk}{(2\pi)^n}\ln[(k^2-e^2\rho_0^2)^{n-3}\times(k^2-m_H^2)(k^2-m_+^2)(k^2-m_-^2)], \tag{15}$$

where m_+^2 and m_-^2 are solutions of the quadratic equation $(k^2)^2-(2e^2\rho_0^2+m_G^2)k^2+e^4\rho_0^4=0$. One notices that the gauge-boson's degree of freedom in $V_{1\text{-}loop}$ has been continued to $n-3$ in order to preserve the relevant Ward identities. For example, this continuation is crucial to ensure the Ward identity which relates the scalar self-energy to the contribution of the tadpole diagram. Our expression for $V_{1\text{-}loop}(\rho_0)$ agrees with previous results obtained in the unitary gauge [14]. One could also calculate the effective potential in the *ghost-free* Lorentz gauge with $L_{gf}=-(1/2\xi)(\partial_\mu B^\mu)^2$. The cancellation of the gauge-parameter (ξ) dependence in the effective potential has been demonstrated in the case of massless scalar QED where $\mu^2=0$ [11,12]. It can be easily extended to the massive case, and the resultant effective potential coincides with Eq. (15). In the Appendix, we will also demonstrate the cancellation of gauge-parameter dependence in the calculation of Higgs-boson self-energy. The obtained self-energy will be shown to coincide with its counterpart obtained from the Landau-DeWitt gauge. We do this not only to show that the Vilkovisky-DeWitt formulation coincides with the ordinary formulation in the Landau-DeWitt gauge, but also to illustrate how it gives rise to identical effective action in spite of beginning with different gauges.

It is instructive to rewrite Eq. (15) as

$$V_{1\text{-}loop}[\rho_0]=\frac{\hbar}{2}\int\frac{d^{n-1}\vec{k}}{(2\pi)^{n-1}}((n-3)\omega_B(\vec{k})+\omega_H(\vec{k})+\omega_+(\vec{k})+\omega_-(\vec{k})), \tag{16}$$

where $\omega_B(\vec{k})=\sqrt{\vec{k}^2+e^2\rho_0^2}$, $\omega_H(\vec{k})=\sqrt{\vec{k}^2+m_H^2}$ and $\omega_\pm(\vec{k})=\sqrt{\vec{k}^2+m_\pm^2}$. One can see that $V_{1\text{-}loop}$ is a sum of the zero-point energies of four excitations with masses $m_B\equiv e\rho_0$, m_H, m_+ and m_-. Since there are precisely four physical degrees of freedom in the scalar QED, we see that the Vilkovisky-DeWitt effective potential does exhibit a correct number of physical degrees of freedom. Such a nice property is not shared by the ordinary effective potential calculated in the *ghost free* Lorentz gauge just mentioned. As will be shown later, the ordinary effective potential in this gauge contains unphysical degrees of freedom.

III. VILKOVISKY-DEWITT EFFECTIVE POTENTIAL OF THE SIMPLIFIED STANDARD MODEL

In this section, we compute the effective potential of the simplified standard model where charged boson fields and all fermion fields except the top-quark field are disregarded. The gauge interactions for the top quark and the neutral scalar bosons are prescribed by the following covariant derivatives [9]:

$$D_\mu t_L=\left(\partial_\mu+ig_LZ_\mu-\frac{2}{3}ieA_\mu\right)t_L,$$

$$D_\mu t_R=\left(\partial_\mu+ig_RZ_\mu-\frac{2}{3}ieA_\mu\right)t_R,$$

$$D_\mu\phi=(\partial_\mu+i(g_L-g_R)Z_\mu)\phi, \tag{17}$$

where Z_μ and A_μ denote the Z boson and the photon respectively; the coupling constants g_L and g_R are given by $g_L=(-g_1/2+g_2/3)$ and $g_R=g_2/3$ with $g_1=g/\cos\theta_W$ and $g_2=2e\tan\theta_W$ respectively. The self-interactions of scalar fields are described by the same potential term as that in Eq. (4). Clearly this toy model exhibits a $U(1)_A\times U(1)_Z$ symmetry where each $U(1)$ symmetry is associated with a neutral gauge boson. The $U(1)_Z$ charges of t_L, t_R and ϕ are related in such a way that the following Yukawa interactions are invariant under $U(1)_A\times U(1)_Z$:

$$L_Y=-y\overline{t_L}\phi t_R-y\overline{t_R}\phi^*t_L. \tag{18}$$

Since Vilkosvisky-DeWitt effective action coincides with the ordinary effective action in the Landau-DeWitt gauge, we thus calculate the effective potential in this gauge, which is defined by the following gauge-fixing terms [17]:

$$L_{gf}=-\frac{1}{2\alpha}\left(\partial_\mu\tilde{Z}^\mu+\frac{ig_1}{2}\eta^\dagger\Phi-\frac{ig_1}{2}\Phi^\dagger\eta\right)^2-\frac{1}{2\beta}(\partial_\mu\tilde{A}^\mu)^2, \tag{19}$$

with $\alpha,\beta\to 0$. We note that $\tilde{A}^\mu$, $\tilde{Z}^\mu$ and η are quantum fluctuations associated with the photon, the Z boson and the scalar boson respectively, i.e., $A^\mu=A^\mu_{cl}+\tilde{A}^\mu$, $Z^\mu=Z^\mu_{cl}+\tilde{Z}^\mu$, and $\phi=\Phi+\eta$ with A^μ_{cl}, Z^μ_{cl} and Φ being the background fields. For computing the effective potential, we take Φ as a space-time-independent constant denoted as ρ_o, and set A^μ_{cl}

$=Z_{cl}^{\mu}=0$. Following the method outlined in the previous section, we obtain the one-loop effective potential

$$V_{VD}(\rho_0)=\frac{\hbar}{2}\int\frac{d^{n-1}\vec{k}}{(2\pi)^{n-1}}((n-3)\omega_Z(\vec{k})+\omega_H(\vec{k})+\omega_+(\vec{k})+\omega_-(\vec{k})-4\omega_F(\vec{k})), \quad (20)$$

where $\omega_i(\vec{k})=\sqrt{\vec{k}^2+m_i^2}$ with $m_Z^2=(g_1^2/4)\rho_0^2$, $m_\pm^2=m_Z^2+\frac{1}{2}(m_G^2\pm m_G\sqrt{m_G^2+4m_Z^2})$ and $m_F^2\equiv m_t^2=y^2\rho_0^2/2$. The Goldstone boson mass m_G is defined as before, i.e., $m_G^2=\lambda(\rho_o^2-2\mu^2)$ with μ being the mass parameter of the Lagrangian. One may notice the absence of photon contributions in the above effective potential. This is not surprising since photons do not couple directly to the Higgs boson.

Performing the integration in Eq. (20) and subtracting the infinities with modified minimal subtraction ($\overline{\rm MS}$) scheme prescription, we obtain

$$V_{VD}(\rho_0)=\frac{\hbar}{64\pi^2}\left(m_H^4\ln\frac{m_H^2}{\kappa^2}+m_Z^4\ln\frac{m_Z^2}{\kappa^2}+m_+^4\ln\frac{m_+^2}{\kappa^2}+m_-^4\ln\frac{m_-^2}{\kappa^2}-4m_t^4\ln\frac{m_t^2}{\kappa^2}\right)-\frac{\hbar}{128\pi^2}\times(3m_H^4+5m_Z^4+3m_G^4+12m_G^2m_Z^2-12m_t^4), \quad (21)$$

where κ is the mass scale introduced in the dimensional regularization. Although $V_{VD}(\rho_0)$ is obtained in the Landau-DeWitt gauge, we should stress that any other gauge with non-vanishing T_{jk}^i should lead to the same result. For later comparisons, let us write down the ordinary one-loop effective potential in the Lorentz gauge [removing the scalar part of Eq. (19)] as follows [2]:

$$V_L(\rho_0)=\frac{\hbar}{2}\int\frac{d^{n-1}\vec{k}}{(2\pi)^{n-1}}((n-1)\omega_Z(\vec{k})+\omega_H(\vec{k})+\omega_a(\vec{k},\alpha)+\omega_b(\vec{k},\alpha)-4\omega_F(\vec{k})), \quad (22)$$

where α is the gauge-fixing parameter and $\omega_{a,b}(\vec{k},\alpha)=\sqrt{\vec{k}^2+m_{a,b}^2(\alpha)}$ with $m_a^2(\alpha)=1/2\cdot(m_G^2+\sqrt{m_G^4-4\alpha m_Z^2m_G^2})$ and $m_b^2(\alpha)=1/2\cdot(m_G^2-\sqrt{m_G^4-4\alpha m_Z^2m_G^2})$. It is easily seen that there are 6 bosonic degrees of freedom in V_L, i.e., two extra degrees of freedom emerge as a result of choosing the Lorentz gauge. In the Landau gauge, which is a special case of the Lorentz gauge with $\alpha=0$, there is still one extra degree of freedom in the effective potential [6]. Since the Landau gauge is adopted most frequently for computing the ordinary effective potential, we shall take $\alpha=0$ in V_L hereafter. Performing the integrations in V_L and subtracting the infinities, we obtain

$$V_L(\rho_0)=\frac{\hbar}{64\pi^2}\left(m_H^4\ln\frac{m_H^2}{\kappa^2}+3m_Z^4\ln\frac{m_Z^2}{\kappa^2}+m_G^4\ln\frac{m_G^2}{\kappa^2}-4m_t^4\ln\frac{m_t^2}{\kappa^2}\right)-\frac{\hbar}{128\pi^2}(3m_H^4+5m_Z^4+3m_G^4-12m_t^4). \quad (23)$$

We remark that V_L differs from V_{VD} except at the point of extremum where $\rho_0^2=2\mu^2$. At this point, one has $m_G^2=0$ and $m_\pm^2=m_Z^2$, which lead to $V_{VD}(\rho_0=2\mu^2)=V_L(\rho_0^2=2\mu^2)$. That $V_{VD}=V_L$ at the point of extremum is a consequence of the Nielsen identity [3] mentioned earlier.

To derive the Higgs boson mass bound from $V_{VD}(\rho_0)$ or $V_L(\rho_0)$, one encounters a breakdown of the perturbation theory at the point of sufficiently large ρ_0 such that, for instance, $(\lambda/16\pi^2)\ln(\lambda\rho_0^2/\kappa^2)>1$. To extend the validity of the effective potential to the large-ρ_0 region, the effective potential has to be improved by the renomalization group (RG) analysis. Let us denote the effective potential as V_{eff} which includes the tree-level contribution and quantum corrections. The renormalization-scale independence of V_{eff} implies the following equation [18,5]:

$$\left(-\mu(\gamma_\mu+1)\frac{\partial}{\partial\mu}+\beta_{\hat{g}}\frac{\partial}{\partial\hat{g}}-(\gamma_\rho+1)t\frac{\partial}{\partial t}+4\right)\times V_{eff}(t\rho_0^i,\mu,\hat{g},\kappa)=0, \quad (24)$$

where μ is the mass parameter of the Lagrangian as shown in Eq. (4), and

$$\beta_{\hat{g}}=\kappa\frac{d\hat{g}}{d\kappa},$$

$$\gamma_\rho=-\kappa\frac{d\ln\rho}{d\kappa},$$

$$\gamma_\mu=-\kappa\frac{d\ln\mu}{d\kappa}, \quad (25)$$

with $\hat{g}$ denoting collectively the coupling constants λ, g_1, g_2 and y; ρ_0^i is an arbitrarily chosen initial value for ρ_0. Solving this differential equation gives

$$V_{eff}(t\rho_0^i,\mu_i,\hat{g}_i,\kappa)=\exp\left(\int_0^{\ln t}\frac{4}{1+\gamma_\rho(x)}dx\right)\times V_{eff}(\rho_0^i,\mu(t,\mu_i),\hat{g}(t,\hat{g}_i),\kappa), \quad (26)$$

with $x=\ln(\rho_0'/\rho_0^i)$ for an intermediate scale ρ_0', and

$$t\frac{d\hat{g}}{dt}=\frac{\beta_{\hat{g}}(\hat{g}(t))}{1+\gamma_\rho(\hat{g}(t))}\ \text{with}\ \hat{g}(0)=\hat{g}_i, \quad (27)$$

$$\mu(t,\mu_i)=\mu_i\exp\left(-\int_0^{\ln t}\frac{1+\gamma_\mu(x)}{1+\gamma_\rho(x)}dx\right). \tag{28}$$

To fully determine V_{eff} at a large ρ_0, we need to calculate the β functions of λ, g_1, g_2 and y, and the anomalous dimensions γ_μ and γ_ρ. It has been demonstrated that the n-loop effective potential is improved by the $(n+1)$-loop β and γ functions [19,20]. Since the effectve potential is calculated to the one-loop order, a consistent RG analysis requires the knowledge of β and γ functions up to a two-loop accuracy. However, as the computations of two-loop β and γ functions are quite involved, we will only improve the tree-level effective potential with one-loop β and γ functions. After all, the main focus of this paper is to show how to obtain a gauge-independent Higgs boson mass bound rather than a detailed calculation of this quantity.

To compute one-loop β and γ functions, we first calculate the renormalization constants $Z_\lambda, Z_{g_1}, Z_{g_2}, Z_y, Z_{\mu^2}$ and Z_ρ, which are defined by

$$\lambda^{bare}=Z_\lambda\lambda,\quad g_1^{bare}=Z_{g_1}g_1,\quad g_2^{bare}=Z_{g_2}g_2,$$
$$y^{bare}=Z_y y,\quad (\mu^2)^{bare}=Z_{\mu^2}\mu^2,\quad \rho^{bare}=\sqrt{Z_\rho}\rho. \tag{29}$$

In the ordinary formalism of the effective action, all of the above renormalization constants except Z_ρ are in fact gauge-independent at the one-loop order in the $\overline{\text{MS}}$ scheme. For Z_ρ, the result given by the commonly adopted Landau gauge differs from that obtained from the Landau-DeWitt gauge. In Appendix A, we shall reproduce Z_ρ obtained in the Landau-DeWitt gauge with the general Vilkovisky-DeWitt formulation. The calculation of various renormalization constants are straightforward. In the $\overline{\text{MS}}$ scheme, we have (we will set $\hbar=1$ from this point on)

$$Z_\lambda=1-\frac{1}{128\pi^2\epsilon'}\left(\frac{3g_1^4}{\lambda}-24g_1^2-\frac{16y^4}{\lambda}+32y^2+160\lambda\right),$$

$$Z_{g_1}=Z_{g_2}=1-\frac{1}{216\pi^2\epsilon'}\left(\frac{27g_1^2}{8}+2g_2^2-3g_1g_2\right),$$

$$Z_y=1+\frac{1}{192\pi^2\epsilon'}(9g_1^2+4g_1g_2-24y^2),$$

$$Z_{\mu^2}=1+\frac{1}{128\pi^2\epsilon'}\left(\frac{3g_1^4}{\lambda}-12g_1^2-\frac{16y^4}{\lambda}+16y^2+96\lambda\right),$$

$$Z_\rho=1+\frac{1}{32\pi^2\epsilon'}(-5g_1^2+4y^2), \tag{30}$$

where $1/\epsilon'\equiv 1/\epsilon+\frac{1}{2}\gamma_E-\frac{1}{2}\ln(4\pi)$ with $\epsilon=n-4$. The one-loop β and γ functions resulting from the above renormalization constants are

$$\beta_\lambda=\frac{1}{16\pi^2}\left(\frac{3}{8}g_1^4-3\lambda g_1^2-2y^4+4\lambda y^2+20\lambda^2\right),$$

$$\beta_{g_1}=\frac{g_1}{4\pi^2}\left(\frac{g_1^2}{16}-\frac{g_1g_2}{18}+\frac{g_2^2}{27}\right),$$

$$\beta_{g_2}=\frac{g_2}{4\pi^2}\left(\frac{g_1^2}{16}-\frac{g_1g_2}{18}+\frac{g_2^2}{27}\right),$$

$$\beta_y=\frac{y}{8\pi^2}\left(y^2-\frac{3g_1^2}{8}+\frac{g_1g_2}{12}\right),$$

$$\gamma_\mu=\frac{1}{2\pi^2}\left(\frac{3\lambda}{4}+\frac{3g_1^4}{128\lambda}-\frac{3g_1^2}{32}-\frac{y^4}{8\lambda}+\frac{y^2}{8}\right),$$

$$\gamma_\rho=\frac{1}{64\pi^2}(-5g_1^2+4y^2). \tag{31}$$

Similar to what was mentioned earlier, all of the above quantities are gauge-independent in the $\overline{\text{MS}}$ scheme except γ_ρ, the anomalous dimension of the scalar field. In the Landau gauge of the ordinary formulation, we have

$$\gamma_\rho=\frac{1}{64\pi^2}(-3g_1^2+4y^2). \tag{32}$$

IV. THE HIGGS BOSON MASS BOUND

The lower bound of the Higgs boson mass can be derived from the vacuum instability condition of the electroweak effective potential [21]. In this derivation, there exists different criteria for determining the instability scale of the electroweak vacuum. The first criterion is to identify the instability scale as the critical value of the Higgs-field strength beyond which the renormalization-group- (RG-) improved tree-level effective potential becomes negative [22–24]. To implement this criterion, the tree-level effective potential is improved by the leading [24] or next-to-leading order [22,23] renormalization group equations, where one-loop or two-loop β and γ functions are employed. Furthermore, one-loop corrections to parameters of the effective potential are also taken into account [23,24]. However, the effect of one-loop effective potential is not considered.

To improve the above treatment, Casas *et al.* [25] considered the effect of RG-improved one-loop effective potential. The vacuum-instability scale is then identified as the value of the Higgs-field strength at which the sum of tree-level and one-loop effective potentials vanishes. In our subsequent analysis, we will follow this criterion except that the one-loop effective potential is not RG improved.

To derive the Higgs boson mass bound, one begins with Eq. (26) which implies

$$V_{tree}(t\rho_0^i,\mu_i,\lambda_i)=\frac{1}{4}\chi(t)\lambda(t,\lambda_i)((\rho_0^i)^2-2\mu^2(t,\mu_i))^2, \tag{33}$$

with $\chi(t)=\exp(\int_0^{\ln t}[4/1+\gamma_\rho(x)]dx)$. Since Eq. (28) implies that $\mu(t,\mu_i)$ decreases as t increases, we then have $V_{tree}(t\rho_0^i,\mu_i,\lambda_i)\approx\frac{1}{4}\chi(t)\lambda(t,\lambda_i)(\rho_0^i)^4$ for a sufficiently large t. Similarly, the one-loop effective potential $V_{1\text{-}loop}(t\rho_0^i,\mu_i,\hat{g}_i,\kappa)$ is also proportional to $V_{1\text{-}loop}(\rho_0^i,\mu(t,\mu_i),\hat{g}(t,\hat{g}_i),\kappa)$ with the same proportional constant $\chi(t)$. Because we shall ignore all running effects in $V_{1\text{-}loop}$, we can take $\hat{g}(t,\hat{g}_i)=\hat{g}_i$ and $\mu(t,\mu_i)=(1/t)\mu_i$ in $V_{1\text{-}loop}$. For a sufficiently large t, $V_{1\text{-}loop}$ can also be approximated by its quartic terms. In the Landau-DeWitt gauge with the choice $\kappa=\rho_0^i$, we obtain

$$\begin{aligned}V_{VD}\approx\frac{(\rho_0^i)^4}{64\pi^2}\Bigg[&9\lambda_i^2\ln(3\lambda_i)+\frac{g_{1i}^4}{16}\ln\left(\frac{g_{1i}^2}{4}\right)\\&-y_i^4\ln\left(\frac{y_i^2}{2}\right)+A_+^2(g_{1i},\lambda_i)\ln A_+(g_{1i},\lambda_i)\\&+A_-^2(g_{1i},\lambda_i)\ln A_-(g_{1i},\lambda_i)\\&-\frac{3}{2}\left(10\lambda_i^2+\lambda_i g_{1i}^2+\frac{5}{48}g_{1i}^4-y_i^4\right)\Bigg],\end{aligned}\tag{34}$$

where $A_\pm(g_1,\lambda)=g_1^2/4+\lambda/2\cdot(1\pm\sqrt{1+g_1^2/\lambda})$. Similarly, the effective potential in the Landau gauge is given by

$$\begin{aligned}V_L\approx\frac{(\rho_0^i)^4}{64\pi^2}\Bigg[&9\lambda_i^2\ln(3\lambda_i)+\frac{3g_{1i}^4}{16}\ln\left(\frac{g_{1i}^2}{4}\right)-y_i^4\ln\left(\frac{y_i^2}{2}\right)\\&+\lambda_i^2\ln(\lambda_i)-\frac{3}{2}\left(10\lambda_i^2+\lambda_i g_{1i}^2+\frac{5}{48}g_{1i}^4-y_i^4\right)\Bigg].\end{aligned}\tag{35}$$

Combining the tree-level and the one-loop effective potentials, we arrive at

$$V_{eff}(t\rho_0^i,\mu_i,\hat{g}_i,\kappa)\approx\frac{1}{4}\chi(t)(\lambda(t,\lambda_i)+\Delta\lambda(\hat{g}_i))(\rho_0^i)^4,\tag{36}$$

where $\Delta\lambda$ represents the one-loop corrections obtained from Eqs. (34) or (35). Let $t_{VI}=\rho_{VI}/\rho_0^i$, the condition for the vacuum instability of the effective potential is then [25]

$$\lambda(t_{VI},\lambda_i)+\Delta\lambda(\hat{g}_i)=0.\tag{37}$$

We note that the couplings $\hat{g}_i$ in $\Delta\lambda$ are evaluated at $\kappa=\rho_0^i$, which can be taken as the electroweak scale. Hence we have $g_{1i}\equiv g/\cos\theta_W=0.67$, $g_{2i}\equiv 2e\tan\theta_W=0.31$, and $y_i=1$. The running coupling $\lambda(t_{VI},\lambda_i)$ also depends upon g_1,g_2 and y through β_λ, and γ_ρ shown in Eq. (31). To solve Eq. (37), we first determine the running behaviors of the coupling constants g_1,g_2 and y. For g_1 and g_2, we have

$$t\frac{d(g_l^2(t))}{dt}=2g_l(t)\frac{\beta_{g_l}(\hat{g}(t))}{1+\gamma_\rho(\hat{g}(t))}\approx\beta_{g_l^2},\tag{38}$$

where $l=1,2$, and the contribution of γ_ρ is neglected in accordance with our leading-logarithmic approximation. Also $\beta_{g_l^2}=g_l^2/2\pi^2\cdot(g_1^2/16-g_1g_2/18+g_2^2/27)$. Although the differential equations for g_1^2 and g_2^2 are coupled, they can be easily disentangled by observing that g_1^2/g_2^2 is a RG-invariant. Numerically, we have $\beta_{g_l^2}=c_lg_l^4$ with $c_1=2.3\times10^{-3}$ and $c_2=1.1\times10^{-2}$. Solving the differential equations gives

$$g_l^{-2}(t)=g_l^{-2}(0)-c_l\ln t.\tag{39}$$

With $g_1(t)$ and $g_2(t)$ determined, the running behavior of y can be calculated analytically [4]. Given $\beta_{y^2}\equiv 2y\beta_y=c_3y^4-c_4g_1^2y^2$ with $c_3=2.5\times10^{-2}$ and $c_4=8.5\times10^{-3}$, we obtain

$$\begin{aligned}y^2(t)=\Bigg[&\left(\frac{g_1^2(t)}{g_{1i}^2}\right)^{c_4/c_1}\left(y_i^{-2}-\frac{c_3}{c_1+c_4}g_{1i}^{-2}\right)\\&+\frac{c_3}{c_1+c_4}g_1^{-2}(t)\Bigg]^{-1}.\end{aligned}\tag{40}$$

Now the strategy for solving Eq. (37) is to make an initial guess on λ_i, which enters into $\lambda(t)$ and $\Delta\lambda$, and repeatedly adjust λ_i until $\lambda(t)$ completely cancels $\Delta\lambda$. For $t_{VI}=10^2$ (or $\rho_0\approx10^4$ GeV) which is the new-physics scale reachable by the CERN Large Hadron Collider (LHC), we find $\lambda_i=4.83\times10^{-2}$ for the Landau-DeWitt gauge, and $\lambda_i=4.8\times10^{-2}$ for the Landau gauge. For a higher instability scale such as the scale of grand unification, we have $t_{VI}=10^{13}$ or $\rho_0\approx10^{15}$ GeV. In this case, we find $\lambda_i=3.13\times10^{-1}$ for both the Landau-DeWitt and Landau gauges. The numerical similarity between the λ_i of each gauge can be attributed to an identical β function for the running of $\lambda(t)$, and a small difference between the $\Delta\lambda$ of each gauge. We recall from Eq. (27) that the evolutions of λ in the above two gauges will be different if the effects of next-to-leading logarithms are taken into account. In that case, the difference between the γ_ρ of each gauge gives rise to different evolutions for λ. For a large t_{VI}, one may expect to see a non-negligible difference between the λ_i of each gauge.

The critical value $\lambda_i=4.83\times10^{-2}$ corresponds to a lower bound for the $\overline{\text{MS}}$ mass of the Higgs boson. Since $m_H=2\sqrt{\lambda}\mu$, we have $(m_H)_{\overline{\text{MS}}}\geqslant 77$ GeV. For $\lambda_i=3.13\times10^{-1}$, we have $(m_H)_{\overline{\text{MS}}}\geqslant 196$ GeV. To obtain the lower bound for the physical mass of the Higgs boson, finite radiative corrections must be added to the above bounds [4]. We will not pursue these finite corrections any further since we are simply dealing with a toy model. However we would like to point out that such corrections are gauge-independent as ensured by the Nielsen identity [3].

V. CONCLUSION

We have computed the one-loop effective potential of an Abelian $U(1)\times U(1)$ model in the Landau-DeWitt gauge, which reproduces the result given by the gauge-independent Vilkovisky-DeWitt formulation. One-loop β and γ functions were also computed to facilitate the RG improvement of the effective potential. A gauge-independent lower bound for the Higgs-boson self-coupling or equivalently the $\overline{\rm MS}$ mass of the Higgs boson was derived. We compared this bound to that obtained using the ordinary Landau-gauge effective potential. The numerical values of both bounds are almost identical due to the leading-logarithmic approximation we have taken. A complete next-to-leading-order analysis should better distinguish the two bounds. This improvement as well as extending the current analysis to the full standard model will be reported in future publications.

Finally we would like to comment on the issue of comparing our result with that of Ref. [5]. So far, we have not found a practical way of relating the effective potentials calculated in both approaches. In Ref. [5], to achieve a *gauge-invariant* formulation, the theory is written in terms of a new set of fields which are related to the original fields through non-local transformations. Taking scalar QED as an example, the new scalar field $\phi'(\vec{x})$ is related to the original field through [6]

$$\phi'(\vec{x})=\phi(\vec{x})\exp\left(ie\int d^3y\vec{A}(\vec{y})\cdot\vec{\nabla}_yG(\vec{y}-\vec{x})\right), \quad (41)$$

with $G(\vec{y}-\vec{x})$ satisfying $\nabla^2G(\vec{y}-\vec{x})=\delta^3(\vec{y}-\vec{x})$. To our knowledge, it does not appear obvious how one might incorporate the above non-local and non-Lorentz-covariant transformation into the Vilkovisky-DeWitt formulation. This is an issue deserving further investigations.

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APPENDIX: THE HIGGS-BOSON SELF-ENERGY AND VILKOVISKY-DEWITT EFFECTIVE ACTION

In this Appendix, we calculate the Higgs-boson self-energy of scalar QED from the Vilkovisky-DeWitt effective action. We will focus on the momentum-dependent part of the self-energy, which is not a part of the effective potential calculated in Sec. II. Furthermore only the infinite part of the self-energy will be calculated. We thus perform the calculation in the symmetry phase of the theory.

We begin with the Lagrangian in Eq. (4) where μ^2 is negative, i.e., $-\mu^2\equiv u^2>0$. In this case, ϕ_1 and ϕ_2 have an identical mass $m_\phi^2=2\lambda u^2$. Let us rename ϕ_1 as ρ and ϕ_2 as χ according to our notation in the symmetry-broken phase. If one follows the background field expansion in Eq. (2), one would expand the QED action by writing $A^\mu=A^\mu_{cl}+B^\mu$, $\rho=\rho_{cl}+\eta_1$, and $\chi=\chi_{cl}+\eta_2$, with A^μ_{cl}, ρ_{cl} and χ_{cl} the classical background fields, and B^μ, η_1 and η_2 the corresponding quantum fluctuations. However, as mentioned earlier, the above quantum fluctuations should be replaced by vectors σ^i in the configuration space. Hence the action in Eq. (2) should be expanded covariantly [7] in powers of σ^i. To simplify our notations, we use $\tilde{B}_\mu$ and $\tilde{\eta}_i$ to denote the new quantum fluctuations. Since we will only calculate the self-energy of ρ, we may take $A^\mu_{cl}=\chi_{cl}=0$ for simplicity. With the covariant expansion, the Lagrangian in Eq. (4) generates the following quadratic terms:

$$\begin{aligned}L_{quad}=&-\frac{1}{4}(\partial_\mu\tilde{B}_\nu-\partial_\nu\tilde{B}_\mu)^2+\frac{1}{2}(\partial_\mu\tilde{\eta}_1)(\partial^\mu\tilde{\eta}_1)\\&+\frac{1}{2}(\partial_\mu\tilde{\eta}_2)(\partial^\mu\tilde{\eta}_2)+e\rho_{cl}(\partial^\mu\tilde{\eta}_2)\tilde{B}_\mu-e\tilde{\eta}_2(\partial^\mu\rho_{cl})\tilde{B}_\mu\\&+\frac{1}{2}e^2\rho_{cl}^2\tilde{B}^\mu\tilde{B}_\mu-\lambda\left[\frac{1}{2}\rho_{cl}^2(3\tilde{\eta}_1^2+\tilde{\eta}_2^2)+u^2(\tilde{\eta}_1^2+\tilde{\eta}_2^2)\right]\\&-\Gamma^{\Phi^l}_{\Phi^m\Phi^n}\frac{\delta S}{\delta\Phi^l}\tilde{\phi}^m\tilde{\phi}^n, \qquad (A1)\end{aligned}$$

where Φ^l and $\tilde{\phi}^l$ denote generically the classical background fields and the quantum fluctuations respectively. We choose the R_ξ background-field gauge with the gauge-fixing term:

$$L_{gf}=-\frac{1}{2\alpha}(\partial_\mu\tilde{B}^\mu-\alpha e\rho_{cl}\tilde{\eta}_2)^2. \qquad (A2)$$

The corresponding Faddeev-Popov Lagrangian is then

$$L_{FP}=\omega^*(-\partial^2-\alpha e^2\rho_{cl}^2)\omega. \qquad (A3)$$

Compared to the usual background-field formalism, the quadratic quantum flucuations L_{quad} contain extra terms proportional to the connection Γ^i_{jk} of the configuration space. These extra terms are crucial for the cancellation of gauge-parameter dependence in the Higgs-boson self-energy. From Eqs. (5), (6), (7) and (8), we calculate those connections which are relevant to the Higgs-boson self-energy. We find

$$\begin{aligned}\Gamma^{\rho(z)}_{A_\mu(x)A_\nu(y)}|_\Phi&=-e^2\rho_{cl}(z)(\partial^\mu_xN^{xz})(\partial^\nu_yN^{yz})\\\Gamma^{\rho(z)}_{\chi(x)\chi(y)}|_\Phi&=e^2N^{xy}(\delta^4(y-z)+\delta^4(x-z))\rho_{cl}(z)\\&\quad-e^4\rho_{cl}(z)N^{zx}N^{zy}\rho_{cl}(x)\rho_{cl}(y)\\\Gamma^{\rho(z)}_{A_\mu(x)\chi(y)}|_\Phi&=e(\partial^\mu_xN^{zx})\delta^4(z-y)-e^3\rho_{cl}(z)\\&\quad\times(\partial^\mu_xN^{xz})N^{zy}\rho_{cl}(y), \qquad (A4)\end{aligned}$$

where

$$N^{xy}=\left\langle x\left|\frac{1}{\partial^2+e^2\rho_{cl}^2(X)}\right|y\right\rangle$$

with $X_\mu|x\rangle=x_\mu|x\rangle$; and the notation $|_\Phi$ denotes evaluating the connection at the classical background fields. The above connections are to be multiplied by $\delta S/\delta\rho|_\Phi\equiv(-\partial^2-2\lambda u^2-\lambda\rho_{cl}^2)\rho_{cl}$ with the space-time variable z integrated over. It is interesting to note that the product of Γ and $\delta S/\delta\rho$ contain terms which are able to generate the Higgs-boson self-energy. For example, in the expression $-\int d^4xd^4y(\Gamma^{\rho(z)}_{A_\mu(x)A_\nu(y)}\delta S/\delta\rho|_\Phi)\tilde{B}_\mu(x)\tilde{B}_\nu(y)$, we can set

$\rho_{cl}=0$ in N^{xz} and N^{yz} and contract the pair of gauge fields. This gives rise to, in the momentum space, the following Higgs-boson self-energy:

$$\Sigma_\rho^{AA}(p^2)=-\frac{\alpha e^2}{8\pi^2}\frac{1}{\epsilon'}p^2, \tag{A5}$$

where $1/\epsilon'\equiv 1/\epsilon+\frac{1}{2}\gamma_E-\frac{1}{2}\ln(4\pi)$ with $\epsilon=n-4$. For $-\int d^4x d^4y(\Gamma^{\rho(z)}_{\chi(x)\chi(y)}\delta S/\delta\rho|_\Phi)\tilde{\eta}_2(x)\tilde{\eta}_2(y)$, we again set $\rho_{cl}=0$ in N^{xy} and contract the pair of scalar fields. We obtain the Higgs-boson self-energy

$$\Sigma_\rho^{\chi\chi}(p^2)=\frac{e^2}{4\pi^2}\frac{1}{\epsilon'}p^2. \tag{A6}$$

Finally, the term $-\int d^4x d^4y(\Gamma^{\rho(z)}_{A_\mu(x)\chi(y)}\delta S/\delta\rho|_\Phi)\tilde{B}_\mu(x)\tilde{\eta}_2(y)$ can produce an effective $\rho_{cl}-\tilde{B}_\mu-\tilde{\eta}_2$ vertex, namely, $[\int d^4p d^4k/(2\pi)^8]\Gamma^\mu(p,k)\rho_{cl}(k)\tilde{B}_\mu(p)\eta_2(-p-k)$, with $\Gamma_\mu(p,k)=i(p^\mu/p^2)\cdot(k^2-2\lambda u^2)$. This vertex can contribute to the Higgs-boson self-energy by contracting with another vertex of the same kind. Similarly, it could contract with an ordinary $\rho_{cl}-\tilde{B}_\mu-\tilde{\eta}_2$ vertex. It turns out that both contractions produce only finite contributions to the momentum-dependent part of the Higgs-boson self-energy. Therefore Eqs. (A5) and (A6) are the only divergent contributions of Γ^i_{jk} to the momentum-dependent part of the Higgs-boson self-energy. These contributions are to be added to the self-energy obtained by contracting a pair of ordinary $\rho_{cl}-\tilde{B}_\mu-\tilde{\eta}_2$ vertices. We find

$$\Sigma_\rho^{ordinary}(p^2)=\frac{e^2}{8\pi^2}\frac{1}{\epsilon'}(3+\alpha)p^2. \tag{A7}$$

From Eqs. (A5), (A6) and (A7), we arrive at

$$\Sigma_\rho(p^2)=\Sigma_\rho^{AA}(p^2)+\Sigma_\rho^{\chi\chi}(p^2)+\Sigma_\rho^{ordinary}(p^2)=\frac{5e^2}{8\pi^2}\frac{1}{\epsilon'}p^2. \tag{A8}$$

We can see that the gauge-parameter dependence of $\Sigma_\rho^{ordinary}(p^2)$ is cancelled by that of $\Sigma_\rho^{AA}(p^2)$. From Eq. (A8), the wave-function renormalization constant of the Higgs boson is found to be

$$Z_\rho=1-\frac{5e^2}{8\pi^2}\frac{1}{\epsilon'}. \tag{A9}$$

This result can be applied to the model in Sec. III with the replacement $e\to -g_1/2$ according to Eq. (17). Hence $Z_\rho=1-(5g_1^2/32\pi^2)(1/\epsilon')$ in that model, which reproduces the relevant part of Eq. (30) calculated in the Landau-DeWitt gauge.

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