Unexpected symmetries in classical moduli spaces

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We analyze the structure of the moduli space of a supersymmetric $SU(5)$ chiral gauge theory with two matter fields in the **10** representation and two fields in the $\overline{\mathbf{5}}$ representation. Inspection of the exact Kähler potential of the classical moduli space shows that the symmetry group of the moduli space is larger than the global symmetry group of the underlying gauge theory. As a consequence, the gauge theory has classical inequivalent vacua which yield identical low energy theories. [S0556-2821(99)06611-4]

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I. INTRODUCTION

Depending on the matter content, supersymmetric gauge theories can have large vacuum degeneracies [1]. In the absence of a superpotential, classical vacua are associated with vacuum expectation values for which the D terms of the scalar potential vanish. In the Wess-Zumino gauge, the D-flat directions contain those points in the vector space of scalar components of the chiral superfields that satisfy the condition

$$D^a = \sum_i \phi_i^\dagger T^a \phi^i = 0, \tag{1}$$

where the sum is over all matter multiplets, ϕ^i is the scalar component of the superfield Φ^i and T^a are the generators of the gauge group in the appropriate representation. In the case where all matter transforms under (anti)fundamental representations of the gauge group, it is relatively simple to construct solutions to Eq. (1), but for theories with matter in tensor representations, the solutions may be rather complex. No standard methods to find the most general solution are available in the latter case. Reference [2] gives an overview of the efforts to parametrize flat directions in various models.

The D-flatness condition Eq. (1) is covariant under the gauge group G and invariant under the global symmetry group H_G of the gauge theory. The manifold of flat directions is therefore covered with $G \otimes H_G$ orbits. Points in the manifold that lie on the same $G \otimes H_G$ orbit are physically equivalent. The analysis of the flat directions is therefore simplified considerably when the redundancy due to gauge and global symmetry transformations is removed.

To this end, the G orbits in the flat direction manifold can be labeled by a finite set of basic holomorphic gauge invariant polynomials $X_n(\phi^i)$ [3–5]. Any holomorphic gauge invariant polynomial in the fields ϕ^i can be written in terms of products and sums of the basic invariants X_n, by virtue of the decomposition rules for the products of representations of the fields ϕ^i. For some theories the invariants X_n are algebraically independent; for others, relations exist among them.

The invariants X_n form the coordinates of the moduli space (the flat direction manifold modulo gauge transformations) The H_G orbits in the moduli space can be labeled by a finite set $\{I_x\}$ of basic, H_G invariant, Hermitian polynomials in terms of X_n and $X_n^\dagger$. Any Hermitian H_G invariant on the moduli space can be expressed as a function of the I_x. The moduli space is, in fact, a Kähler manifold. Its Kähler potential, induced by the Kähler potential of the gauge theory, is defined by $K_M[I_x(X_n^\dagger, X_n)] = \phi_i^\dagger \phi^i$ for every point in the flat direction manifold.

When the X_n are promoted to chiral superfields, the moduli space becomes equivalent to a supersymmetric chiral sigma model [4,6]. This sigma model describes the low energy limit of the underlying gauge theory if the gauge symmetry is completely broken—the effective, classical theory describing the low energy limit of the gauge theory built on the classical vacuum with expectation values $\langle \phi^i \rangle$ is equivalent to the sigma model when it is expanded around the expectation values $\langle X_n \rangle = X_n(\langle \phi^i \rangle)$. This effective theory, which describes the interactions of the massless degrees of freedom, can also be obtained directly by integrating out the massive vector multiplets[1] in the gauge theory.

Nonperturbative effects can change the classical picture of the moduli space dramatically [2,7,8]. In some cases, a dynamically generated superpotential lifts the moduli space, in other cases, classical constraints among the moduli fields are modified, and in still other cases the structure of the moduli space remains unchanged. Holomorphy and symmetries severely constrain the form of dynamically generated superpotentials. Unfortunately, modifications to the Kähler potential are less well understood.

In some situations, however, corrections to the classical Kähler potential are small. This is for example the case in models with calculable dynamical supersymmetry breaking, where the vacuum expectation values of the scalar fields are much larger than the dynamical scale of the gauge theory.

By construction, the Kähler potential of the classical moduli space is invariant under global symmetry group H_G of the underlying gauge theory. By a detailed analysis of the moduli space of a chiral $SU(5)$ gauge theory with two antisymmetric tensors and two anti-fundamentals, we will show that the symmetry group H_M of the moduli space can be

[1]At the classical level, this means considering tree diagrams with only massless degrees of freedom at the external lines, and contracting internal propagators of massive degrees of freedom in the limit $p^2/M^2 \to 0$.

larger than H_G. When a superpotential is added and nonperturbative effects are taken into account, this $SU(5)$ theory is one of the classic models with calculable dynamical supersymmetry breaking [2,9–13].

The fact that H_M is larger than H_G has some interesting consequences. Classical vacua which are not related by gauge and global symmetry transformations still give rise to the same effective theory in the low energy limit. Moreover, the unbroken symmetry group in the effective theory extends the unbroken symmetry group of the full gauge theory. We have verified that the extended symmetry is not a consequence of a custodial symmetry. Moreover, as we calculated the *exact* classical Kähler potential, it is not a consequence of a truncation either.

In Sec. III, we present a detailed analysis of the moduli space of the $SU(5)$ theory. However, we first discuss a simple, well-known, vectorlike theory with $SU(3)$ gauge symmetry [4] in Sec. II; this serves to illustrate our methods, and to emphasize the main point of this paper by contrast—as the symmetry group of the moduli space of the $SU(3)$ model coincides with the global symmetry group of the underlying gauge theory.

II. SUPERSYMMETRIC QCD WITH TWO FLAVORS

We consider supersymmetric QCD with three colors and two flavors [4]. The quark chiral superfields, which are denoted by Q^i_a, and $\overline{Q}^\alpha_i$, transform as **3** and $\overline{\mathbf{3}}$ under $SU(3)$. Here $i=1,2,3$ is the color index, and $a=1,2$ and $\alpha=1,2$ are flavor indices. The global symmetry group[2] H_G of the theory—the relevant symmetry group at the classical level—is $SU(2)_Q\otimes SU(2)_{\overline{Q}}\otimes U(1)_Q\otimes U(1)_{\overline{Q}}\otimes U(1)_R$. Under H_G the quark superfields transform as $Q^i_a\sim(2,1,1,0,0)$ and $\overline{Q}^\alpha_i\sim(1,2,0,1,0)$. The scalar components of the chiral superfields do not transform under $U(1)_R$, and therefore this factor cannot be spontaneously broken by expectation values of the scalar fields.

The nonanomalous subgroup H_{NA} of H_G—the relevant symmetry group at the quantum level—is $SU(2)_Q\otimes SU(2)_{\overline{Q}}\otimes U(1)_B\otimes U(1)_{R'}$. Under H_{NA} the quark superfields transform as $Q^i_a\sim(2,1,1,-1/2,)$ and $\overline{Q}^\alpha_i\sim(1,2,-1,-1/2)$.

The flat directions of the theory are solutions to the equation

$$Q^{\dagger a}_i Q^j_a-\overline{Q}^{\dagger j}_\alpha \overline{Q}^\alpha_i=c\,\delta^j_i. \tag{2}$$

Here c is, *a priori*, an arbitrary real constant. However, it turns out there are only solutions for $c=0$. Any solution to Eq. (2) can be obtained from the solution $Q^1_1=\overline{Q}^1_1=a$, $Q^2_2=\overline{Q}^2_2=b$, with all other components vanishing, by applying appropriate gauge and global symmetry transformations. For generic values of the real parameters a and b, the gauge group is completely broken. Eight of the twelve chiral superfields are eaten to give mass to the vector multiplets. As a consequence, the number of moduli fields is four and the moduli space is eight dimensional. The unbroken global symmetry group is $U(1)\otimes U(1)\otimes U(1)_R$. The moduli space is therefore spanned by the two parameters a and b, and six of the nine parameters of H_G transformations.

The basic holomorphic gauge invariants for this theory are $M^\alpha_a=\overline{Q}^\alpha_i Q^i_a$, transforming as $(2,2,1,1,0)$ under H_G. These four meson fields form the coordinates of the moduli space, and their vacuum expectation values can be written in the form $M^1_1=m_1$, $M^2_2=m_2$ and $M^2_1=M^1_2=0$ by H_G transformations.

The basic Hermitian structures—invariant under the global symmetry transformations and constructed out of the meson fields—are $I_1=M^{\dagger a}_\alpha M^\alpha_a$ and $I_2=M^{\dagger a}_\alpha M^{\dagger b}_\beta M^\beta_a M^\alpha_b$, with the range of I_2 limited by the inequality $1/2I_1^2\leq I_2\leq I_1^2$. The *exact* induced Kähler potential of the classical moduli space is defined as $K_M[I_1(M^\dagger,M),I_2(M^\dagger,M)]\equiv Q^{\dagger a}_i Q^i_a+\overline{Q}^{\dagger i}_\alpha \overline{Q}^\alpha_i$, and a simple calculation gives

$$K_M=2\sqrt{\frac{1}{2}I_1+\frac{1}{2}\sqrt{2I_2-I_1^2}}+2\sqrt{\frac{1}{2}I_1-\frac{1}{2}\sqrt{2I_2-I_1^2}}. \tag{3}$$

This Kähler potential is invariant under the global symmetry group H_G of the underlying gauge theory by construction. As is conventional, K_M is not invariant under any other symmetries, so that the symmetry group H_M of the moduli space is equal to H_G. As will become clear in the next section, however, even though H_M always contains H_G, it can in fact be larger.

The Kähler potential of the moduli space is derived in terms of the scalar components of the superfields. However, when the the moduli fields M^α_a are promoted to superfields, a supersymmetric sigma model ensues. The low energy limit of the classical gauge theory constructed on the vacuum with expectation values $\langle Q^i_a\rangle$ and $\langle\overline{Q}^\alpha_i\rangle$ is equivalent to the sigma model with vacuum expectation values $\langle M^\alpha_a\rangle=\langle\overline{Q}^\alpha_i\rangle\langle Q^i_a\rangle$.

The H_G orbits that cover the moduli space can be labeled by $\{a,b\}$, $\{m_1,m_2\}$, or $\{I_1,I_2\}$. Points in the moduli space that lie on the same orbit yield physically equivalent classical vacua. The orbits, in turn, can be grouped into strata. Different orbits that belong to the same stratum yield vacua that are physically inequivalent, but qualitatively similar. Such vacua yield the same symmetry breaking pattern and the same degeneracies in the mass spectrum, but the masses are quantitatively different. The strata can be categorized as follows.

(i) For generic H_G orbits, labeled by generic values of $\{I_1,I_2\}$, the little group is $U(1)\otimes U(1)\otimes U(1)_R$. The gauge symmetry is completely broken.

(ii) For orbits with $I_2=1/2I_1^2$, $(b=\pm a;\ m_1=\pm m_2)$ the little group is $SU(2)\otimes U(1)\otimes U(1)_R$. The gauge symmetry is completely broken.

(iii) For orbits with $I_2=I_1^2$, $(b=0;\ m_2=0)$ the little group is $U(1)\otimes U(1)\otimes U(1)\otimes U(1)_R$. In the sigma model, the metric derived from the Kähler potential K_M is singular.

[2]According to our conventions for R symmetry, the charge of the scalar component of a chiral superfield is R, whereas the charge of the fermionic component is $R-1$. The gaugino has charge 1.

Moreover, in the gauge theory the gauge group is broken to $SU(2)$ and therefore, the low energy theory should include the massless gauge multiplets.

(iv) When $I_1=0$, none of the gauge and global symmetries are broken.

The classical picture of the moduli space is altered dramatically by nonperturbative effects. The nonanomalous global symmetry group H_{NA} of the gauge theory allows a unique, nonperturbative superpotential [4] of the form

$$W_{np}=\frac{\Lambda^7}{\overline{Q}_i^\alpha Q_a^i \overline{Q}_j^\beta Q_b^j \epsilon_{\alpha\beta}\epsilon^{ab}}. \tag{4}$$

Explicit instanton calculations in the semiclassical approximation [14,15] show that such an effective superpotential is indeed generated and that Λ is the dynamical scale of the gauge theory. The F-term contributions to the scalar potential completely lift the D-flat directions. The scalar potential does not have a minimum, tends to zero only at infinity, and renders the theory unstable. However, the scalar potential is stabilized if a mass term of the form

$$W_m=m_\alpha^a \overline{Q}_i^\alpha Q_a^i \tag{5}$$

is added to the superpotential. If the scale of the masses m_α^a is much smaller than the dynamical scale Λ, then the vacuum expectation values of the scalar fields are much larger than Λ and the theory is weakly coupled. It is in this limit that the classical Kähler potential is relevant. The theory below the dynamical scale can be described in terms of the moduli fields M_a^α, with Kähler potential K_M and superpotential

$$W=\frac{\Lambda^7}{M_a^\alpha M_b^\beta \epsilon_{\alpha\beta}\epsilon^{ab}}+m_\alpha^a M_a^\alpha. \tag{6}$$

The vacuum energy vanishes, and supersymmetry is not broken in this theory.

III. CHIRAL $SU(5)$ THEORY

The chiral supersymmetric $SU(5)$ gauge theory we discuss in this section contains two matter fields transforming under the **10** representation of $SU(5)$, and two fields transforming under the $\overline{\mathbf{5}}$ representation. These matter fields are denoted by the two index antisymmetric tensors T_a^{ij}, and $\overline{F}_i^\alpha$, where $i,j=1,\ldots,5$ are gauge indices, and $a=1,2$ and $\alpha=1,2$ are flavor indices. With this matter content, the theory is anomaly free and asymptotically free.

The global symmetry group H_G of the theory is $SU(2)_T \otimes SU(2)_{\overline{F}} \otimes U(1)_T \otimes U(1)_{\overline{F}} \otimes U(1)_R$. Under H_G, the matter fields transform as $T_a \sim (1,2,1,0,0)$ and $\overline{F}^\alpha \sim (2,1,0,1,0)$. The scalar components of the chiral superfields do not transform under $U(1)_R$. Their vacuum expectation values therefore do not break this symmetry. Under the nonanomalous subgroup of H_G, $SU(2)_T \otimes SU(2)_{\overline{F}} \otimes U(1)_A \otimes U(1)_{R'}$, the matter fields transform as $T_a \sim (1,2,1,1)$ and $\overline{F}^\alpha \sim (2,1,-3,-4)$.

The D-flat directions of the theory are solutions to the equation

$$T_{ij}^{a\dagger} T_a^{ik} - \overline{F}_\alpha^{k\dagger} \overline{F}_j^\alpha = c\,\delta_j^k, \tag{7}$$

where c is an arbitrary real constant. In Refs. [2,9,11,13], some incomplete families of solutions to Eq. (7) were presented. Here, we give the most general solution which of course includes the previously found families. Any solution to Eq. (7) can be obtained from a four-parameter solution through gauge and global symmetry transformations. This four-parameter solution takes the form $T_2^{12}=a$, $T_2^{34}=b$, $\overline{F}_1^1=c$, $\overline{F}_5^1=d$, and

$$\begin{aligned}
T_1^{13}&=\frac{c}{b}\sqrt{a^2-c^2}\sqrt{\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}},\\
T_1^{45}&=\frac{a}{b}\sqrt{a^2-c^2}\sqrt{\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}},\\
T_2^{23}&=\frac{c}{\sqrt{a^2-c^2}}\sqrt{b^2-(a^2-c^2)},\\
T_2^{25}&=-\frac{cd}{a},\\
T_2^{45}&=\frac{d}{ba}\sqrt{a^2-c^2}\sqrt{b^2-(a^2-c^2)},\\
\overline{F}_3^1&=-\frac{a}{\sqrt{a^2-c^2}}\sqrt{b^2-(a^2-c^2)},\\
\overline{F}_2^2&=\frac{c}{b}\sqrt{b^2-(a^2-c^2)}\sqrt{\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}},\\
\overline{F}_4^2&=-\sqrt{a^2-c^2}\sqrt{\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}}.
\end{aligned} \tag{8}$$

All other components vanish, and a, b, c, and d are real parameters. For generic values of $\{a,b,c,d\}$, the gauge symmetry is completely broken. Therefore, twenty four of the thirty chiral superfields are eaten to give masses to the vector multiplets, leaving six moduli fields to function as coordinates for the twelve dimensional moduli space. The global symmetry group H_G is broken to $U(1)_R$. In terms of the fundamental fields, the moduli space is spanned by the four parameters $\{a,b,c,d\}$ of the solution given in Eq. (8), and eight of the nine parameters of H_G transformations. The basic holomorphic gauge invariants for this theory are given by

$$\begin{aligned}
X_a&=\epsilon_{\alpha\beta}\overline{F}_i^\alpha \overline{F}_j^\beta T_a^{ij},\\
J_a^\alpha&=\epsilon_{ijklm}\overline{F}_n^\alpha T_a^{ij} T_b^{kl} T_c^{mn}\epsilon^{bc}.
\end{aligned} \tag{9}$$

Under H_G, these holomorphic gauge invariants transform as $X_a \sim (1,2,1,2,0)$ and $J_a^\alpha \sim (2,2,3,1,0)$. By suitable H_G transformations, the vacuum expectation values of the basic holomorphic gauge invariants can be written as $X_1=x_1$, $X_2=x_2$, $J_1^1=j_1$, $J_2^2=j_2$ and $J_1^2=J_2^1=0$, with x_1, x_2, j_1 and j_2 real parameters. In fact, the expectation values of the holomorphic gauge invariants for the four-parameter solution, given in Eq. (8), already have this form:

$$X_1=2\frac{ad}{b}(a^2-c^2)\left(\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}\right),$$

$$X_2=2\frac{a^3}{b}\sqrt{b^2-(a^2-c^2)}\left(\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}\right)^{3/2},$$

$$J_1^1=12\frac{a^2c^2}{b^2}(a^2-c^2)\left(\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}\right)^2,$$

$$J_1^2=0,$$

$$J_2^1=0,$$

$$J_2^2=-12a^2(a^2-c^2)\left(\frac{b^2}{a^2-c^2}+\frac{d^2}{a^2}\right). \tag{10}$$

The holomorphic invariants X_a and J_a^α provide the coordinates for the moduli space. A completely H_G invariant description of the moduli space can be given in terms of the four Hermitian invariants

$$I_1=X^{a\dagger}X_a,$$

$$I_2=J_\alpha^{a\dagger}J_a^\alpha,$$

$$I_3=X^{a\dagger}J_\beta^{b\dagger}X_bJ_a^\beta,$$

$$I_4=J_\alpha^{a\dagger}J_\beta^{b\dagger}J_a^\beta J_b^\alpha, \tag{11}$$

where the range of I_4 is limited to $1/2I_2^2\leqslant I_4\leqslant I_2^2$, and the range of I_3 is limited by $(2I_3-I_1I_2)^2\leqslant(2I_4-I_2^2)I_1^2$. The moduli space is thus covered by H_G orbits, labeled by $\{a,b,c,d\}$, $\{x_1,x_2,j_1,j_2\}$, or $\{I_1,I_2,I_3,I_4\}$. In our previous work [13], the exact Kähler potential[3] of the classical moduli space was derived. Invariance under H_G dictates that the Kähler potential has the functional form

$$K_M(X^\dagger,X,J^\dagger,J)=K_M(I_1,I_2,I_3,I_4). \tag{12}$$

Defining

$$A=125I_1,$$

$$B=\frac{25}{9}\left(\sqrt{\frac{1}{2}I_2+\frac{1}{2}\sqrt{2I_4-I_2^2}}+\sqrt{\frac{1}{2}I_2-\frac{1}{2}\sqrt{2I_4-I_2^2}}\right), \tag{13}$$

and

$$p=2\sqrt{B}\cos\left(\frac{1}{3}\arccos\frac{A}{B^{3/2}}\right), \tag{14}$$

the Kähler potential of the moduli space is given by

$$K_M=\frac{3}{10}\left(p+\frac{B}{p}\right). \tag{15}$$

The metric derived from this Kähler potential is singular if $I_4=I_2^2$. Curiously, K_M does not depend on I_3. As a consequence—and this illustrates the central point of this paper—the symmetry group H_M of the moduli space $SU(2)_X\otimes SU(2)_1\otimes SU(2)_2\otimes U(1)_X\otimes U(1)_J\otimes U(1)_R$ is larger than the global symmetry group H_G of the underlying gauge theory. The moduli fields transform under H_M as $X_a\sim(2,1,1,1,0,0)$ and $J_a^\alpha\sim(1,2,2,0,1,0)$. The $U(1)_R$ factor in H_M is the same factor that appears in H_G. Only fermions transform under this symmetry, and it does not play any role in the discussion below. We will therefore suppress this factor from here on.

Generic H_M orbits in the moduli space, labeled by $\{I_1,I_2,I_4\}$, contain one-parameter families of H_G orbits, labeled by I_3. In particular, the points

$$X_1=x\cos\phi,$$

$$X_2=x\sin\phi,$$

$$J_1^1=j_1,$$

$$J_1^2=0,$$

$$J_2^1=0,$$

$$J_2^2=j_2, \tag{16}$$

for fixed values of $\{x,j_1,j_2\}$, and varying ϕ, are equivalent in the moduli space, as ϕ corresponds to the parameter of an $SU(2)_X$ rotation. H_M orbits can therefore also be labeled by $\{x,j_1,j_2\}$, and the H_G orbits contained in an H_M orbit can be labeled by ϕ.

When the moduli fields are promoted to superfields, a supersymmetric sigma model results. The low energy limit of the gauge theory built on the classical vacuum with expectation values $\langle T_a^{ij}\rangle$ and $\langle\overline{F}_i^\alpha\rangle$ is equivalent to the sigma model with vacuum expectation values

$$\langle X_a\rangle=\epsilon_{\alpha\beta}\langle\overline{F}_i^\alpha\rangle\langle\overline{F}_j^\beta\rangle\langle T_a^{ij}\rangle$$

and

[3]The Kähler potential of the moduli space $K_M(I_1,I_2,I_3,I_4)=1/2T_{ij}^{a\dagger}T_a^{ij}+\overline{F}_\alpha^{i\dagger}\overline{F}_i^\alpha$ for all values of the parameters $\{a,b,c,d\}$ of the four-parameter solution to the D-flatness equation.

$$\langle J_a^\alpha\rangle=\epsilon_{ijklm}\langle\overline{F}_n^\alpha\rangle\langle T_a^{ij}\rangle\langle T_b^{kl}\rangle\langle T_c^{mn}\rangle\epsilon^{bc}.$$

The extended symmetry of the moduli space has two important consequences. First, vacua of the gauge theory corresponding to fixed values of $\{x,j_1,j_2\}$, but varying ϕ, are physically inequivalent. In particular, the masses[4] of the vector multiplets are a function of ϕ. In fact, while for generic values of ϕ gauge and global symmetries are completely broken, for the special values of $\phi=0$ and $\phi=\pi/2$ there is a remaining global $U(1)$ symmetry. However, all vacua of the sigma model with fixed values of $\{x,j_1,j_2\}$ and arbitrary value of ϕ, either generic or special, are equivalent. Therefore, the low energy limit of the gauge theory, which is obtained by integrating out the massive vector multiplets in the limit $p^2/M^2\to 0$, is identical for each value of ϕ. Physically inequivalent vacua of the gauge theory, with distinct mass spectra and possibly even distinct global symmetry breaking patterns, yield the same low energy theory. Second, for generic vacua, the global symmetry group of the gauge theory is broken to $U(1)_R$. However, the symmetry group H_M of the moduli space is broken to $U(1)\otimes U(1)\otimes U(1)_R$. Therefore, the low energy limit of the gauge theory has a larger symmetry group than expected from the global symmetry breaking pattern of the full gauge theory.

The three-parameter solution to the D-flatness condition Eq. (7), obtained by imposing the condition $b^2=a^2-c^2$ on the four-parameter solution given in Eq. (8), corresponds to arbitrary $\{I_1,I_2,I_4\}$ and $I_3=0$, or, alternatively, arbitrary $\{x,j_1,j_2\}$ and $\phi=0$. This three-parameter solution, therefore, contains a representative point on all H_M orbits in the moduli space. However, it does not contain a representative point on all H_G orbits. Therefore, the corresponding classical vacua yield all physically inequivalent low energy theories, yet not all physically inequivalent classical gauge theories.

We will describe the moduli space in terms of strata of H_M and H_G orbits in turn. The first approach lends itself for the study of all inequivalent low energy theories, while the latter is more suitable for the study of all inequivalent classical gauge theories.

As explained before, H_M orbits are labeled by either $\{I_1,I_2,I_4\}$ or $\{x,j_1,j_2\}$. For generic orbits, labeled by generic values of $\{x,j_1,j_2\}$, H_M is broken to $U(1)\otimes U(1)$. One of the $U(1)$ factors is a subgroup of $SU(2)_X\otimes U(1)_X$; the other, a subgroup of $SU(2)_1\otimes SU(2)_2\otimes U(1)_J$. The number of broken symmetry generators, 9, is larger than the number of moduli fields, 6, and therefore some of the corresponding Goldstone bosons are non-doubled. Apart from the generic stratum, there are strata for which the little group is larger. The strata can be classified as follows.

(i) $I_1=0,\ I_2=0$; $(x=0,\ j_1=0,\ j_2=0)$. The metric is singular and there is no spontaneous symmetry breaking. Therefore, the little group is $SU(2)_X\otimes SU(2)_1\otimes SU(2)_2\otimes U(1)_X\otimes U(1)_J$. The multiplets transform as (2,0,0,1,0) and (1,2,2,0,1).

(ii) $I_2=0$; $(j_1=0,\ j_2=0)$. The metric is singular, and the little group is $U(1)\otimes SU(2)_1\otimes SU(2)_2\otimes U(1)_J$. The multiplets transform as (0,2,2,1), (0,1,1,0) and (1,1,1,0).

(iii) $I_1=0,\ I_4=I_2^2$; $(x=0,\ j_2=0)$. The metric is singular, and the little group is $SU(2)_X\otimes U(1)\otimes U(1)\otimes U(1)_X$. The multiplets transform as (2,0,0,1), (1,0,−2,0), (1,−1,1,0), (1,−1,−1,0), and (1,0,0,0).

(iv) $I_1=0,\ I_4=\frac{1}{2}I_2^2$; $(x=0,\ j_1=\pm j_2)$. The little group is $SU(2)_X\otimes SU(2)\otimes U(1)_X$. The multiplets transform as (2,1,1), (1,3,0), and (1,1,0).

(v) $(I_1=0;x=0)$. The little group is $SU(2)_X\otimes U(1)\otimes U(1)_X$. Two multiplets transform as (1,0,0), while the remaining multiplets transform as (2,0,1), (1,−1,0), and (1,1,0).

(vi) $I_4=I_2^2$; $(j_2=0)$. The metric is singular, and the little group is $U(1)\otimes U(1)\otimes U(1)$. Two multiplets transform as (0,0,0), while the remaining multiplets transform as (0,0,−2) (1,0,0), (0,−1,1), and (0,−1,−1).

(vii) $I_4=\frac{1}{2}I_2^2$; $(j_1=\pm j_2)$. The little group is $U(1)\otimes SU(2)$. Two multiplets transform as (0,1), while the remaining multiplets transforms as (0,3) and (1,1).

(viii) Generic $I_1,\ I_2,\ I_4$; (generic $x_1,\ j_1,\ j_2$). The little group is $U(1)\otimes U(1)$. Three multiplets transform as (0,0), while the remaining multiplets transform as (1,0), (0,−1) and (0,1).

H_G orbits can be labeled by $\{I_1,I_2,I_3,I_4\}$, $\{x_1,x_2,j_1,j_2\}$, or $\{a,b,c,d\}$. For each stratum, we indicate the subgroup of H_G which remains unbroken, and also the remaining subgroup of the gauge group in case the gauge symmetry is not completely broken.

(i) $I_1=0,\ I_2=0$; $(x_1=0,\ x_2=0,\ j_1=0,\ j_2=0)$. The gauge and global symmetries remain unbroken.

(ii) $I_2=0$; $(x_2=0,\ j_1=0,\ j_2=0)$. The unbroken global symmetry group is $U(1)\otimes SU(2)_{\overline{F}}\otimes U(1)$. The gauge symmetry is broken to $SU(3)$. The solution $T_1^{12}=a,\ \overline{F}_1^1=a,\ \overline{F}_2^2=a$, with $x_1=2a^3$, contains representative points of the orbits in this stratum.

(iii) $I_1=0,\ I_4=I_2^2$; $(x_1=0,\ x_2=0,\ j_2=0)$. The remaining global symmetry group is $U(1)\otimes U(1)\otimes U(1)$. The gauge symmetry is broken to $SU(2)$. The solution $T_1^{12}=a,\ T_2^{45}=a,\ \overline{F}_4^1=a$, with $j_1=12a^4$ contains representative points of orbits in this stratum.

(iv) $I_1=0,\ I_4=1/2I_2^2$; $(x_1=0,\ x_2=0,\ j_1=\pm j_2)$. The unbroken global symmetry group is $SU(2)\otimes U(1)$, and the gauge symmetry is completely broken. The solution $T_1^{12}=T_1^{34}=T_2^{15}=T_2^{24}=\overline{F}_1^1=\overline{F}_4^2=a$, with $j_1=-j_2=12a^4$, contains representative points of orbits in this stratum.

(v) $I_1=0$; $(x_1=0,\ x_2=0)$. The remaining global symmetry group is $U(1)\otimes U(1)$, and the gauge symmetry is completely broken. The solution $T_1^{12}=a,\ T_1^{34}=T_2^{15}=\sqrt{a^2+b^2}$,

[4] The mass spectrum of the vector multiplets, which we studied numerically, displays some unusual features. For generic values of $\{x,j_1,j_2,\phi\}$ both the gauge and global symmetries of the gauge theory are completely broken. However, the spectrum contains four degenerate pairs of masses, and one degenerate quintuplet. Moreover, even though the spectrum changes with ϕ for fixed values of $\{x,j_1,j_2\}$, the sum of the squares of the masses and the mass of the degenerate quintuplet remain independent of ϕ.

$T_2^{24}=b$, $\overline{F_1^1}=a$ and $\overline{F_4^2}=b$, with $j_1=12a^2(a^2+b^2)$ and $j_2=-12b^2(a^2+b^2)$, contains representative points of orbits in this stratum. The solution presented in Ref. [9] also contains representative points of orbits in this stratum.

(vi) $I_3=I_1I_2$, $I_4=I_2^2$; $(x_2=0,\ j_2=0)$. The unbroken global symmetry group is $U(1)\otimes U(1)$, and the gauge symmetry broken to $SU(2)$. The solution $T_1^{12}=a$, $T_1^{45}=T_2^{13}=b$, $\overline{F_1^1}=a$ and $\overline{F_2^2}=\sqrt{a^2-b^2}$, with $x=2a^2\sqrt{a^2-b^2}$, $j_1=12a^2b^2$ and $j_2=0$, contains representative points of orbits in this stratum.

(vii) $(2I_3-I_1I_2)^2=I_1^2(2I_4-I_2^2)$; $(x_2=0)$. The remaining global symmetry group is $U(1)$, and the gauge symmetry is completely broken. The flat directions presented in Refs. [2,11] contain representative points on the orbits in this stratum. As shown in Ref. [13], the classical vacuum of the $SU(5)$ model with calculable supersymmetry breaking lies on an orbit in this stratum with the property $j_1=\pm j_2$. In terms of H_G orbits, this additional condition does not lead to a larger little group.

(viii) $I_4^2=I_2^2$; $(j_2=0)$. The remaining global symmetry group is $U(1)$, and the gauge symmetry is broken to $SU(2)$.

(ix) Generic I_1, I_2, I_3, I_4; (generic x_1, x_2, j_1, j_2). Both global and gauge symmetries are completely broken.

Even though every H_G orbit is contained in an H_M orbit, not every stratum of H_G orbits is completely contained in a stratum of H_M orbits.

As in the $SU(3)$ model discussed in Sec. I, nonperturbative effects completely change the classical picture of the moduli space. A nonperturbative effective superpotential

$$W_{np}=\frac{\Lambda^{11}}{J_a^\alpha J_b^\beta \epsilon_{\alpha\beta}\epsilon^{ab}}, \tag{17}$$

generated by instantons, lifts the vacuum degeneracy completely. However, instead of a mass term, which is not consistent with the chiral nature of the $SU(5)$ theory, a renormalizable Yukawa-type interaction in the superpotential can be introduced to stabilize the scalar potential. As described in Refs. [9,10,12,13], if the coupling constant of this Yukawa term is sufficiently small, the theory below the dynamical scale of the gauge interactions is a supersymmetric sigma model, which has X_a and J_a^α as coordinates, K_M as Kähler potential, and

$$W=\frac{\Lambda^{11}}{J_a^\alpha J_b^\beta \epsilon_{\alpha\beta}\epsilon^{ab}}+\lambda X_1 \tag{18}$$

as the superpotential. In contrast to the $SU(3)$ model, the vacuum energy does not vanish, and therefore supersymmetry is broken. The light mass spectrum, as calculated in Refs. [12,13] displays some degeneracies which cannot be explained by the symmetry breaking pattern of the global symmetry group of the gauge theory including the superpotential. However, as a consequence of the H_M invariance of the Kähler potential, the symmetry group of the sigma model extends the global symmetry group of the full gauge theory. In particular, the sigma model is invariant under $SU(2)_1\otimes SU(2)_2$ transformations. The degeneracies in the light spectrum square with the breaking pattern of the extended symmetry group of the sigma model.

IV. CONCLUSIONS

We have presented a detailed study of the classical moduli space of the $SU(5)$ gauge theory with two antisymmetric tensors and two antifundamentals. We found that the symmetry group H_M of the classical moduli space extends the global symmetry group H_G of the gauge theory. We analyzed the moduli space in terms of orbits of both symmetry groups.

The extended symmetry of the moduli space has two main consequences. Physically inequivalent classical vacua of the gauge theory may have identical low energy limits, and the effective models that describe the massless degrees of freedom in the low energy limit have a symmetry group that is larger than the unbroken subgroup of H_G. Even though nonperturbative effects completely lift the classical moduli space, a remnant of the extended symmetry group of the Kähler potential is the origin of degeneracies in the mass spectrum of the calculable $SU(5)$ model with dynamical supersymmetry breaking.

The extended symmetry of the classical moduli space is traced to the fact that the Kähler potential does not depend on an Hermitian invariant consistent with the global symmetry group of the gauge theory. We calculated the mass spectrum of the gauge theory for vacua that are related by H_M transformations but not by H_G transformations, and we found that the mass spectrum of the massive vector multiplets differs. This assured us that the additional symmetry of the moduli space is not realized as a symmetry of the full gauge theory. In fact, the same evidence also eliminates the possibility that just the scalar potential is invariant under the extended symmetry.

As an aside, the degeneracies in the spectrum of the massive vector multiplets pose an intriguing question. In a generic point of the moduli space, all global and gauge symmetries are broken, and therefore no degeneracies are expected. However, the existence of a degenerate quintuplet hints at some kind of symmetry.

Returning to the question of the extended symmetry of the classical moduli space, we cannot completely rule out the possibility that the full gauge theory, or the just the scalar potential, is invariant under some symmetry other than any of the extended symmetry transformations of the classical moduli space, maybe even a discrete symmetry, that we are unaware of. If such a symmetry exists and if it forbids the absent terms in the Kähler potential, then the extended symmetry of the classical moduli space that we have found would be coincidental.

If the latter scenario is not realized, it is possible to take the point of view that the classical moduli spaces of supersymmetric chiral gauge theories with matter in tensor representations have complicated structure, and that calculating their Kähler potential provides an apt tool to understand this structure. However, we find such a perspective somewhat unsatisfying and still feel that it is worthwhile to seek a fundamental principle that allows the determination of the symmetries of the classical moduli space without an explicit calculation of the Kähler potential.

Finally, we want to address the question whether the classical moduli spaces of other supersymmetric gauge theories have extended symmetries. Nontrivial flavor structure and matter transforming under nonfundamental representations of the gauge group seem to be prerequisites. However, with such matter content, the parametrization of generic flat directions often is prohibitively complicated, and an explicit calculation of the Kähler potential of the classical moduli space is impossible. This is particularly the case when the matter content is chosen so that the gauge symmetry is nonanomalous, although this does not seem to be required in a study of classical moduli spaces.

Looking at closely related models, the SU(5) model with one generation—one antisymmetric tensor and one antifundamental—has no flat directions. The model with three generations has twenty one moduli fields and its inequivalent classical vacua are labeled by twenty four parameters. Parametrizing generic flat directions for this model is a forbidding task. Nevertheless, the structure of the classical moduli space is of interest: When nonperturbative effects are taken into account, the model is in an *s*-confining phase [16], and the structure of its classical moduli space is conjectured to be unmodified.

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