Spin-momentum correlations in inclusive semileptonic decays of polarized Λ_b baryons

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We consider spin-momentum correlations between the spin of the bottom baryon Λ_b and the momenta of its decay products in its inclusive semileptonic decay. We define several polar and azimuthal spin-momentum correlation measures in different event coordinate systems. The values of the spin-momentum correlation measures are calculated up to $\mathcal{O}(1/m_b^2)$ using the standard OPE und HQET methods. Some of the measures turn out to be sufficiently large to make them good candidates for a determination of the polarization of the Λ_b in e.g. Z decays. [S0556-2821(99)07313-0]

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I. INTRODUCTION

A few years ago the ALEPH Collaboration measured the polarization of bottom baryons Λ_b's originating from Z decays [1]. The ALEPH Collaboration quoted a value for the polarization of $P=-0.23^{+0.24}_{-0.20}\pm 0.08$ which is significantly smaller than what would be expected theoretically in the standard model [$P=-(0.60$–$0.70)$] [2]. Recently a new measurement of the polarization has become available from the OPAL Collaboration [3]. They obtain the result $P=-0.56^{+0.20}_{-0.13}\pm 0.09$ which is in agreement with theoretical expectations.

The measurement of the ALEPH Collaboration is based on the observation of Bonvicini and Randall [4] that, with negatively polarized Λ_b's, the spectra of the decay electrons and antineutrinos become harder and softer relative to unpolarized decay, respectively, and that the fragmentation dependence of $b\to\Lambda_b$ practically drops out in the ratio $y=\langle E_l\rangle/\langle E_{\bar\nu}\rangle$. In a previous paper we explored possible improvements on such spectra related polarization measures [5]. A promising candidate measure is, among others, the ratio $y_2=\langle E_l^2\rangle/\langle E_{\bar\nu}^2\rangle$, a measurement of which may help to reduce the errors in the original ALEPH analysis.

The method used by the OPAL Collaboration [3] is to compare the observed distribution of the ratio E_l/E_ν against a simulation of this ratio using a JETSET Monte Carlo event generator. It is perhaps worth mentioning that the distribution of this ratio is sensitive to the precise shape of the $b\to\Lambda_b$ fragmentation function [5], which is not the case with the ratios $y_n=\langle E_l^n\rangle/\langle E_{\bar\nu}^n\rangle$. A modified method was proposed in [5] which avoids this problem, wherein the fragmentation dependence is eliminated between the two ratios $\langle E_l/E_{\bar\nu}\rangle$ and $\langle E_{\bar\nu}/E_l\rangle$.

In this paper we explore possibilities to determine the polarization of the Λ_b through angular spin-momentum correlations of the spin of the Λ_b and the momenta of its decay products in its inclusive semileptonic decays (the results of a preliminary version of the present work have been presented in [6]). We work in the rest frame of the decaying Λ_b throughout and define various polarization measures which we compute up to $\mathcal{O}(1/m_b^2)$ in the heavy mass expansion using the standard operator product expansion (OPE) and heavy quark effective theory (HQET) approach to inclusive semileptonic decays developed in [7–10]. The $O(\alpha_s)$ radiative corrections to some of these asymmetry parameters (B_1,B_3) have been previously computed [13,14].

The analysis of the spin-momentum correlation measures in Sec. II makes use of so-called helicity systems, in which the plane spanned by the three final state momenta $\vec p_X$, $\vec p_l$ and $\vec p_\nu$ (event plane) is in the (x,z) plane. The orientation of the polarization vector $\vec P$ is specified by two angles (ϑ,φ) for which we compute the angular decay distributions. In Sec. III we do the same exercise for so-called transversity systems, where the event plane defines the (x,y) coordinate plane.

II. SPIN-MOMENTUM CORRELATIONS IN THE HELICITY SYSTEM

As we are analyzing the decay $\Lambda_b\to X_c(p_X)+l^-(p_l)+\overline{\nu_l}(p_\nu)$ in the rest frame of the Λ_b, the three-momenta $\vec p_X,\vec p_l$ and $\vec p_\nu$ lie in a plane — the event plane. It is then a matter of choice how to orient the event coordinate system relative to the event plane and thereby relative to the polarization vector of the Λ_b. In this section we will discuss so-called helicity systems in which the z axis is in the event plane. It is then convenient to define three coordinate systems according to the orientation of the z axis. Also one has to specify the orientation of the x axis for which one has two possible choices in each system. We thus define our coordinate systems as

$$\begin{aligned}
&\text{system 1:}\quad \vec p_l\| z,\quad a:\ (\vec p_{\bar\nu})_x\geqslant 0\quad b:\ (\vec p_X)_x\geqslant 0\\
&\text{system 2:}\quad \vec p_X\| z,\quad a:\ (\vec p_l)_x\geqslant 0\quad b:\ (\vec p_{\bar\nu})_x\geqslant 0\\
&\text{system 3:}\quad \vec p_{\bar\nu}\| z,\quad a:\ (\vec p_X)_x\geqslant 0\quad b:\ (\vec p_l)_x\geqslant 0.
\end{aligned}\tag{2.1}$$

In this paper we shall always work in systems 1a, 2a and 3a such that $\vec p_{\bar\nu}$, $\vec p_l$ and $\vec p_X$, respectively, have positive x components. When using systems 1b, 2b and 3b the sign of the

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coefficient B in the angular decay distribution defined in Eq. (2.2) below remains unchanged while the sign of the coefficient C changes as can be seen by making the transformation $\cos\varphi \to \cos(\varphi+\pi) = -\cos\varphi$.

It should be clear that the choice of the z axis in the event plane is optional. Other possible choices would be to take the directions bisecting any two of the three momenta directions $\vec{p}_X, \vec{p}_l, \vec{p}_\nu$, etc. The above choice has been made for experimental convenience.

In generic form the five-fold decay distribution (differential in $q_0, q^2, \cos\theta, \cos\vartheta$ and φ) reads

$$\frac{d\Gamma}{dq_0 dq^2 d\cos\theta d\cos\vartheta d\varphi} = \frac{1}{4\pi}\Gamma_b\left\{\frac{d\hat{\Gamma}_A}{d\hat{q}_0 d\hat{q}^2 d\cos\theta} + P\left(\frac{d\hat{\Gamma}_B}{d\hat{q}_0 d\hat{q}^2 d\cos\theta}\cos\vartheta + \frac{d\hat{\Gamma}_C}{d\hat{q}_0 d\hat{q}^2 d\cos\theta}\sin\vartheta\cos\varphi\right)\right\} \tag{2.2}$$

where

$$\Gamma_b = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \tag{2.3}$$

is the reference rate of the decay into three massless final particles. Other symbols appearing in Eq. (2.2) are defined as follows. The energy and the invariant mass squared of the virtual boson are denoted by q_0 and q^2 respectively, with corresponding reduced quantities $\hat{q}_0 = q_0/m_b$ and $\hat{q}^2 = q^2/m_b^2$. The polar angle of the lepton l^- in the $(l^-, \overline{\nu_l})$ rest frame relative to the direction of $\vec{p}_X$ is denoted by θ. There is one unpolarized reduced rate function $d\hat{\Gamma}_A$ and two polarized rate functions $d\hat{\Gamma}_B$ and $d\hat{\Gamma}_C$. We shall sometimes also employ the notation

$$\frac{d\hat{\Gamma}_I}{d\hat{q}_0 d\hat{q}^2 d\cos\theta} = I(\hat{q}_0, \hat{q}^2, \cos\theta), \quad I = A, B, C, \tag{2.4}$$

with a corresponding notation for the once, twice and thrice integrated forms. The polar angle ϑ and the azimuthal angle φ define the orientation of the polarization vector $\vec{P}$ in the helicity system as drawn in Fig. 1. Finally, $P = |\vec{P}|$ is the magnitude of the polarization of the Λ_b.

In Eq. (2.2) we have chosen the set of phase space variables $(\hat{q}_0, \hat{q}^2, \cos\theta)$. One could have equally well chosen the set $(\hat{q}_0, \hat{q}^2, y = 2E_l/m_b)$ where E_l denotes the energy of the lepton. Using the relation

$$y = -\hat{p}\cos\theta + \hat{q}_0 \tag{2.5}$$

one has

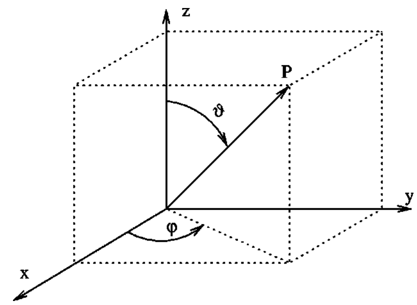

FIG. 1. Helicity coordinate system defining the polar angle ϑ and the azimuthal angle φ. The decay plane is in the (x,z) plane.

$$\frac{d\hat{\Gamma}}{d\hat{q}_0 d\hat{q}^2 dy} = -\hat{p}\frac{d\hat{\Gamma}}{d\hat{q}_0 d\hat{q}^2 d\cos\theta} \tag{2.6}$$

where $\hat{p} = \sqrt{\hat{q}_0^2 - \hat{q}^2}$. However, the choice of variables $(\hat{q}_0, \hat{q}^2, \cos\theta)$ has technical advantages when calculating the $\mathcal{O}(1/m_b^2)$ contributions to the rate expressions. This can be seen as follows. The absorptive parts of the OPE expansion give rise to higher order derivatives of the δ function, of the form

$$\delta^{(n)}\left(\hat{q}_0 - \frac{1}{2}(1-\rho+\hat{q}^2)\right) \tag{2.7}$$

where $\rho = m_c^2/m_b^2$. When doing the q_0 integration the derivatives of the δ function can be shifted to the integrand using partial integration, plus possible surface term contributions. When using the $(\hat{q}_0, \hat{q}^2, \cos\theta)$ set of phase-space variables, the surface term contributions are identically zero [15], whereas there are nonvanishing surface term contributions in $(\hat{q}_0, \hat{q}^2, y)$ phase space. In particular for $m_l \neq 0$ the surface term contributions can become technically quite involved in the latter case and lead to spurious singularities which have to be treated with care [16]. Thus, when using the $(\hat{q}_0, \hat{q}^2, \cos\theta)$ set of variables the $\hat{q}_0$ integration can easily be done.

Next we turn to the remaining $\hat{q}^2$ and $\cos\theta$ integrations. It turns out that the $\cos\theta$ dependence of the unpolarized rate function $A(\hat{q}^2, \cos\theta)$ and the polarized rate functions $B(\hat{q}^2, \cos\theta), C(\hat{q}^2, \cos\theta)$ is particularly simple in system 2 and can easily be integrated. The $\cos\theta$ dependence is so simple in this system since it is determined by bilinear forms of the matrix elements of the Wigner $d^1_{mm'}(\theta)$ function. The requisite $\cos\theta$ integrations can easily be done and one has [15]

$$\frac{d\hat{\Gamma}_A}{d\hat{q}^2} = 4\hat{p}[-2\hat{q}^4 + \hat{q}^2(1+\rho) + (1-\rho)^2](1-K_b) \tag{2.8}$$

$$\frac{d\hat{\Gamma}_B}{d\hat{q}^2} = 8\hat{p}^2(1+\varepsilon_b)(2\hat{q}^2+\rho-1) - K_b\left(4\hat{q}^6 + \frac{2}{3}\hat{q}^4 - 6\hat{q}^4\rho + \frac{8}{3}\hat{q}^2(1-\rho) - 2(1-\rho)^3\right) \tag{2.9}$$

TABLE I. Values of spin independent reduced rates $\Gamma_A := A$ and spin dependent rates $\Gamma_{B_i} := B_i$ and $\Gamma_{C_i} := C_i$ in the helicity systems $i=1,2,3$ for different values of the mass ratio $\rho = m_c^2/m_b^2$. Also shown are asymmetry parameters α_i and γ_i. In brackets are shown the changes induced by the inclusion of $\mathcal{O}(1/m_b^2)$ contributions corresponding to the values $K_b = 0.01$ [16,11] and $\varepsilon_b = -\frac{2}{3}K_b$ [12].

ρ	0.081	0.091	0.101	0
A	$0.554(1-K_b)$	$0.516(1-K_b)$	$0.481(1-K_b)$	$1-K_b$
B_{1a}	$-0.147(1+\varepsilon_b-K_b)$	$-0.134(1+\varepsilon_b-K_b)$	$-0.122(1+\varepsilon_b-K_b)$	$-\frac{1}{3}(1+\varepsilon_b-K_b)$
C_{1a}	$0.413(1+\varepsilon_b-K_b)$	$0.386(1+\varepsilon_b-K_b)$	$0.360(1+\varepsilon_b-K_b)$	$\frac{8\pi}{35}(1+\varepsilon_b-K_b)$
α_{1a}	$-0.265(1+\varepsilon_b)$	$-0.259(1+\varepsilon_b)$	$-0.253(1+\varepsilon_b)$	$-\frac{1}{3}(1+\varepsilon_b)$
γ_{1a}	$0.586(1+\varepsilon_b)$	$0.587(1+\varepsilon_b)$	$0.589(1+\varepsilon_b)$	$-\frac{2\pi^2}{35}(1+\varepsilon_b)$
B_{2a}	$-0.231(1+\varepsilon_b)$	$-0.219(1+\varepsilon_b)$	$-0.207(1+\varepsilon_b)$	$-\frac{1}{3}(1+\varepsilon_b)-\frac{5}{9}K_b$
	$+0.397K_b$	$+0.383K_b$	$+0.368K_b$	
	$(+0.005)$	$(+0.005)$	$(+0.005)$	
C_{2a}	$-0.382(1+\varepsilon_b)$	$-0.355(1+\varepsilon_b)$	$-0.329(1+\varepsilon_b)$	$-\frac{8\pi}{35}(1+\varepsilon_b)+\frac{8\pi}{21}K_b$
	$+0.446K_b$	$+0.405K_b$	$+0.369K_b$	
	$(+0.007)$	$(+0.006)$	$(+0.006)$	
α_{2a}	-0.418	-0.424	-0.430	$-\frac{1}{3}(1+\varepsilon_b)-\frac{8}{9}K_b$
	$(+0.006)$	$(+0.006)$	$(+0.006)$	
γ_{2a}	-0.542	-0.540	-0.538	$-\frac{2\pi^2}{35}(1+\varepsilon_b)+\frac{4\pi^2}{105}K_b$
	$(+0.004)$	$(+0.004)$	$(+0.004)$	
B_{3a}	$A(1+\varepsilon_b)$	$A(1+\varepsilon_b)$	$A(1+\varepsilon_b)$	$1+\varepsilon_b-K_b$
C_{3a}	$-0.898K_b$	$-0.827K_b$	$-0.761K_b$	$-\frac{64\pi}{105}K_b$
α_{3a}	$1+\varepsilon_b$	$1+\varepsilon_b$	$1+\varepsilon_b$	$1+\varepsilon_b$
γ_{3a}	$-1.274K_b$	$-1.258K_b$	$-1.243K_b$	$-\frac{16\pi^2}{105}K_b$

$$\frac{d\hat{\Gamma}_C}{d\hat{q}^2} = 3\pi\hat{p}\sqrt{\hat{q}^2}\left((1+\varepsilon_b)(\hat{q}^2+\rho-1) + \frac{2}{3}K_b(\hat{q}^2-\rho+1)\right) \tag{2.10}$$

where $\hat{p} = \frac{1}{2}\sqrt{\hat{q}^4 - 2\hat{q}^2(1+\rho)+(1-\rho)^2}$, K_b is the mean kinetic energy of the heavy quark in the Λ_b baryon and ε_b is a spin dependent forward matrix element on the Λ_b. They are defined by [9]

$$K_b = -\frac{1}{2M_B}\left\langle \Lambda_b(v)\left|\bar{b}\frac{(iD)^2}{2m_b^2}b\right|\Lambda_b(v)\right\rangle_{\text{spin aver.}},$$

$$\langle\Lambda_b(v)|\bar{b}\gamma_\mu\gamma_5 b|\Lambda_b(v)\rangle = (1+\varepsilon_b)\bar{u}\gamma_\mu\gamma_5 u. \tag{2.11}$$

We will use in the numerical evaluations below the value $K_b = 0.01$, following from the QCD sum rule calculation of [11]. The parameter ε_b will be taken to saturate the model-independent inequality $\varepsilon_b \leqslant -\frac{2}{3}K_b$ [12]. As discussed in this reference, this inequality is saturated if certain double insertions of the chromomagnetic operator can be neglected; QCD sum rule calculations indicate that this is a good approximation.

The final $\hat{q}^2$ integration has to be done in the limits $0 \leqslant \hat{q}^2 \leqslant (1-\sqrt{\rho})^2$. The integration is simple for $A(\hat{q}^2)$ and $B(\hat{q}^2)$, but the integration of $C(\hat{q}^2)$ leads to hypergeometric functions because of the extra square root factor $\sqrt{\hat{q}^2}$. One obtains

$$\hat{\Gamma}_A = (1-8\rho+8\rho^3-\rho^4-12\rho^2\log\rho)(1-K_b) \tag{2.12}$$

$$\hat{\Gamma}_{B_2} = (1+\varepsilon_b)\left(-\frac{1}{3}-30\rho^2-\frac{40}{3}\rho^3+\rho^4+\frac{32}{3}\rho\sqrt{\rho}(1+3\rho)\right) + K_b\left[-\frac{5}{9}-40\rho-\frac{110}{3}\rho^2+\frac{64}{9}\rho^3-\rho^4+\frac{32}{3}\sqrt{\rho}\times\left(1+\frac{17}{3}\rho\right)\right] \tag{2.13}$$

$$\hat{\Gamma}_{C_2}=\frac{3\pi^2}{32}\Bigg\{(1+\varepsilon_b)\Bigg[(1-\sqrt{\rho})^6(1+\sqrt{\rho})_2F_1\left(-\frac{1}{2},\frac{5}{2};4;z\right)$$
$$-2(1-\sqrt{\rho})^5(1+\sqrt{\rho})^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2};3;z\right)\Bigg]$$
$$+\frac{2}{3}K_b\Bigg[(1-\sqrt{\rho})^6(1+\sqrt{\rho})_2F_1\left(-\frac{1}{2},\frac{5}{2};4;z\right)$$
$$+2(1-\sqrt{\rho})^5(1+\sqrt{\rho})^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2};3;z\right)\Bigg]\Bigg\} \qquad (2.14)$$

where the hypergeometric function in Eq. (2.14) is defined as usual by

$$_2F_1(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \mathrm{d}x\, x^{b-1}(1-x)^{c-b-1}\times(1-zx)^{-a} \qquad (2.15)$$

and its argument is

$$z=\left(\frac{1-\sqrt{\rho}}{1+\sqrt{\rho}}\right)^2. \qquad (2.16)$$

In Table I we list the values of $\hat{\Gamma}_A := A, \hat{\Gamma}_{B_2} := B_2$ and $\hat{\Gamma}_{C_2} := C_2$ for four different values of the mass ratio ρ, including the nonperturbative $O(1/m_b^2)$ corrections. In the brackets we list the numerical value of the $\mathcal{O}(1/m_b^2)$ corrections, which are very small and amount to 1–2% of the tree-level contribution. The three choices of the mass ratio squared $\rho = m_c^2/m_b^2$ in Table I are taken from the discussion in [5]. The value $\rho=0$ is relevant for the $b\to u$ transitions and also relevant for a comparison with the well-known results in μ decay.

Next we calculate the reduced rate functions in system 1. It is clear that the unpolarized rate function A is the same in both systems, and that the polarized rate functions B and C in the two systems are related to each other. In fact, it is not difficult to see that the relation between the two sets of polarized rate functions is given by

$$\begin{pmatrix} B_{1a}(\hat{q}^2,\cos\theta) \\ C_{1a}(\hat{q}^2,\cos\theta)\end{pmatrix}=\begin{pmatrix}\cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12}\end{pmatrix}\begin{pmatrix} B_{2a}(\hat{q}^2,\cos\theta) \\ C_{2a}(\hat{q}^2,\cos\theta)\end{pmatrix} \qquad (2.17)$$

where θ_{12} is the (polar) angle between $\vec{p}_X$ and $\vec{p}_l$. This angle can be related to $\hat{q}^2$ and $\cos\theta$ by

$$\cos\theta_{12}=(\hat{q}_0\cos\theta-\hat{p})/(2\hat{E}_e). \qquad (2.18)$$

From Eq. (2.17) it is evident that the $\cos\theta$ dependence of the polarized rate functions in system 1 is somewhat more complicated than that in system 2.

We do not pursue this possible line of approach any further here but compute the rate functions A, B_1 and C_1 directly in the kinematic configuration at hand. Expressed in terms of the hadronic and leptonic tensors $W_{\mu\nu}$, $L_{\mu\nu}$, the rate functions are given by

$$A+P(B_1\cos\vartheta+C_1\sin\vartheta\cos\varphi)=12L_{\mu\nu}W^{\mu\nu}\,\mathrm{d}x\mathrm{d}y\mathrm{d}\hat{q}^2. \qquad (2.19)$$

This differential rate is evaluated most conveniently by integrating first over $\hat{q}^2$ within the limits $(0,xy)$ and subsequently over the neutrino energy $x=(1-\rho-y,x_0)$, with $x_0=1-\rho/(1-y)$. Let us first list the results of these two integrations which agree with the corresponding results obtained in [16]. One has

$$\frac{\mathrm{d}\hat{\Gamma}_A}{\mathrm{d}y}=2y^2x_0^2[-3y+6+x_0(y-3)]+K_b\frac{4y^3}{(1-y)^2}\times\Big(-(y-2)y-2x_0(y^2-3y+5)+x_0^2(2y^2-8y+15)-\frac{2}{3}x_0^3(y^2-5y+10)\Big) \qquad (2.20)$$

$$\frac{\mathrm{d}\hat{\Gamma}_{B_1}}{\mathrm{d}y}=2(1+\varepsilon_b)y^2x_0^2[-3y+x_0(y+1)]+K_b\frac{4y^3}{(1-y)^2}\times\Big(-y^2-2x_0y(y-4)+x_0^2(2y^2-6y-5)-\frac{2}{3}x_0^3(y^2-2y-5)\Big) \qquad (2.21)$$

$$\frac{\mathrm{d}\hat{\Gamma}_{C_1}}{\mathrm{d}y}=(1+\varepsilon_b)\frac{3}{2}\pi y^2\sqrt{1-y}\,x_0^2(2-x_0)+K_b\frac{\pi}{8}\frac{y^3}{(1-y)^{3/2}}\times[16y+8x_0(y-10)+6x_0^2(-5y+20)+5x_0^3(3y-10)]. \qquad (2.22)$$

While the calculation of the rate functions $A(y)$ and $B_1(y)$ is rather straightforward one encounters certain singular expressions in the case of $C_1(y)$. In [16] a method for dealing with these problems has been proposed (see also [17]). In the following we present an alternative treatment, which offers perhaps a better perspective on the physical origin of these singularities. After inserting the OPE result for the hadronic tensor into the rate formula (2.19) one obtains

$$C_1(x,y,\hat{q}^2)=\sin\theta_{e\nu}[F\delta(\hat{q}^2-\hat{q}_0^2)+G\delta'(\hat{q}^2-\hat{q}_0^2)+H\delta''(\hat{q}^2-\hat{q}_0^2)], \qquad (2.23)$$

with $\hat{q}_0^2=x+y+\rho-1$ and $\cos\theta_{e\nu}=1-2\hat{q}^2/(xy)$. The integration over $\hat{q}^2$ can be performed straightforwardly with the result

$$C_1(x,y)=\theta(xy-\hat{q}_0^2)\,\theta(\hat{q}_0^2)\left\{\sin\theta_{e\nu}(F-G'+H'')-\frac{2}{xy}\frac{\cos\theta_{e\nu}}{\sin\theta_{e\nu}}(G-2H')-\frac{4}{x^2y^2}\frac{1}{\sin^3\theta_{e\nu}}H\right\}_{\hat{q}^2=\hat{q}_0^2}$$
$$-\frac{1}{(1-y)^2}[\sin\theta_{e\nu}H]_{\hat{q}^2=xy}\,\delta'(x-x_0)+[\sin\theta_{e\nu}H]_{\hat{q}^2=0}\,\delta'(x-1+\rho+y)$$
$$+\frac{1}{1-y}\left\{\sin\theta_{e\nu}(G-H')-\frac{2}{xy}\frac{\cos\theta_{e\nu}}{\sin\theta_{e\nu}}H\right\}_{\hat{q}^2=xy}\delta(x-x_0)-\left\{\sin\theta_{e\nu}(G-H')-\frac{2}{xy}\frac{\cos\theta_{e\nu}}{\sin\theta_{e\nu}}H\right\}_{\hat{q}^2=0}$$
$$\times\delta(x-1+\rho+y). \tag{2.24}$$

The primes on G,H denote differentiation with respect to $\hat{q}^2$. The difficulty with this expression is that the last two surface terms are divergent, since $\sin\theta_{e\nu}=0$ for $\hat{q}^2=0$ and $\hat{q}^2=xy$. Therefore the result (2.24) is ill defined as it stands and must be defined in some way. We choose to do this by imposing a cut off ε on the angle $\theta_{e\nu}$ such that $\theta_{e\nu}=(\varepsilon,\pi-\varepsilon)$.

Such a cut off is implicit in any experimental extraction of the rate function C. At $\theta_{e\nu}=0$ and π the decay products are collinear in the decay rest frame and consequently the orientation of the decay plane is undetermined. Therefore C is practically undefined at this kinematic point, which has to be excluded from the analysis.

In our calculation this cutoff is implemented by integrating only over $\hat{q}^2$ within the limits $\hat{q}^2_{\min}=xy\varepsilon^2/4$, $\hat{q}^2_{\max}=xy(1-\varepsilon^2/4)$. The limits on the neutrino energy x will have to be modified too, as follows:

$$x'_{\min}=\frac{1-y-\rho}{1-y\varepsilon^2/4}=(1-y-\rho)+y(1-\rho-y)\frac{1}{4}\varepsilon^2+O(\varepsilon^4) \tag{2.25}$$

$$x'_{\max}=\frac{1-y-\rho}{(1-y)+y\varepsilon^2/4}=x_0-\frac{y(1-\rho-y)}{(1-y)^2}\frac{1}{4}\varepsilon^2+O(\varepsilon^4). \tag{2.26}$$

With this regularization the boundary terms in Eq. (2.24) give poles of the form $1/\varepsilon$. These poles are cancelled after integration over x by similar singular terms arising from the last term in the first line of Eq. (2.24). The H term is singular at the end points of the x interval, as can be seen explicitly from the expression for $\sin\theta_{e\nu}$

$$[\sin\theta_{e\nu}]_{\hat{q}^2=\hat{q}_0^2}=\frac{2\sqrt{1-y}}{xy}\sqrt{(x_0-x)(x-1+\rho+y)}. \tag{2.27}$$

Integration of the H term in Eq. (2.24) over x within the limits (2.25), (2.26) will give, as mentioned, $1/\varepsilon$ poles which exactly cancel those present in the boundary terms. Therefore the correct result for $C_1(y)$ is obtained, in the limit of a vanishingly small cut off $\varepsilon\ll 1$, by simply ignoring the surface terms and evaluating the integral over x of the first line in Eq. (2.24) in a minimal subtraction prescription, which subtracts the $1/\varepsilon$ poles arising from the modified integration limits (2.25), (2.26).

The y integration has to be done in the limits $0\leqslant y\leqslant 1-\rho$. Of relevance are only the spin dependent rate functions $d\Gamma_{B_1}$ and $d\Gamma_{C_1}$ since the result of integrating the spin independent piece $d\Gamma_A$ can be checked to reproduce the result Eq. (2.12). One obtains

$$\hat{\Gamma}_{B_1}=(1+\varepsilon_b-K_b)\left(-\frac{1}{3}+4\rho+12\rho^2-\frac{44}{3}\rho^3-\rho^4+(12+8\rho)\rho^2\log\rho\right) \tag{2.28}$$

$$\hat{\Gamma}_{C_1}=\frac{8\pi}{35}(1+\varepsilon_b-K_b)[1-7\rho+35\rho^2+35\rho^3-\rho^{5/2}\times(56+8\rho)]. \tag{2.29}$$

where $\hat{\Gamma}_{B_1}:=B_1$ and $\hat{\Gamma}_{C_1}:=C_1$. In Table I we list the numerical values for the reduced spin dependent rate functions $\hat{\Gamma}_{B_1}:=B_1$ and $\hat{\Gamma}_{C_1}:=C_1$ for the same four values of $\rho=m_c^2/m_b^2$. The discussion of the numerical results will be deferred to until after the corresponding results in system 3 are written down.

The spin dependent rate functions in system 3 can be obtained using similar methods. The simplest way to treat this case is by exchanging the electron and neutrino momenta in the lepton tensor $L_{\mu\nu}$. We obtain the following results:

$$\frac{d\hat{\Gamma}_{B_3}}{dx}=(1+\varepsilon_b)\left(\frac{12\rho^2}{1-x}-12x^3-24x^2\rho+12x^2-12x\rho^2-12\rho^2\right)+K_b\left(\frac{8(1-y_0)^2}{1-x}-12y_0^2+12y_0(2+\rho)-20x^3+12x\rho^2+16\rho^2-12\rho-12\right) \tag{2.30}$$

$$\frac{d\hat{\Gamma}_{C_3}}{dx} = -K_b 4\pi x^2 y_0^2 \sqrt{1-x}. \tag{2.31}$$

In these expressions $y_0 = 1-\rho/(1-x)$ denotes the maximum value taken by the electron energy at a given neutrino energy x.

The integrated angular rate functions in system 3 can be easily obtained by integration in the x interval $[0,1-\rho]$. They are given by

$$\hat{\Gamma}_{B_3} = (1+\varepsilon_b - K_b)(1-8\rho+8\rho^3-\rho^4-12\rho^2\log\rho) \tag{2.32}$$

$$\hat{\Gamma}_{C_3} = -K_b \frac{64\pi}{105}[1-14\rho-35\rho^2+\rho^{3/2}(35+14\rho-\rho^2)], \tag{2.33}$$

where again $\hat{\Gamma}_{B_3} := B_3$ and $\hat{\Gamma}_{C_3} := C_3$.

It is noteworthy that the polar analyzing power α in system 3 takes the maximal value of 1 for the leading order free quark decay contribution. This has been made manifest in Table I by rewriting the angular rate B_{3a} in terms of the unpolarized rate function A dropping a $\mathcal{O}(1/m_b^4)$ contribution. The fact that $\alpha = 1$ can be understood by rewriting the $(V-A)(V-A)$ form of the matrix element into a $(S+P)(S-P)$ form with the help of the Fierz transformation of the second type [18]. One obtains, in this way,

$$\begin{aligned}
&[\overline{u(c)}\gamma^\mu(1-\gamma_5)u(b)][\overline{u(l^-)}\gamma_\mu(1-\gamma_5)v(\overline{\nu})] \\
&\quad = 2[\overline{u(c)}(1+\gamma_5)C\overline{u}^T(l^-)][v^T(\overline{\nu})C^{-1}(1-\gamma_5)u(b)] \\
&\quad = 2[\overline{u(c)}(1+\gamma_5)u(l^-)][\overline{v(\overline{\nu})}(1-\gamma_5)u(b)]
\end{aligned} \tag{2.34}$$

with C the charge conjugation matrix. In the Fierz-rearranged form it is clear that the b spin is aligned with the spin direction of the $\overline{\nu}$ which points along its momentum direction. Thus the polar angle dependence in system 3 is given by $1+\cos\vartheta$, corresponding to a maximal polar analyzing power in this system. Note that this argument is independent of the value of the charm quark mass, such that the maximal value of this asymmetry parameter is obtained for any value of the mass ratio ρ. This can be seen directly from comparing Eqs. (2.12) and (2.32) where the ρ-dependent coefficients of the free quark decay contribution (and the K_b contribution for that matter) can be seen to be equal to one another.

As mentioned above, the positivity of the decay rate for any values of (ϑ,φ) requires the asymmetry parameter B_3 to be smaller than or equal to 1. From this and the result (2.32) for this parameter one can obtain the constraint $\varepsilon_b \leqslant 0$ on the nonperturbative matrix element ε_b to leading order in α_s. This is compatible with, although less stringent than, the inequality $\varepsilon_b \leqslant -\frac{2}{3}K_b$ obtained in [12] from a zero recoil sum rule.

The azimuthal asymmetry vanishes at leading order in $1/m_b$ since the polar asymmetry takes the maximal value of 1 in system 3. This can be understood by noting that the positivity of the differential rate requires the combination $|B|^2 + |C|^2\cos^2\phi$ to be smaller than the unity for any ϕ. This gives that C must vanish if $B=1$.

TABLE II. Fully integrated angular coefficients B_i and C_i $(i=1,2,3)$ for the processes $b\to c+e^-+\overline{\nu}_e$, $\overline{b}\to\overline{c}+e^++\nu_e$, $c\to s+e^++\nu_e$ and $\overline{c}\to\overline{s}+e^-+\overline{\nu}_e$ for the three coordinate systems $1a$, $2a$ and $3a$ defined in Eq. (2.1). Results for systems $1b$, $2b$ and $3b$ can be obtained using $B_{ib}=B_{ia}$ and $C_{ib}=-C_{ia}$.

	$1a$	$2a$	$3a$	$1a$	$2a$	$3a$
$b\to c+e^-+\overline{\nu}_e$	B_{1a}	B_{2a}	B_{3a}	C_{1a}	C_{2a}	C_{3a}
$\overline{b}\to\overline{c}+e^++\nu_e$	$-B_{1a}$	$-B_{2a}$	$-B_{3a}$	$-C_{1a}$	$-C_{2a}$	$-C_{3a}$
$c\to s+e^++\nu_e$	B_{3a}	B_{2a}	B_{1a}	$-C_{3a}$	$-C_{2a}$	$-C_{1a}$
$\overline{c}\to\overline{s}+e^-+\overline{\nu}_e$	$-B_{3a}$	$-B_{2a}$	$-B_{1a}$	C_{3a}	C_{2a}	C_{1a}

When comparing the results of systems 1 and 2 one notes the equality of the free quark decay (FQD) angular rate functions $B_1=B_2$ and $C_1=-C_2$ when $\rho=0$, i.e. the case relevant for $b\to u$ transitions. This can be seen to be a consequence of the fact that the u quark and the electron are Fierz symmetric partners in the decay. In the case of mass degeneracy as for the case discussed here, the FQD decay distributions are symmetric under exchange of the two and thus are the same in systems 1 and 2. The minus sign in the relation $C_1=-C_2$ comes about because one is comparing system 1_a with system 2_b when exchanging the Fierz partners.

Up to now we have given results for the fully integrated rate coefficients B_i and C_i $(i=1,2,3)$ for the process $b\to c+e^-+\overline{\nu}_e$ apart from the rate function A which is the same in all systems. In Table II we give the corresponding coefficients for the processes $\overline{b}\to\overline{c}+e^++\nu_e$, $c\to s+e^++\nu_e$ and $\overline{c}\to\overline{s}+e^-+\overline{\nu}_e$. They can be obtained from the CP invariance of the interaction and the symmetry under the exchange $e^-\to\nu_e$ when going from $b\to c$ to $c\to s$ in the integrated rate formula or by direct calculation. In the latter case we have used a general $V+xA$ interaction in order to obtain a nonvanishing result for the "C_{3a}" entries. In addition, when going from $b\to c$ to $c\to s$ and $\overline{c}\to\overline{s}$ one has to replace $\rho=m_c^2/m_b^2$ by $\rho=m_s^2/m_c^2$.

As concerns the numerical values of the polarization dependent contributions for $\rho=0$ we want to draw an analogy to muon decay. For this purpose we arrange the decay products in muon decay in the same weak isospin order as in the $b\to u$ case. One has

$$b \to \overbrace{u + l^-}\, + \bar{\nu}_l \quad \Longleftrightarrow \quad \mu^- \to \overbrace{\nu_\mu + e^-}\, + \bar{\nu}_e \tag{2.35}$$

where we have drawn the braces connecting the Fierz partners for added emphasis. From comparing the two decays the value $B_1=B_2=-1/3$ in Table I should be well familiar from μ decay, when the electron mass is neglected. The result $C_1=-C_2=8\pi/35$ has not been widely publicized in μ decay for the obvious reason that its determination requires an

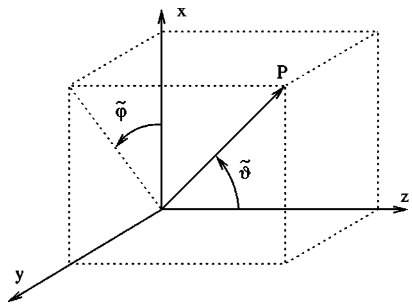

FIG. 2. Transversity coordinate system defining the polar angle $\tilde{\vartheta}$ and the azimuthal angle $\tilde{\varphi}$. The decay plane is in the (x,y) plane.

azimuthal measurement which cannot be done in μ decay because of the two undetected neutrinos in the final state.

Instead of analyzing the full (ϑ,φ) two-fold angular decay distribution one can reduce the two-fold distribution to single angle decay distributions by doing either the ϑ integration or the φ integration. One obtains

$$\frac{d\hat{\Gamma}}{d\cos\vartheta}\propto 1+\alpha_P P\cos\vartheta \tag{2.36}$$

and

$$\frac{d\hat{\Gamma}}{d\varphi}\propto 1+\gamma_P P\cos\varphi \tag{2.37}$$

where, in terms of the angular coefficients A,B and C, the polar and azimuthal asymmetry parameters are given by $\alpha_P=B/A$ and $\gamma_P=\pi C/(4A)$, respectively. Note that the asymmetry parameters lie in the following intervals:

$$-1\leqslant\alpha_P\leqslant 1 \quad \text{and}$$
$$-\frac{3\pi^2}{32\sqrt{2}}\leqslant\gamma_P\leqslant\frac{3\pi^2}{32\sqrt{2}} \quad \left(\frac{3\pi^2}{32\sqrt{2}}\simeq 0.654\right). \tag{2.38}$$

Table I also contains the numerical values of the asymmetry parameters α_P and γ_P.

III. SPIN-MOMENTUM CORRELATIONS IN THE TRANSVERSITY SYSTEM

In the transversity coordinate systems the event plane is in the (x,y) plane. The orientation of the polarization vector $\vec{P}$ is specified by the polar angle $\tilde{\vartheta}$ and $\tilde{\varphi}$ as drawn in Fig. 2. The relation between the transversity angles $(\tilde{\vartheta},\tilde{\varphi})$ and the helicity angles (ϑ,φ) can be easily seen to be given by

$$\cos\tilde{\vartheta}=\sin\vartheta\sin\varphi \tag{3.1}$$

$$\sin\tilde{\vartheta}\sin\tilde{\varphi}=\sin\vartheta\cos\varphi \tag{3.2}$$

$$\sin\tilde{\vartheta}\cos\tilde{\varphi}=\cos\vartheta. \tag{3.3}$$

TABLE III. Transversity system asymmetries in system $i=1,2,3$ for two typical values of the quark mass ratio $\rho=m_c^2/m_b^2$. The nonperturbative $1/m_b^2$ corrections have been neglected.

ρ	0.091	0
$\tilde{\gamma}_{P_1}$	0.622	0.622
β_1	1.237	1.136
$\tilde{\gamma}_{P_2}$	0.635	0.622
β_2	−1.018	−1.136
$\tilde{\gamma}_{P_3}$	0.785	0.785
β_3	0	0

Correspondingly one has the two-fold angular decay distribution

$$\frac{d\hat{\Gamma}}{d\cos\tilde{\vartheta}\,d\tilde{\varphi}}=\Gamma_b[A+P\sin\tilde{\vartheta}(B\cos\tilde{\varphi}+C\sin\tilde{\varphi})] \tag{3.4}$$

after internal three-fold phase space integration. We shall not discuss the two-fold angular decay distribution in the transversity system any further but immediately turn to the single angle decay distributions. Again there are the two possibilities of integrations over $\tilde{\varphi}$ or over $\cos\tilde{\vartheta}$.

Integrating Eq. (3.4) over $\tilde{\varphi}$ leads to a flat $\cos\tilde{\vartheta}$ distribution while integrating over $\cos\tilde{\vartheta}$ one obtains

$$\frac{d\hat{\Gamma}}{d\cos\tilde{\varphi}}\propto 1+\tilde{\gamma}_P\cos(\tilde{\varphi}-\beta) \tag{3.5}$$

where

$$\tilde{\gamma}_P=\frac{\sqrt{B^2+C^2}}{4A}\pi \tag{3.6}$$

and the phase angle β is given by

$$\beta=\arcsin\frac{C}{\sqrt{B^2+C^2}}. \tag{3.7}$$

In Table III we list the values of the asymmetry parameter $\tilde{\gamma}_P$ and the phase angle β for two typical values of $\rho=m_c^2/m_b^2$ in systems 1, 2 and 3.

IV. CONCLUSIONS

We presented in this paper a study of the spin-momentum correlations in Λ_b decays, including nonperturbative corrections of order $1/m_b^2$. Numerically the nonperturbative corrections to the various inclusive asymmetries are rather small, of the order of or smaller than 1%. This is very similar to the situation encountered for radiative QCD corrections to spin-momentum correlations, which were computed in [13]. They were found to be smaller than 1% in all cases of practical interest, due to a cancellation in the ratio of polarized and unpolarized decay rates respectively. One concludes therefore that the free-quark decay model can be expected to give accurate results for the asymmetries considered. The results of our paper could be expected to be useful in measuring the polarization of Λ_b produced in e^+e^- annihilation through their semileptonic decay products, complementing the methods already proposed in [4,5].

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