Chiral quark soliton model with Pauli-Villars regularization

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The Pauli-Villars regularization scheme is often used for evaluating parton distributions within the framework of the chiral quark soliton model with the inclusion of the vacuum polarization effects. Its simplest version with a single subtraction term should, however, be taken with some caution, since it does not fully get rid of divergences contained in scalar and psuedoscalar quark densities appearing in the soliton equation of motion. To remedy this shortcoming, we propose here its natural extension, i.e., the Pauli-Villars regularization scheme with multisubtraction terms. We also carry out a comparative analysis of the Pauli-Villars regularization scheme and more popular proper-time one. It turns out that some isovector observables such as the isovector magnetic moment of the nucleon are rather sensitive to the choice of the regularization schemes. In the process of tracing the origin of this sensitivity, a noticeable difference of the two regularization schemes is revealed. [S0556-2821(99)02913-6]

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I. INTRODUCTION

The recent calculations of nucleon-parton distributions within the chiral quark soliton model (CQSM) exclusively utilize the so-called Pauli-Villars regularization scheme [1–6]. This is to be contrasted with the fact that most of the past calculations of the nucleon static observables were carried out by using the proper-time regularization scheme [7–9]. There are some reasons for it. The first reason is mainly technical. For obtaining parton distributions, one needs to evaluate the nucleon matrix elements of the quark bilinear operators which are nonlocal in two space-time coordinates. The problem is that we have no unanimous idea about how to generalize the proper-time scheme for the reguralization of such unusual quantities. The second but more positive reason for using the Pauli-Villars regularization scheme has been advocated by Diakonov *et al.* [1,2]. They emphasize that this regularization scheme preserves certain general properties of parton distributions such as positivity, factorization properties, sum rules, etc., which are easily violated by other regularization schemes such as the proper-time one. (Still another choice for introducing regularization into the model is to use the momentum-dependent constituent quark mass motivated from the instanton picture of QCD vacuum [10]. This possibility has been investigated in a recent paper by Golli *et al.* [11]. A physical motivation and a general discussion on the regularization of the model can be found in a recent review on the chiral quark soliton model by Diakonov [12].)

Recently, there was a controversial debate on the stability of soliton solutions in the CQSM regularized with the Pauli-Villars subtraction scheme [13,14]. It seems that the problem has been settled by now, since stable soliton solutions seem to exist at any rate if Pauli-Villars regularization is applied to quark seas only, not to the discrete bound state sometimes called the valence quark orbital. Unfortunately, this is not the end of the story. In fact, soliton solutions of the CQSM with use of the Pauli-Villars regularization scheme were obtained many years ago by Döring *et al.* [15]. (To be more precise, the model used by them is not the CQSM but the Nambu–Jona-Lasinio model. In fact, they were forced to impose an *ad hoc* nonlinear constraint for scalar and pseudoscalar meson fields at a later stage of manipulation. Otherwise, they would not have obtained any convergent solutions [16].) The fact that the single-subtraction Pauli-Villars scheme cannot regularize the vacuum quark condensate was already noticed in an earlier paper [17] as well as in this paper [15]. To remove this divergence, which is necessary for obtaining a finite gap equation, Döring *et al.* propose to add some counterterms, which depend on the meson fields, to the original effective action. It is very important to recognize that this procedure is not workable within the CQSM, since their counterterms reduce to mere constants under the chiral circle condition which we impose from the very beginning. Thus, one must conclude that the simplest Pauli-Villars scheme with the single-subtraction term is unable to fully get rid of the divergence of the vacuum quark condensate at least in the nonlinear model. One should take this fact seriously, because it brings about trouble also in the physics of the soliton sector. To understand it, one has only to remember the fact that the scalar quark density appearing in the soliton equation of motion is expected to approach a finite and nonzero value characterizing the vacuum quark condensate as the distance from the soliton center becomes large [18]. This necessarily means that the scalar quark density appearing in the soliton equation of motion cannot also be free from divergences.

The purpose of the present study is then twofold. On the one hand, we want to show that the single-subtraction Pauli-Villars scheme is not a fully satisfactory regularization

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scheme, and that at least one more subtraction term is necessary for a consistent regularization of the effective theory. This will be made convinced through the formal discussion given in Sec. II and also the explicit numerical results shown in Sec. III A. On the other hand, we also want to know the regularization-scheme dependence of the CQSM through a comparative analysis of typical static observables of the nucleon predicted by the two regularization schemes, i.e., the Pauli-Villars one and the proper-time one. A discussion of this second issue will be given in Sec. III B. We then summarize our conclusion in Sec. IV.

II. PAULI-VILLARS REGULARIZATION SCHEME

We begin with the effective Lagrangian of the chiral quark model with an explicit chiral symmetry breaking term as

$$\mathcal{L}_{CQM}=\mathcal{L}_0+\mathcal{L}', \tag{1}$$

where $\mathcal{L}_0$ denotes the chiral symmetric part [19] given by

$$\mathcal{L}_0=\bar{\psi}[i\partial\!\!\!/-MU^{\gamma_5}(x)]\psi \tag{2}$$

with

$$U^{\gamma_5}(x)=e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)/f_\pi}=\frac{1+\gamma_5}{2}U(x)+\frac{1-\gamma_5}{2}U^\dagger(x), \tag{3}$$

while

$$\mathcal{L}'=\frac{1}{4}f_\pi^2 m_\pi^2\,\mathrm{tr}[U(x)+U^\dagger(x)-2] \tag{4}$$

is thought to simulate a small deviation from the chiral-symmetric limit. Here the trace in Eq. (4) is to be taken with respect to flavor indices. From the fundamental viewpoint, this form of effective action may not be enough to fully take account of the effect of explicit chiral symmetry violation in QCD. Still we would expect that the above Lagrangian provides us with some qualitative information about the effect of explicit chiral symmetry breaking, though the main interest of the present study is not to see it. (Naturally, one could have taken an alternative choice that introduces explicit chiral-symmetry-breaking effect in the form of quark mass term. We did not do so, because of the reason explained in the Appendix.)

The idea of the Pauli-Villars regularization can most easily be understood by examining the form of the effective meson action derived from Eq. (1) with the help of the standard derivative expansion:

$$S_{eff}[U]=S_f[U]+S_m[U], \tag{5}$$

where

$$\begin{aligned}S_f[U]&=-iN_c\mathrm{Sp}\ln(i\partial\!\!\!/-MU^{\gamma_5})\\&=\int d^4x\{4N_cM^2I_2(M)\,\mathrm{tr}(\partial_\mu U\partial^\mu U^\dagger)\\&\quad+\text{higher derivative terms}\},\end{aligned} \tag{6}$$

$$S_m[U]=\int d^4x\frac{1}{4}f_\pi^2 m_\pi^2\,\mathrm{tr}[U(x)+U^\dagger(x)-2]. \tag{7}$$

In Eq. (6), the coefficient

$$I_2(M)\equiv -i\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M^2)^2} \tag{8}$$

of the pion kinetic term diverges logarithmically. In fact, by introducing a ultraviolet cutoff momentum α that should eventually be made infinity, one finds that

$$I_2(M)\sim\frac{1}{16\pi^2}\{\ln\alpha^2-\ln M^2-1\}. \tag{9}$$

This logarithmic divergence can be removed if one introduces a regularized action as follows:

$$S_{eff}^{reg}[U]=S_f^{reg}[U]+S_m[U], \tag{10}$$

where

$$S_f^{reg}[U]\equiv S_f[U]-\left(\frac{M}{M_{PV}}\right)^2 S_f^{M_{PV}}[U]. \tag{11}$$

Here $S_f^{M_{PV}}$ is obtained from $S_f[U]$ with M replaced by the Pauli-Villars regulator mass M_{PV}. Further requiring that the above regularized action reproduce the correct normalization for the pion kinetic term, one obtains the condition

$$\frac{N_cM^2}{4\pi^2}\ln\left(\frac{M_{PV}}{M}\right)^2=f_\pi^2, \tag{12}$$

which can be used to fix the regulator mass M_{PV}. Once the effective action is regularized, the static soliton energy should be a finite functional of the soliton profile $F(r)$ under the standard hedgehog ansatz $U(\boldsymbol{x})=\exp[i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)]$. Since the soliton equation of motion is obtained from the stationary condition of the static energy against the variation of $F(r)$, everything seems to be going well with the above single-subtraction Pauli-Villars regularization procedure. Unfortunately, this is not the case. To understand what the problem is, we first recall the fact that the scalar quark density appearing in the soliton equation of motion is expected to approach a finite and nonzero constant characterizing the vacuum quark condensate as the distance from the soliton center becomes large [18]. [This is a natural consequence of our demand that both of the soliton ($B=1$) and vacuum ($B=0$) sectors must be described by the same (or single) equation of motion.] On the other hand, it has been known that the vacuum quark condensate contains quadratic divergences that cannot be removed by the single-subtraction Pauli-Villars scheme [15,17]. This then indicates that the scalar quark density appearing in the soliton equation of motion cannot also be free from divergences.

To get rid of all the troublesome divergences, we propose here to increase the number of subtraction terms, thereby starting with the following action

$$S_{eff}^{reg}[U]=S_f^{reg}[U]+S_m[U], \tag{13}$$

where

$$S_f^{reg}[U]\equiv S_f[U]-\sum_{i=1}^{N} c_i S_f^{\Lambda_i}[U], \tag{14}$$

with N being the number of subtraction terms. The logarithmic divergence of the original action is removed if the condition

$$1-\sum_{i=1}^{N} c_i\left(\frac{\Lambda_i}{M}\right)^2=0 \tag{15}$$

is satisfied. Similarly, the normalization condition (12) is replaced by

$$\frac{N_c M^2}{4\pi^2}\sum_{i=1}^{N} c_i\left(\frac{\Lambda_i}{M}\right)^2 \ln\left(\frac{\Lambda_i}{M}\right)^2=f_\pi^2. \tag{16}$$

The single-subtraction Pauli-Villars scheme corresponds to taking $N=1,\Lambda_1=M_{PV}$, and $c_1=(M/M_{PV})^2$. This is naturally the simplest case that satisfies both conditions (15) and (16).

To derive soliton equation of motion, we must first write down a regularized expression for the static soliton energy. Under the hedgehog ansatz $\boldsymbol{\pi}(\boldsymbol{x})=f_\pi \hat{\boldsymbol{r}}F(r)$ for the background pion fields, it is obtained in the form

$$E_{static}^{reg}[F(r)]=E_f^{reg}[F(r)]+E_m[F(r)], \tag{17}$$

where the meson part is given by

$$E_m[F(r)]=-f_\pi^2 m_\pi^2\int d^3x[\cos F(r)-1], \tag{18}$$

while the fermion (quark) part is given as

$$E_f^{reg}[F(r)]=E_{val}+E_{vp}^{reg}, \tag{19}$$

with

$$E_{val}=N_c E_0, \tag{20}$$

$$E_{vp}^{reg}=N_c\sum_{n<0}(E_n-E_n^{(0)})-\sum_{i=1}^{N} c_i N_c \times\sum_{n<0}(E_n^{\Lambda_i}-E_n^{(0)\Lambda_i}). \tag{21}$$

Here E_n are the quark single-particle energies, given as the eigenvalues of the static Dirac Hamiltonian in the background pion fields:

$$H|n\rangle=E_n|n\rangle, \tag{22}$$

with

$$H=\frac{\boldsymbol{\alpha}\cdot\nabla}{i}+\beta M[\cos F(r)+i\gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\sin F(r)], \tag{23}$$

while the energies $E_n^{(0)}$ denote the energy eigenvalues of the vacuum Hamiltonian given by Eq. (23) with $F(r)=0$ or $U=1$. Equation (19) means that the quark part of the static energy is given as a sum of the contribution of the discrete bound-state level and that of the negative energy Dirac continuum. The latter part is regularized by subtracting from the Dirac sea contribution a linear combination of the corresponding sum evaluated with the regulator mass Λ_i instead of the dynamical quark mass. [$E_n^{\Lambda_i}$ in these subtraction terms are the eigenenergies of the Dirac Hamiltonian (23) with M replaced by Λ_i and with the same background pion field.]

Now the soliton equation of motion is obtained from the stationary condition of $E_{static}^{reg}[F(r)]$ with respect to the variation of the profile function $F(r)$:

$$0=\frac{\delta E_{static}[F(r)]}{\delta F(r)}=4\pi r^2\{-M[S(r)\sin F(r) -P(r)\cos F(r)]+f_\pi^2 m_\pi^2\sin F(r)\}, \tag{24}$$

which gives

$$F(r)=\arctan\left(\frac{P(r)}{S(r)-f_\pi^2 m_\pi^2/M}\right). \tag{25}$$

Here $S(r)$ and $P(r)$ are regularized scalar and pseudoscalar densities given as

$$S(r)=S_{val}(r)+\sum_{n<0}S_n(r)-\sum_{i=1}^{N} c_i\frac{\Lambda_i}{M}\sum_{n<0}S_n^{\Lambda_i}(r), \tag{26}$$

$$P(r)=P_{val}(r)+\sum_{n<0}P_n(r)-\sum_{i=1}^{N} c_i\frac{\Lambda_i}{M}\sum_{n<0}P_n^{\Lambda_i}(r), \tag{27}$$

with

$$S_n(r)=\frac{N_c}{4\pi}\int d^3x\langle n|\boldsymbol{x}\rangle\gamma^0\frac{\delta(|\boldsymbol{x}|-r)}{r^2}\langle\boldsymbol{x}|n\rangle, \tag{28}$$

$$P_n(r)=\frac{N_c}{4\pi}\int d^3x\langle n|\boldsymbol{x}\rangle i\gamma^0\gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\frac{\delta(|\boldsymbol{x}|-r)}{r^2}\langle\boldsymbol{x}|n\rangle, \tag{29}$$

and $S_{val}(r)=S_{n=0}(r)$ and $P_{val}(r)=P_{n=0}(r)$, while $S_n^{\Lambda_i}(r)$ and $P_n^{\Lambda_i}(r)$ are the corresponding densities evaluated with the regulator mass Λ_i instead of the dynamical quark mass M. As usual, a self-consistent soliton solution is obtained in an iterative way. First by assuming an appropriate (though arbitrary) soliton profile $F(r)$, the eigenvalue problem of the Dirac Hamiltonian is solved. Using the resultant eigenfunctions and their associated eigenenergies, one can calculate the regularized scalar and pseudoscalar quark densities $S(r)$ and $P(r)$. Equation (25) can then be used to obtain a new soliton profile $F(r)$. The whole procedure above is repeated with this new profile $F(r)$ until the self-consistency is satisfied.

Now we recall an important observation made before. The scalar quark density $S(r)$ at spatial infinity $r=\infty$ with respect to the soliton center should coincide with the scalar quark density in the vacuum ($B=0$) sector, which is nothing but the familiar vacuum quark condensate (per unit volume) $\langle\bar{\psi}\psi\rangle_{vac}$. That is, the following simple relation must hold:

$$\langle\bar{\psi}\psi\rangle_{vac} = \frac{1}{V}\int S(r=\infty)d^3r = S(r=\infty). \qquad (30)$$

(Later, this relation will be checked numerically.) What we must do now is to find necessary conditions for the subtraction constants c_i and Λ_i in the multisubtraction Pauli-Villars scheme to make the vacuum quark condensate finite. This can be achieved by examining the expression of the vacuum quark condensate obtained consistently with the soliton equation of motion:

$$\langle\bar{\psi}\psi\rangle_{vac}^{reg}=\langle\bar{\psi}\psi\rangle_{vac}-\sum_{i=1}^{N} c_i\left(\frac{\Lambda_i}{M}\right)\langle\bar{\psi}\psi\rangle_{vac}^{\Lambda_i}, \qquad (31)$$

where

$$\langle\bar{\psi}\psi\rangle_{vac}=-4N_cM\int\frac{d^3k}{(2\pi)^3}\frac{1}{E_k^{(0)}}, \qquad (32)$$

with $E_k^{(0)}=(k^2+M^2)^{1/2}$, while $\langle\bar{\psi}\psi\rangle_{vac}^{\Lambda_i}$ are obtained from $\langle\bar{\psi}\psi\rangle_{vac}$ with the replacement of M by Λ_i. Using the integration formula

$$\int^{\alpha}\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{k^2+M^2}}=\frac{1}{8\pi^2}\{2\alpha^2-M^2\ln\alpha^2+(1-2\ln 2)M^2+M^2\ln M^2\}, \qquad (33)$$

with α being a ultraviolet cutoff momentum, we obtain

$$\begin{aligned}\langle\bar{\psi}\psi\rangle_{vac}^{reg}=-\frac{N_cM}{2\pi^2}\Bigg\{&\left[1-\sum_{i=1}^{N}c_i\left(\frac{\Lambda_i}{M}\right)^2\right]2\alpha^2\\&-\left[M^2-\sum_{i=1}^{N}c_i\left(\frac{\Lambda_i}{M}\right)^2\Lambda_i^2\right]\ln\alpha^2\\&+\left[M^2-\sum_{i=1}^{N}c_i\left(\frac{\Lambda_i}{M}\right)^2\Lambda_i^2\right](1-2\ln 2)+M^2\ln M^2\\&-\sum_{i=1}^{N}c_i\left(\frac{\Lambda_i}{M}\right)^2\Lambda_i^2\ln\Lambda_i^2\Bigg\}, \qquad (34)\end{aligned}$$

which clearly shows that $\langle\bar{\psi}\psi\rangle_{vac}$ contains quadratic and logarithmic divergences as α going to infinity. These divergences can, respectively, be removed if the subtraction constants are chosen to satisfy the following conditions:

$$M^2-\sum_{i=1}^{N}c_i\Lambda_i^2=0, \qquad (35)$$

$$M^4-\sum_{i=1}^{N}c_i\Lambda_i^4=0. \qquad (36)$$

Using the first of these conditions, the finite part of $\langle\bar{\psi}\psi\rangle_{vac}$ can also be expressed as

$$\langle\bar{\psi}\psi\rangle_{vac}=\frac{N_cM^3}{2\pi^2}\sum_{i=1}^{N}c_i\left(\frac{\Lambda_i}{M}\right)^4\ln\left(\frac{\Lambda_i}{M}\right)^2. \qquad (37)$$

It is now obvious that the single-subtraction Pauli-Villars scheme cannot satisfy both conditions (35) and (36) simultaneously. Although the quadratic divergence may be removed, the logarithmic divergence remains in $\langle\bar{\psi}\psi\rangle_{vac}$ and consequently also in $S(r=\infty)$ in view of relation (30). To get rid of both these divergences, we need at least two subtraction terms, which contain four parameters c_1,c_2 and Λ_1,Λ_2. The strategy for fixing these parameters is as follows. First by solving the two equations (35) and (36) with $N=2$ for c_1 and c_2, we obtain

$$c_1=\left(\frac{M}{\Lambda_1}\right)^2\frac{\Lambda_2^2-M^2}{\Lambda_2^2-\Lambda_1^2}, \qquad (38)$$

$$c_2=-\left(\frac{M}{\Lambda_2}\right)^2\frac{\Lambda_1^2-M^2}{\Lambda_2^2-\Lambda_1^2}, \qquad (39)$$

which constrains the values of c_1 and c_2, once Λ_1 and Λ_2 are given. For determining Λ_1 and Λ_2, we can then use two conditions (16) and (37), which amounts to adjusting the normalization of the pion kinetic term and the value of vacuum quark condensate.

III. NUMERICAL RESULTS AND DISCUSSION

A. Single- versus double-subtraction Pauli-Villars regularization

The most important parameter of the CQSM is the dynamical quark mass M, which plays the role of the quark-pion coupling constant, thereby controlling basic soliton properties. Throughout the present investigation, we use the value $M=400$ MeV favored from previous analyses of static baryon observables. In the case of the single-subtraction Pauli-Villars scheme, the regulator mass M_{PV} is uniquely fixed to be $M_{PV}=570.86$ MeV by using the normalization condition (12) for the pion kinetic term, and there is no other adjustable parameter in the model. In the case of the double-subtraction Pauli-Villars scheme, we have four regularization parameters c_1,c_2,Λ_1, and Λ_2. From the divergence-free conditions (35) and (36), c_1 and c_2 are constrained as Eqs. (38) and (39), while Λ_1 and Λ_2 are determined from Eqs.

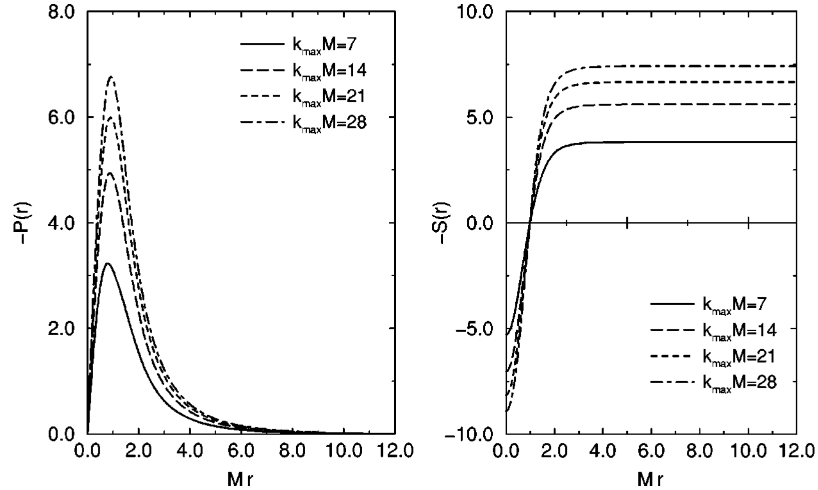

FIG. 1. The k_{max} dependence of the scalar quark density $S(r)$ and the pseudoscalar density $P(r)$ in the single-subtraction Pauli-Villars scheme.

(16) and (37) with $f_\pi = 93\,\text{MeV}$ and $\langle \overline{\psi}\psi \rangle_{vac} = -(286.6\,\text{MeV})^3$. In spite of their nonlinearity, the two conditions (16) and (37) are found to uniquely fix the two parameters Λ_1 and Λ_2 within the physically acceptable range of parameters. The solution that we found is

$$c_1 = 0.445, \quad c_2 = -0.00612,$$

$$\Lambda_1 = 630.01\,\text{MeV}, \quad \Lambda_2 = 1642.13\,\text{MeV}. \tag{40}$$

As usual, all the numerical calculations are carried out by using the so-called Kahana-Ripka basis [20]. Following them, the plane-wave basis, introduced as a set of eigenstates of the free Hamiltonian $H_0 = \boldsymbol{\alpha}\cdot\nabla/i + \beta M$, is discretized by imposing an appropriate boundary condition for the radial wave functions at the radius D chosen to be sufficiently larger than the soliton size. The basis is made finite by including only those states with the momentum k as $k < k_{max}$. The eigenvalue problem (22) is then solved by diagonalizing the Dirac Hamiltonian H in the above basis. We are thus able to solve the self-consistent Hartree problem and also to calculate any nucleon observables with full inclusion of the sea-quark degrees of freedom. If the theory is consistently regularized, final answers must be stable against increase of k_{max} and D (especially against the increase of k_{max}). Now we show in Fig. 1 the k_{max} dependence of the theoretical pseudoscalar and scalar quark densities in the single-subtraction Pauli-Villars scheme. These curves are obtained for a fixed value of D as $MD = 12$. The corresponding k_{max} dependence of the quark densities in the double-subtraction Pauli-Villars scheme is shown in Fig. 2. Comparing the two figures, one immediately notices that the quark densities obtained in the single-subtraction Pauli-Villars scheme do not cease to increase in magnitude as k_{max} grows. Undoubtedly, this must be a signal of logarithmic divergences contained in $S(r=\infty)$ [and generally also in $P(r)$ and $S(r)$]. On the other hand, in the case of the double-subtraction Pauli-Villars scheme, the magnitudes of $P(r)$ and $S(r)$ are seen to grow much more slowly. To convince ourselves more clearly of the above qualitative difference of

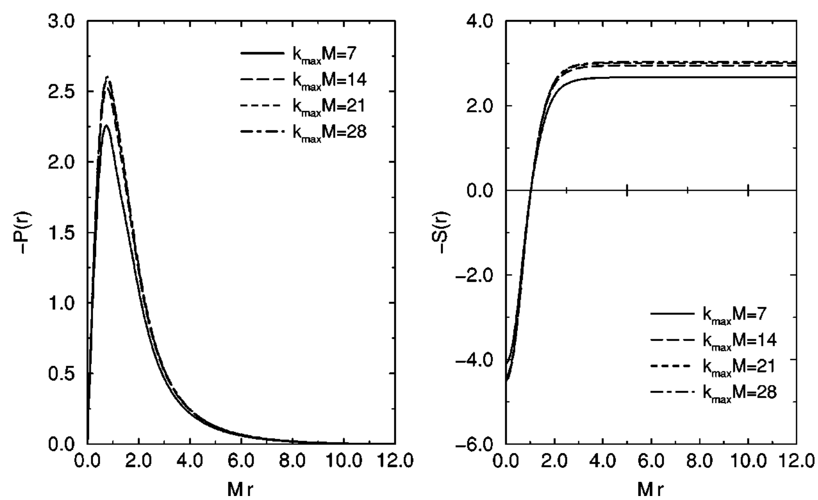

FIG. 2. The k_{max} dependence of the scalar quark density $S(r)$ and the pseudoscalar density $P(r)$ in the double-subtraction Pauli-Villars scheme.

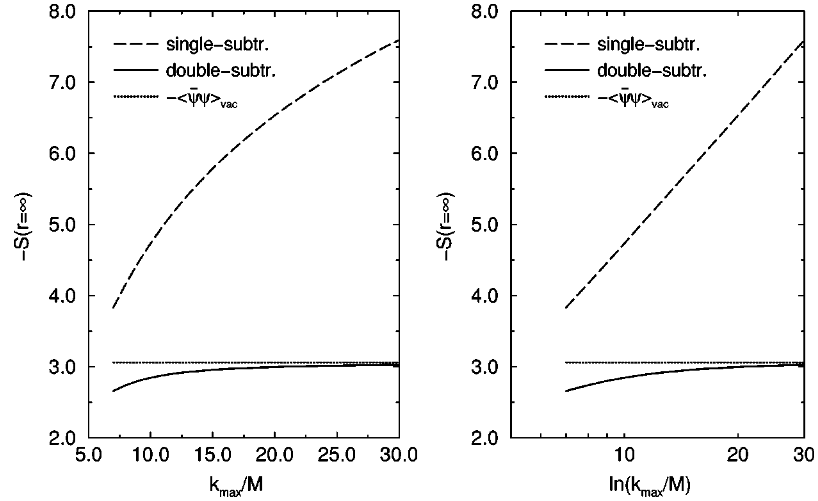

FIG. 3. The scalar quark densities at the spatial infinity $S(r=\infty)$ as functions of k_{max}/M and as functions of $\ln(k_{max}/M)$ in the single- and double-subtraction Pauli-Villars schemes.

the two regularization schemes, we plot in Fig. 3 the value of $S(r=\infty)$, i.e., the scalar quark density at spatial infinity, as functions of k_{max} and also as functions of $\ln(k_{max}/M)$. Contrary to the case of the single-subtraction scheme in which a clear signal of logarithmic divergence is observed, the value of $S(r=\infty)$ obtained in the double-subtraction scheme is seen to converge to some limiting value. Although the rate of this convergence is rather slow, it appears that this limiting value certainly coincides with the prescribed value of the vacuum quark condensate $\langle\bar{\psi}\psi\rangle_{vac}=-(286.6\,\mathrm{MeV})^3=-3.062\,\mathrm{fm}^{-3}$.

Now that one is convinced of the fact that the naive Pauli-Villars scheme with the single-subtraction term contains a logarithmic divergence in the quark densities appearing in the soliton equation of motion, one may come to the following question. Why could the authors of Ref. [12] obtain self-consistent soliton solutions despite the presence of the above-mentioned divergences? The answer lies in the method of obtaining a self-consistent soliton profile in the nonlinear model (not in the original Nambu–Jona-Lasinio model). After evaluating the pseudoscalar and scalar quark densities with some (large but) finite model space (especially with finite k_{max}), a new profile function $F(r)$ to be used in the next iterative step is obtained from Eq. (25). Since $P(r)$ and $S(r)$ appear, respectively, in the numerator and denominator of the argument of arctangents, it can happen that the logarithmic divergences contained in both $P(r)$ and $S(r)$ are offset from each other. (We point out that the effect of the term $f_\pi^2 m_\pi^2/M$ accompanying the scalar quark density is rather small, anyway.) In fact, Fig. 4 shows the k_{max} dependence of the self-consistent profile function $F(r)$ in both the single-subtraction scheme and the double-subtraction scheme. One sees that the resultant $F(r)$ is quite stable against an increase of k_{max} even in the single-subtraction scheme, in spite of the fact that it shows logarithmically divergent behavior for both $P(r)$ and $S(r)$. Undoubtedly, this is the reason why the authors of [15] succeeded in obtaining a self-consistent soliton profile $F(r)$ despite the di-

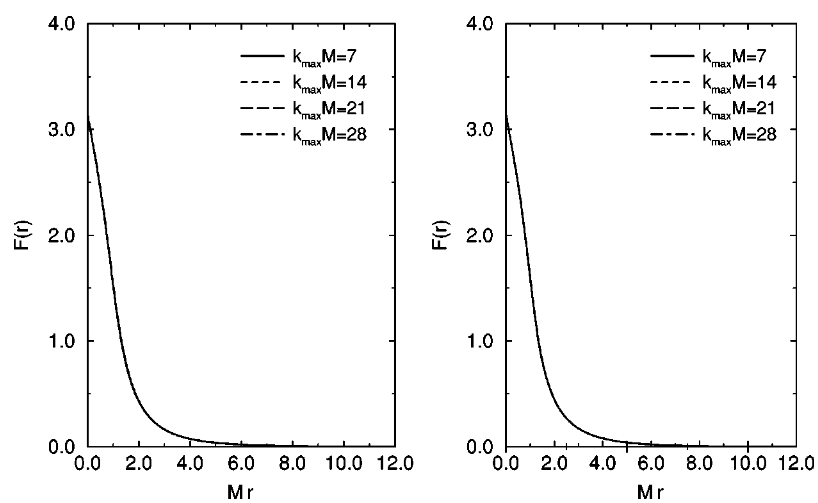

FIG. 4. The k_{max} dependence of the self-consistent soliton profiles $F(r)$ in the single- and double-subtraction Pauli-Villars schemes. The curves with different k_{max} are almost indistinguishable.

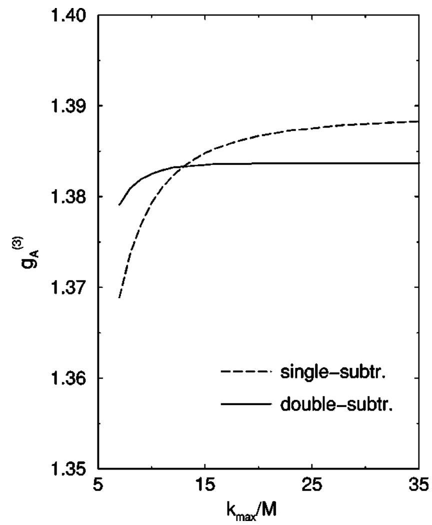

FIG. 5. The k_{max} dependence of the nucleon isovector axial-charges $g_A^{(3)}$ in the single- and double-subtraction Pauli-Villars schemes.

vergences remaining in each of $P(r)$ and $S(r)$. Because of this fortunate accident, self-consistent soliton profiles $F(r)$ in the nonlinear model can be obtained with a good accuracy by using a modest value of k_{max} not only for the double-subtraction scheme but also for the single-subtraction one, and besides the resultant $F(r)$ are not much different in these two schemes. This also applies to most nucleon observables which depend only on $F(r)$ and have no direct dependence on $S(r)$ and/or $P(r)$. The previous calculation of parton distributions with use of the single-subtraction Pauli-Villars scheme may be justified in this sense. To verify the validity of this expectation, we investigate the k_{max} dependence of a typical nucleon observable which contains only a logarithmic divergence, i.e., the isovector axial-vector coupling constant $g_A^{(3)}$. Figure 5 show the k_{max} dependence of $g_A^{(3)}$ in the single- and double-subtraction Pauli-Villars regularization schemes. One sees that this quantity certainly shows a tendency of convergence in both regularization schemes, though the rate of convergence in the double-subtraction scheme is much faster than for the scalar and pseudoscalar densities in the same regularization scheme. Nonetheless, one must be very careful if one is interested in nucleon observables, which have a direct dependence on $S(r)$ or $P(r)$. The most important nucleon observable, which falls into this category, is the nucleon scalar charge (or the quark condensate in the nucleon) given by

$$\langle N|\bar{\psi}\psi|N\rangle \equiv \int d^3r[S(r)-S(r=\infty)]. \tag{41}$$

The superiority of the double-subtraction scheme to the single-subtraction one must be self-explanatory in this case, since this quantity is convergent only in the former scheme.

B. Pauli-Villars versus proper-time regularization

How to introduce an ultraviolet cutoff into our effective chiral theory is a highly nontrivial problem. Diakonov *et al.* advocated the Pauli-Villars subtraction scheme as a ''good'' regularization scheme for evaluating leading-twist parton distribution functions of the nucleon within the chiral quark soliton model [1,2]. The reason is that it preserves several general properties of the parton distributions (such as positivity, factorization properties, sum rules, etc.), which can easily be violated by a naive ultraviolet regularization. On the other hand, Schwinger's proper-time regularization has most frequently been used for investigating low energy nucleon properties within the chiral quark soliton model [7–9]. One might then wonder how these predictions obtained by using the proper-time regularization scheme are altered if one uses the Pauli-Villars one.

Before entering into this discussion, we think it useful to recall some basic properties of the proper-time regularization scheme. In this scheme, the regularized effective meson action takes the same form as Eq. (10) except that $S_f^{reg}[U]$ is now given in the form

$$S_f^{reg}[U] = \frac{1}{2} i N_c \int_0^\infty \frac{d\tau}{\tau} \varphi(\tau)\, \mathrm{Sp}(e^{-\tau D^\dagger D} - e^{-\tau D_0^\dagger D_0}), \tag{42}$$

with

$$D = i\partial\!\!\!/ - MU^{\gamma_5}, \quad D_0 = i\partial\!\!\!/ - M. \tag{43}$$

The regularization function $\varphi(\tau)$ is introduced so as to cut off ultraviolet divergences which now appear as a singularity at $\tau=0$. For determining it, we can use a similar criterion as what was used in the Pauli-Villars scheme. That is, we require that the regularized theory reproduce the correct normalization of the pion kinetic term as well as the empirical value of the vacuum quark condensate. This gives two conditions [21]

$$\frac{N_c M^2}{4\pi^2} \int_0^\infty \frac{d\tau}{\tau} \varphi(\tau) e^{-\tau M^2} = f_\pi^2, \tag{44}$$

$$\frac{N_c M}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^2} \varphi(\tau) e^{-\tau M^2} = \langle \bar{\psi}\psi \rangle_{vac}. \tag{45}$$

Schwinger's original choice corresponds to taking

$$\varphi(\tau) = \theta\left(\tau - \frac{1}{\Lambda^2}\right), \tag{46}$$

with Λ being a physical cutoff energy. However, this simplest choice cannot satisfy the two conditions (44) and (45) simultaneously. Then, we use here a slightly more complicated form as

$$\varphi(\tau) = c\,\theta\left(\tau - \frac{1}{\Lambda_1^2}\right) + (1-c)\,\theta\left(\tau - \frac{1}{\Lambda_2^2}\right), \tag{47}$$

which contains three parameters c, Λ_1, and Λ_2 [22]. Although the above two conditions are not enough to uniquely fix the above three parameters, we find that solution sets (c,Λ_1,Λ_2) lie only in a small range of parameter space and that this slight difference of regularization parameters hardly affects the soliton properties. We use the following set of parameters in the numerical investigation below:

$$c=0.720,\quad \Lambda_1=412.79\,\mathrm{MeV},\quad \Lambda_2=1330.60\,\mathrm{MeV}. \tag{48}$$

Within the framework of the chiral quark soliton model, which assumes slow collective rotation of a hedgehog soliton as

$$U^{\gamma_5}(\boldsymbol{x},t)=A(t)U_0^{\gamma_5}(\boldsymbol{x})A^\dagger(t),\quad A(t)\subset \mathrm{SU}(2), \tag{49}$$

the nucleon matrix element of any quark bilinear operator $\bar{\psi}O\psi$ is given as a perturbative series in the collective angular velocity operator Ω defined by

$$\Omega=iA^\dagger(t)\frac{d}{dt}A(t). \tag{50}$$

It is shown below that a noteworthy difference between the proper-time regularization and the Pauli-Villars one appears at the zeroth order term in Ω. We recall that, in both schemes, the $O(\Omega^0)$ contribution to this matrix element is given as

$$\langle O\rangle^{\Omega^0}=\int \mathcal{D}A\Psi^{(J)*}_{M_JM_T}[A]\langle O\rangle^{\Omega^0}_A\Psi^{(J)}_{M_JM_T}[A], \tag{51}$$

with

$$\langle O\rangle^{\Omega^0}_A=\langle O\rangle^{\Omega^0}_{val}+\langle O\rangle^{\Omega^0}_{vp}, \tag{52}$$

where $\Psi^{(J)}_{M_JM_T}[A]$ is a wave function describing the collective rotational motion. In Eq. (53),

$$\langle O\rangle^{\Omega^0}_{val}=N_c\langle 0|\tilde{O}|0\rangle,\quad \text{with}\quad \tilde{O}=A^\dagger OA, \tag{53}$$

represents the contribution of the discrete bound-state level called the valence-quark one. Within the Pauli-Villars scheme, the contribution of the Dirac continuum can be given in either of the following two forms:

$$\begin{aligned}\langle O\rangle^{\Omega^0}_{vp}&=N_c\sum_{n<0}\langle n|\tilde{O}|n\rangle-\text{Pauli-Villars subtraction}\\&=-N_c\sum_{n\geq 0}\langle n|\tilde{O}|n\rangle-\text{Pauli-Villars subtraction.}\end{aligned} \tag{54}$$

Note that the first form is given as a sum over the occupied single-quark levels, while the second form given as a sum over the nonoccupied levels. The equivalence of the two expressions follows from the identity

$$0=\mathrm{Sp}\tilde{O}=\sum_{n<0}\langle n|\tilde{O}|n\rangle+\sum_{n\geq 0}\langle n|\tilde{O}|n\rangle, \tag{55}$$

which holds for most operators including the isovector magnetic moment operator investigated below, if it is combined with the fact that a similar identity holds also for the corresponding Pauli-Villars subtraction terms. The situation is a little different for the proper-time regularization scheme. The regularized Dirac sea contribution in this scheme is given in the following form [8]:

$$\langle O\rangle^{\Omega^0}_{vp}=-\frac{N_c}{2}\sum_{n=all}\mathrm{sgn}(E_n)g(E_n)\langle n|\tilde{O}|n\rangle, \tag{56}$$

with

$$g(E_n)=\frac{1}{\sqrt{\pi}}\int_0^\infty\frac{d\tau}{\sqrt{\tau}}|E_n|e^{-\tau E_n^2}. \tag{57}$$

To compare this with the corresponding expression in the Pauli-Villars scheme, it is convenient to rewrite it as

$$\langle O\rangle^{\Omega^0}_{vp}=\frac{1}{2}\left\{N_c\sum_{n<0}g(E_n)\langle n|\tilde{O}|n\rangle-N_c\sum_{n\geq 0}g(E_n)\langle n|\tilde{O}|n\rangle\right\}. \tag{58}$$

One sees that here the answer is given as an average of the two expressions, i.e., the one given as a sum over the occupied levels and the others given as a sum over the nonoccupied levels. (This feature is a consequence of the starting covariant expression for an operator expectation value in the proper-time scheme.) However, contrary to the previous case in which ultraviolet regularization is introduced in the form of the Pauli-Villars subtraction, now there is no reason to

TABLE I. The static soliton energy in the proper-time regularization scheme and the (double-subtraction) Pauli-Villars one. E_{val} and $E^{reg}_{v.p.}$, respectively, stand for the valence quark contribution and the Dirac sea one to the fermionic energy, while E_m represents the mesonic part of the energy. The sum of these three parts gives the total static energy E^{reg}_{static}.

	E_{val} [MeV]	$E^{reg}_{v.p.}$ [MeV]	E_m [MeV]	E^{reg}_{static} [MeV]
Proper-time ($m_\pi=138$ MeV)	633.0	617.6	37.2	1287.9
Pauli-Villars ($m_\pi=138$ MeV)	447.6	569.2	51.3	1068.1
Proper-time ($m_\pi=0$ MeV)	555.6	688.6	0	1244.2
Pauli-Villars ($m_\pi=0$ MeV)	351.5	655.4	0	1006.9

TABLE II. The quark spin content of the nucleon $\langle\Sigma_3\rangle$ in the proper-time regularization scheme and the Pauli-Villars one.

	$\langle\Sigma_3\rangle_{val}$	$\langle\Sigma_3\rangle_{v.p.}$	$\langle\Sigma_3\rangle$
Proper-time (m_π=138 MeV)	0.484	0.005	0.489
Pauli-Villars (m_π=138 MeV)	0.391	0.008	0.399
Proper-time (m_π=0 MeV)	0.374	0.007	0.380
Pauli-Villars (m_π=0 MeV)	0.286	0.011	0.298

believe that the above two terms give the same answer. In fact, the introduction of the energy dependent cutoff factor $g(E_n)$ generally breaks the equivalence of the two expressions because of the spectral asymmetry of the positive- and negative-energy levels induced by the background pion field of hedgehog form.

Now we start a comparative analysis of the two regularization schemes on the basis of the numerical results. For reference, we also solve the soliton equation of motion in the chiral limit. By assuming no (or at least weak) m_π dependence of $\langle\bar{\psi}\psi\rangle_{vac}$ appearing in Eqs. (16) and (37), this calculation can be done by setting $m_\pi=0$ in Eq. (18) and (25) without changing the sets of regularization parameters given in Eq. (40) and (48). Since the method of cutting off the ultraviolet component is totally different for the two regularization schemes, it naturally affects solutions of the soliton equation of motion. Although the detailed contents of the soliton energy are highly model-dependent concepts and are not direct observables, they are anyhow very sensitive to this difference of the self-consistent solutions. Table I shows this comparison. Comparing the answers of the two regularization schemes, one finds that the Pauli-Villars scheme leads to a more strongly deformed soliton, which means a deeper binding of the discrete valence level and larger vacuum polarization energy. One sees that the total soliton energy is lower for the Pauli-Villars scheme than for the proper-time scheme. One also observes that the soliton energy is very sensitive to the pion mass. When one goes from the finite pion mass case to the chiral limit, one obtains much lower soliton energy.

Probably, the most important observable which has strong sensitivity to the above difference of the self-consistent solutions is the flavor-singlet axial charge or the quark spin content of the nucleon $\langle\Sigma_3\rangle$. The theoretical predictions for this quantities in the two regularization schemes are shown in Table II. In evaluating this quantity, we did not introduce any regularization, because it is related to the imaginary part of the (Euclidean) effective action and is convergent itself. This means that the difference between the two schemes purely comes from that of the self-consistent solutions. One sees that the Pauli-Villars scheme leads to smaller quark spin content. The reason can easily be understood. Within the framework of the chiral-quark soliton model, the rest of the nucleon spin is carried by the orbital angular momentum of quark fields and this latter portion increases as the deformation of the soliton becomes larger [8]. A similar tendency is also observed when one goes from the finite pion mass case to the chiral limit.

There are different kinds of nucleon observables, which contain (potential) logarithmic divergence and thus depend directly on how they are regularized. Most typical are the $O(\Omega^0)$ contribution to the isovector axial-vector coupling constant $g_A^{(3)}$ and the isovector magnetic moment μ_V of the nucleon. Let us first show the results for the isovector magnetic moment, since it turns out to have stronger dependence on the choice of the regularization scheme. Table III shows the $O(\Omega^0)$ contribution to the isovector magnetic moment. For each regularization scheme, the second column represents the answer obtained with the occupied expression, while the third column gives the answer obtained with the nonoccupied one. In the case of the Pauli-Villars scheme, the equivalence of the two expressions is nicely confirmed by the explicit numerical calculation. In the case of the proper-time scheme, however, we encounter quite a different situation. First, the answer obtained with the occupied expression is about 30% larger than the corresponding answer of the Pauli-Villars scheme, while the answer obtained with the nonoccupied expression is about 80% smaller than the answer obtained with the occupied one. Since the final answer of the proper-time scheme is given as an average of the occupied and nonoccupied expressions, the consequence is that the prediction of the proper-time scheme for the $O(\Omega^0)$ contribution to μ_V is about 14% smaller than the corresponding prediction of the Pauli-Villars scheme. (See the fourth column of Table III.) Note that the difference between the two regularization schemes becomes much more drastic when one goes to the chiral limit. This is due to the fact that the

TABLE III. The $O(\Omega^0)$ contributions to the isovector magnetic moment of the nucleon in the proper-time regularization scheme and the Pauli-Villars one. The second column represents for the valence quark contribution. The third and fourth columns stand for the answers for the vacuum polarization contributions, respectively, obtained with occupied and nonoccupied formulas, while the fifth column gives the average of the two answers. The total $O(\Omega^0)$ contributions are shown in the sixth column.

	$\mu_{val}^{(3)}(\Omega^0)$		$\mu_{v.p.}^{(3)reg}(\Omega^0)$		$\mu^{(3)}(\Omega^0)$
		Occupied	Nonoccupied	Average	
Proper-time (m_π=138 MeV)	1.611	1.312	0.210	0.761	2.372
Pauli-Villars (m_π=138 MeV)	1.762	0.996	0.996	0.996	2.759
Proper-time (m_π=0 MeV)	1.623	1.908	0.588	1.248	2.875
Pauli-Villars (m_π=0 MeV)	1.810	1.738	1.738	1.738	3.547

TABLE IV. The final predictions for the isovector magnetic moment of the nucleon, given as sums of the $O(\Omega^0)$ and $O(\Omega^1)$ contributions.

	$\mu^{(3)}(\Omega^0)$	$\mu^{(3)}(\Omega^1)$	$\mu^{(3)}(\Omega^0+\Omega^1)$
Proper-time (m_π=138 MeV)	2.372	1.072	3.445
Pauli-Villars (m_π=138 MeV)	2.759	1.211	3.970
Proper-time (m_π=0 MeV)	2.875	1.032	3.907
Pauli-Villars (m_π=0 MeV)	3.547	1.182	4.729

$O(\Omega^0)$ vacuum polarization contribution to the isovector magnetic moment is extremely sensitive to the pion mass effect such that it is much larger in the chiral limit.

Before comparing our theoretical predictions with the observed isovector magnetic moment of the nucleon, we must take account of the $O(\Omega^1)$ contribution, too, since it is known to give a sizable correction to the leading-order result [23,24]. Although we do not go into details here, it turns out that this $O(\Omega^1)$ piece is not so sensitive to the difference of the regularization scheme as the $O(\Omega^0)$ piece is. The reason is that this $O(\Omega^1)$ term is given as a double sum over the occupied levels and the nonoccupied ones and the formula has some symmetry under the exchange of these two types of single-quark orbitals [25]. The final predictions for the nucleon isovector magnetic moment obtained as a sum of the $O(\Omega^0)$ and $O(\Omega^1)$ contributions are shown in Table IV. After all, the prediction of the Pauli-Villars scheme is about 15% larger than that of the proper-time scheme and a little closer to the observed moment. The effect is much more drastic in the chiral limit. The prediction of the Pauli-Villars scheme is about 20% larger than that of the proper-time scheme and nearly reproduces the observed isovector magnetic moment of the nucleon, i.e., $\mu^{(3)}_{expt}\simeq 4.71$.

Finally, we show in Table V the predictions for the isovector axial charge of the nucleon obtained as a sum of the $O(\Omega^0)$ and $O(\Omega^1)$ contributions. Also for this quantity, there are some detailed differences between the predictions of the two regularization schemes. Nonetheless, the final answers for $g_A^{(3)}$ turn out to be not so sensitive to the difference of the regularization schemes as compared with the case of the isovector magnetic moment. Besides, one also notices that the finite pion mass effect hardly influences the final prediction for this particular quantity.

IV. CONCLUSION

In summary, the single-subtraction Pauli-Villars regularization scheme, which is often used in evaluating nucleon structure functions within the framework of the CQSM, cannot be regarded as a fully consistent regularization scheme in that it still contains ultraviolet divergences in the scalar and psuedoscalar quark densities appearing in the soliton equation of motion. However, these divergences can easily be removed by increasing the number of subtraction terms from 1 to 2. After this straightforward generalization, the effective theory is totally divergence free. Especially, both the vacuum quark condensate and the isoscalar piece of the nucleon scalar charge become finite now. Nonetheless, we find that, owing to the accidental cancellation explained in the text, one can obtain a finite soliton profile $F(r)$ even in the single-subtraction scheme, and besides the resultant soliton solution is not extremely different from the corresponding one obtained in the double-subtraction scheme. Furthermore, it turns out that, for most nucleon observables, which contain only the logarithmic divergence, the predictions of the two regularization schemes are not much different. The previous calculations of quark distribution functions with use of the single-subtraction Pauli-Villars regularization scheme would be justified in this sense.

We have also carried out a comparative analysis of typical nucleon observables based on the Pauli-Villars regularization scheme and the proper-time one. A nice property of the Pauli-Villars regularization scheme, which is not possessed by the proper-time one, is that it preserves a nontrivial symmetry of the original theory, i.e., the equivalence of the occupied and nonoccupied expressions for $O(\Omega^0)$ contributions to nucleon observables. The improvement obtained for the isovector magnetic moment of the nucleon was shown to be related to this favorable property of the Pauli-Villars regularization scheme. How to introduce an ultraviolet cutoff into an effective low energy model should in principle be predictable from the underlying QCD dynamics. For lack of precise information about it, however, phenomenology must provide us with an important criterion for selecting regularization schemes. The regularization scheme based on the Pauli-Villars subtraction appears to be a good candidate also in this respect.

TABLE V. The final predictions for the isovector axial-coupling constant of the nucleon, given as sums of the $O(\Omega^0)$ and $O(\Omega^1)$ contributions.

	$g_A^{(3)}(\Omega^0)$	$g_A^{(3)}(\Omega^1)$	$g_A^{(3)}(\Omega^0+\Omega^1)$
Proper-time (m_π=138 MeV)	0.848	0.412	1.260
Pauli-Villars (m_π=138 MeV)	0.976	0.408	1.384
Proper-time (m_π=0 MeV)	0.921	0.348	1.269
Pauli-Villars (m_π=0 MeV)	1.054	0.344	1.398

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APPENDIX: EFFECTIVE LAGRANGIAN WITH CURRENT QUARK MASS

The effective Lagrangian of the chiral-quark model with nonzero current quark mass is given by

$$\mathcal{L}=\bar{\psi}[i\partial\!\!\!/-MU^{\gamma_5}(x)-m_0]\psi. \tag{A1}$$

We want to explain below what complication arises when regularizing the above Lagrangian in the Pauli-Villars subtraction scheme. Let us begin with the unregularized effective meson action corresponding to the above Lagrangian:

$$S_{eff}=-\frac{1}{2}iN_c\{\mathrm{Sp}\ln D^\dagger D-\mathrm{Sp}\ln D_0^\dagger D_0\}, \tag{A2}$$

where

$$D^\dagger D=\partial^2+\bar{M}^2+m_0M(U^{\gamma_5}+U^{\gamma_5^+}-2)+iM\partial\!\!\!/U^{\gamma_5}, \tag{A3}$$

$$D_0^\dagger D_0=\partial^2+\bar{M}^2. \tag{A4}$$

Here

$$\bar{M}\equiv M+m_0 \tag{A5}$$

denotes the quark mass in the physical vacuum ($U^{\gamma_5}=1$). As will become clear soon, the fact that both M and $\bar{M}$ appear in Eq. (A3) makes the regularization in the Pauli-Villars scheme rather complicated. Before discussing why it is so, let us first show that the regularization in the proper-time scheme can be done without any problem. The regularization function $\varphi(\tau)$ in this scheme is usually determined by the combined use of the derivative expansion and the perturbative expansion in m_0. The condition that reproduces the correct normalization of the pion kinetic term is given by

$$\frac{N_cM^2}{4\pi^2}\int_0^\infty\frac{d\tau}{\tau}\varphi(\tau)e^{-\tau\bar{M}^2}=f_\pi^2. \tag{A6}$$

On the other hand, the correct normalization of the pion mass term is reproduced if the following condition is satisfied [21]:

$$m_0\frac{N_cM}{2\pi^2f_\pi^2}\int_0^\infty\frac{d\tau}{\tau^2}\varphi(\tau)e^{-\tau\bar{M}^2}=m_\pi^2. \tag{A7}$$

We also require that the vacuum quark condensate be finite, which gives

$$\frac{N_c\bar{M}}{2\pi^2}\int_0^\infty\frac{d\tau}{\tau^2}\varphi(\tau)e^{-\tau\bar{M}^2}=-\langle\bar{\psi}\psi\rangle_{vac}. \tag{A8}$$

What is noticeable here is that a common integral appears in Eqs. (A7) and (A8), thereby ensuring, irrespective of the form of $\varphi(\tau)$, the identity

$$m_0\langle\bar{\psi}\psi\rangle_{vac}=-\left(\frac{\bar{M}}{M}\right)f_\pi^2m_\pi^2 \tag{A9}$$

or, equivalently,

$$m_0\langle\bar{\psi}\psi\rangle_{vac}=-\left(1+\frac{m_0}{M}\right)f_\pi^2m_\pi^2. \tag{A10}$$

Except for the higher order correction in m_0/M, this is essentially the celebrated Gell-Mann–Oakes–Renner relation [18]. After all, the cutoff function $\varphi(\tau)$ in the proper-time scheme can be determined so as to satisfy the two conditions (A6) and (A7), for a given value of m_0 or $\langle\bar{\psi}\psi\rangle_{vac}$, which is consistent with Eq. (A10).

Next, we turn to the Pauli-Villars scheme. The regularized action in this scheme is defined by

$$S_{eff}^{reg}=S_f[U]-\sum_{i=1}^N c_iS_f^{\Lambda_i}[U], \tag{A11}$$

with

$$S_f[U]=-iN_c\,\mathrm{Sp}\ln(i\partial\!\!\!/-MU^{\gamma_5}-m_0), \tag{A12}$$

while $S_f^{\Lambda_i}[U]$ is obtained from $S_f[U]$ with the replacement of the dynamical quark mass M by the regulator mass Λ_i. Now we can proceed as before. In order to reproduce the pion kinetic term, we need the following two conditions:

$$M^2-\sum_{i=1}^N c_i\Lambda_i^2=0, \tag{A13}$$

$$\frac{N_c}{4\pi^2}\sum_{i=1}^N c_i\Lambda_i^2\ln\left(\frac{\bar{\Lambda}}{\bar{M}}\right)^2=f_\pi^2, \tag{A14}$$

with the definition $\bar{\Lambda}_i\equiv\Lambda_i+m_0$. Here, the first condition is necessary for removing logarithmic divergence. Next, the pion mass term is reproduced under the following conditions:

$$M-\sum_{i=1}^N c_i\Lambda_i=0, \tag{A15}$$

$$M\bar{M}^2-\sum_{i=1}^N c_i\Lambda_i\bar{\Lambda}_i^2=0, \tag{A16}$$

$$m_0\frac{N_c\bar{M}^3}{2\pi^2}\sum_{i=1}^N c_i\left(\frac{\Lambda_i}{\bar{M}}\right)\left(\frac{\bar{\Lambda}_i}{\bar{M}}\right)^2\ln\left(\frac{\bar{\Lambda}_i}{\bar{M}}\right)^2=-f_\pi^2m_\pi^2. \tag{A17}$$

Here, the first and second conditions are, respectively, for removing quadratic and logarithmic divergences. Finally, the finite value of the vacuum quark condensate is obtained with the conditions

$$M\bar{M}-\sum_{i=1}^N c_i\Lambda_i\bar{\Lambda}_i=0, \tag{A18}$$

$$M\bar{M}^3-\sum_{i=1}^{N} c_i\Lambda_i\bar{\Lambda}_i^3=0, \tag{A19}$$

$$\frac{N_c\bar{M}^3}{2\pi^2}\left(\frac{\bar{M}}{M}\right)\sum_{i=1}^{N} c_i\left(\frac{\Lambda_i}{\bar{M}}\right)\left(\frac{\bar{\Lambda}_i}{\bar{M}}\right)^3\ln\left(\frac{\bar{\Lambda}_i}{\bar{M}}\right)^2=\langle\bar{\psi}\psi\rangle_{vac}. \tag{A20}$$

Among the five conditions (A13), (A15), (A16), (A18), and (A19), which are to remove ultraviolet divergences, not all of them are independent. In fact, one can easily verify that they reduce to the following four conditions:

$$\sum_{i=1}^{N} c_i\Lambda_i=M, \tag{A21}$$

$$\sum_{i=1}^{N} c_i\Lambda_i^2=M^2, \tag{A22}$$

$$\sum_{i=1}^{N} c_i\Lambda_i^3=M^3, \tag{A23}$$

$$\sum_{i=1}^{N} c_i\Lambda_i^4=M^4. \tag{A24}$$

Here we point out that different powers of $\bar{\Lambda}_i/\bar{M}$ appear in Eqs. (A17) and (A20). [We recall that these two conditions (A17) and (A20), respectively, correspond to Eqs. (A7) and (A8) in the proper-time regularization scheme.] This appears to originate from the fact that relation (A17) is obtained from the perturbative expention in m_0, thereby containing the parameter m_0 with mass dimension, while Eq. (A20) does not. As a consequence, the Gell-Mann–Oakes–Renner relation does not follow automatically, i.e., for an arbitrary choice of the regularization parameters c_i and Λ_i. This is in contrast to the proper-time regularization scheme in which the same identity holds irrespective of the form of the cutoff function $\varphi(\tau)$.

To sum up, it seems that consistent regularization of the effective Lagrangian (A1) with the finite current quark mass within the framework of the Pauli-Villars subtraction scheme demands that four finiteness conditions (A21), (A22), (A23), and (A24) and three normalization conditions (A14), (A17), and (A20) be satisfied. This means that we need at least four subtraction terms with eight parameters (c_i and Λ_i with $i=1,\ldots,4$). We are not yet sure whether there is a reasonable set of parameters which satisfies all the above conditions.

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