Off-diagonal distributions fixed by diagonal partons at small x and ξ

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We show that the off-diagonal (or skewed) parton distributions are completely determined at small x and ξ by the (conventional) diagonal partons. We present predictions which can be used to estimate the off-diagonal distributions at small x and ξ at any scale. [S0556-2821(99)02113-X]

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I. INTRODUCTION

Precision data are becoming available for hard scattering processes whose description requires knowledge of off-diagonal (or so-called ''skewed'') parton distributions. Particularly relevant processes are the diffractive production of vector mesons and of high E_T jets in high energy electron-proton collisions.

We shall use the ''off-forward'' distributions

$$H(x,\xi)\equiv H(x,\xi,t,\mu^2)$$

with support $-1\leqslant x\leqslant 1$ introduced by Ji [1–3], with the minor difference that the gluon $H_g=xH_g^{\mathrm{Ji}}$. They depend on the momentum fractions $x_{1,2}=x\pm\xi$ carried by the emitted and absorbed partons at each scale μ^2 and on the momentum transfer variable $t=(p-p')^2$; see Fig. 1. The values of t and $\xi=(x_1-x_2)/2$ do not change as we evolve the parton distributions up in the scale μ^2. That is t and ξ lie outside the evolution. In the limit $\xi\to 0$ the distributions reduce to the conventional diagonal distributions

$$H_q(x,0)=\begin{cases} q(x) & \text{for } x>0,\\ -\bar{q}(-x) & \text{for } x<0,\end{cases}$$

$$H_g(x,0)=xg(x). \tag{1}$$

Detailed reviews of off-diagonal distributions can be found, for example, in Refs. [3–5].

It is usual to anticipate that the ξ dependence of H is controlled by the non-perturbative starting (input) distribution at some low scale $\mu^2=Q_0^2$. However here we wish to explore the possibility that, in the small $x,\xi\ll 1$ region, the ''skewed'' off-diagonal behavior comes mainly from the evolution. Indeed we expect this to be the case. At each step of the evolution the momentum fraction carried by parton i ($i=1,2$) decreases. So when the evolution length is sufficiently large [i.e. $\ln(Q^2/Q_0^2)\gg 1$], the important values of $x\sim x_0$ of the input ($\mu^2=Q_0^2$), which control the behavior in the $x\sim\xi$ domain at the high scale ($\mu^2=Q^2$), will satisfy $x_0\gg\xi$. Clearly we can neglect the ξ dependence in the x_0 region and start evolving from purely diagonal partons with $x_1=x_2$.

Here we demonstrate how, in the phenomenologically important small ξ region (for $t\to 0$), the off-diagonal distributions are determined unambiguously in terms of the small x behavior of the (conventional) diagonal partons which is known from experiment. We therefore have the attractive possibility to include data described by off-diagonal distributions in a global parton analysis to better constrain the small x behavior of the diagonal distributions.

II. OFF-DIAGONAL DISTRIBUTIONS IN TERMS OF CONFORMAL MOMENTS

In terms of the operator product expansion (OPE) the evolution of the off-diagonal distributions may be viewed as the renormalisation of the matrix elements $O_N=\langle p'|\hat{O}_N|p\rangle$ of the conformal (Ohrndorf [6]) operators, where p and p' are the momenta of the incoming and outgoing protons. For the quark, the leading twist operator $\hat{O}_N$ is of the form

$$\hat{O}_N^q=\sum_{k=0}^{N}\binom{N}{k}\binom{N+2}{k+1}\partial_L^k\partial_R^{N-k} \tag{2}$$

where the derivatives ∂_L and ∂_R act on the left and right quarks in Fig. 1. As a consequence the quark matrix element takes the form

$$O_N^q=\int_{-1}^{1}dx R_N^q(x_1,x_2)H_q(x,\xi) \tag{3}$$

with $x_{1,2}=x\pm\xi$, where the polynomials [7]

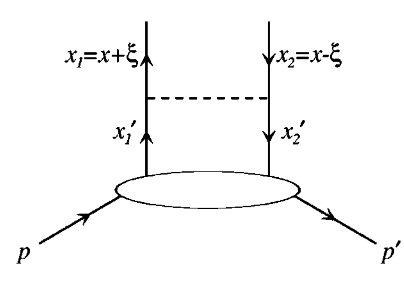

FIG. 1. A schematic diagram showing the variables for the off-diagonal parton distribution $H(x,\xi)$ where $x_{1,2}=x\pm\xi$.

$$R_N^q = \sum_{k=0}^{N} \binom{N}{k} \binom{N+2}{k+1} x_1^k x_2^{N-k}. \tag{4}$$

In other words the polynomials $R_N(x_1,x_2)$ are the basis which specifies the conformal moments O_N. In the diagonal limit, with $x_1 = x_2$, Eq. (3) reduces to the well-known moments

$$M_N^q = \int_0^1 x^N q(x) dx. \tag{5}$$

Unlike the common x^N basis in the diagonal case, the gluon and quark polynomial bases differ from each other. For the gluon we have

$$R_N^g = \sum_{k=0}^{N} \binom{N}{k} \binom{N+4}{k+2} x_1^k x_2^{N-k}, \tag{6}$$

to be compared with the quark polynomials of Eq. (4).

Recall that the off-diagonal distributions are symmetric in ξ [2,3]:

$$H_i(x,\xi) = H_i(x,-\xi) \tag{7}$$

with $i=q$ or g. This is just the left-right or $x_1 \leftrightarrow x_2$ symmetry of Fig. 1. In terms of the x variable the symmetry relations are

$$H_q^s(x,\xi) = -H_q^s(-x,\xi),$$

$$H_q^{ns}(x,\xi) = H_q^{ns}(-x,\xi),$$

$$H_g(x,\xi) = H_g(-x,\xi) \tag{8}$$

for the quark singlet, non-singlet and gluon respectively.

The conformal moments O_N have the advantage that they are not mixed, at least at LO, during the off-diagonal evolution, but simply get multiplicatively renormalized[1]

$$O_N(Q^2) = O_N(Q_0^2) \left(\frac{Q^2}{Q_0^2} \right)^{\gamma_N} \tag{9}$$

with the same anomalous dimension γ_N as in the diagonal case. The problem of how to restore the analytic off-diagonal distribution $H(x,\xi)$ from knowledge of its conformal moments $O_N(\xi)$ of Eq. (3) has been solved recently by Shuvaev [8]. The prescription is as follows. We first calculate an auxiliary function

$$f(x',\xi;t) \equiv f(x') = \int \frac{dN}{2\pi i} (x')^{-N} O_N(\xi)/R_N(1,1) \tag{10}$$

using a simple Mellin transform, where for simplicity of presentation we shall omit the arguments ξ, t and μ^2 of f. Next we perform the convolution

$$H(x,\xi) = \int_{-1}^{1} dx' \mathcal{K}(x,\xi;x') f(x'), \tag{11}$$

where, for quarks, the kernel is given by

$$\mathcal{K}_q(x,\xi;x') = -\frac{1}{\pi |x'|} \mathrm{Im} \int_0^1 ds (1 - y(s)x')^{-3/2} \tag{12}$$

with

$$y(s) = \frac{4s(1-s)}{x + \xi(1-2s)}. \tag{13}$$

To gain insight into the Shuvaev prescription we repeat that, from a theoretical OPE point of view, it is best to analyze experimental data for processes described by off-diagonal distributions in terms of the conformal moments O_N of Eq. (3) which diagonalize the (LO) evolution. However, phenomenologically it is more convenient to work in terms of the off-diagonal parton distributions themselves. The Shuvaev transform (10) and (11) performs the necessary inverse of Eq. (3) at any fixed ξ, t and μ^2; that is it enables $H(x,\xi)$ to be constructed from $O_N(\xi)$. So far this is just a mathematical procedure. The crucial physical step is to relate the auxiliary function $f(x')$ directly to the diagonal partons. It is easy to show, for $\xi \ll 1$, that $f(x')$ in fact reduces to a diagonal parton distribution. Indeed the conformal moments may be expressed in the form

$$O_N(\xi) = \sum_{k=0}^{[(N+1)/2]} O_{Nk} \xi^{2k}, \tag{14}$$

which embodies the ''polynomial condition'' that the power of ξ should be at most of the order of $N+1$. For $\xi \ll 1$ we have

$$O_N(\xi) \simeq O_{N0} = O_N(0). \tag{15}$$

Now, up to the trivial normalization factor $R_N(1,1)$, the diagonal moment $O_N(0)$ is equal to the x^N moment of the diagonal parton distribution. So for $\xi \ll 1$ we can put $f_q(x') = q(x')$ in Eq. (11), and then use Eq. (12) to determine the off-diagonal distribution $H_q(x,\xi)$ in terms of the conventional quark distribution. In this limit the kernel $\mathcal{K}$ just becomes a non-trivial representation of the delta function $\delta(x - x')$.

Since Eq. (12) is a principal value integration, the apparent singularity at $y(s)x' = 1$ is not a problem. However, for computation purposes, it is convenient to first weaken this singularity in the s integration by integrating by parts. Then Eqs. (11) and (12) become

$$H_q(x,\xi) = \int_{-1}^{1} dx' \left[\frac{2}{\pi} \mathrm{Im} \int_0^1 \frac{ds}{y(s)\sqrt{1 - y(s)x'}} \right] \frac{d}{dx'} \left(\frac{q(x')}{|x'|} \right). \tag{16}$$

Here we have used the properties that $q(x') \to 0$ as $x' \to 1$ and that

[1]For simplicity we take the coupling α_S to be fixed. The generalization to running α_S is straightforward.

$$q^s(x')=-q^s(-x'), \quad q^{ns}(x')=q^{ns}(-x'), \tag{17}$$

see Eq. (8). Note that, for small ξ, we can identify the auxiliary function $f(x')$ of Eq. (10) with the diagonal partons at any scale, as the same anomalous dimensions γ_N control both the diagonal and off-diagonal evolution.

So far we have neglected the t dependence and set $t=0$. However from the sum rule [3] we know that the t dependence of the first conformal moment is given by the proton form factor $G(t)$,

$$O_{N=0}(t)=\langle p'|\hat{O}_0|p\rangle \propto G(t). \tag{18}$$

In fact it is natural to assume that all the moments are proportional to $G(t)$

$$O_N(t)=O_N(t=0)G(t) \tag{19}$$

and simply multiply Eq. (11) by $G(t)$ to restore the t dependence of the distributions. Another argument in favor of such a factorization is the form of the Mellin integration (10) where, for small x, the saddle point is located near the singularity at $N=0$ which comes from the behavior of the singlet anomalous dimension, $\gamma_N \propto 1/N$. Thus the dominant contribution comes from $O_{N=0}$ which is indeed proportional to $G(t)$, and due to the polynomial condition (14) does not depend on ξ at all [3].

The formula for the gluon is a little different to that for the quarks. The reason is that in the off-diagonal case the functions H_q and $H_g=xH_g^{\rm Ji}$ form the singlet multiplet which is multiplicatively renormalized. The additional x in the gluon reveals itself as an extra factor of $x+\xi(1-2s)$ in the kernel. Thus for the gluon, in place of Eq. (16), we have

$$H_g(x,\xi;t)=xH_g^{\rm Ji}=\int_{-1}^{1}dx'\left[\frac{2}{\pi}{\rm Im}\int_0^1\frac{ds\big(x+\xi(1-2s)\big)}{y(s)\sqrt{1-y(s)x'}}\right]\times\frac{d}{dx'}\left(\frac{g(x')}{|x'|}\right)G(t). \tag{20}$$

III. PREDICTIONS OF THE OFF-DIAGONAL DISTRIBUTIONS FOR SMALL x AND ξ

We see that Eqs. (16) and (20) completely determine the behavior of the off-diagonal distributions in the small x,ξ domain in terms of the diagonal distributions. In fact by making the physically reasonable small x assumption that the diagonal partons are given by

$$xq(x)=N_q x^{-\lambda_q}, \quad xg(x)=N_g x^{-\lambda_g} \tag{21}$$

we can perform the x' integration analytically.[2] We obtain

$$H_i(x,\xi;t)=N_i\frac{\Gamma\left(\lambda+\frac{5}{2}\right)}{\Gamma(\lambda+2)}\frac{2}{\sqrt{\pi}}\int_0^1 ds[x+\xi(1-2s)]^p\times\left[\frac{4s(1-s)}{x+\xi(1-2s)}\right]^{\lambda_i+1}G(t) \tag{22}$$

with $i=q$ or g, and where $p=0$ and 1 for quarks and the gluon respectively.

At first sight it appears that for singlet quarks (where $\lambda_q>0$ and $p=0$) we face a strong singularity in integral (22) when the term $D\equiv x+\xi(1-2s)\to 0$ in the denominator. Fortunately the singlet quark distribution is antisymmetric in x. To obtain the imaginary part of the integral (16) we must choose $x'>0$ for $D>0$ and $x'<0$ for $D<0$. Therefore we must treat Eq. (22) as a principal value integral and take the difference between the $D\to 0+$ and $D\to 0-$ limits. Thus the main singularity is cancelled and Eq. (22) becomes integrable for any $\lambda_q<1$.

Note that the dominant contribution to the x' integrations of Eqs. (16) and (20) comes from the region of small $x'\sim x,\xi$. Indeed with the input given by Eq. (21), the integral for the quark distribution has a strong singularity at small x'

$$I_q\sim\int dx'(x')^{-\lambda_q-3}{\rm Im}\left(\frac{1}{y(s)\sqrt{1-y(s)x'}}\right). \tag{23}$$

However when we take the imaginary part, the x' integration is cut-off by the theta function $\theta(x'-1/y(s))$ at

$$x'=1/y(s)\sim x+\xi(1-2s). \tag{24}$$

So we obtain the small ξ behavior $I_q\sim\xi^{-\lambda_q-1}$, and the distribution (16) has the form

$$H_q(x,\xi)=\xi^{-\lambda_q-1}F_q(x/\xi). \tag{25}$$

Similarly it follows that $H_g=\xi^{-\lambda_g}F_g(x/\xi)$.

The predictions for the off-diagonal distributions are shown in Fig. 2. In diagrams (a)–(c) we show the ratio R to the diagonal distribution in the form

$$R=\frac{H(x,\xi)}{H(x+\xi,0)}, \tag{26}$$

and so the only free parameter is λ, the exponent which fixes the $x^{-\lambda}$ behavior of the input diagonal partons, as in Eq. (21). Notice that on account of Eq. (25) the ratios R at small x and ξ are a function of only the ratio of the variables x/ξ.

The ratios R of Eq. (26) are the relevant ratios. For example, high energy diffractive $q\bar{q}$ electroproduction is described by two gluon exchange with

[2]We use the substitution $z=1/x'y(s)$ and note that

$$\int_0^1 dz z^{\lambda+3/2}(1-z)^{-1/2}=\frac{\Gamma\left(\lambda+\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(\lambda+3)}.$$

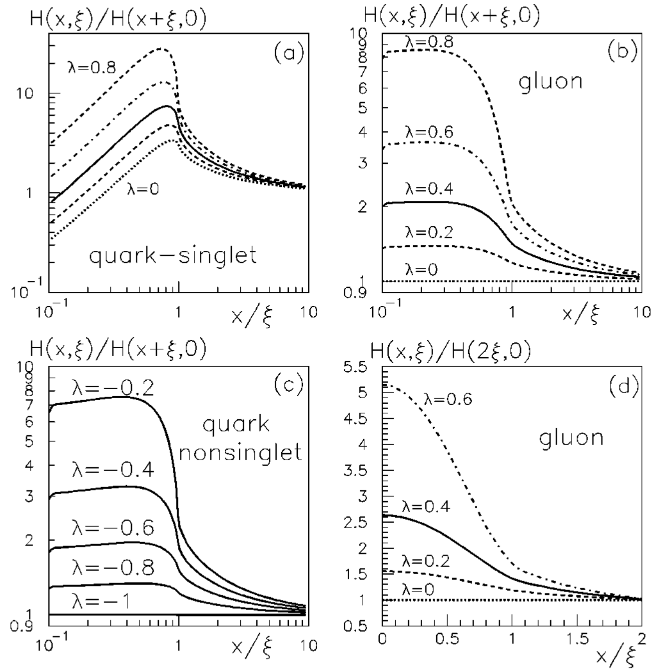

Fig.2

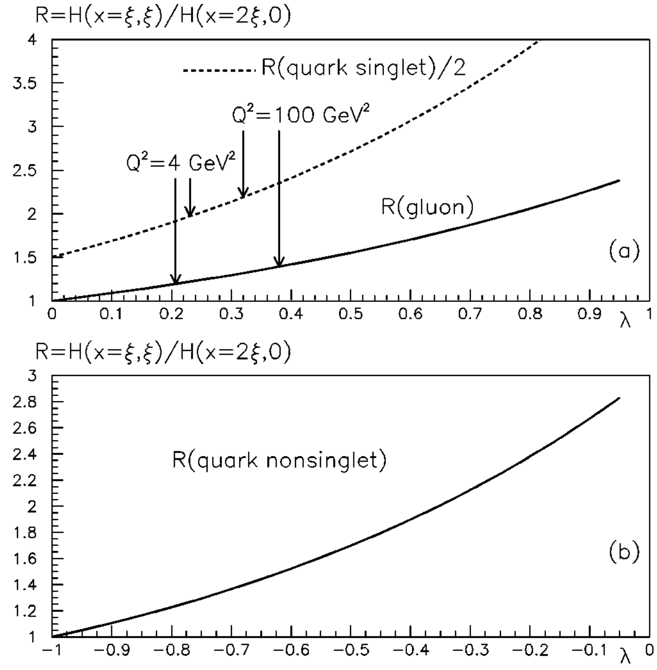

Fig.3

FIG. 2. Predictions at small x and ξ for the ratio of off-diagonal $[H(x,\xi)]$ to diagonal $[H_q(\bar{x},0)=f_q(\bar{x}),H_g(\bar{x},0)=\bar{x}f_g(\bar{x})]$ parton distributions. The diagonal partons are taken to have the form $xf(x)=Nx^{-\lambda}$. Plots (a), (b) and (c) show the quark singlet, gluon and quark non-singlet ratios taking $\bar{x}=x+\xi$ as the argument of the diagonal partons. Plot (d) shows the gluon ratio again but versus a linear scale and with argument $\bar{x}=2\xi$ so as to display the x behavior of $H_g(x,\xi)$.

FIG. 3. The off-diagonal to diagonal ratio, R, at small x and ξ versus the power λ which specifies the $x^{-\lambda}$ behavior of the input diagonal parton as in Eq. (21). Note that the quark singlet ratio has been divided by 2. The vertical arrows indicate the values of λ found in a global parton analysis [9] at $Q^2=4$ and 100 GeV^2.

$$x_1 \simeq (Q^2+M^2_{q\bar{q}})/W^2 \gg x_2, \tag{27}$$

where W is the center-of-mass energy of the proton and the photon of virtuality Q^2. A common approximation is to describe the process in terms of the diagonal gluon $x_1 g(x_1)$, sampled at $x_1=x+\xi$. In this case the inclusion of off-diagonal effects will enhance the cross section by a factor of R_g^2, where R_g is evaluated at $x/\xi=1$, see Fig. 2(b) or 2(d).

For $x \gg \xi$ we see that the off-diagonal to diagonal ratios, R, tend to unity, as expected. Moreover, due to the $x \to -x$ antisymmetry property (8), we see that the quark singlet vanishes as $x \to 0$. Also for a flat input gluon, $xg(x) \to$ constant as $x \to 0$ (that is $\lambda_g=0$), we see that R_g does not depend on ξ at all. The same is true for the quarks, but now when $q(x) \to$ constant, that is when $\lambda_q=-1$, as seen in the $R_q^{ns}=1$ result of Fig. 2(c).

All the scale dependence of the distributions is hidden in the Q^2 behavior of the powers $\lambda(Q^2)$. The position of the saddle point $N=\lambda$ in the Mellin integral (10) moves to the right in the complex N plane as Q^2 increases and so the off-diagonal ''enhancement'' increases; in other words R increases with Q^2. A particular example is the double logarithm approximation when, in the singlet sector, the saddle point

$$N=\lambda_g(Q^2) \simeq \sqrt{(\alpha_S/\pi)\ln(1/x)\ln(Q^2/Q_0^2)}. \tag{28}$$

In Fig. 2(d) we show the off-diagonal gluon distribution again, but now using a (more detailed) linear scale and comparing with the diagonal distribution $H(\bar{x},0)$ taken at fixed $\bar{x}=2\xi$, so as to avoid the extra x dependence coming from the diagonal gluon in the denominator of the R_g ratio. This demonstrates that the extra x dependence is responsible for the slight decrease observed in R_g of Fig. 2(b) as $x \to 0$, and that the decrease is not due to the behavior of $H_g(x,\xi)$.

The behavior of the ratios at $x=\xi$ are explicitly

$$R=\frac{H(\xi,\xi)}{H(2\xi,0)}=\frac{2^{2\lambda+3}}{\sqrt{\pi}}\,\frac{\Gamma(\lambda+5/2)}{\Gamma(\lambda+3+p)},$$

where $p=0$ for quarks and $p=1$ for gluons. The ratios are plotted in Fig. 3 as a function of λ. The vertical arrows shown on the plot indicate the values of λ_g and λ_q obtained from the gluon and sea quark distributions at $Q^2=4$ and 100 GeV^2 of a recent global (diagonal) parton analysis [9]. The plot can be used to find the enhancement of the cross section for the high energy diffractive electroproduction of vector mesons arising from off-diagonal parton effects. The enhancement is given by R_g^2 where R_g is the value of the gluon ratio at $x=\xi$, which is shown in Fig. 3, at the appropriate scale, that is at the appropriate value of $\lambda_g(Q^2)$. For instance, for the photoproduction of J/ψ and Υ at the DESY ep collider HERA the enhancement is about $(1.15)^2$ and

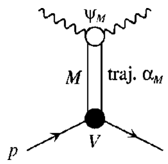

FIG. 4. Meson (M) Regge exchange indicating the structure of the off-diagonal contribution of Eq. (32).

$(1.32)^2$ respectively,[3] if we use a scale $M_V^2/4$, where M_V is the mass of the vector meson.

From Figs. 2 and 3 we see that the off-diagonal or "skewed" effect (the ratio R) is much stronger for singlet quarks than for gluons. The explanation is straightforward. At low x the distributions are driven by the double leading logarithmic evolution of the gluon distribution. At each step of the evolution the momentum fractions x_i are strongly ordered ($x_1' \gg x_1, x_2' \gg x_2$ on Fig. 1). For gluons it is just the "last splitting function" $P_{gg}(x_2, x_2'; \xi)$ which generates the main ξ dependence, or skewedness, of the distribution. However for the sea or singlet quarks it is necessary to produce a quark with the help of P_{qg} at the last splitting. The splitting function P_{qg} has no logarithmic $1/z = x_2'/x_2$ singularity and so x_2 is the order of x_2'. Consequently both the splitting functions $P_{qg}(x_2, x_2'; \xi)$ and $P_{gg}(x_2', x_2''; \xi)$ generate the asymmetry of the off-diagonal distribution. Hence, at low x, the singlet quark has a much stronger off-diagonal effect than the gluon.

IV. DISCUSSION

In order to conclude that the conformal moments allow us to use the diagonal partons to uniquely determine the off-diagonal partons at small x and ξ, including also their normalization and Q^2 behavior, it is necessary to consider some further points.

First we could worry that in the analytical continuation in N of the conformal moments,

$$O_N = \sum_k \xi^{2k} O_{Nk} \quad \text{with} \quad 2k < N+1, \tag{29}$$

the higher ($k \geq 1$) terms will generate a singularity at $N > \lambda + 2k$. In such a case the small x, ξ contribution would be driven by this singularity. However we show that such a singularity to the right of $N = \lambda + 2k$ cannot occur. From the structure of the polynomials $R_N(x_1, x_2)$ of Eqs. (4) and (6), it is clear that there are no such singularities for integer $N > 2k$. On the other hand a singularity at non-integer $N = \beta + 2k$ would generate a function $f(x')$ of Eq. (10) which depends on the ratio $\xi^{2k}/x'^{\beta+2k}$. After the convolution (11) we would obtain a distribution which violates the polynomial condition [3]

$$\int dx x^N H(x,\xi) = \sum_{k=0}^{[(N+1)/2]} A_k \xi^{2k}, \tag{30}$$

which comes from Lorentz invariance (and the tensor structure of the operators). Thus the higher ξ^2 terms (with $k \geq 1$) in Eq. (14) should die out with decreasing ξ.

A second consideration is that, from a formal point of view, we may add to the off-diagonal distribution any function

$$\Delta H(x,\xi) = g(x,\xi)\,\theta(\xi - |x|) \tag{31}$$

which exists only in the time-like Efremov-Radyushkin-Brodsky-Lepage (ERBL) region [12] with $|x| < \xi$. Such a contribution ΔH remains in the ERBL region during evolution. However ΔH disappears as $\xi \to 0$ and so it cannot be restored purely from diagonal partons. A physical way to model such an ERBL contribution is to consider t channel meson (M) exchanges of Fig. 4. The contribution ΔH is then given by the leading twist wave function ψ_M of the meson multiplied by the corresponding Regge exchange amplitude

$$\Delta H^{\text{Reggeon}} = \psi_M(x/\xi, Q^2)\,\xi^{-\alpha_M(t)} V(\mu^2 = Q_0^2; \xi; t). \tag{32}$$

The appropriate exchange is the f_2 meson which, in the constituent quark model, is formed from a P-wave $q\bar{q}$ state with $J^{PC} = 2^{++}$. The Regge factor $\xi^{-\alpha_M}$ is the analogue of the $x^{-\eta}$ (or $\xi^{-\eta}$) factor in the non-singlet quark distribution $H^{ns} \sim x^{-\eta}$; in our notation of Eq. (21) and Fig. 2 with $xq^{ns} \sim x^{-\lambda_{ns}}$ we have $\lambda_{ns} = \eta - 1$. Phenomenologically we expect that $\eta \sim \alpha_M(0) \sim 0.5$. The key factor in Eq. (32) is V which specifies the coupling of the Reggeon to the proton. From Regge phenomenology the vertex factor V was extracted for the diagonal case where the ERBL domain does not exist. Let us try to estimate a possible ERBL contribution to the off-diagonal distributions. The value of the pion-nucleon Σ-term at low scales determines the number of current quarks and antiquarks in the nucleon to be [13]

$$\langle N | \bar{q}q | N \rangle \simeq 8. \tag{33}$$

Allowing for valence quarks, this implies that the average number of $q\bar{q}$ pairs is about 2.5. At such low scales the partons are distributed more or less uniformly in the whole $(-1,1)$ interval and so the probability to find two partons in the ERBL domain $(-\xi, \xi)$ is of the order of ξ^2. Such a ΔH is a negligible $O(\xi^2)$ contribution at small ξ in agreement with our decomposition of the conformal moments.

So far our distributions enable us to calculate the imaginary part of the amplitude, say for Compton scattering[4] with incoming and outgoing photon virtualities $q^2 = -Q^2$ and

[3] In practice the diagonal distributions have more complicated forms than that assumed in Eq. (21). For instance if we were to input $xg \sim x^{-\lambda_g}(1-x)^6$ in Eq. (20) and to perform the x' integration numerically then we find R_g increases from 1.32 to 1.41 for Υ photoproduction at HERA where $x \simeq 0.01$; the change in R_g occurs because the x sampled by the HERA data is not sufficiently small. $R_g^2 \simeq 2$ is in agreement with the previous estimates of the enhancement due to off-diagonal effects [10,11].

[4] To be specific we consider the case with $t \leq 0$, $q^2 \leq 0$ and $q'^2 \leq 0$.

$q'^2=-Q'^2$. At small x and ξ it turns out that the real part of the amplitude may be calculated easily using a dispersion relation in the center-of-mass energy squared $W_s^2=(p+q)^2$. Let us consider the cut in the right-half W_s plane, that is the discontinuity for $W_s^2>0$. For fixed t, Q^2 and Q'^2, the ratio $r=x/\xi$ is fixed as well, since $(x+\xi)/(x-\xi)=Q^2/Q'^2$. Thus the energy squared may be written

$$W_s^2=(1-x)\frac{Q^2}{x+\xi}=(1-r\xi)\frac{Q^2}{1+r}\frac{1}{\xi}. \quad (34)$$

However we must take into account the cuts in both the right and left half-planes, that is the s and u channel cuts. The left-hand cut corresponds to the u channel process (obtained by the interchange $p\leftrightarrow -p'$) with energy squared

$$W_u^2=-(1+x)\frac{Q^2}{x+\xi}. \quad (35)$$

The unpolarized deeply virtual Compton amplitude is the sum of the s- and u-channel terms, $A=A_s+A_u$, and appears to have even signature, that is A is crossing symmetric. Strictly speaking at large x and ξ there is some asymmetry (since $W_u^2\neq -W_s^2$), which may be considered as the odd signature contribution and should be treated appropriately in the dispersion integral. However the situation is particularly simple at small $x\ll 1$, where $(1\pm x)\simeq 1$. Then we may write the whole amplitude $A\propto(W^2)^\lambda$, with the help of the even signature factor

$$\mathcal{S}^+=\frac{1}{2}(1+(-1)^\lambda), \quad (36)$$

in the form

$$A=i\,\mathrm{Im}A\left(\frac{1+e^{-i\pi\lambda}}{1+\cos\pi\lambda}\right). \quad (37)$$

Moreover for small λ we have

$$\mathrm{Re}A\simeq\frac{\pi\lambda}{2}\mathrm{Im}A. \quad (38)$$

Strictly speaking the conformal moments O_N only renormalize multiplicatively, as in Eq. (9), at leading order (LO). Due to a conformal anomaly at next-to-leading (NLO) the moment O_N mixes, on evolution, with moments $O_{N'}$ with $N'<N$ [14]. The mixing is taken into account by a matrix $\boldsymbol{B}_{NN'}$, which obeys its own evolution equation [15]. Of course the mixing is absent in the diagonal case when $\xi\to 0$, whereas for non-zero ξ we have

$$O_N^{\mathrm{NLO}}=\sum_{N'=0}^{N}\boldsymbol{B}_{NN'}O_{N'}^{\mathrm{NLO\,(diag)}}\xi^{N-N'} \quad (39)$$

where $O_N, O_{N'}$ and $\boldsymbol{B}_{NN'}$ all depend on $\alpha_S(Q^2)$. Thus in the small $\xi\ll 1$ limit we can safely use expressions (16) and (20) for $H(x,\xi)$ even at NLO.

In summary, in the low ξ region we can use expressions (16) and (20) to reliably predict the off-diagonal distributions $H(x,\xi)$ in terms of the diagonal partons at any scale. All that is required is a two-fold integration. The expected accuracy is of the order of ξ^2. As a specific example we assumed in Eq. (21) that the diagonal partons had a power-like $x^{-\lambda}$ behavior for small x. In this case one integration can be done analytically and we have even simpler expressions for H_q and H_g, see Eq. (22). The results are shown in Figs. 2 and 3 and allow the off-diagonal distributions to be determined for any small x,ξ values at any scale. One important consequence is that data for processes, which are described by off-diagonal distributions, can be included in a global analysis to better constrain the low x behavior of the (conventional) diagonal partons.

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