Final state interactions in $D^0 \to K^0 \overline{K^0}$

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It is believed that the production rate of $D^0 \to K^0 \overline{K^0}$ is almost solely determined by final state interactions (FSIs) and hence provides an ideal place to test FSI models. Here we examine model calculations of the contributions from s-channel resonance $f_J(1710)$ and t-channel exchange to FSI effects in $D^0 \to K^0 \overline{K^0}$. The contribution from s-channel $f_0(1710)$ is small. For the t-channel FSI evaluation, we employ the one-particle-exchange model and Regge model, respectively. The results from the two methods are roughly consistent with each other and can reproduce the large rate of $D^0 \to K^0 \overline{K^0}$ reasonably well. [S0556-2821(99)00213-1]

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I. INTRODUCTION

The importance of the final state interactions (FSIs) in hadronic processes has been recognized for a long time. Recently its applications in D and B decays have attracted the extensive interest and attention of theorists. In many decay modes FSIs may play a crucial role.

Here the FSI refers to soft rescattering processes at the hadronic level [1]. Since all FSI processes concern nonperturbative QCD and cannot be reliably evaluated with any well-established theoretical framework, we have to rely on phenomenological models to analyze FSI effects in certain reactions. The chiral Lagrangian approach has proved to be reliable for evaluating hadronic processes, but there are too many free parameters which are determined by the fitting data, so that its applications are highly constrained. Therefore, we have tried to look for some simplified models which can give rise to a reasonable estimation of FSIs.

The decay $D^0 \to K^0 \overline{K^0}$ is a very interesting mode. Pham [2] and Lipkin [3] noticed the important role of FSIs in this production long time ago. A direct $D^0 \to K^0 \overline{K^0}$ can only occur via a W-boson exchange based on a quark-diagram analysis [4], and moreover, since the Cabibbo-Kobayashi-Maskawa (CKM) entries for $c\bar{u} \to d\bar{d}$ and $s\bar{s}$ have opposite signs, the reaction must be proportional to an SU(3) breaking. Therefore according to the common knowledge obtained by studying B and D decays, direct $D^0 \to K^0 \overline{K^0}$ is much more suppressed than $D^0 \to K^+ K^-$. However, the data show that $B(D^0 \to K^0 \overline{K^0}) \sim (6.5 \pm 1.8) \times 10^{-4}$ and $B(D^0 \to K^+ K^-) \sim (4.27 \pm 0.16) \times 10^{-3}$ [5]. Obviously, the $D^0 \to K^0 \overline{K^0}$ is realized through inelastic final state interactions. Namely, $K^0 \overline{K^0}$ is not a direct product of D^0 decay, but is a secondary one from other hadrons (mesons, mainly) which have a larger direct production rate in D^0 decays, via hadronic rescattering. Hence the decay $D^0 \to K^0 \overline{K^0}$ provides an ideal place to test model calculations of FSIs.

Around the $m_D = 1.86$ GeV energy region, there is an abundant spectrum of resonances. The s-channel resonance contribution can be very important. However, only those with appropriate quantum numbers ($J^{PC} = 0^{++}$) can contribute to the FSIs for the $D^0 \to K^0 \overline{K^0}$ mode. According to PDG tables [5], there is only one 0^{++} resonance with mass sufficiently close to m_D, i.e., the 0^{++} component in $f_J(1710)$. In this work, we evaluate the s-channel contribution by only accounting for $f_J(1710)$ and the rest is attributed to t-channel exchange. For t-channel exchange, we consider two approaches. One is the one-particle-exchange (OPE) model, specifically here single-meson exchange, while another is the Regge pole model. In fact, the Regge trajectories contain all nonperturbative QCD effects, but from another angle, its leading term is exactly the exchange of a meson with appropriate quantum numbers. The calculation with the single-meson-exchange scenario is obviously much simpler and straightforward. Moreover, some theoretical uncertainties are included in an off-shell form factor which modifies the effective vertices and therefore can compensate for residue effects which exist in a precise Regge pole model. This compensation can at least be of the same accuracy as the Regge pole model with several free parameters. One can trust that the results obtained in the two approaches should be qualitatively consistent, even though not exactly equal. Our later numerical results confirm this assertion.

In Sec. II, we give the formulations for s- and t-channel FSI effects. For the t-channel case, both the one-particle-exchange model and the Regge pole model are used. The numerical results and a discussion are given in Sec. III.

II. FORMULATIONS

The direct decay amplitudes of $D^0 \to VV'$ and $D^0 \to PP'$, where $V(V')$ and $P(P')$ denote vector and pseudoscalar mesons, are given in many papers in the literature and we will follow the conventions of [6].

A. s-channel resonance contribution

Even though the spectrum is abundant at the m_D region, only the 0^{++} component of $f_J(1710)$ can make substantial contributions to the s-channel FSIs. However, the 0^{++} component of $f_J(1710)$ is still not well determined [5]. We use the data of $f_0(1710)$ for our later calculations. It is expected that the errors obtained are within the error tolerance region of the present data.

To lowest order, the effective coupling of f_0 to VV' and PP' (V,P are vector and pseudoscalar), which is of concern here, can be of the form

$$L_I = g\phi^+\phi f \quad \mathrm{PP'f}, \tag{1}$$

$$L_I = g' A_\mu A^\mu f \quad \mathrm{VV'f}. \tag{2}$$

With these Lagrangians, the effective coupling constants g and g' are obtained by fitting the branching ratios of f_0 to VV' or PP'.

The effective weak decay Hamiltonion for our process is

$$\begin{aligned} H_{eff} = \frac{G_F}{\sqrt{2}} \{ & V_{us}V_{cs}^*[c_1(\overline{s}c)_{V-A}(\overline{u}s)_{V-A} \\ & + c_2(\overline{s}s)_{V-A}(\overline{u}c)_{V-A}] \\ & + V_{ud}V_{cd}^* \quad [c_1(\overline{d}c)_{V-A}(\overline{u}d)_{V-A} \\ & + c_2(\overline{d}d)_{V-A}(\overline{u}c)_{V-A}]\}, \end{aligned} \tag{3}$$

where V_{us}, V_{cs}, V_{ud}, V_{cd} are the CKM matrix entries, and $V-A$ represents $\gamma_\mu(1-\gamma_5)$.

The amplitude of the decay $D^0 \to K^+K^-$ is

$$\begin{aligned} & A(D^0 \to K^+K^-) \\ & = \frac{G_F}{\sqrt{2}} V_{us}V_{cs}^* a_1 \langle K^+K^- | (\overline{s}c)_{V-A}(\overline{u}s)_{V-A} | D^0 \rangle \\ & = \frac{G_F}{\sqrt{2}} V_{us}V_{cs}^* a_1 [-f_K F_0^{DK}(m_K^2)(m_D^2 - m_K^2)], \end{aligned} \tag{4}$$

where $a_1 = c_1 + (1/N_c)c_2$. Nonfactorization effects are neglected here.

In terms of these effective couplings, the amplitude of $D^0 \to K^0 \overline{K}^0$ with the s-channel resonance $f_0(1710)$ contribution can be written as

$$\begin{aligned} A^{FSI} = & \sum_{all\ MM'} \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4 \\ & \times (p_1 + p_2 - p_B) A(D^0 \to M + M') \\ & \times g_{MM'} \cdot g_{KK} \frac{i}{s^2 - m_{f_0}^2 + i m_{f_0}\Gamma_{tot}} \kappa, \end{aligned} \tag{5}$$

where $i/(s^2 - m_{f_0}^2 + i m_{f_0}\Gamma_{tot})$ is the relativistic Breit-Wigner resonance propagator for $f_0(1710)$, s is the total c.m. energy square, and κ is an isospin factor. The sum over the weak amplitudes $A(D^0 \to MM')$ includes all possible states. The physical picture is shown in Fig. 1. For intermediate

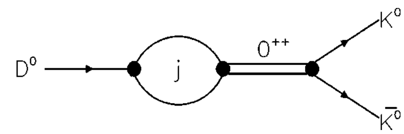

FIG. 1. The s-channel resonance FSI contribution. j represents the intermediate states.

mesons other than $f_0(1710)$, their propagators may provide a suppression factor $1/(m_D^2 - m^2)$ which would wash out their contributions.

Thus it is easy to derive

$$\begin{aligned} & A(D^0 \to K^+K^- \to f_0(1710) \to K^0\overline{K}^0) \\ & = \frac{G_F}{\sqrt{2}} \frac{g_{KKf}^2}{32\pi} V_{us}V_{cs}^* a_1 f_K F_0^{DK}(m_D^2 - m_K^2) \\ & \times \left(1 - \frac{4m_K^2}{m_D^2}\right)^{1/2} \frac{(m_D^2 - m_f^2) - i\Gamma_f m_f}{(m_D^2 - m_f^2)^2 + \Gamma_f^2 m_f^2}. \end{aligned} \tag{6}$$

For the vector meson case, we have

$$\begin{aligned} & A(D^0 \to \rho^+\rho^- \to f_0(1710) \to K^0\overline{K}^0) \\ & = \frac{G_F}{\sqrt{2}} \frac{g_{KKf} \cdot g_{\rho\rho f}}{16\pi} V_{ud}V_{cd}^* f_K \sqrt{\frac{1}{3}} a_1 \left(1 - \frac{4m_\rho^2}{m_D^2}\right)^{1/2} \\ & \times \Bigg[m_\rho(m_D + m_\rho) f_\rho A_1 \left(2 + \frac{(m_D^2 - 2m_\rho^2)^2}{4m_\rho^4}\right) \\ & - \frac{2m_\rho}{m_D + m_\rho} f_\rho A_2 \left(m_D^2 - \frac{m_D^4}{2m_\rho^2} + \frac{m_D^4(m_D^2 - 2m_\rho^2)}{8m_\rho^4}\right) \Bigg] \\ & \times \frac{(m_D^2 - m_f^2) - i\Gamma_f m_f}{(m_D^2 - m_f^2)^2 + \Gamma_f^2 m_f^2}, \end{aligned} \tag{7}$$

where all F_0^{DK}, a_1, A_1, A_2, etc., are defined according to the conventions in [6].

For $A(D^0 \to \pi^+\pi^- \to f_0(1710) \to K^0\overline{K}^0)$, we need to replace g_{KKf}^2, $V_{us}V_{cs}^*$, m_K^2 in the expression by $g_{KKf}g_{\pi\pi f}$, $V_{ud}V_{cd}^*$, m_π^2. For other intermediate states such as $K^{*+}K^{*-}$ and $\rho^+\rho^-$, we have no data about their branching ratios of $f_0(1710)$; so we do not consider their s-channel contribution at present.

B. t-channel contribution: The OPE model

In the OPE model, a single t-channel (the same as u-channel) virtual particle is exchanged (see Fig. 2), and it is natural to assume that the lightest particle with proper quantum number dominates.

The exchange scenario has been studied in the $D \to VP$ and $B \to \pi K$ cases [7,8]. The effective vertices of strong interaction for the rescattering process, such as $g_{KK^*\pi}, g_{\rho KK}$, etc., are gained from data provided the flavor SU(3) symmetry holds. However, since the t-channel-exchanged particles

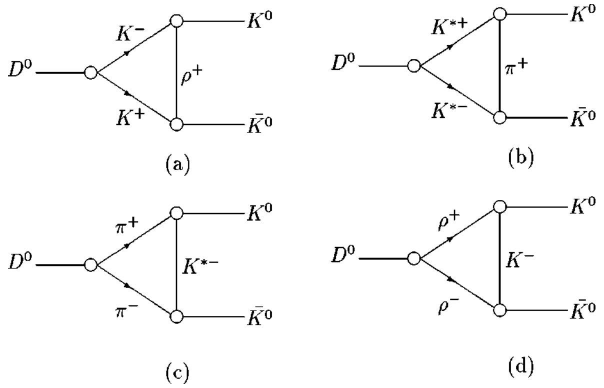

FIG. 2. Final state interactions in $D^0 \to K^0\overline{K^0}$ due to one-particle exchange.

P and V are off their mass shell, a phenomenological form factor $F(\Lambda)=(\Lambda^2-m^2/(\Lambda^2-t)$ is introduced to compensate the off-shell effect at the vertices [9]. It is noted that Λ is a parameter which takes values between 1.2–2.0 GeV in the normal sense. As pointed out in last section, the value of the parameter Λ would smear the errors caused by assuming the dominance of one-particle exchange.

Obviously, for the $D^0 \to K^0\overline{K^0}$ final state, the PV intermediate state is forbidden. Meanwhile, we also ignore contributions from intermediate states with more than two mesons or baryons, which are definitely much smaller.

There are two key aspects to making the concerned processes substantial. First the direct production amplitude of $D \to PP$ or VV must be large enough, and the second, the scattering amplitude of PP (or VV)$\to K^0\overline{K^0}$ is not small. It depends on the effective couplings and how far the propagating meson deviates from its mass shell. Since the scattering PP (or VV)$\to K^0\overline{K^0}$ is, in general, an inelastic process, the absolute values of the amplitudes are smaller than unity.

First, let us study which channels of $D \to PP$ or VV are substantially large. Here let us just make some order estimations of the amplitudes before doing concrete calculations.

Based on the quark diagrams, definitely $D^0 \to K^+K^-$, $K^{*+}K^{*-}$, $\pi^+\pi^-$, $\rho^+\rho^-$ have larger amplitudes because they are realized via so-called external W emission [4], which is much larger than other mechanisms.

For $D^0 \to \pi^0\pi^0$ or $\rho^0\rho^0$, even though they can happen via internal W emission, the amplitudes are about 3 times smaller than external W emission as

$$\sqrt{\frac{\Gamma(D^0 \to \pi^0\pi^0)}{\Gamma(D^0 \to \pi^+\pi^-)}}=\left(\frac{a_2}{\sqrt{2}a_1}\right)\sim 0.3 \sim \sqrt{\frac{\Gamma(D^0 \to \rho^0\rho^0)}{\Gamma(D^0 \to \rho^+\rho^-)}}.$$

Therefore, in our later calculations, we neglect contributions from such intermediate states.

1. $D^0 \to PP \to K^0\overline{K^0}$ case

Here we present the formulas for $D^0 \to K^+K^- \to K^0\overline{K^0}$ as an example and a similar expression can be written down for $D^0 \to \pi^+\pi^- \to K^0\overline{K^0}$. In this case the exchanged meson is $\rho^\pm$.

It is believed [7,8] that a single particle exchange in the t channel would make dominant contributions to the FSIs. For the $D^0 \to K^+K^- \to K^0\overline{K^0}$ process shown in Fig. 2, K^+, K^-, and the t-channel-exchanged ρ form a triangle diagram. As a matter of fact, for a pure FSI process, we only need to evaluate the absorptive part of the triangle. Definitely, the dispersive part of this loop can be calculated in terms of the dispersion relation [9], and generally it is expected to be of the same order as the absorptive part of the loop.

According to the Cutkosky rule, we make cuts to let K^+, K^- be on shell, leaving $\rho^\pm$ to be off shell. At the $K^+\rho^\pm K^0$ vertex the effective Hamiltonian is $g_{\rho KK}\epsilon_\mu(p_{K^+}+p_{K^0})^\mu$.

The amplitude of $D^0 \to K^+K^- \to K^0\overline{K^0}$ is

$$\begin{aligned}A^{FSI}=&\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-p_B)\\&\times A(D^0 \to K^+K^-)g^2_{\rho KK}(p_1+p_3)^\mu(p_2+p_4)^\nu\\&\times\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m^2_\rho}\right)\frac{i}{k^2-m^2_\rho}F(k^2)\\=&\int_{-1}^{1}d(\cos\theta)\frac{|\overrightarrow{p_1}|}{16\pi m_D}g^2_{\rho KK}\frac{iF(k^2)}{k^2-m^2_\rho}H\cdot A(D^0 \to K^+K^-),\end{aligned}\tag{8}$$

where $H=-(p_1\cdot p_2+p_1\cdot p_4+p_2\cdot p_3+p_3\cdot p_4)$ and $A(D^0 \to K^+K^-)$ is the direct weak production amplitude. We have set $p_1=p_{K^+}$, $p_2=p_{K^-}$, $p_3=p_{K^0}$, $p_4=p_{\bar{K}^0}$, and θ is the angle between $\overrightarrow{p_1}$ and $\overrightarrow{p_3}$. Here k is the four-momentum of the exchanged particle ρ, and $k^2=(p_1-p_3)^2=m_1^2+m_3^2-2E_1E_3+2|\overrightarrow{p_1}||\overrightarrow{p_3}|\cos\theta$.

The form factor $F(k^2)$ in Eq. (14) is an off-shell form factor for the vertices ρKK. Because the effective coupling constant $g_{\rho KK}$ is obtained from the data where the three particles are all on shell, while in our case the exchanged ρ meson is off shell, a compensation form factor is needed. We take $F(p_\rho^2)=[(\Lambda^2-m_\rho^2)/(\Lambda^2-p_\rho^2)]^2$ as suggested in Ref. [7],

2. $D^0 \to VV \to K^0\overline{K^0}$ case

The case for intermediate states of two vector mesons (VV) has been studied in $B \to \rho K^* \to \pi K$ processes [8]. It is shown that the VV intermediate states give a significant contribution to final state interactions. Here we take $D^0 \to K^{*+}K^{*-} \to K^0\overline{K^0}$ as an example, while the expression for $D^0 \to \rho^+\rho^- \to K^0\overline{K^0}$ is a close analogue.

The amplitude for $D^0 \to K^{*+}K^{*-}$ decay is

$$A(D^0 \to K^{*+}K^{*-})=\frac{G_F}{\sqrt{2}}V_{us}V^*_{cs}a_1\cdot M^{K^{*+}K^{*-}},\tag{9}$$

where

$$M^{K^{*+}K^{*-}} \equiv \langle K^{*+}|(\overline{u}s)_{V-A}|0\rangle\langle K^{*-}|(\overline{s}c)_{V-A}|D^0\rangle = m_{K^*}(m_D+m_{K^*})f_{K^*}A_1^{DK^*}(m_{K^*}^2)(\epsilon_{K^{*+}}\cdot\epsilon_{K^{*-}}) - \frac{2m_{K^*}}{m_D+m_{K^*}}f_{K^*}A_2^{DK^*}(m_{K^*}^2)(\epsilon_{K^{*+}}\cdot p_D) \times(\epsilon_{K^{*-}}\cdot p_D) - i\frac{2m_{K^*}}{m_D+m_{K^*}}f_{K^*}V^{DK^*} \times(m_{K^*}^2)\epsilon_{\mu\nu\rho\sigma}\epsilon_{K^{*+}}^{\mu}\epsilon_{K^{*-}}^{\nu}p_{K^{*+}}^{\rho}p_{K^{*-}}^{\sigma}. \quad (10)$$

Unlike the $D^0\to K^+K^-\to K^0\overline{K}^0$ case, the t-channel-exchanged particle in $D^0\to K^{*+}K^{*-}\to K^0\overline{K}^0$ is $\pi^{\pm}$. Since it is the lightest meson of the right quantum number, it should give rise to the largest contribution. The amplitude for the final state interaction of the process $D^0\to K^{*+}K^{*-}\to K^0\overline{K}^0$ is

$$A^{FSI} = \frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-p_B) \times A(D^0\to K^{*+}K^{*-})\langle K^{*+}K^{*-}|S|K^0\overline{K}^0\rangle = \int_{-1}^{1}d(\cos\theta)\frac{|\vec{p_1}|}{16\pi m_D}\frac{iF(p_{\pi^0}^2)}{(p_{\pi^0}^2-m_{\pi^0}^2)} \times\Bigg[m_{K^*}(m_D+m_{K^*})f_{K^*}A_1^{DK^*}(m_{K^*}^2)\cdot H_1 - \frac{2m_{K^*}}{m_D+m_{K^*}}m_D^2 f_{K^*}A_2^{DK^*}(m_{K^*}^2)\cdot H_2\Bigg], \quad (11)$$

where S is the S matrix of the strong interaction, θ is the angle between $\vec{p_1}$ and $\vec{p_3}$, and

$$H_1 = 4g_{K^*K\pi}^2\Bigg[(p_3\cdot p_4) - \frac{(p_2\cdot p_3)(p_2\cdot p_4)}{m_2^2} - \frac{(p_1\cdot p_3)(p_1\cdot p_4)}{m_1^2} + \frac{(p_1\cdot p_2)(p_2\cdot p_3)(p_2\cdot p_4)}{m_1^2m_2^2}\Bigg]$$

$$H_2 = 4g_{K^*K\pi}^2\Bigg[(p_3^0m_4^0) - \frac{(p_2^0p_3^0)(p_2\cdot p_4)}{m_2^2} - \frac{(p_1^0p_4^0)(p_1\cdot p_3)}{m_1^2} + \frac{(p_1^0p_2^0)(p_2\cdot p_3)(p_2\cdot p_4)}{m_1^2m_2^2}\Bigg]. \quad (12)$$

As mentioned above, the expression for $A^{FSI}(D^0\to\rho^+\rho^-\to K^0\overline{K}^0)$ is similar; the only distinction is that for $D^0\to\rho^+\rho^-\to K^0\overline{K}^0$, the exchanged particle is $K^{\pm}$ instead.

3. *t-channel contribution: The Regge pole model*

The principles of Regge theory are [10] that (i) the scattering amplitudes are analytic functions of the angular momentum J, (ii) a particle of mass m and spin σ will be on a Regge trajectory $\alpha(t)$ (where t is the Mandelstam invariant parameters) and $\sigma=\alpha(m^2)$, and (iii) the partial-wave amplitude has a pole of the form $1/[J-\alpha(t)]$. It is suggested that the Regge theory provides a very simple and economical description of the total cross section at the high energy region [11].

The invariant amplitude for the scattering of particles with helicities λ_i from the Regge phenomenology is [10]

$$\mathcal{M}_{i\to f}^{\lambda_1\lambda_2;\lambda_3\lambda_4} = -\left(\frac{-t}{s_0}\right)^{\lambda/2}\frac{e^{-i\pi\alpha(t)}+\mathcal{J}}{2\sin\pi\alpha(t)}\gamma_{\lambda_3\lambda_4}^{\lambda_1\lambda_2}\left(\frac{s}{s_0}\right)^{\alpha(t)}, \quad (13)$$

where s and t are Mandelstam invariants, and $\lambda=|\lambda_3-\lambda_1|+|\lambda_4-\lambda_2|$. Here $\mathcal{J}$ is the signature for Regge trajectory. For the Pomeron and π trajectory, $\mathcal{J}=+1$; for the ρ and K^* trajectory, $\mathcal{J}=-1$. This expression corresponds to an asymptotic behavior when $s\gg s_0$ and s_0 is a scale parameter. In most of the literature, s_0 is taken as 1 GeV^2. This Regge asymptotic behavior works very well in the energy region $\sqrt{s}\geqslant 5$ GeV. We extend the energy region to $\sqrt{s}=m_D$. The extension is reliable because we have accounted for the s-channel resonance $f_0(1710)$ contribution separately, while contributions from the remaining resonances can be treated as a smooth function of s which is determined by the crossed t-channel exchange [12] and it is the fundamental of the Regge pole theory. Thus we can assume that there would not be a large deviation from the Regge asymptotic behavior. $\gamma(t)$ is a residue function. The linear Regge trajectory as an approximation is taken for our calculations $\alpha(t)=\alpha_0+\alpha' t$. Here α' is nearly a universal parameter for all Regge trajectories (except for the Pomeron), $\alpha'\approx 0.9$. $\alpha_0=0.5$ for ρ and ω trajectories, $\alpha_0=0.3$ for K^* trajectories, $\alpha_0=0$ for π trajectories, and $\alpha_0=-0.3$ for K trajectories. But in our calculation we have adopted the approximation $\alpha_0=0.5$ for ρ and K^* trajectories and $\alpha_0=0$ for π and K trajectories in order to carry out dispersion integration analytically.

We take the $D^0\to K^{*+}K^{*-}\to K^0\overline{K}^0$ as an example and for the other intermediate states the expressions are similar.

First, we rewrite the helicity amplitude of $D^0\to VV$ decay in a convenient form [13]

$$A_{\lambda_1\lambda_2} = \langle V_1(k_1,\lambda_1)V_2(k_2,\lambda_2)|H_w|D^0(p)\rangle = \epsilon_\mu^*(k_1,\lambda_1)\epsilon_\nu^*(k_2,\lambda_2)\Bigg[ag^{\mu\nu}+\frac{b}{m_1m_2}p^\mu p^\nu + i\frac{c}{m_1m_2}\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}p_\beta\Bigg], \quad (14)$$

where λ_1, λ_2 are the helicity of V_1, V_2, and ϵ_μ, ϵ_ν are the polarization vectors of V_1, V_2. From Eq. (10), the above factors for decay $D^0\to K^{*+}K^{*-}$ are

TABLE I. FSI amplitudes from s-channel contributions of $f_0(1710)$.

Decay mode	A^{FSI} (GeV)
$D^0\to K^+K^-\to K^0\overline{K^0}$	$(-0.24-i0.53)\times10^{-7}$
$D^0\to\pi^+\pi^-\to K^0\overline{K^0}$	$(0.13+i0.32)\times10^{-7}$
Total	$(-0.11-i0.21)\times10^{-7}$

$$a=\frac{G_F}{\sqrt{2}}V_{us}^*V_{cs}a_1(m_D+m_{K^*})m_{K^*}f_{K^*}A_1^{DK^*}(m_{K^*}^2),$$

$$b=-\frac{G_F}{\sqrt{2}}V_{us}^*V_{cs}a_1\frac{2m_{K^*}^3}{(m_D+m_{K^*})}f_{K^*}A_2^{DK^*}(m_{K^*}^2),$$

$$c=-\frac{G_F}{\sqrt{2}}V_{us}^*V_{cs}a_1\frac{2m_{K^*}^3}{(m_D+m_{K^*})}f_{K^*}V^{DK^*}(m_{K^*}^2). \quad (15)$$

In the rest frame of the D^0, K^{*+}, and K^{*-} have the same helicity. According to [13], there are three independent helicity amplitudes

$$\begin{cases} A_{++}=-a+\sqrt{x^2-1}c, \\ A_{--}=-a-\sqrt{x^2-1}c, \\ A_{00}=-xa-(x^2-1)b, \end{cases} \quad (16)$$

where $x\equiv k_1k_2/m_{K^*}^2=(m_D^2-2m_{K^*}^2)/2m_{K^*}^2$.

The discontinuity of amplitude for the final state interaction of $D^0\to K^{*+}K^{*-}\to K^0\overline{K^0}$ is

$$\begin{aligned}\mathrm{Disc}A^{FSI}=&\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4\\ &\times(p_1+p_2-p_B)A(D^0\to K^{*+}K^{*-})_{\lambda\lambda}\\ &\times\mathcal{M}^{\lambda\lambda;00}(K^{*+}K^{*-}\to K^0\overline{K^0}),\end{aligned} \quad (17)$$

where λ is the helicity of the intermediate state K^*. The discontinuity of this amplitude precisely corresponds to the absorptive part of the hadronic triangle (see Fig. 2) for the one-particle-exchange case where K^{*+},K^{*-} are on shell. For the rescattering of $K^{*+}K^{*-}\to K^0\overline{K^0}$, the exchange trajectory is π. The helicity amplitudes A_{++}, A_{--}, A_{00} all contribute to the same helicity state $\{00\}$ of $K^0\overline{K^0}$:

$$\begin{aligned}\mathrm{Disc}A^{FSI}=&\frac{\sqrt{1-4m_{K^*}^2/m_D^2}}{16\pi s}\int_{t_{min}}^{t_{max}}dt(A_{++}\mathcal{M}^{++;00}\\ &+A_{--}\mathcal{M}^{--;00}+A_{00}\mathcal{M}^{00;00})\\ =&\epsilon_\pi\left(\frac{s}{s_0}\right)^{\alpha_0-1},\end{aligned} \quad (18)$$

where ϵ_π represents the value except the factor $(s/s_0)^{\alpha_0-1}$ and is calculated numerically.

The full amplitude of the final state interaction of $D^0\to K^{*+}K^{*-}\to K^0\overline{K^0}$ can be obtained by using the dispersion relation.

$$A^{FSI}=\frac{\epsilon_\pi}{\pi}\int_{4m_{K^*}^2}^{\infty}ds\frac{(s/s_0)^{\alpha_0-1}}{(s-m_D^2)}=\frac{\epsilon_\pi}{\pi m_D^2}\ln\left(1-\frac{m_D^2}{4m_{K^*}^2}\right). \quad (19)$$

For the process $D^0\to\rho^+\rho^-\to K^0\overline{K^0}$, the leading trajectory of rescattering is K. For the processes $D^0\to K^+K^-\to K^0\overline{K^0}$ and $D^0\to\pi^+\pi^-\to K^0\overline{K^0}$, the leading trajectories of rescattering are ρ and K^*, respectively.

III. NUMERICAL RESULTS AND DISCUSSION

To reproduce the experimental data $B(D^0\to K^0\overline{K^0})\sim6.5\times10^{-4}$, we need the amplitude to be $|A(D^0\to K^0\overline{K^0})|\sim3.35\times10^{-7}$ GeV. Now we examine numerical results of various FSI amplitudes.

For the s-channel contribution, we take the experimental data [5] as input: $m_f=1710$ MeV, $\Gamma_{tot}=133\pm14$ MeV, $B(K\overline{K})=\Gamma_{K\overline{K}}/\Gamma_{tot}=0.38$, $B(\pi\pi)=\Gamma_{\pi\pi}/\Gamma_{K\overline{K}}=0.39$. Since other channels of $f_0(1710)$ decays have not been measured yet, we do not include their contribution in this numerical estimation. We expect that they will give similar contribution as K^+K^- and $\pi^+\pi^-$ modes. The numerical results of the s-channel $f_0(1710)$ contributions are tabulated in Table. I.

One can notice that contributions from the K^+K^- and $\pi^+\pi^-$ intermediate states interfere destructively, because $V_{cd}\approx-V_{us}$. The sum of two contributions is small com-

TABLE II. FSI amplitudes from t-channel contributions in the OPE model.

Decay mode	A^{FSI} (GeV)		
	$\Lambda=1.2$ GeV	$\Lambda=1.6$ GeV	$\Lambda=2.0$ GeV
$D^0\to K^+K^-\to K^0\overline{K^0}$	$-i1.52\times10^{-7}$	$-i3.23\times10^{-7}$	$-i4.52\times10^{-7}$
$D^0\to\pi^+\pi^-\to K^0\overline{K^0}$	$i1.02\times10^{-7}$	$i3.11\times10^{-7}$	$i4.89\times10^{-7}$
$D^0\to K^{*+}K^{*-}\to K^0\overline{K^0}$	$i4.37\times10^{-7}$	$i5.89\times10^{-7}$	$i6.91\times10^{-7}$
$D^0\to\rho^+\rho^-\to K^0\overline{K^0}$	$-i1.79\times10^{-7}$	$-i3.02\times10^{-7}$	$-i3.98\times10^{-7}$
Total	$i2.08\times10^{-7}$	$i2.75\times10^{-7}$	$i3.30\times10^{-7}$

TABLE III. FSI amplitudes in Regge pole models.

Decay mode	A^{FSI} (GeV)	
	Model I	Model II
$D^0\to K^+K^-\to K^0\overline{K^0}$	$(-0.31-i2.61)\times10^{-7}$	$(-1.06-i2.18)\times10^{-7}$
$D^0\to\pi^+\pi^-\to K^0\overline{K^0}$	$(0.38+i3.17)\times10^{-7}$	$(-1.47+i2.38)\times10^{-7}$
$D^0\to K^{*+}K^{*-}\to K^0\overline{K^0}$	$(-1.13+i0.07)\times10^{-7}$	$(-4.08+i2.75)\times10^{-7}$
$D^0\to\rho^+\rho^-\to K^0\overline{K^0}$	$(1.0-i0.67)\times10^{-7}$	$(2.80-i1.12)\times10^{-7}$
Total	$(-0.06-i0.04)\times10^{-7}$	$(-3.81+i1.83)\times10^{-7}$

pared with what experimental data need. However, if the parameters of $f_0(1710)$ change [14], the *s*-channel contributions may become more important. For a more precise estimation, we shall wait for further experimental information on 0^{++} resonances near M_D.

For the OPE model and the Regge pole model, we take $c_1=1.26$, $c_2=-0.51$ [7]; decay constants [5,15] $f_\pi=0.13$ GeV, $f_K=0.16$ GeV, $f_\rho=0.221$ GeV, $f_{K^*}=0.221$ GeV; and form factors [6,15]

$$F_0^{D\pi}(0)=0.692,\ \ A_1^{D\rho}(0)=0.775,\ \ A_2^{D\rho}(0)=0.923,$$

$$F_0^{DK}(0)=0.762,\ \ A_1^{DK^*}(0)=0.880,\ \ A_2^{DK^*}(0)=1.147.$$

For the OPE model, the effective strong coupling constants are given in [7]: $g_{K^*K\pi}=5.8$ and $g_{\rho\pi\pi}=\sqrt{2}g_{\rho KK}=6.1$. The parameter Λ in the off-shell form factor $F(k^2)$ varies in a range of 1.2–2.0 GeV [9]. In Table II, we tabulate the results corresponding to three cases: $\Lambda=1.2$ GeV, $\Lambda=1.6$ GeV, $\Lambda=2.0$ GeV.

Here three points are worthy of note. (1) The process $D^0\to K^{*+}K^{*-}\to K^0\overline{K^0}$ has the largest contribution. The reason is because the exchanged particle is the lightest meson, the pion. This conclusion is the same as in [7,8]. (2) The predicted amplitude of the process $D^0\to K^{*+}K^{*-}\to K^0\overline{K^0}$ is not very sensitive to the choice of the parameter Λ. On the contrary, for the other three processes, the amplitudes are more sensitive to the choice. As is well known, the OPE model is more applicable when the virtual exchanged particle is close to its mass shell. In fact, the heavier the particle is, or the further it is off its mass shell, then the more sensitive the amplitude is to the parameter Λ. (3) We only calculate the absorptive part which gives the imaginary part of the FSI amplitudes only. It gives the correct order of magnitude of the FSI effects. The dispersive real part of the FSI amplitudes can be calculated in terms of the dispersion relation [9] with additional parameters, and generally it is of the same order of magnitude as the absorptive part. So the OPE model can reproduce FSI effects rather well.

For the Regge pole model, two different treatments of the residue function $\gamma(t)$ are assumed. Model I assumes constant residue functions γ [16]: $\gamma^2_{\pi\pi\rho}=\sqrt{2}\gamma_{KK\rho}=\sqrt{s_0}2Y^2_{\pi p}/Y_{pp}\approx72$. Model II takes $\gamma(t)$ to make Eq. (13) the same as in the OPE model for t near the mass squared of the leading exchanged particle. The numerical results are listed in Table III.

Comparing the imaginary part in Regge pole models with the OPE results in Table II, the biggest difference is for $D\to VV\to PP$ modes in model I. Model I is the conventional approximation of the Regge model for high energies. It is obviously not a good approximation for the M_D energy region. We found that the main problem is that the high energy approximation for the *t*-dependent couplings, $(-t/s_0)^{\lambda/2}$ in Eq. (13), is not good for $VV\to PP$ at the M_D energy. If we replace the $(-t/s_0)^{\lambda/2}\gamma(t)$ in Eq. (13) by the corresponding effective couplings in the OPE model, then we get model II which gives results roughly consistent with OPE results.

In summary, for the *t*-channel FSI contributions to $D^0\to K^0\overline{K^0}$, the OPE model and Regge pole model with a proper treatment of $\gamma(t)$ (model II) are roughly consistent with each other and can reproduce the experimental data reasonably well. They may be used to estimate *t*-channel FSI effects for other *D*-decay channels. The Regge pole model assuming a constant $\gamma(t)$ (model I) is not suitable for the M_D energy region. The *s*-channel FSI contribution from known 0^{++} resonances is small.

The situation of FSIs for *B*-meson decays should be different. There is an *s*-dependent suppression factor $(s/s_0)^{\alpha(t)}$ in the Regge pole model. The discontinuity of the final state interaction amplitude is proportional to $(s/s_0)^{\alpha_0-1}$. For inelastic rescattering for which the exchange trajectory $\alpha_0<1$, the discontinuity of the final state interaction amplitude decreases as the energy increases. This predicts that the final state interaction will be small in the high energy region. There is no such *s*-dependent suppression factor in the OPE model. At high energies, *t*-channel exchange of heavier particles will become more important. The *s*-independent off-shell form factors in the OPE model may be not enough to compensate for these effects. We will continue our study in the *B* region and the results will be published elsewhere.

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