P wave to S wave pion transitions of charmed baryons

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The spin-flavor constituent quark model symmetry of the light diquark system is used to analyze single pion transitions of heavy baryon states. A light-front constituent quark model calculation shows that the strength of the single pion couplings of the charmed baryon ground state to the antisymmetric P wave states are suppressed with respect to those of the symmetric multiplet. We also investigate the constituent quark masses dependence of P wave to S wave strong coupling constants and decay rates. [S0556-2821(99)06511-X]

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I. INTRODUCTION

In the last decade, the heavy quark effective theory [1] has been used extensively to analyze decay processes involving heavy baryons [2,3]. However, because of a lack of experimental data, most of these studies have been primarily concentrated on ground-state charm and bottom hadrons. The main observation is that, in the heavy quark limit, the $SU(2)$ flavor and spin symmetries relate the form factors or coupling constants of different transitions among heavy hadron states. Using this heavy quark symmetry (HQS), for instance, semileptonic decays of $\Lambda_b \rightarrow \Lambda_c$ are described by a single form factor and $\Sigma_b^{(*)} \rightarrow \Sigma_c^{(*)}$ required two independent form factors [2,4]. Therefore, semileptonic transitions among ground state (S wave) heavy baryons are described in terms of only three form factors.

To go beyond these predictions, however, one needs to incorporate other symmetries manifested by the light degrees of freedom. The three form factors describe semileptonic decays of S wave heavy baryons are instead determined by a single universal function. This result is a consequence of the spin-flavor $SU(4)$ symmetry which arises in the baryon sector in the large-N_c limit [5]. Similar conclusions can be achieved using a constituent quark model with the underlying $SU(2N_f) \times O(3)$ light diquark symmetry [6]. Moreover, these symmetries can, also, be used to significantly reduce the number of the current induced transition form factors for P wave to S wave heavy baryon states [6,7].

The mechanism of strong and electromagnetic decays of heavy baryons are quite different from those in weak current-induced transitions. In the former, the heavy quark is not active and the transitions are solely that of the light diquark system. This is contrary to semileptonic decays where the transitions are induced by the heavy quark. Instead of treating the pion as a genuine $q\bar{q}$ state, it is assumed that there exist an effective operator which couples the pion to each of the light quarks in the heavy baryon. This assumption was also employed in most of the noncharmed $SU(6) \times O(3)$ or $SU(6)_W \times O(3)$ quark models [8].

Applications of the constituent quark model symmetries to analyze strong transitions among heavy baryon states, in the heavy quark limit, were presented in Refs. [7,9] using the heavy hadron chiral perturbation theory (HHCPT) and in a covariant $SU(2N_f) \times O(3)$ approach [10–12]. The main achievement obtained so far is that, ground state heavy baryons one-pion decays are described by a single coupling constant. However, two couplings are required to determine transitions from each of the two P wave multiplets to the ground state. The strength of these couplings are usually determined from the experimental data which are not available at present for most of the charmed baryon resonances. Therefore, a model calculation of these couplings should give at least some indication of the qualitative nature of single pion transitions.

The s-wave coupling f_s and d-wave coupling f_d correspond to $f_{\Lambda_{c1}\Sigma_c\pi}$ and $f_{\Lambda_{c1}^*\Sigma_c\pi}$, respectively, for charmed baryons were calculated in Ref. [13] using light-front (LF) wave functions. These couplings are sufficient to describe single pion transitions from the symmetric P wave multiplet to the S wave ground state. The purpose of this paper is to use the same (LF) wave functions to calculate the couplings $f_{\Sigma_{c1}\Sigma_c\pi}$ and $f_{\Sigma_{c1}^*\Sigma_c\pi}$ which describe the transitions from the antisymmetric P wave multiplet to the ground state of charmed baryons. Also, we shall investigate the dependence of the single pion charmed baryons decay rates on the constituent quark masses.

In Sec. II we use the $SU(2N_f) \times O(3)$ spin wave functions to derive the constituent quark model relations for the single pion transitions of heavy baryon states. We shall devote Sec. III to the LF calculation of the single pion transition coupling constants $f_{\Sigma_{c1}\Sigma_c\pi}$ and $f_{\Sigma_{c1}^*\Sigma_c\pi}$, as well as to investigate the sensitivity of the decay rates on the constituent quark masses. Finally, we summarize our analysis in a concluding Sec. IV.

II. THE $SU(2N_f) \times O(3)$ DIQUARK SYMMETRY

In the heavy quark limit both the heavy quark quantum numbers and the spin and parity of the diquark system are conserved. Therefore, it is possible and even more convenient to use the diquark quantum numbers to classify heavy baryons which will simplify analyzing transitions among their states. The light diquark system in heavy baryons may be usefully classified in terms of the well known $SU(2N_f) \times O(3)$ symmetry, where N_f represents the number of light flavor and σ denotes the spin.

The light diquark system in a heavy baryon state is represented by the $SU(6)$ tensor $(\Phi)_{AB}$, with $A \equiv (\alpha, a)$ and

TABLE I. Heavy quark symmetry predictions for S wave to S wave and P wave to S wave single pion transitions.

$S\to S$ transitions	Coupling factors	$K\to S$ transitions	Coupling factors	$k\to S$ transitions	Coupling factors
$\Sigma_Q^{(*)}\to\Lambda_Q$	$f_p^{(1)}$	$\Lambda_{QK1}\to\Sigma_Q^{(*)}$	$f_{s,d}^{(1)}$	$\Sigma_{Qk1}\to\Sigma_Q^{(*)}$	$f'^{(1)}_{s,d}$
$\Sigma_Q^{(*)}\to\Sigma_Q^{(*)}$	$f_p^{(2)}$	$\Sigma_{QK0}\to\Lambda_Q$	$f_s^{(2)}$	$\Lambda_{Qk0}\to\Lambda_Q$	$f'^{(2)}_s$
		$\Sigma_{QK1}\to\Sigma_Q^{(*)}$	$f_{s,d}^{(3)}$	$\Lambda_{Qk1}\to\Sigma_Q^{(*)}$	$f'^{(3)}_{s,d}$
		$\Sigma_{QK2}\to\Lambda_Q$	$f_d^{(4)}$	$\Lambda_{Qk2}\to\Lambda_Q$	$f'^{(4)}_d$
		$\Sigma_{QK2}\to\Sigma_Q^{(*)}$	$f_d^{(5)}$	$\Lambda_{Qk2}\to\Sigma_Q^{(*)}$	$f'^{(5)}_d$

$B\equiv(\beta,b)$ representing the light quarks spin and flavor quantum numbers. The (Λ-type) and (Σ-type) diquark spin wave functions $(\phi)_{\alpha\beta}$ are antisymmetric and symmetric, respectively, when interchanging the Dirac indices α and β. As far as flavor symmetry is concerned, the (Λ-type) baryons transform as an antitriplet while the (Σ-type) baryons transform as a sextet representations of $SU(3)$. The wave functions are antisymmetric and symmetric, respectively, under the permutation of the flavor indices a and b. To represent the orbital excitations we use the relative momenta $K=(1/\sqrt{6})(p_1+p_2-2p_3)$ and $k=(1/\sqrt{2})(p_1-p_2)$ which are symmetric and antisymmetric, respectively, under interchange of the constituent light quark momenta p_1 and p_2.

For the ground state, the tensor $(\Phi)_{AB}$ is symmetric under the A and B permutation and transforms as **21** irreducible representation of $SU(6)$. However, for P wave states, $(\Phi)_{AB}$ can be written in terms of two irreducible $SU(6)$ representations of definite symmetries. The first (K multiplet), with angular excitation $l_K=1$, is symmetric and transforms as **21** while the second (k multiplet), with angular excitation $l_k=1$, is antisymmetric and transforms as **15** irreducible representation of $SU(6)$. The diquark system total wave function $(\Psi)_{AB}$ with parity $P=(-1)^{(l_K+l_k)}$, however, is constructed to be symmetric with respect to the overall flavor $\times$spin$\times$orbital symmetry. These symmetry properties are of crucial importance and will be used later on when evaluating transition matrix elements. For detailed analysis about the normalization and other properties of these functions the reader is refered to Ref. [14].

HQS predicts that S wave to S wave single pion transitions involve two p-wave coupling constants. Furthermore, transitions from each of the K and k multiplets down to the ground state are determined in terms of seven other couplings which consist of three s-wave and four d-wave couplings for each. The parity conserved single pion transitions and their couplings are listed in Table I. The matrix elements M^π of these transitions are explicitly given in [10].

The general form of M^π can be written as

$$M^\pi=\mathcal{H}^{\mu_1\cdots\mu_{j_1}}_{\nu_1\cdots\nu_{j_2}}\mathcal{M}^{\nu_1\cdots\nu_{j_2}}_{\mu_1\cdots\mu_{j_1}}, \tag{1}$$

where $\mathcal{H}$ and $\mathcal{M}$ represents the heavy quark and the diquark transition matrix elements, respectively, and j_1 and j_2 are the diquark spin degrees of freedom in the initial and final heavy baryons. In pion transitions, the heavy quark is not active and is moving with the heavy baryon velocity v, therefore, $\mathcal{H}$ is simply a product of Dirac spinor $u(v)$ or the superfield $\psi^{\mu_1\cdots\mu_j}(v)$ and their antispinors. The superfield $\psi^{\mu_1\cdots\mu_j}(v)$, with j being an integer representing the spin of the diquark system, stands for the doublet spin wave functions corresponding to the two heavy quark symmetry nearly degenerate states with spins $(j\pm 1/2)$. On the other hand, the diquark transition matrix elements $\mathcal{M}^{\nu_1\cdots\nu_{j_2}}_{\mu_1\cdots\mu_{j_1}}$ are, in general, tensors of rank j_1+j_2 and are constructed from the pion momentum $q_{\perp\mu}=q_\mu-vqv_\mu$, $g_{\perp\mu\nu}=g_{\mu\nu}-v_\mu v_\nu$ and $\varepsilon_{\mu\nu\rho\delta}$ to ensure parity conservation.

To reduce the number of the various HQS strong couplings of Table I we need to write the decay matrix elements $\mathcal{M}$ in terms of the $SU(2N_f)\times O(3)$ light diquark covariant spin wave functions and appropriate transition operators. The transition amplitudes for a diquark transition $\{j_1\}\to\{j_2\}+\pi$ are given by

$$\mathcal{M}^{\nu_1\nu_2\cdots\nu_{j_2}}_{\mu_1\mu_2\cdots\mu_{j_1}}=(\overline{\phi}^{\nu_1\nu_2\cdots\nu_{j_2}})^{\alpha\beta}(\mathcal{O}')^{\alpha'\beta'}_{\alpha\beta}(\phi_{\mu_1\mu_2\cdots\mu_{j_1}})_{\alpha'\beta'}. \tag{2}$$

Here, $\mathcal{O}'$ is an effective operator and we have, explicitly, written the Dirac indices $\alpha^{(\prime)}$ and $\beta^{(\prime)}$ as well as the Lorentz indices μ_i and ν_i. However, the flavor indices has been omitted, which can easily be included in the decay rates. The diquark spin wave functions $(\phi)_{\alpha\beta}$ are generally given in terms of $[\chi^0]_{\alpha\beta}$ and $[\chi^1_\mu]_{\alpha\beta}$ the spin-0 and spin-1 projection operators, respectively. They are antisymmetric and symmetric, respectively, when interchanging α and β and have the form

$$\{\chi^0\}_{\alpha\beta}=\{(\not{v}+1)\gamma_5 C\}_{\alpha\beta}, \quad [\chi^{1,\mu}]_{\alpha\beta}=[(\not{v}+1)\gamma^\mu_\perp C]_{\alpha\beta}. \tag{3}$$

Here, the charge conjugation operator $C=i\gamma_0\gamma_2$ and $\gamma_{\perp\mu}=\gamma_\mu-\not{v}v_\mu$. The diquark spin wave functions $(\phi)_{\alpha\beta}$ are explicitly given in Table II. For heavy baryon resonances, they are also functions of the relative momenta K and k.

It is more convenient, especially when analyzing transitions among higher resonance states, to rewrite the diquark system spin wave functions $(\phi)_{\alpha\beta}$ such that the angular momentum excitations K and k are factorized which are then included in the transition effective operator, i.e.,

$$(\phi_{\mu_1\mu_2\cdots\mu_j})_{\alpha\beta}=(\phi^{\lambda_1\lambda_2\cdots\lambda_L}_{\mu_1\mu_2\cdots\mu_j})_{\alpha\beta}p_{\lambda_1}p_{\lambda_2}\cdots p_{\lambda_L}, \tag{4}$$

TABLE II. Covariant diquark spin wave functions of *S* wave and *P* wave heavy baryons. A traceless symmetric tensor is represented by $\{A_\mu B_\nu\}^0=\frac{1}{2}A_\mu B_\nu+\frac{1}{2}A_\nu B_\mu-\frac{1}{3}ABg_{\mu\nu}$.

	j^P	$\phi_{\alpha\beta}$
S wave states		
Λ_Q	0^+	$(\chi^0)_{\alpha\beta}$
Σ_Q	1^+	$(\chi^{1,\mu})_{\alpha\beta}$
Symmetric *P* wave states		
Λ_{QK1}	1^-	$(\chi^0 K_\perp^\mu)_{\alpha\beta}$
Σ_{QK0}	0^-	$\frac{1}{\sqrt{3}}(\chi^{1,\mu}K_{\perp\mu})_{\alpha\beta}$
Σ_{QK1}	1^-	$\frac{i}{\sqrt{2}}(\varepsilon_{\mu\nu\rho\delta}\chi^{1,\nu}K_\perp^\rho v^\delta)_{\alpha\beta}$
Σ_{QK2}	2^-	$\frac{1}{2}(\{\chi^{1,\mu_1}K_\perp^{\mu_2}\}^0)_{\alpha\beta}$
Antisymmetric P wave states		
Σ_{Qk1}	1^-	$(\chi^0 k_\perp^\mu)_{\alpha\beta}$
Λ_{Qk0}	0^-	$\frac{1}{\sqrt{3}}(\chi^{1,\mu}k_{\perp\mu})_{\alpha\beta}$
Λ_{Qk1}	1^-	$\frac{i}{\sqrt{2}}(\varepsilon_{\mu\nu\rho\delta}\chi^{1,\nu}k_\perp^\rho v^\delta)_{\alpha\beta}$
Λ_{Qk2}	2^-	$\frac{1}{2}(\{\chi^{1,\mu_1}k_\perp^{\mu_2}\}^0)_{\alpha\beta}$

where each of the p_{λ_i}, corresponding to either *K* or *k*, represents one unit of the light diquark angular momentum excitations L_i. Therefore, the matrix elements in Eq. (2) take the general form

$$\mathcal{M}^{\nu_1\nu_2\cdots\nu_{j_2}}_{\mu_1\mu_2\cdots\mu_{j_1}}=(\overline{\phi}^{\nu_1\nu_2\cdots\nu_{j_2}}_{\eta_1\eta_2\cdots\eta_{L_2}})^{\alpha\beta}(\mathcal{O}^{\eta_1\eta_2\cdots\eta_{L_2}}_{\lambda_1\lambda_2\cdots\lambda_{L_1}}(q_\perp))^{\alpha'\beta'}_{\alpha\beta}\times(\phi^{\lambda_1\lambda_2\cdots\lambda_{L_1}}_{\mu_1\mu_2\cdots\mu_{j_1}})_{\alpha'\beta'}, \tag{5}$$

where, L_1 and L_2 being, respectively, the diquark initial and final states angular excitations. To evaluate the matrix elements of Eq. (5), one needs to drop out the *K* and *k* factors from the diquark spin functions ϕ of Table II which are absorbed in the effective operator $\mathcal{O}^{\eta_1\eta_2\cdots\eta_{L_2}}_{\lambda_1\lambda_2\cdots\lambda_{L_1}}(q_\perp)$. The explicit form of the operator $\mathcal{O}^{\eta_1\eta_2\cdots\eta_{L_2}}_{\lambda_1\lambda_2\cdots\lambda_{L_1}}(q_\perp)$ will also depend on the partial wave involved in the decay process.

In the constituent quark model the pion is emitted by one of the light quarks so that the leading contributions to the quark-pion effective operator $\mathcal{O}^{\eta_1\eta_2\cdots\eta_{L_2}}_{\lambda_1\lambda_2\cdots\lambda_{L_1}}(q_\perp)$ are those from one-body operators. It was shown that in the $1/N_c$ expansion, two-body emission operators are nonleading for pion couplings to *S* wave heavy baryons and can be neglected in the constituent quark model approach [15,16]. Moreover, an explicit numerical analysis of the *P* wave to *S* wave strong decays [15] showed that the coefficients of the two-body operators are small.[1] The most general one-body transition operator can be written in the form

$$\mathcal{O}^{\eta_1\eta_2\cdots\eta_{L_2}}_{\lambda_1\lambda_2\cdots\lambda_{L_1}}(q_\perp)=\frac{f_l^{(\prime)}}{2}[(\gamma^\sigma\gamma_5)\otimes(\mathbb{1})\pm(\mathbb{1})\otimes(\gamma^\sigma\gamma_5)]R^{\eta_1\eta_2\cdots\eta_{L_2}}_{\sigma\lambda_1\lambda_2\cdots\lambda_{L_1}}(q_\perp), \tag{6}$$

where f_l and f_l' are *l*-wave constituent quark model coupling constant, they, respectively, correspond to the operator with the $+$ and $-$ signs in Eq. (6). The tensor *R* is constructed to project out the appropriate partial wave amplitude.

To be more specific, the partial amplitude involved in *S* wave to *S* wave transitions ($L_1=L_2=0$) is *p* wave so that we can write

$$R_\sigma(q_\perp)=f_p^{(\prime)}q_{\perp\sigma}. \tag{7}$$

For *P* wave to *S* wave transitions ($L_1=1$ and $L_2=0$) there are *s*-wave and *d*-wave amplitudes with couplings $f_s^{(\prime)}$ and $f_d^{(\prime)}$. Therefore, we have

$$R_{\sigma\lambda}(q_\perp)=f_s^{(\prime)}g_{\sigma\lambda}, \tag{8}$$

$$R_{\sigma\lambda}(q_\perp)=f_d^{(\prime)}T_{\sigma\lambda}, \tag{9}$$

here, the traceless symmetric tensor $T_{\sigma\lambda}$ is given by

$$T_{\sigma\lambda}(q_\perp)=q_{\perp\sigma}q_{\perp\lambda}-\frac{1}{3}q_\perp^2 g_{\perp\sigma\lambda}. \tag{10}$$

Having constructed the transition operators $\mathcal{O}$ we can now proceed to calculate the single pion matrix elements in Eq. (5). As mentioned before, $(\Phi)_{AB}$ is symmetric when interchanging *A* and *B* for both *S* wave and K-multiplet *P* wave and is antisymmetric for the *k*-multiplet *P* wave. Therefore, one can easily show that the diquark matrix elements of the transition operator with the negative sign in Eq. (6) vanishes for both *S* wave to *S* wave and *P* wave *K*-multiplet to *S* wave single pion decays. On the other hand for *k*-multiplet *P* wave to *S* wave transitions, it is the operator with the positive sign in Eq. (6) which vanishes. One thus concludes that, *S* wave to *S* wave transitions are determined by a single coupling and transitions from each multiplet of the *P* wave to *S* wave pion decays are determined by *s*-wave and *d*-wave constituent coupling constants.

Now, with the help of Table II and Eqs. (7)–(9), the nonvanishing matrix elements of Eq. (5) among $SU(6)\times O(3)$ diquark states can easily be calculated. One finds that, *S* wave to *S* wave HQS couplings are related to the single constituent quark coupling constant f_p by

[1]The authors of Ref. [16], however, show that two-body operators for *S* wave pion couplings to *P* wave states are not suppressed with respect to one-body operators in large-N_c expansion. A study of the contribution of these operators to pion transitions among heavy baryon states will be postponed to a future work.

$$f_p^{(1)}=f_p^{(2)}=f_p. \tag{11}$$

Similar analysis shows that in P wave (K-multiplet) to S wave transitions one gets the following relations:

$$f_s^{(1)}=f_s,\ \ f_s^{(2)}=\sqrt{3}f_s,\ \ f_s^{(3)}=-\sqrt{2}f_s, \tag{12}$$

$$f_d^{(1)}=f_d,\ \ f_d^{(3)}=\frac{1}{\sqrt{2}}f_d,\ \ f_d^{(4)}=f_d,\ \ f_d^{(5)}=-f_d. \tag{13}$$

For P wave (k-multiplet) to S wave transitions we have an identical relations of Eq. (12) with the replacement

$$f_s^{(i)}\rightarrow f_s'^{(i)},\ \ f_d^{(i)}\rightarrow f_d'^{(i)},\ \ f_s\rightarrow f_s',\ \text{and}\ \ f_d\rightarrow f_d'. \tag{14}$$

Therefore, in the $SU(6)\times O(3)$ symmetry only five independent couplings are required to describe S wave to S wave and P wave to S wave single pion transitions. The constituent quark model relations in Eqs. (11)–(14) are in agreement, after taking into account the different normalizations, with corresponding results obtained using HHCPT [7]. This analysis can be generalized easily to include transitions from higher resonances.

III. SINGLE PION COUPLING STRENGTHS

To predict transition decay rates between heavy baryon states one needs first to calculate the constituent quark couplings f_p, $f_s^{(\prime)}$, and $f_d^{(\prime)}$. In a recent paper [13], we have used LF quark model wave functions to calculate f_p, f_s, and f_d for charmed baryons. To complete the analysis of P wave to S wave transitions we shall calculate f_s' and f_d' using the same constituent quark model. These two couplings determine one-pion transitions of antisymmetric multiplet to the ground state heavy baryon, and for charmed baryons they correspond to $f_{\Sigma_{c1}\Sigma_c\pi}$ and $f_{\Sigma^*_{c1}\Sigma_c\pi}$. Following Ref. [13], the one pion coupling constants can be written in terms of LF matrix elements of the strong transition current $\hat{j}_\pi(q)$ between LF heavy baryon helicity states. Working in the Drell-Yan frame and using the LF spinors as well as some of their matrix elements between Dirac matrices, presented in Ref. [13], one gets for the p-wave coupling

$$g_{\Sigma_c\Lambda_c\pi}=-\frac{2\sqrt{3M_{\Lambda_c}M_{\Sigma_c}}}{(M^2_{\Sigma_c}-M^2_{\Lambda_c})}\langle\Lambda_c(P',\uparrow)|\hat{j}_\pi(0)|\Sigma_c(P,\uparrow)\rangle. \tag{15}$$

For the s-wave and d-wave couplings of the S wave to the symmetric P wave multiplets we have

$$f_{\Lambda_{c1}\Sigma_c\pi}=\langle\Sigma_c(P',\uparrow)|\hat{j}_\pi(0)|\Lambda_{c1}(P,\uparrow)\rangle \tag{16}$$

and

$$f_{\Lambda^*_{c1}\Sigma_c\pi}=\frac{3\sqrt{2}}{(M_{\Lambda^*_{c1}}-M_\Sigma)^2}\frac{M^2_{\Lambda^*_{c1}}}{(M^2_{\Lambda^*_{c1}}-M^2_{\Sigma_c})}\times\left\langle\Sigma_c(P',\uparrow)|\hat{j}_\pi(0)|\Lambda^*_{c1}\left(P,\frac{1}{2}\right)\right\rangle, \tag{17}$$

and for those of the S wave to the antisymmetric P wave multiplets one has

$$f_{\Sigma_{c1}\Sigma_c\pi}=\langle\Sigma_c(P',\uparrow)|\hat{j}_\pi(0)|\Sigma_{c1}(P,\uparrow)\rangle \tag{18}$$

and

$$f_{\Sigma^*_{c1}\Sigma_c\pi}=\frac{3\sqrt{2}}{(M_{\Sigma^*_{c1}}-M_\Sigma)^2}\frac{M^2_{\Sigma^*_{c1}}}{(M^2_{\Sigma^*_{c1}}-M^2_{\Sigma_c})}\times\left\langle\Sigma_c(P',\uparrow)|\hat{j}_\pi(0)|\Sigma^*_{c1}\left(P,\frac{1}{2}\right)\right\rangle. \tag{19}$$

Similar expressions for the couplings of heavy baryons involving strange quarks such as $\Xi_c^{(*)}$ and $\Xi_{c1}^{(*)}$ can also be written. In the LF formalism the total baryon spin-momentum distribution function can be written in the following general form:

$$\Psi(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)=\chi(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)\,\psi(x_i,\mathbf{p}_{\perp i}). \tag{20}$$

Here, $\chi(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)$ and $\psi(x_i,\mathbf{p}_{\perp i})$ represent the spin and momentum distribution functions, respectively. Assuming factorization of longitudinal and transverse momentum distribution functions, one can write

$$\psi(x_i,\mathbf{p}_{\perp i})=\prod_{i=1}^{3}\delta(x_i-\overline{x_i})\exp\left[-\frac{\mathbf{k}_\rho^2}{2\alpha_\rho^2}-\frac{\mathbf{K}_\lambda^2}{2\alpha_\lambda^2}\right], \tag{21}$$

with

$$\mathbf{k}_\rho=\frac{1}{\sqrt{2}}(\mathbf{p}_{\perp 1}-\mathbf{p}_{\perp 2}),\ \ \mathbf{K}_\lambda=\frac{1}{\sqrt{6}}(\mathbf{p}_{\perp 1}+\mathbf{p}_{\perp 2}-2\mathbf{p}_{\perp 3}). \tag{22}$$

The longitudinal momentum distribution functions are approximated by Dirac-delta functions which are peaked at the constituent quark longitudinal momenta mean values $\overline{x_i}=m_i/M$. This assumption is justified since in the weak binding [17] and the valence [18] approximations, the constituent quarks actually move with the same velocity inside the baryon. In the heavy quark limit, the longitudinal momentum fractions $x_Q\rightarrow 1$ and $x_{\{1,2\}}\rightarrow 0$ which cause the conventional LF momentum distribution functions to be ill defined at the end points. This is part of the motivation in choosing harmonic oscillator functions rather than the conventional LF ones to describe the momentum dependence.

The heavy baryon spin wave functions, which are the LF generalization of the conventional constituent quark model spin-isospin functions, are given by

$$\chi_{\Sigma_Q} = \overline{u(p_1,\lambda_1)}[(\not P + M_\Sigma)\gamma_\perp^\mu]\nu(p_2,\lambda_2)\overline{u(p_3,\lambda_3)} \times \gamma_{\perp\mu}\gamma_5 u(P,\lambda). \quad (23)$$

The $\frac{1}{2}^-$ and $\frac{3}{2}^-$ lowest laying *P* wave states, which are degenerate in the heavy quark limit, have covariant spin wave functions of the form

$$\chi_{B(1/2^-)} = \overline{u(p_1,\lambda_1)}[(\not P + M_B)\gamma_5] \times \nu(p_2,\lambda_2)\overline{u(p_3,\lambda_3)}\not Q\gamma_5 u(P,\lambda) \quad (24)$$

and

$$\chi_{B*(3/2^-)} = \overline{u(p_1,\lambda_1)}[(\not P + M_{B*})\gamma_5] \times \nu(p_2,\lambda_2)\overline{u(p_3,\lambda_3)}Q_\mu u^\mu(P,\lambda). \quad (25)$$

Here $Q=K$ for the symmetric *P* wave states Λ_{QK1} (Ξ_{QK1}) and Λ^*_{QK1} (Ξ^*_{QK1}) and one has $Q=k$ for the antisymmetric *P* wave states Σ_{Qk1} and Σ^*_{Qk1}. These are functions of the longitudinal momentum fraction $x_i = p_i^+/P^+$, the transverse momentum $\mathbf{p}_{\perp i}$ and the helicity λ_i of the constituent quarks in a baryon with momentum P and helicity λ.

The numerical values for the constituent quark masses and the oscillator couplings are taken to be $m_u = m_d = 0.33$ GeV, $m_s = 0.48$ GeV, $m_c = 1.51$ GeV, $\alpha_\rho = 0.40$ GeV, and $\alpha_\lambda = 0.52$ GeV. The charmed baryon masses $M_{\Sigma_c} = 2.45$ GeV, $M_{\Sigma_{c1}} = 2.668$, and $M_{\Sigma^*_{c1}} = 2.701$. Here, the Σ_{c1} and Σ^*_{c1} masses are taken to be 0.075 GeV above the Λ_{c1} and Λ^*_{c1} masses, respectively, as predicted by the quark model [19,20]. Using the LF momentum distribution functions Eq. (21) and the spin functions Eqs. (23)–(25), the two charmed baryons strong couplings $f_{\Sigma_{c1}\Sigma_c\pi}$ and $f_{\Sigma^*_{c1}\Sigma_c\pi}$ are calculated to be

$$f_{\Sigma_{c1}\Sigma_c\pi} = 0.09, \quad f_{\Sigma^*_{c1}\Sigma_c\pi} = 0.91 \ \text{GeV}^{-2}. \quad (26)$$

The single pion decay rates of the two lowest laying states of the antisymmetric multiplet $\Sigma_{c1}(2.668) \to \Sigma_c$ and $\Sigma^*_{c1}(2.701) \to \Sigma_c$ can now be predicted:

$$\Gamma_{\Sigma_{c1}\to\Sigma_c+\pi} = 0.20 \ \text{MeV}, \quad \Gamma_{\Sigma^*_{c1}\to\Sigma_c+\pi} = 0.005 \ \text{MeV}. \quad (27)$$

These results show that the single pion decay rates of the antisymmetric *P* wave multiplet to the ground state are suppressed with respect to the corresponding rates of the symmetric *P* wave multiplet calculated in Ref. [13] which are typically of the order of 1 MeV. One thus concludes that, measurement of these decay rates from the antisymmetric multiplet (*k* multiplet) to the ground state will be difficult.

As the case in all model calculations the numerical results depend on the values of the parameters used in the model. In our LF model the free parameters are the oscillator couplings α_ρ and α_λ which, in general, depend on the constituent quark masses. The investigation of this dependence will complete the analysis of charmed baryons *P* wave to *S* wave single pion transitions. Unfortunately, there is no standard procedure to fix the values of the quark masses and the oscillator couplings, instead different approaches gives different sets of values. In general, the fitting of hadron spectroscopic properties help to determine most of the quark model free parameters. Most of the nonrelativistic and relativistic quark models assume that both the masses and the oscillator confining parameters are of the order of Λ_{QCD}. However, the exact numerical values in a model may differ by up to 120 MeV depending whether they are obtained from fitting the hadron spectrum or their decay properties [23,24].

To make our analysis systematic we adopt a simple nonrelativistic dependence, usually used in the *uds* basis [19,20], of the harmonic oscillator size parameters on the constituent quark masses

$$\alpha_\rho = (3m_\rho\kappa)^{1/4}, \quad \alpha_\lambda = (3m_\lambda\kappa)^{1/4}, \quad (28)$$

with

$$m_\rho = m \ \text{and} \ m_\lambda = 3mm_c/(2m+m_c). \quad (29)$$

The constituent quark masses m and m_c and the scale parameters α_ρ and α_λ may be obtained from fitting the static properties of baryons. If one takes $\alpha_\rho = \alpha_\lambda = 0.41$ GeV, used to fit baryon masses and some decay rates [21], the value of the oscillator constant κ is fixed to be $\kappa = 0.029$ GeV^3 where the light quark mass is $m = 0.33$ GeV, commonly used in nonrelativistic quark models, and which is larger than those assumed by some relativistic models.

The *P* wave to *S* wave single pion coupling constants $g_{\Sigma_c\Lambda_c\pi}$, $f_{\Lambda_{c1}\Sigma_c\pi}$, and $f_{\Lambda^*_{c1}\Sigma_c\pi}$, defined in Eqs. (15),(16), and $f_{\Sigma_{c1}\Sigma_c\pi}$ and $f_{\Sigma^*_{c1}\Sigma_c\pi}$ defined in Eqs. (18) and (19), will be calculated for different values of the light quark masses. The masses $m_u = m_d = m$ are taken to be $m = 0.22$, 0.28, and 0.34 GeV, we also take $m_s = m + 0.15$ GeV. This range covers the values usually used in nonrelativistic [21,22] and relativistic quark models [23,24]. The Λ_c, $\Sigma_c^{(*)}$, and $\Xi_c^{(*)}$ masses are those quoted by the Particle Data Group [25]. We notice that for values less than $m = 0.22$ GeV and more than $m = 0.34$ GeV the integrals are numerically unstable and thus will be excluded. Table III shows the sensitivity to light quark masses of the pion coupling strengths among *S* wave baryons and those of the ground-state to the *P* wave orbital excited states calculated using Eqs. (15)–(19). The *S* wave to the symmetric *P* wave couplings have also been calculated in Ref. [26] using a relativistic three quark model. They found that

$$g_{\Sigma_c\Lambda_c\pi} = 8.88 \ \text{GeV}^{-1}, \quad f_{\Lambda_{c1}\Sigma_c\pi} = 0.52, \quad f_{\Lambda^*_{c1}\Sigma_c\pi} = 22 \ \text{GeV}^{-2} \quad (30)$$

and

$$g_{\Xi^*_c\Xi_c\pi} = 8.34 \ \text{GeV}^{-1}, \quad f_{\Xi^*_{c1}\Xi^*_c\pi} = 0.32, \quad f_{\Xi^*_{c1}\Xi'_c\pi} = 20 \ \text{GeV}^{-2}, \quad (31)$$

TABLE III. P wave to S wave strength couplings dependence on the constituent light quark masses. The value of the oscillator constant is taken to be $\kappa = 0.029\ \mathrm{GeV}^3$.

	$m=0.22$	$m=0.28$	$m=0.34$
p-wave couplings			
$g_{\Sigma_c\Lambda_c\pi}$ (GeV^{-1})	6.16	6.43	6.86
$g_{\Xi_c^*\Xi_c\pi}$ (GeV^{-1})	6.02	6.21	6.48
s-wave couplings			
$f_{\Lambda_{c1}\Sigma_c\pi}$	0.53	0.64	0.80
$f_{\Xi_{c1}^*\Xi_c^*\pi}$	0.32	0.44	0.58
$f_{\Sigma_{c1}\Sigma_c\pi}$	0.06	0.07	0.09
d-wave couplings			
$f_{\Lambda_{c1}^*\Sigma_c\pi}$ (GeV^{-2})	36.6	50.2	65.1
$f_{\Xi_{c1}^*\Xi_c'\pi}$ (GeV^{-2})	24.2	32.0	40.5
$f_{\Sigma_{c1}^*\Sigma_c\pi}$ (GeV^{-2})	0.60	0.75	0.91

which are close to our results obtained using a quark mass $m=0.22$ GeV. In Table IV, we present predictions for the decay rates calculated using the following relations:

$$\Gamma_{(p\text{-wave})} = f_p^2 I^2 \frac{|\vec{q}|^3}{6\pi}\frac{M_{B_Q}}{M_{B_Q'}}, \tag{32}$$

$$\Gamma_{(s\text{-wave})} = f_s^2 I^2 \frac{|\vec{q}|}{2\pi}\frac{M_{B_Q}}{M_{B_Q'}}, \tag{33}$$

$$\Gamma_{(d\text{-wave})} = f_d^2 I^2 \frac{|\vec{q}|^5}{18\pi}\frac{M_{B_Q}}{M_{B_Q'}}, \tag{34}$$

with I is an appropriate $SU(3)$ flavor factor and $|\vec{q}|$ being the pion momentum in the rest frame of the decaying baryon B_Q. For comparison, Table IV also includes results obtained using a relativistic three quark model [26] and the available experimental data [25].

IV. SUMMARY AND CONCLUSIONS

We have used the $SU(2N_f)\times O(3)$ symmetry of the light diquark system to reduce the number of HQS coupling factors of heavy baryon single-pion decays. Assuming one-body interactions, it is shown that five couplings are required to describe single pion transition among S and P wave states in the heavy quark limit. These result are obtained using covariant spin wave functions for the light diquark system and they agree with the HHCPT predictions [7].

We also calculate the two independent couplings $f_{\Sigma_{c1}\Sigma_c\pi}$ and $f_{\Sigma_{c1}^*\Sigma_c\pi}$ using a LF quark model wave functions. These

TABLE IV. Single pion decay rates of charmed baryon states for different values of the light quark mass m. The values of Γ^* are taken from Ref. [26] which are in MeV as well as the calculated decay rates Γ and the light quark mass m.

$B_Q \to B_Q'\pi$	$\Gamma_{m=220}$	$\Gamma_{m=280}$	$\Gamma_{m=340}$	Γ^*	$\Gamma_{\text{expt.}}$
p-wave transitions					
$\Sigma_c^+ \to \Lambda_c\pi^0$	1.39	1.51	1.72	3.63 ± 0.27	
$\Sigma_c^0 \to \Lambda_c\pi^-$	1.28	1.40	1.59	2.65 ± 0.19	
$\Sigma_c^{++} \to \Lambda_c\pi^+$	1.34	1.46	1.67	2.85 ± 0.19	
$\Sigma_c^{*0} \to \Lambda_c\pi^-$	10.15	11.06	12.58	21.21 ± 0.81	$13.0^{+3.7}_{-3.0}$
$\Sigma_c^{*++} \to \Lambda_c\pi^+$	10.50	11.44	13.03	21.99 ± 0.87	$17.9^{+3.8}_{-3.2}$
$\Xi_c^{*0} \to \Xi_c^0\pi^0$	0.44	0.46	0.51	1.01 ± 0.15	<5.5
$\Xi_c^{*0} \to \Xi_c^+\pi^-$	1.08	1.15	1.25	2.11 ± 0.29	
$\Xi_c^{*+} \to \Xi_c^0\pi^+$	0.91	0.97	1.05	1.78 ± 0.33	<3.1
$\Xi_c^{*+} \to \Xi_c^+\pi^0$	0.56	0.60	0.65	1.26 ± 0.17	
s-wave transitions					
$\Lambda_{c1}(2593) \to \Sigma_c^0\pi^+$	1.08	1.59	2.47	0.83 ± 0.09	$<0.86^{+0.73}_{-0.56}$
$\Lambda_{c1}(2593) \to \Sigma_c^+\pi^0$	0.72	1.05	1.64	0.98 ± 0.12	$\Gamma_{\Lambda_{c1}}=3.6^{+2.0}_{-1.3}$
$\Lambda_{c1}(2593) \to \Sigma_c^{++}\pi^-$	0.90	1.30	2.04	0.79 ± 0.09	$<0.86^{+0.73}_{-0.56}$
$\Xi_{c1}^*(2815) \to \Xi_c^{*0}\pi^+$	0.74	1.39	2.41	0.91 ± 0.03	$\Gamma_{\Xi_{c1}^*}<2.4$
$\Xi_{c1}^*(2815) \to \Xi_c^{*+}\pi^0$	0.36	0.68	1.19	0.48 ± 0.02	
$\Sigma_{c1}(2670) \to \Sigma_c\pi$	0.085	0.11	0.19		
d-wave transitions					
$\Lambda_{c1}^*(2625) \to \Sigma_c^0\pi^+$	0.22	0.42	0.71	0.080 ± 0.009	
$\Lambda_{c1}^*(2625) \to \Sigma_c^+\pi^0$	0.20	0.38	0.64	0.095 ± 0.012	$\Gamma_{\Lambda_{c1}^*}<1.9$
$\Lambda_{c1}^*(2625) \to \Sigma_c^{++}\pi^-$	0.21	0.40	0.67	0.076 ± 0.009	
$\Xi_{c1}^*(2815) \to \Xi_c^{0\prime}\pi^+$	0.45	0.79	1.27	0.46 ± 0.39	
$\Xi_{c1}^*(2815) \to \Xi_c^{+\prime}\pi^0$	0.23	0.40	0.64	0.25 ± 0.21	
$\Sigma_{c1}^*(2701) \to \Sigma_c\pi$	0.001	0.003	0.004		

couplings determine the one pion transitions of charmed baryons from the antisymmetric excited state (k multiplet) to the ground state. The LF calculations show that the strong decay rates among these states are suppressed with respect to the corresponding transitions from the symmetric multiplet. This suppression might be due to the fact that the $(\mathbf{15},1^-)$ multiplet lies higher in energy than the $(\mathbf{21},1^-)$. In fact, the k multiplet is at least 75 MeV above the K multiplet as predicted by the quark model [19,20]. Here, (α,L^P) denotes the diquark $SU(6)$ supermultiplet representation α and its orbital excitation L with parity P. However, there might be still some contributions to the width of charmed baryon states forming this multiplet due to transitions involving other mesons or from electromagnetic decay channels. Since these channels are usually small, one thus concludes that the width measurement of the antisymmetric multiplet P wave charmed baryon resonances might be more difficult and will require much more effort.

Finally, as can be seen from Table III, P wave to S wave single pion coupling strengths are sensitive to changes in the light quark masses and the oscillator parameters. There is at least a 25% reduction in the value of the coupling strengths calculated using values for the light quark masses commonly used in relativistic models compared to those obtained using the nonrelativistic once. This feature has also been observed studying the proton spin in Ref. [27], analyzing strong decays of D^* mesons in a light front quark model [28], and in Ref. [29] calculating the axial coupling g_A using a relativistic three quark model formulated on the light cone. These relativistic effects, which might be due to Melosh rotation generated by the constituent quarks internal transverse momentum, are important when analyzing hadron decay form factors.

We conclude noting that decay rates among ground state charmed baryons and most of those from P wave to S wave are within the experimental data. Furthermore, decay rates for s-wave transitions, predicted for values of light quark mass $m=0.22$ GeV, are in agreement with those calculated in Ref. [26]. We also notice that, the d-wave couplings are very sensitive to the value of the masses of baryons involved in the decay process. One thus should take the predicted rates with more caution since there are still large uncertainties in the measurements of these masses. Table III shows that, single pion couplings of $\Xi_c^{(*)}$ baryons are in general smaller than those involving Λ_c or Σ_c within the same multiplet. We would like to mention that the inclusion of two-body and higher operators, which will break the spin-flavor symmetry, may increase the number of coupling factors required to describe P wave to S wave strong transitions. It would be interesting to go beyond heavy quark limit by including $1/m_c$ corrections as well as the mixing of states to form the physical baryon wave functions which might also modify the decay rate predictions by up to 15%.

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